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Abstract

The purpose of this paper is to study optimal congestion taxes in a time allocation framework. This makes it possible to distinguish taxes on inputs in the production of car trips and taxes on transport as an activity. Moreover, the model allows us to consider the implications of treating transport as a demand, derived from other activities. We extend several well known tax rules from the public finance literature and carefully interpret the implications for the optimal tax treatment of passenger transport services. The main findings of the paper are the following. First, if governments are limited to taxing market inputs into transport trip production, the time allocation framework (i) provides an argument for taxing congestion below marginal external cost, (ii) implies a favourable tax treatment for time-saving devices such as GPS, and (iii) provides a previously unnoticed argument for public transport subsidies. Second, if the government has access to perfect road pricing that directly taxes transport as an activity, all previous results disappear. Third, in the absence of perfect road pricing, the activity-specific congestion attracted by employment centres, by shopping centres or by large sports and cultural events should be corrected via higher taxes on market inputs in these activities (e.g., entry tickets, parking fees, etc.).

JEL: D62, H21, R41
Key words: Optimal taxation, congestion, time-commodity substitution, derived demand

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1. Introduction

In this paper, we consider optimal taxes for passenger transport services in a time allocation framework, allowing for potential substitution between market inputs and time in the production of trips. We first derive optimal tax rules under the assumption that the government cannot directly tax car transport, but only has access to taxes on the market inputs into car trip production (gasoline, maintenance, etc.). We then compare the results with optimal tax rules when the government does have the possibility to impose direct taxes on car transport via, e.g., a system of road pricing. To facilitate the interpretation of our findings, we relate the optimal tax expressions to several well known tax rules that have shaped economists' thinking about optimal commodity taxes, including the Diamond-Mirrlees (1971a,b) efficiency theorem, Ramsey (1927) taxation and the Corlett-Hague (1953) rule. Moreover, the time allocation framework allows us to study the implications of treating transport demand as a derived demand from other activities. The derived-demand nature of transport is often emphasized, but the relevant implications for the optimal taxation of congestion have not formally been analyzed.

In the sixties, a series of seminal papers and monographs -- including, among others, Walters (1961), Mohrung and Harwitz (1962), Strotz (1965), Marchand (1968) and Vickrey (1969) -- initiated a large literature on congestion pricing. This literature has been extended in several directions. For example, a variety of second best considerations were incorporated into the analysis (e.g., Verhoef, Nijkamp and Rietveld (1996), Small and Yan (2001)), the implications of the interaction of transport policies with the labour market for congestion tolls were explicitly recognized (Parry and Bento (2001), Van Dender (2003)), and the consequences of agglomeration economies for congestion policies were carefully studied (see, e.g., Safirova (2002)). Moreover, several authors emphasized the importance of congestion for trip scheduling decisions by road users and investigated the implications of schedule delay for congestion policies (Small (1982), Arnott, de Palma and Lindsey (1993)). Finally, the role of other pricing instruments has received considerable attention, in case optimal road tolls can for some reason not be implemented (Fullerton and Mohr (2003), Parry and Small (2005), De Borger and Mayeres (2007)).

Although the literature surveyed above has greatly increased our understanding of the congestion problem and how to deal with it, it has not considered the problem within a formal time allocation framework. The purpose of the current paper is to reconsider the problem of optimal congestion taxes within the time allocation setting originally developed by Becker (1965), and recently embedded in an optimal tax framework by Kleven (2000, 2004). We have two specific reasons for doing so. First, existing models of optimal congestion taxes typically assume that the time it takes to make a particular car trip depends on congestion levels but that, conditional on traffic levels, travel time is exogenous to the individual road user (see, among many others, Verhoef, Nijkamp and Rietveld (1996), Mayeres and Proost (1997), Parry and Bento (2001)). However, recent technological developments suggest the existence of additional substitution possibilities between money and time. Indeed, drivers can invest in time-saving devices such as GPS and ATIS; spending on these market inputs reduces the time it takes to make a trip. Moreover, Verhoef and Rouwendal (2004) have questioned the exogeneity of the traditional speed-flow relation on other grounds. They argue that, if one interprets travel speed as 'average' speed over an entire trip, drivers do have the opportunity to optimally select speed at given traffic flows. The available empirical evidence in fact widely supports the existence of a trade-off between time and monetary spending. For example, fuel use has been empirically found to be directly related to speed over quite a relevant range (see, e.g. Rouwendal (1996)). Furthermore, Fosgerau (2005) reports that, conditional on traffic levels, drivers with better cars drive faster. In addition, it is well known that people spend time looking for a parking spot in order to save on parking costs (Anderson and De Palma (2004)). Although the importance of these substitution possibilities between money and time deserves further empirical analysis, the question arises whether such potential substitution has implications for the optimal tax treatment of passenger transport, given the presence of road congestion[1].

A second reason for reconsidering congestion taxes in a time allocation framework is that, although all introductory textbooks on transport economics (see, e.g., Button (1993)) start out by emphasizing the derived nature of transport demand, existing models have not formally exploited this derived-demand nature. They either treat transport demand as a final demand (e.g., Verhoef et al. (1996)), or they assume

[1] Nielsen (2007) does use a time allocation framework to study optimal taxes in the presence of externalities. However, he assumes fixed proportions between time and commodity inputs in the production of all activities and focuses on atmospheric externalities that do not affect time use. Moreover, he does not study the distinction between taxing market inputs and direct taxes on transport as an activity. Finally, he does not analyze the implications of treating transport as derived from activities.

perfect complementarity with labour supply (e.g., Parry and Bento (2001)). The Becker-Kleven time allocation framework is the ideal vehicle to deal with transport as derived from various activities, and to study the role of taxes on transport and on other market inputs in transport-using activities under different assumptions on the nature of congestion (e.g., is it the joint consequence of multiple activities, or is it activity-specific?) and on the available tax instruments (e.g., is an activity-specific congestion toll possible or not?). As we will show, taking account of the derived-demand nature of transport has a number of highly intuitive implications that, although probably not surprising to specialists in the field, have never formally been derived before.

The main findings of this paper are easily summarized as follows. First, if governments are limited to taxing market inputs (such as gasoline, car maintenance, car accessories, etc.) into the production of car trips, the time allocation framework provides an argument for taxing congestion below the marginal external congestion cost. It also implies a favourable tax treatment for time-saving devices such as GPS and it suggests reducing public transport fares. Second, however, all these results disappear if road pricing is available. Market inputs in transport production should then neither be taxed nor subsidized. The intuition is that the road toll does not distort the choice of time versus commodity inputs in trip production, so that the extra stimulus of lower taxes on time-saving devices disappears. Similarly, the argument in favour of lower public transport fares disappears. We will argue that these findings are easily understood by reconsidering several famous results in the public economics literature in a household production framework, and allowing for congestion. Third, we show that explicitly treating transport demand as derived from activities implies a useful role for taxes on other market inputs in transport-using activities whenever optimal activity-specific congestion tolls are not possible. Not surprisingly, the tax structure raises taxes on inputs of activities that generate a lot of transport. The results imply, in the absence of perfect road pricing, partially correcting the congestion attracted by employment centres, by shopping centres or by large sports and cultural events via higher taxes on market inputs in these activities (e.g., parking, entry tickets, etc.).

The paper has several obvious limitations. First, the focus on time allocation and congestion implies that we ignore other externalities, such as pollution and accident risks. Environmental pollution could be easily introduced but does not affect the findings of the current paper. Ignoring accident risks is not entirely innocuous, however, as it is well known that accident risk and congestion are not unrelated (see, e.g., Verhoef and Rouwendal (2004)). Moreover, insurance against accident risks raises issues of moral hazard, in the sense that drivers may take less care in avoiding accidents and the associated damage than they would in the absence of insurance. Moral hazard has clear implications for the optimal tax treatment of driving that are ignored in the current paper. For example, since driving cannot be taxed directly, Arnott and Stiglitz (1986) suggest taxing complements and subsidizing substitutes to driving to cope with moral hazard. They argue in favour of taxing gasoline and cars (to reduce driving), subsidizing maintenance (to make driving less risky) and, assuming that moral hazard is less pronounced for other modes than for car use, subsidizing alternative modes. Second, the time allocation framework we use throughout the paper is a direct extension of Becker (1965) and Kleven (2000, 2004), despite the criticism of, e.g., Boadway and Gahvari (2006) and Gahvari (2007)[2]. These authors make the useful distinction between 'labour substitutes' (goods for which consumption time yields negative utility) and 'leisure substitutes' (time yields positive utility). They argue that the Becker-Kleven model implicitly assumes that all taxed commodities are labour substitutes, and that the optimal tax rules look quite different if goods happen to be leisure substitutes[3]. However, for the large majority of trips people do consider the time spent in transport as unpleasant, so that this restriction seems rather innocuous for our purposes. Moreover, the Becker-Kleven activity approach is especially attractive from another perspective, highly relevant for this paper: unlike the Boadway-Gahvari model, it allows for substitution in the production of transport activities, and it provides a simple and direct way to study transport demand as derived from the demand for particular other activities. A third important limitation of this paper is the set of tax instruments considered. Throughout the paper, we follow the standard Ramsey approach. This implies that we exclude

[2] Gahvari and Yang (1993) were the first to derive optimal tax rules in a model that explicitly takes into account that the consumption of many goods requires time. Assuming exogenously given time requirements for each of the consumption goods, they reconsider a number of well known tax results previously studied in the public economics literature (e.g., the optimality of uniform taxes, the Corlett and Hague (1953) rule, etc.). In an apparently independent effort, Kleven (2004) directly built upon the seminal paper by Becker (1965). He assumed consumers care about activities rather than goods, and allowed for possible substitution between commodities and time in the production of activities. If no substitution is allowed, the two models can be shown to be equivalent (Gahvari (2007)).

[3] Assuming fixed proportions, they show that goods for which consumption time is unpleasant should be taxed at a higher rate.

lump-sum taxes (such as head taxes) and focus on linear commodity taxes[4]. Moreover, we ignore heterogeneity and distributive concerns. Finally, there are no income taxes in the model[5].

To keep the analysis as transparent as possible, we study the role of the time dimension of transport trips for optimal taxes and the implications of transport as derived from other activities sequentially. The structure of the paper is therefore the following. In Section 2 we present the simplest version of the time allocation model. Transport is viewed as an activity that requires time and commodity inputs and that generates a congestion externality. It is assumed that the government is restricted to taxing market inputs. Section 3 reconsiders the optimal commodity tax structure when the government does have the instruments to tax the transport activity directly, through a system of road pricing. In Section 4 we formulate a model that treats transport demand as derived from other activities, and we discuss the implications for dealing with congestion. Finally, Section 5 concludes.

2. Time allocation, congestion and optimal taxation of market goods

In this section, we consider optimal taxation of market goods in the presence of congestion externalities, using the Becker-Kleven time allocation framework. Transport is treated as an activity that requires both commodity inputs (fuel, maintenance, etc.) and time, and it causes congestion. Throughout this section, we

[4] Although it is not obvious to exclude head taxes on informational grounds, we follow the traditional Ramsey literature and focus on distortionary commodity taxes only. Further restricting the analysis to linear taxes seems plausible. Nonlinear commodity taxes would induce consumers to arbitrage, i.e., those with low marginal prices could buy in bulk and resell to those with higher marginal prices. Moreover, implementing nonlinear commodity taxes would require detailed information on individual consumption of goods. Although this may not be impossible to obtain for some goods – including both non-transport goods (e.g., electricity consumption) and some transport services (e.g., information on individual car kilometres) – for most commodities only anonymous aggregate transactions are observable.

[5] The study of optimal commodity taxes with heterogeneous users and distributive concerns was initiated in the seminal paper by Diamond (1975); see Gahvari (2007) for an extension to a time allocation framework. Integrating indirect and direct taxation, Atkinson and Stiglitz (1976) argued that differential commodity taxes are not needed in the presence of an optimal nonlinear income tax provided commodities are weakly separable from leisure in utility. Recently, it has been shown that this result holds even if the income tax is not optimal (see, most forcefully, Kaplow (2006)). The author therefore argues that nonlinear income taxation (focusing on the trade off between redistribution and labour supply distortions) can be isolated from commodity taxation (focusing on distortions due to relative commodity prices); for further discussion on this ‘new’ view on optimal taxation, see Kaplow (2004, 2006). If the weak separability assumption between regular commodities and leisure does not hold, there still is a role for differentiated commodity taxes (see, e.g., Cremer, Pestieau and Rochet (2001)).

assume that it is not feasible to tax activities directly; only the market inputs in activity production can be taxed.

For pedagogic reasons, we start in subsection 2.1 with a basic setup that assumes there is just one market input and one time input in the production of car trips. Optimal tax rules for this case are derived and interpreted in subsection 2.2. The extension to multiple car inputs is then discussed in subsection 2.3. We relate the results to several classic findings from the optimal tax literature, and we interpret the policy implications for the optimal taxation of passenger transport services. Although the derivations required to show our results are not difficult, to save space we relegate them to appendices.

2.1. Congestion, time allocation and optimal taxation: the basic model

The set-up of the most basic version of the model is as follows. Let the consumer be interested in (*n+2*) activities. There are *n* regular consumptive activities Z_i (*i=1,2,….,n*), activity Z_c is car transport, expressed, e.g., in kilometres travelled. Finally, Z_0 is pure leisure. Preferences are described by the direct utility function

$$u(Z_0, Z_1, ..., Z_n, Z_c) \tag{1}$$

Leisure is assumed not to require market goods; hence, Z_0 is directly expressed in time units. It remains untaxed [6]. Following the original papers by Becker (1965) and DeSerpa (1971), we specify the production of activities from market goods and time as follows:

$$\begin{aligned} Z_i &= Z_i(X_i, T_i), \quad \forall\, i = 1, 2, ..., n \\ Z_c &= Z_c(X_c, T_c) \end{aligned}$$

[6] Note that the assumption that leisure remains untaxed is not just an innocuous normalization issue; it imposes an important constraint on the available tax instruments (see, e.g., Dixit and Munk (1977) and Munk (1980); for a recent survey on the implications of various different normalizations for optimal commodity taxes, see Munk (2006)). Indeed, if all commodities are taxable, the optimal tax structure is just a uniform tax on all commodities. Since this is the equivalent of a lump-sum tax, the tax structure is first-best. This result, in a slightly adapted version, carries over to the time allocation model considered here. The optimal tax structure in the absence of pure leisure is to make taxes, as a percentage of the generalized price (that is the monetary price plus the time cost of consuming the good), uniform for all activities (see Boadway and Gahvari (2006), Gahvari (2007), Kleven (2004)). The presence of untaxed leisure does imply a role for differential taxes on the remaining taxed goods. As noted above, under some strong and unrealistic restrictions on the structure of preferences, optimal commodity taxes remain uniform even in the presence of untaxed leisure. For further details and discussion see, among others, Sandmo (1974), Atkinson and Stiglitz (1976), Deaton (1981), Besley and Jewitt (1995), and Gahvari (2007).

Assuming that all activities *i=1,2,….,n* and *c* are produced under constant returns to scale we have

$$X_i = x_i Z_i, \quad T_i = t_i Z_i, \quad \forall\, i = 1,2,....,n$$
$$X_c = x_c Z_c, \quad T_c = t_c Z_c$$

where the x_i and t_i are the unit requirements of market goods and time per unit of activity Z_i. Note that they depend on input prices, unless production is fixed proportions, see below.

The consumer faces both a budget and a time constraint. They are given by, respectively:

$$\sum_{i=1}^{n} p_i X_i + p_c X_c = wT_W + y \tag{2}$$

$$Z_0 + \sum_{i=1}^{n} T_i + T_c + T_W = \overline{T} \tag{3}$$

where the p_i's are unit prices of the respective market goods, T_W is working time, *w* is the wage rate and *y* is non-wage income[7].

An advantage of Becker's formulation is that the constant returns to scale assumption allows the consumption and activity production decisions to be separated. The production decisions involve minimizing, for each activity, the unit costs of production; consumption decisions then follow from maximizing utility, where the budget restriction is defined in terms of generalized prices and income.

Consider the production decision. Minimizing unit costs for all regular activities Z_i (*i=1,…..n*)

$$\underset{x_i,t_i}{Min} \quad p_i x_i + wt_i \quad s.t. \quad Z_i(x_i, t_i) = 1$$

leads to the optimal input use of goods and time, denoted $x_i(p_i, w), t_i(p_i, w)$, respectively. The optimal generalized price of activity *i* is given by

$$Q_i(p_i, w) = p_i x_i(p_i, w) + wt_i(p_i, w) \tag{4}$$

Moreover, Shephard's lemma applies to the unit cost function:

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[7] Solving the utility maximization problem subject to the two constraints immediately implies that the resource value of time equals the wage (Becker (1965)). Note that there is a large literature on the value of time, indicating that time values deviate from the wage rate in more realistic settings (see the survey by Jara-Diaz (2008), among many others). Moreover, the resource value does not equal the value of an exogenous reduction in travel time (see, e.g., De Donnea (1971), Jara-Diaz (2008), Calfee and Winston (1998)). The point made in this paper is unaffected by assuming more realistic time values.

$$\frac{\partial Q_i(p_i,w)}{\partial p_i} = x_i(p_i,w); \frac{\partial Q_i(p_i,w)}{\partial w} = t_i(p_i,w) \tag{5}$$

Next consider car transport trips. They are also produced using market and time inputs and, as argued in the introduction, there are good reasons to allow for some substitution possibilities. Fixed proportions are, therefore, treated as a special case. Car trips differ from other activities, however, in that (assuming fixed road capacity) the overall demand for transport activity Z_c by all N consumers causes congestion. Total demand is denoted:

$$E = NZ_c \tag{6}$$

As is standard in the transport economics literature, an individual consumer takes the traffic level E as given when making decisions. Cost minimizing behaviour, conditional on the traffic level, then yields the optimal input choice of goods and time per unit of transport (kilometre or trip) as follows:

$$x_c(p_c,w;E), t_c(p_c,w;E)$$

The generalized price of transport is given by:

$$Q_c(p_c,w;E) = p_c x_c(p_c,w;E) + w t_c(p_c,w;E) \tag{7}$$

Shephard's lemma yields

$$\frac{\partial Q_c(p_c,w;E)}{\partial p_c} = x_c(p_c,w;E); \frac{\partial Q_c(p_c,w;E)}{\partial w} = t_c(p_c,w;E) \tag{8}$$

Note that congestion raises the generalised price of transport, so $\frac{\partial Q_c(p_c,w;E)}{\partial E} > 0$.

The consumption decisions of consumers, substituting (3) in (2) and using (4) and (7), follow from the optimisation problem:

$$\underset{Z_0,Z_1...,Z_n,Z_c}{Max} \quad u(Z_0,Z_1,...,Z_n,Z_c)$$

$$s.t. \sum_{i=1}^{n} Q_i(p_i,w)Z_i + Q_c(p_c,w;E)Z_c + wZ_0 = w\overline{T} + y$$

This yields demands for all activities as functions of generalized prices:

$$Z_i\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,w;E),w,w\overline{T}+y\right], \quad \forall i = 0,1,....,n$$

$$Z_c\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,w;E),w,w\overline{T}+y\right]$$

The associated indirect utility function $v(.)$ is:

$$v\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,w;E),w,w\overline{T}+y\right].$$

2.2. Optimal tax rules

In this subsection, we report the optimal tax rules and carefully interpret the implications for the optimal taxation of passenger transport services. It is assumed that the government maximises the welfare of the standard individual subject to a budget restriction:

$$\underset{p_1,\dots,p_n,p_c}{Max}\; v\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,w,E),w,w\overline{T}+y\right]$$

$$s.t.\quad \left(\sum_{i=1}^{n}(p_i-q_i)*x_i(p_i,w)*Z_i(.)\right)+(p_c-q_c)*x_c(p_c,w)*Z_c(.)=R$$

In this formulation, the demand for activities has been specified above, and R is required government revenue[8]. The q_i $(i=1,....,n)$ and q_c are the pre-tax producer prices of the non-transport and the transport market inputs, respectively.

The optimal tax rules are derived in Appendix 1. To interpret these rules let us, as is common in the optimal tax literature, start by assuming zero cross-elasticities between all activities $i=1,....,n$. In the standard commodity tax model this assumption yields the inverse elasticity rule, probably the best known version of the more general Ramsey (see Ramsey (1927)) rule[9]. This second-best tax rule implies that commodities should be taxed inversely proportional to the own price elasticity of demand. More price-sensitive commodities are to be taxed at relatively lower rates in order to minimize distortions: the deadweight loss (the loss in consumer surplus minus the tax revenue that goes to the government) of a given tax on such goods is much larger than for less price elastic goods.

The equivalent of the inverse elasticity rules for the time allocation model with congestion can be written as follows (see Appendix 1):

$$\frac{p_j-q_j}{p_j}=\frac{\phi}{\alpha_{x_j}\hat{\varepsilon}_{jj}-\sigma_j\alpha_{t_j}} \qquad \textit{for all } j \tag{9}$$

$$\frac{p_c-q_c}{p_c}=\frac{\phi}{\alpha_{x_c}\hat{\varepsilon}_{cc}-\sigma_c\alpha_{t_c}}+\frac{MECC}{p_cx_c}\left[\frac{\hat{\varepsilon}_{cc}\alpha_{x_c}}{\hat{\varepsilon}_{cc}\alpha_{x_c}-\sigma_c\alpha_{t_c}}\right] \tag{10}$$

[8] Note that we have normalized $N=1$ without loss of generality; it would drop out of the first-order conditions anyway.

[9] With non-zero cross elasticities, the Ramsey rule argues for a tax structure that yields equal percentage reductions in the consumption of all taxed commodities. For more details see, for example, Diamond (1975) and Boadway and Gahvari (2006).

In these expressions, the $\hat{\varepsilon}_{jj}$ *(j=1,…,n; c)* denote the own compensated elasticities of demand for activity *j* with respect to its generalized price, and $\alpha_{x_j} = \dfrac{p_j x_j}{Q_j}$; $\alpha_{t_j} = \dfrac{wt_j}{Q_j}$ *(j=1,…,n;c)* are the shares of market goods and time in the generalized price of *j*. Furthermore, σ_j is the substitution elasticity between commodity and time inputs in the production of activity *j*, and *MECC* is the marginal external cost of congestion associated with an increase in transport activity Z_c. Finally, ϕ is a parameter that reflects the difference between the net marginal social utility of income and the shadow price of the government's budget restriction. The requirement for the government to raise a given revenue and the unavailability of a lump-sum tax instrument imply that the cost of funds exceeds the social value of income, so that $\phi < 0$. All parameters appearing in (9)-(10) are discussed in detail in Appendix 1.

Interpretation of the tax rules is easy. The rule for non-transport market goods (9) is just a standard Ramsey tax rule adjusted for time allocation[10]. To see this, note that the time allocation model induces a distinction between the compensated price elasticities of activities with respect to their generalized prices (the $\hat{\varepsilon}_{jj}$) and the compensated demand elasticities of market inputs X_j with respect to their market price p_j. In Appendix 1 we show that:

$$\frac{\partial \hat{X}_j}{\partial p_j}\frac{p_j}{\hat{X}_j} = \alpha_{x_j}\hat{\varepsilon}_{jj} - \sigma_j \alpha_{t_j}$$

Using this in expression (9) it becomes clear that, just like in the standard Ramsey tax model, the optimal tax rule implies taxes that are inversely proportional to the price elasticity of demand for the market good being taxed. However, the time allocation setting offers two further insights. First, (9) implies that, for given generalized price elasticities $\hat{\varepsilon}_{jj}$, taxes are lower for market inputs into activity production that easily substitute for time (high σ_j). This substitution possibility makes the demand for the market input more elastic, hence the lower tax. Second, tax rule (9) plausibly suggests higher taxes on market goods that are used in highly time-intensive activities. This is easiest to see by assuming zero substitution between market inputs and time. It then

[10] This rule has also been derived in Kleven (2000), the working paper version of Kleven (2004). In the published version, he focuses on Leontief household production, not allowing substitution between money and time in activity production.

immediately follows that the tax is inversely proportional to the factor share of market goods. Noting that the factor shares sum to one, it immediately follows that a higher time input share raises the optimal tax. Conditional on a given activity elasticity $\hat{\varepsilon}_{jj}$, more time-intensive production makes market commodity demand less elastic.

The tax rule (10) for market inputs in the production of transport trips consists, as usual in the case of externalities (see, e.g., Bovenberg and Goulder (1996)), of a Ramsey component and a 'Pigovian' externality component. The former is the same as for the other activities. Compared to standard optimal tax models, the Pigovian component differs on two accounts. First, since the activity Z_c causes congestion but only the market input X_c can be taxed, it is the externality per unit of market input that is taxed. Second, if time and market goods are at least to some extent substitutable in transport trip production ($\sigma_c \neq 0$), then the Pigouvian component is below *MECC*. Only a fraction of the *MECC* per unit of market input is charged to consumers; this fraction will be smaller the larger the time-intensiveness of transport production and the larger the substitution possibilities. The intuition is again that, if market inputs can be substituted for time in producing transport trips, the lower tax on market goods stimulates the use of a less time-consuming technology in the production of transport trips. These time savings are welfare-improving.

Next consider the case with non-zero cross elasticities. As argued extensively by, e.g., Munk (1980, 2006), the optimal tax structure is a compromise between, on the one hand, limiting the distortion in consumption of non-leisure activities and, on the other hand, discouraging the consumption of the untaxed activity. The tax rules obtained allow us to reflect on a wide range of relevant issues, including the taxation of transport as opposed to other activities, the optimal taxation of different transport modes, and taxation of different trip purposes (e.g., commuting, shopping, etc.) by the same mode. An appropriate instrument to illustrate the implications of assuming non-zero elasticities is to derive the equivalent of the Corlett-Hague (1953) rule, which has been highly influential in the public economics literature for thinking about optimal tax design. The original rule was derived in a model with two standard goods plus leisure, and it strongly emphasized the role of the complementarity of goods with leisure for optimal tax rules.

Consider, therefore, the time allocation model used above for the case where the consumer cares about leisure and two activities. The equivalent of the Corlett-

Hague rule for our model with congestion is derived in Appendix 1, see expression (A1.9). For pedagogic reasons, however, it is instructive to first consider the implications in the absence of congestion. For two arbitrary activities (*1,2*), it follows from Appendix 1 that the adapted Corlett-Hague rule can then be written as:

$$\frac{\dfrac{p_1 - q_1}{p_1}}{\dfrac{p_2 - q_2}{p_2}} = \frac{\left[\hat{\varepsilon}_{22} + \hat{\varepsilon}_{11} + \hat{\varepsilon}_{10} - \sigma_2 \dfrac{\alpha_{t_2}}{\alpha_{x_2}}\right]}{\left[\hat{\varepsilon}_{22} + \hat{\varepsilon}_{11} + \hat{\varepsilon}_{20} - \sigma_1 \dfrac{\alpha_{t_1}}{\alpha_{x_1}}\right]} \left(\frac{\alpha_{x_2}}{\alpha_{x_1}}\right) \tag{11}$$

In this expression, the $\hat{\varepsilon}_{10}, \hat{\varepsilon}_{20}$ are the compensated cross price elasticities of demand for the two activities with respect to leisure, respectively.

The standard Corlett-Hague rule is obtained by assuming time inputs in activity production are zero (hence $\alpha_{t_1} = \alpha_{t_2} = 0; \alpha_{x_1} = \alpha_{x_2} = 1$). The rule then implies that the relative optimal taxes (as a fraction of final prices) only depend on the cross price elasticities of the goods with leisure. Therefore, goods that are more complementary with leisure should be taxed higher, and goods that are less complementary, or goods that substitute for leisure, should be taxed less. This has some obvious implications for the taxation of transport, even making abstraction of congestion. For example, interpret the first good as non-commuting transport and the second as commuting transport; the rule can then be interpreted as providing an argument for taxing commuting (a complement to labour, a substitute for leisure) at a lower rate than other travel purposes, such as pleasure trips, visits to friends or family, etc. Implementation would of course require distinguishing taxes on commuting versus other trip purposes; however, this is straightforward by providing income tax deductions for commuting. Empirical support for this tax differentiation according to trip purpose is given by, e.g., Van Dender (2003).

If we introduce time, but assume there is no substitution between inputs ($\sigma_1, \sigma_2 = 0$), we obtain the rule reported in, among others, Kleven (2004, Proposition 4) and Boadway and Gahvari (2006, p. 1857). Noting that $\alpha_{x_j} + \alpha_{t_j} = 1 \quad (j = 1,2)$, the rule now implies (conditional on cross effects with leisure) that more time-intensive activities should be taxed at relatively higher rates. In a transport context, one interpretation is to have relatively high taxes on transport compared to many other activities, as transport is generally quite time-intensive. A second interpretation is to treat the two activities as transport trips by two different transport modes. In that case, (11) suggests that the most time-intensive mode should be taxed higher. Loosely speaking, this could be interpreted as saying that fast modes should be taxed less than slow modes (of course, still making abstraction of external costs) so that, for example, car and rail should be taxed relatively less than bus. However, one should be careful with this interpretation: what matters is the factor share of time, not just the absolute time input.

Introducing substitution between market goods and time in activity production ($\sigma_1 \neq 0, \sigma_2 \neq 0$) further implies (see (11)) to lower the relative tax on market goods that easily substitute for time. For example, assume that the existence of time saving devices in car transport allow substituting time and money by investing in GPS, etc., and that (because of fixed time schedules) such substitution is not possible for public transport, then this suggests lowering the relative tax on car transport.

Finally, introducing congestion complicates the adapted Corlett-Hague rule substantially. It is given as (A1.9) in Appendix 1 for the case of two transport modes: activity *1* are trips by an uncongested public transport mode (say, rail), activity *c* are car trips. To interpret the rule and to isolate the implications of congestion most clearly, let us eliminate other complications: assume zero income effects, and assume the cost of funds equals the marginal utility of income. These assumptions imply $\phi = 0$. Moreover, assume no input substitution possibilities for rail transport, hence σ_1=0. In the standard model, these assumptions produce the first-best optimal tax structure in which the externality-generating good is taxed at marginal external cost, and all other commodity taxes (here rail) are zero. However, the time allocation model now yields the following (see Appendix 1):

$$\frac{p_1 - q_1}{p_1} = -\frac{1}{\Delta}\left\{\frac{MECC}{Q_c} \quad \hat{\varepsilon}_{1c}\sigma_c\alpha_{t_c} \quad \right\}$$

$$\frac{p_c - q_c}{p_c} = \frac{1}{\Delta}\left\{\frac{MECC}{Q_c}\left[\alpha_{x_1} \quad \hat{\varepsilon}_{cc}\hat{\varepsilon}_{11} - \hat{\varepsilon}_{c1}\hat{\varepsilon}_{1c} \quad \right]\right\}$$

where $\Delta = \alpha_{x_1}\alpha_{x_c} \quad \hat{\varepsilon}_{11}\hat{\varepsilon}_{cc} - \hat{\varepsilon}_{c1}\hat{\varepsilon}_{1c} \quad - \alpha_{x_1}\hat{\varepsilon}_{11}\sigma_c\alpha_{t_c} > 0$. Assuming that rail and car transport are substitutes ($\hat{\varepsilon}_{c1} > 0$), and using the definition of Δ, it immediately follows that:

$$\frac{p_1 - q_1}{p_1} < 0 \qquad\qquad \frac{p_c - q_c}{p_c} < \frac{MECC}{p_c x_c} \tag{12}$$

Consistent with previous findings, note that the market input in car transport is taxed at less than marginal external cost. Moreover, a direct consequence is that public transport is subsidized. The reason is that, if substitutability exists, taxing car inputs below external cost not only stimulates the production of transport in a less time-intensive way, but it also reduces the generalized price for car trips, raising demand and congestion. Given positive cross price effects, a standard second-best argument then suggests to correct this by reducing the public transport fare.

It is useful to summarize the implications of the results discussed in this section for the optimal taxation of passenger transport modes. First, the time allocation framework obviously implies higher taxes on market goods that produce car trips to correct for congestion. More importantly, however, conditional on congestion, it suggests a more favourable tax treatment of car use as compared to the traditional framework. The existence of time-saving devices such as GPS or Advanced Traveller Information Systems (ATIS) implies potential substitution possibilities between money and time. This reduces both the Ramsey and the Pigovian components of the optimal tax rule, implying a lower tax. Second, interpreting the results in terms of the relative treatment of private and public transport, we also find relatively low taxes on car use, provided that the time cost share of car trips is lower than for public transport. Moreover, this effect is strengthened to the extent that substitution exists in the production of car trips, but not in public transport. Third, substitution possibilities also suggest public transport subsidies, because the inability to tax time inputs means that car transport is not charged the full marginal external cost. Fourth, the argument in favour of taxing commuting less than non-commuting transport clearly not only survives in a time allocation setting, it is actually strengthened to the extent that the time cost share of commuting is plausibly higher than for non-commuting trips. Finally, interpreting the results to compare the tax treatment of different public transport modes, the model gives arguments for taxing bus use higher than rail or metro use. Buses are slower and they cause more external costs than rail or metro.

Two important remarks conclude this subsection. First, it has been argued that the importance of the share of the market inputs in the generalized price of activities (see, for example, (11)) gives the time allocation framework some informational advantages over the standard commodity tax framework, making it easier to implement (see, e.g., Kleven (2004)). Under some specific conditions, the tax rules can be shown to depend only on factor shares, and elasticities are not even needed.[11] In terms of passenger transport taxation, this would be a great advantage, because the time cost shares for various transport services are much easier to determine based on observable information than, for example, various cross elasticities with leisure. Unfortunately, the argument is not compelling, because the conditions required to base optimal taxes on factor shares only are highly unrealistic, so that elasticity information is still needed in practice. Second, note that we assumed linear taxes, we ignored heterogeneity and distributive concerns. Even in the absence of congestion, introducing heterogeneity and allowing for nonlinear taxes does not lead to rules that are straightforward to interpret; moreover, as argued above, implementing nonlinear commodity taxes is not realistic on informational grounds. Allowing for distributive issues and heterogeneity (in earning ability) in a model of linear commodity taxes implies that, although the direct relation between time shares and optimal taxes has to be qualified, they continue to play a crucial role in the tax structure (see Gahvari (2007) for details).

2.3. Extension to multiple car inputs

The previous subsection suggested relatively low taxes on market inputs in transport if this induces time savings. We briefly extend the model to multiple car inputs. Not surprisingly, this immediately implies that time-saving devices, such as GPS, should be taxed at low rates, or even be subsidized.

To show this, let Z_c again be car trips, but assume that a car trip is produced using *m* market inputs such as the vehicle, fuel, maintenance, potential extras such as GPS, etc. Moreover, the trip requires time. We then have:

$$Z_c(X_c^1, X_c^2, ..., X_c^m, T_c)$$

where the X_c^l ($l = 1, ..., m$) denote the various car inputs. Note that time is aggregated, the distinction between driving time, parking time, etc. is not explicitly made. It easily follows that the generalised price of a unit of transport activity can then be written as:

[11] For example, if leisure is separable in utility, and the subutility function of the regular activities is homothetic, then information on the factor shares and on the demand for leisure are sufficient to design the optimal tax structure (Kleven (2004), Gahvari (2007)).

$$Q_c(p_c^1, p_c^2,..., p_c^m, w, E) = \sum_{k=1}^{m} p_c^k x_c^k(p_c^1, p_c^2,..., p_c^m, w, E) + wt_c(p_c^1, p_c^2,..., p_c^m, w, E)$$

In Appendix 2, we derive the optimal tax rules for all market inputs *j* in regular activities and for the *m* market inputs into car trip production. The tax rule for commodity inputs in regular activities is obviously the same as in the previous subsection. The tax rules for the transport market inputs imply that, in general, inputs that are complementary to time are taxed at higher rates than inputs that are substitutes for time. To see this, assume for simplicity that there are just two market inputs. In Appendix 2, we then show the following result:

$$\frac{\dfrac{p_c^1 - q_c^1}{p_c^1}}{\dfrac{p_c^2 - q_c^2}{p_c^2}} = \frac{\left[\eta_{11}^c + \eta_{22}^c\right] + w\dfrac{\partial t_c}{\partial p_c^1}\dfrac{1}{x_c^1}}{\left[\eta_{11}^c + \eta_{22}^c\right] + w\dfrac{\partial t_c}{\partial p_c^2}\dfrac{1}{x_c^2}} \qquad (13)$$

This is a 'Corlett-Hague type' rule for the pricing of car inputs. Relative tax rates are largely driven by their effect on the time use per trip. To see the implications, let the first input be a time-saving device such as GPS. We then have $\dfrac{\partial t_c}{\partial p_c^1} > 0$: making GPS more expensive reduces demand for this device and hence raises the time used in a standard transport trip. Assume for simplicity that the second input is neutral with respect to time ($\dfrac{\partial t_c}{\partial p_c^2} = 0$). Expression (13) then immediately implies that time-saving devices will be taxed at lower rates than other car inputs. If the time-saving effect is substantial, these devices can easily be subsidized in the optimal tax structure.

3. Time allocation and road pricing

In the previous section, we argued that the time allocation framework provides an argument (i) for taxing congestion below marginal external cost, (ii) for a favourable tax treatment of time-saving devices such as GPS, (iii) for reducing public transport fares. These results were based on the assumption that the government is limited to taxing market inputs into activities. In this section, we treat transport as an activity that can be directly taxed. Whereas direct taxation of activities seems infeasible for most other activities, in the case of transport it makes perfect sense in the future, when direct taxation of trips or kilometres travelled can be achieved through systems of road pricing. Introducing road pricing is often preferred by economists over, e.g., fuel taxes, because only time-differentiated road tolls are efficient instruments to tackle congestion (Small and Verhoef (2007), De Borger and Proost (2001)): fuel taxes cannot differentiate between peak and off peak periods. In this paper, however, we do not focus on time-differentiation, but point out some differences between taxing transport inputs (fuel, etc.) and transport activities (road tolls) that follow from assuming a time-allocation framework.

Suppose, therefore, that the government is not limited to taxing transport via the market inputs used, but that it can also directly tax the activity Z_c. Denote the per kilometre tax by θ_c. Budget and time restrictions now read

$$\sum_{i=1}^{n} p_i X_i + p_c X_c + \theta_c Z_c = wT_W + y$$

$$Z_0 + \sum_{i=1}^{n} T_i + T_c + T_W = \overline{T} \ .$$

For non-transport goods, the analysis of the previous case (subsections 2.2) still applies; nothing changes. For car transport, the generalised price now equals:

$$Q_c(p_c, \theta_c, w, E) = p_c x_c(p_c, w, E) + wt_c(p_c, w, E) + \theta_c$$

In Appendix 3 we show that, again assuming zero cross-price elasticities between activities:

$$\frac{p_j - q_j}{p_j} = \frac{\phi}{\alpha_{x_j}\varepsilon_{jj} - \sigma_j \alpha_{t_j}} \tag{14}$$

$$p_c = q_c \qquad\qquad \theta_c = MECC + \frac{\phi}{\varepsilon_{cc}} Q_c \tag{15}$$

Interpretation is straightforward. The tax rules for market inputs in activities other than transport are the same as before, see (9). As transport is concerned, note that inputs in car trip production remain untaxed, and only the direct tax on the activity is used to control congestion.

That inputs in car production remain untaxed is not surprising; it follows from appropriate reinterpretation of the Diamond-Mirrlees (1971a,b) efficiency theorem. This was originally derived in a setting with strict separation of consumption decisions (by households) and production decisions (by firms) and showed that, if all commodities are taxable, second best optimal taxation retains production efficiency, so that taxes on inputs in production are not needed. One implication for the transport sector was that, in the absence of externalities, freight transport should remain untaxed. In the household production framework used in this paper, we similarly have separation between household production and consumption decisions; the possibility to tax the consumption activity directly implies that the input tax becomes unnecessary. Moreover, the Pigouvian component of the optimal road toll is now just the *MECC*. This contrasts with the results of the previous section, where we found a Pigovian tax component below *MECC*. The lower input tax reduced the use of market inputs and raised the relative use of time. However, the road toll θ_c does not generate the extra distortion on the time allocation of the consumer and is, therefore, more efficient.

Of course, the advantage of road pricing identified here comes additional to previous arguments in favour of road pricing given in the literature. For example, a time-differentiated toll is more efficient than a fuel tax because the latter cannot differentiate between peak and off peak; similarly, it can be differentiated by link on a network. Moreover, with a fuel tax the cost per kilometre varies with fuel efficiency, whereas congestion does not.

The policy implications of the different results in Sections 2 and 3 may be quite relevant. In the previous section, we found that considerations of time allocation provided an extra argument for lower fares for public transport; moreover, they implied an argument in favour of subsidies for time-saving equipment (GPS, ATIS, etc.). Both arguments disappear if road pricing is available. Market inputs should neither be taxed nor subsidized, including time-saving devices such as GPS. Subsidy arguments for time-saving devices only make sense when road pricing, for whatever reason, is not a feasible policy instrument. The intuition is that a road toll does not distort the choice of time versus commodity inputs in trip production, so that the extra stimulus of lower taxes on time-savings disappears.

4. Transport as derived demand: optimal taxation and activity-specific congestion

Transport is one of the most obvious examples of a derived demand (Button (1993)). Road and public transport users demand transport services because they are interested in particular activities such as shopping, going to work, visiting friends, attending cultural attractions or sport events, etc. Despite this obvious observation, models dealing with optimal taxation of transport externalities typically treat transport as a final demand.

In this section, we use the Becker-Kleven activity approach and ask whether the derived-demand nature has any implications for the optimal tax treatment of congestion. Note that it is not at all obvious that it does. For example, if congestion is the joint consequence of demands for a variety of different activities but an optimal road toll is available, there is no reason to expect tax rules to differ in any substantial way from the standard rules obtained for a model with multiple trip purposes (for such a model, see Van Dender (2003)). Similarly, if congestion is activity-specific (because, e.g., transport demands associated with shopping, working, etc. use different parts of the network, or they take place at different times of the day) but activity-specific tolls are possible, the derived-demand nature of transport again does not seem to have strong implications[12]. Finally, treating transport as an input into the production of activities bears quite some resemblance to the treatment of freight transport (an input in production), and it is well known that optimal congestion tolls reflect marginal external cost if perfect tolling instruments are available (see, e.g., Calthrop et al (2007))[13].

The derived demand nature of transport does make a difference in cases where no perfect toll instrument is available to tackle congestion. For example, consider situations where congestion is activity-specific. This may be quite realistic in practice. For example, think of shopping as an activity. Since shopping is often concentrated at large shopping centres, the activity leads to activity-specific congestion: shoppers driving towards the large shopping malls cause congestion in the neighbourhood of these malls, and the time losses are imposed on other shoppers. Similar stories can be told in the case of agglomeration of the workforce at employment centres, traffic related to concerts and sport manifestations, etc. Activity-specific tolls are not often

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[12] In the first case, one easily shows that congestion is captured by the road toll, and that the commodity tax rules are independent of externalities. In the second case, one finds standard congestion tax rules where the Pigouvian component reflects activity-specific marginal external cost, and commodity input taxes are again independent of externalities. Formal proofs are available from the author.

[13] If no perfect tolling instrument is available, then combinations of taxes or subsidies on other inputs combined with an output tax on the freight-using outputs can be welfare-optimal (Fullerton and Mohr (2003)). Of course, in the current paper, the equivalent of output taxes (taxes on final activities such as shopping) is not feasible, but taxes or subsidies on inputs in the production of activities are.

possible in such cases: at best, transport is taxed using an optimal uniform toll that does not differentiate between activities; at worst, a suboptimal toll or fuel tax exists that is rather vaguely related to external congestion costs. In what follows, we use the activity-based approach of this paper to show that in such cases there may be a useful role for taxes on the other market inputs into these activities to correct for congestion. Although the results to be derived are highly intuitive, they cannot be derived in a standard setting that treats transport as a final demand.

We consider the simplest possible setup to derive our results. We limit the analysis to leisure and two activities; nothing is gained by considering more. Both activities require car transport as an input. Let

$$Z_1 = Z_1(X_1, T_1, C_1), \quad X_1 = x_1 Z_1; T_1 = t_1 Z_1; C_1 = c_1 Z_1$$

$$Z_2 = Z_2(X_2, T_2, C_2), \quad X_2 = x_2 Z_2; T_2 = t_2 Z_2; C_2 = c_2 Z_2$$

where C_i is the total use of car transport in the production of activity *i*, and c_i is the number of kilometres needed per unit of activity *i*. Of course, x_i, t_i, c_i depend on input prices if we allow for possible substitution between inputs. We do assume in this section (to focus clearly on the issue of derived demand) that transport itself is produced in fixed proportions, so that the distinction between commodity taxes and activity taxes that we discussed in the previous section can be avoided.

The transport tax per kilometre is denoted θ_c. Note that it is assumed to be uniform, i.e., it is not activity-specific. However, we do allow for the possibility that congestion is activity-specific; it may be generated by the traffic associated with a given activity and impose time losses only on other participants in this activity:

$$E_1 = NC_1 = Nc_1 Z_1; \quad E_2 = NC_2 = Nc_2 Z_2 \tag{16}$$

This seems a reasonable description of at least some activities; people going to shopping centres cause local congestion there, but do not interact with people attending an evening concert, and vice versa. It follows that the generalized price of one unit of car transport is also activity-specific; it is given by:

$$P_c^i(p_c, \theta_c, w, E_i) = p_c x_c(E_i) + w t_c(E_i) + \theta_c \tag{17}$$

The unit cost of Z_i can be obtained from

$$\underset{x_i, t_i, c_i}{Min}\; p_i x_i + w t_i + \left[P_c^i(p_c, \theta_c, w, E_i) \right] c_i \quad s.t.\, Z_i = 1$$

This leads to the cost minimizing input uses

$$x_i(p_i, w, P_c^i), \quad t_i(p_i, w, P_c^i), \quad c_i(p_i, w, P_c^i) \tag{18}$$

and the generalized unit price of activity *i*:

$$Q_i(p_i, w, P_c^i). \tag{19}$$

For later reference, note that by Shephard's lemma and the definition of the generalized transport price:

$$\frac{\partial Q_i}{\partial \theta_c} = c_i; \frac{\partial Q_i}{\partial p_i} = x_i; \frac{\partial Q_i}{\partial w} = t_i; \frac{\partial Q_i}{\partial E_i} = c_i \frac{\partial Q_c}{\partial E_i}. \tag{20}$$

Finally, consumer behaviour produces activity demands

$$Z_i\left[Q_1(p_1, w, P_c^1), Q_2(p_2, w, P_c^2), w\overline{T} + y\right]$$

and indirect utility:

$$v\left[Q_1(p_1, w, P_c^1), Q_2(p_2, w, P_c^2), w\overline{T} + y\right]$$

The optimal tax problem can be formulated as:[14]

$$\underset{p_1, p_2, \theta_c}{Max}\ v\left[Q_1(p_1, w, P_c^1), Q_2(p_2, w, P_c^2), w\overline{T} + y\right]$$

$$\begin{aligned} s.t. \quad & (p_1 - q_1) * x_1(.) * Z_1\ . + (p_2 - q_2) * x_2(.) * Z_2\ . \\ & + \theta_c *\ c_1(.)Z_1(.) + c_2(.)Z_2(.)\ = R \end{aligned}$$

The tax rules are derived in Appendix 4. The general results are complex; however, the most important insights can be obtained by focusing on the case where $\phi = 0$. We find the following rules:

$$\frac{p_1 - q_1}{p_1} = \frac{\left(\varepsilon_{11}\alpha_{c_1} + \eta_{cx}^1 \dfrac{c_1 P_c^1}{p_1 x_1}\right)}{P_c^1\ \ \varepsilon_{11}\alpha_{x_1} + \eta_{xx}^1}(1-\delta)(MECC_1 - MECC_2) \tag{21}$$

$$\frac{p_2 - q_2}{p_2} = \frac{\left(\varepsilon_{22}\alpha_{c_2} + \eta_{cx}^2 \dfrac{c_2 P_c^2}{p_2 x_2}\right)}{P_c^2\ \ \varepsilon_{22}\alpha_{x_2} + \eta_{xx}^2}\delta(MECC_2 - MECC_1) \tag{22}$$

$$\theta_c = \delta(MECC_1) + (1-\delta)(MECC_2) \tag{23}$$

In these expressions, $0 < \delta < 1$. Moreover,

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[14] The government has in principle four instruments to control congestion and generate revenues, viz. , p_1, p_2, p_c, θ_c. However, given our assumptions, it is obvious that with the toll θ_c available, there is no need to tax car inputs, hence $p_c - q_c = 0$.

$$\alpha_{x_i} = \frac{p_i x_i}{Q_i}, \alpha_{c_i} = \frac{P_c^i c_i}{Q_i}$$

are the cost shares of market goods and transport in the generalized price of activity *i*. Finally,

$$\eta_{xx}^i = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}, \eta_{cx}^i = \frac{\partial c_i}{\partial p_i}\frac{p_i}{c_i}, \eta_{xc}^i = \frac{\partial x_i}{\partial P_c^i}\frac{P_c^i}{x_i}, \eta_{cc}^i = \frac{\partial c_i}{\partial P_c^i}\frac{P_c^i}{c_i}$$

are own and cross price elasticities of input demands (market inputs *x* and car transport *c*) in the production of a given activity *i*.

Interpretation of the tax rules (21)-(22)-(23) is as follows. First, not surprisingly, the optimal toll (23) is a weighted average of the two marginal external costs, the weight δ depending in a complex way on various elasticities (see Appendix 4). Second, however, (21)-(22) suggest that due to the absence of activity-specific congestion tolls, the optimal taxes on commodity inputs in activities (e.g., commodities at shopping centres, tickets at theatres or sports events, etc.) do play a role in dealing with congestion. To see the implications, make the mild assumption that the cross effects of transport demand with respect to the price of other commodity inputs in an activity (the η_{cx}^1, η_{cx}^2) are not too highly positive, so that the numerators of (21)-(22) are negative. Further arbitrarily assume that $MECC_1 > MECC_2$; i.e., the first activity attracts much more transport and generates more congestion. Then expressions (21)-(22) immediately imply:

$$\frac{p_1 - q_1}{p_1} > 0 \, ; \quad \frac{p_2 - q_2}{p_2} < 0$$

In other words, the tax structure will take account of heavy congestion attracted by the first activity by making market inputs into this activity more expensive; the opposite holds for the market input in the second activity. This makes sense: the optimal uniform road toll implies that congestion associated with the first activity is taxed too low, congestion at the second activity is over-taxed.

In practice, the results suggest that, in the absence of perfect toll instruments, the congestion generated by particular activities (congestion which typically occurs at very specific points in time and space) is corrected via, for example, high parking fees at shopping and employment centres, higher taxes on shops at shopping malls, higher taxes on ticket prices at concert halls and sports stadiums, etc. Doing so raises the generalized activity price, reduces demand and, hence, reduces congestion. Note that the above results were derived for a setting where $\phi = 0$. However, incorporating budgetary considerations (and hence Ramsey terms will appear, see Appendix 4) does not change the above insights: the message is that taxes on other market inputs in congestion-prone activities are actively used to correct for congestion externalities.

Finally, market input taxes will play an even more explicit role if for some reason (political or otherwise) the uniform tax on transport is not optimal. Assume, e.g., that the transport tax is fixed at a suboptimal level $\bar{\theta}_c$. The tax structure of the inputs in transport-using activities will then be used to further 'correct' for the suboptimal congestion tax. To see this, assume again there are only two activities and, as before, let cross-elasticities be zero to facilitate the interpretation. Simple algebra shows the relative taxes on the commodity inputs in both activities to be as follows (Appendix 4):

$$\frac{p_1 - q_1}{p_1} = \frac{\phi - \left(\frac{\bar{\theta}_c - MECC_1}{P_c^1}\right)\left[\alpha_{c_1}\varepsilon_{11} + \frac{c_1 P_c^1}{p_1 x_1}\eta_{cx}^1\right]}{\alpha_{x_1}\varepsilon_{11} + \eta_{xx}^1} \tag{24}$$

$$\frac{p_2 - q_2}{p_2} = \frac{\phi - \left(\frac{\bar{\theta}_c - MECC_2}{P_c^2}\right)\left[\alpha_{c_2}\varepsilon_{22} + \frac{c_2 P_c^2}{p_2 x_2}\eta_{cx}^2\right]}{\alpha_{x_2}\varepsilon_{22} + \eta_{xx}^2} \tag{25}$$

The intuition is obvious. Assume the transport tax is below marginal external cost for both activities. The tax structure will then charge higher taxes on market inputs into activities that generate much transport or on inputs that easily substitute for transport. For example, the tax structure raises the tax $p_1 - q_1$ on the first activity to the extent that α_{c_1} is large (transport is a large share of total activity cost), or that have a large downward effect on transport use (η_{cx}^1<0 and large in absolute value).

The results of this section suggest tackling the congestion generated by particular activities by raising taxes on market inputs into these activities. This could imply, for example, high parking fees at shopping and employment centres, at cultural and sports manifestations, etc. Doing so raises the generalized activity price, reduces demand and, hence, reduces congestion. Provided that a substantial fraction of visitors come by car (if not, this policy may be less desirable because they do not discriminate between users of different modes), it could also mean relatively high taxes on shops at shopping malls, higher taxes on ticket prices at concert halls and sports stadiums, etc.

5. Conclusions

The purpose of this paper was to study the optimal tax treatment of congestion in a formal time allocation setting. By extending several well known optimal tax rules we derived the following results. First, assuming that the government is restricted to taxing market inputs into transport trip production (fuel, maintenance, etc.), the time allocation framework provides an argument for taxing congestion below marginal external cost. The Pigouvian tax element is below marginal external cost and is a declining function of the importance of time in trip production and of the substitution possibilities between market goods and time. Market goods that substitute for time (such as, e.g., GPS and some ATIS technology) should be taxed at a lower rate than other inputs (such as fuel), or even be subsidized. Moreover, substitution possibilities between time and market goods in the production of car trips also provide an argument for subsidizing public transport. Second, however, in case the government can use road pricing to directly tax transport, we show that the Pigouvian tax component equals marginal external cost. In line with maintaining household production efficiency, it is no longer optimal to subsidize time-saving equipment, and the argument for reducing public transport fares disappears. Intuitively, the road toll does not distort the choice of time versus commodity inputs in trip production, so that the extra stimulus of lower taxes on time-saving devices or on public transport is no longer needed. Third, we show that explicitly treating transport demand as derived from activities implies a useful role for taxes on other market inputs in transport-using activities whenever optimal activity-specific congestion tolls are not possible. Among others, the results imply, in the absence of perfect road pricing, partially correcting the congestion attracted by employments centres, by shopping centres or by large sports and cultural events partially via higher taxes on market inputs in these activities (e.g., parking, entry tickets, etc.).

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Appendix 1

Consider the problem:

$$\underset{p_1,\ldots,p_n,p_c}{Max}\ v\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,w,E),w,w\overline{T}+y\right]$$

$$s.t.\quad \left(\sum_{i=1}^{n}(p_i-q_i)*x_i(p_i,w)*Z_i(.)\right)+(p_c-q_c)*x_c(p_c,w)*Z_c(.)=R$$

The first-order condition for an arbitrary market input price p_j in a non-transport activity can be written as:

$$\begin{aligned}&\frac{\partial v}{\partial Q_j}\frac{\partial Q_j}{\partial p_j}+\frac{\partial v}{\partial Q_c}\frac{\partial Q_c}{\partial E}\frac{dE}{dp_j}\\&+\gamma\left\{\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial Z_i}{\partial Q_j}\frac{\partial Q_j}{\partial p_j}+\left[(p_j-q_j)\frac{\partial x_j}{\partial p_j}+x_j\right]Z_j+(p_c-q_c)x_c\frac{\partial Z_c}{\partial Q_j}\frac{\partial Q_j}{\partial p_j}\right\}\\&+\gamma\left\{\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial Z_i}{\partial Q_c}\frac{\partial Q_c}{\partial E}+(p_c-q_c)\left[x_c\frac{\partial Z_c}{\partial Q_c}\frac{\partial Q_c}{\partial E}+Z_c\frac{\partial x_c}{\partial E}\right]\right\}\frac{dE}{dp_j}=0\end{aligned}\qquad \text{(A1.1)}$$

where γ is the shadow price of the government's budget constraint.

To simplify this expression, it is instructive to first look at the externality in more detail. Consider the effect of an increase in an arbitrary price p_j (*j=1,2,…,n; c*) on the traffic level *E*. Differentiating

$$E=NZ_c$$

and using the demand function for car transport we easily derive:

$$\frac{dE}{dp_j}=\frac{N\left[\frac{\partial Z_c}{\partial Q_j}\frac{\partial Q_j}{\partial p_j}\right]}{1-\beta}\qquad \text{(A1.2)}$$

where $\beta=N\frac{\partial Z_c}{\partial Q_c}\frac{\partial Q_c}{\partial E}$ is the feedback effect of congestion on demand (see Sandmo (2000), Mayeres and Proost (1997)). Note that the feedback effect dampens the direct effect of a price change whenever β is negative: more congestion increases the generalised price of car transport which itself reduces demand for transport and, therefore, congestion.

Without loss of generality, we have normalized *N=1*. Using Roy's identity, Shephard's lemma (5), and (A1.2), the condition (A1.1) can be rewritten as:

$$-\lambda Z_j x_j + \gamma \left\{ \sum_{i=1}^{n} (p_i - q_i) x_i x_j \frac{\partial Z_i}{\partial Q_j} + \left[(p_j - q_j) \frac{\partial x_j}{\partial p_j} + x_j \right] Z_j + (p_c - q_c) x_c x_j \frac{\partial Z_c}{\partial Q_j} \right\} \\ - \gamma * MECC * x_j \frac{\partial Z_c}{\partial Q_j} = 0 \tag{A1.3}$$

where λ is the private marginal utility of income, and *MECC* is the full marginal external cost of an increase in activity Z_c. It is given by:

$$MECC =$$

$$-\frac{1}{\gamma} \left\{ \frac{\partial v}{\partial Q_c} \frac{\partial Q_c}{\partial E} + \gamma \left[\sum_{i=1}^{n} (p_i - q_i) x_i \frac{\partial Z_i}{\partial Q_c} \frac{\partial Q_c}{\partial E} + (p_c - q_c)(x_c \frac{\partial Z_c}{\partial Q_c} \frac{\partial Q_c}{\partial E} + Z_c \frac{\partial x_c}{\partial E}) \right] \right\} \frac{1}{1-\beta}$$

Note that this is a very general definition, capturing budgetary and feedback effects of increases in transport flows. Congestion does not only have a direct welfare effect (i.e., it raises the generalised price of transport and, therefore, reduces utility), but it also induces changes in tax revenues (the generalised price change of transport affects all demand functions and all tax revenues). Moreover, the definition of the *MECC* takes into account the demand feedback. Finally, utility effects are transformed into monetary terms by dividing by the shadow price of the budget constraint.

Rearranging (A1.3), and dividing by $\gamma Z_j x_j$, yields

$$\sum_{i=1}^{n} (p_i - q_i) x_i \frac{\partial Z_i}{\partial Q_j} \frac{1}{Z_j} + (p_j - q_j) \frac{\partial x_j}{\partial p_j} \frac{1}{x_j} + (p_c - q_c - \frac{MECC}{x_c}) x_c \frac{\partial Z_c}{\partial Q_j} \frac{1}{Z_j} = \frac{\lambda - \gamma}{\gamma}$$

Finally, use the Slutsky equation to transform price effects in compensated terms. This leads to:

$$\sum_{i=1}^{n} (p_i - q_i) x_i \frac{\partial \tilde{Z}_i}{\partial Q_j} \frac{1}{Z_j} + (p_j - q_j) \frac{\partial x_j}{\partial p_j} \frac{1}{x_j} + (p_c - q_c - \frac{MECC}{x_c}) x_c \frac{\partial \hat{Z}_c}{\partial Q_j} \frac{1}{Z_j} = \phi \tag{A1.4}$$

In this expression, $\hat{Z}_i$ refers to compensated demands, and

$$\phi = \frac{\lambda - \gamma}{\gamma} + \left\{ \sum_{i=1}^{I} (p_i - q_i) x_i \frac{\partial Z_i}{\partial y} + (p_c - q_c - \frac{MECC}{x_c}) x_c \frac{\partial Z_c}{\partial y} \right\}$$

is a parameter, the same in all tax expressions, that reflects the difference between the marginal social utility of income, corrected for external costs, and the shadow price of the government budget constraint. For the marginal social utility we refer to, among others, Diamond (1975)).

A completely analogous procedure applies to the first order condition for the price of market goods used in transport production p_c. We find:

$$\sum_{i=1}^{n}(p_i - q_i)x_i \frac{\partial \hat{Z}_i}{\partial Q_c}\frac{1}{Z_c} + (p_c - q_c)\frac{\partial x_c}{\partial p_c}\frac{1}{x_c} + (p_c - q_c - \frac{MECC}{x_c})x_c \frac{\partial \hat{Z}_c}{\partial Q_c}\frac{1}{Z_c} = \phi \qquad \text{(A1.5)}$$

Assuming zero cross-elasticities between all activities we can rewrite the resulting expressions as:

$$(\frac{p_j - q_j}{p_j})p_j x_j \frac{\partial \hat{Z}_j}{\partial Q_j}\frac{1}{Z_j} + (\frac{p_j - q_j}{p_j})\frac{\partial x_j}{\partial p_j}\frac{p_j}{x_j} = \phi \qquad \text{for all } j \quad \text{(A1.6)}$$

$$(\frac{p_c - q_c - \frac{MECC}{x_c}}{p_c})p_c x_c \frac{\partial \hat{Z}_c}{\partial Q_c}\frac{1}{Z_c} + (\frac{p_c - q_c}{p_c})\frac{\partial x_c}{\partial p_c}\frac{p_c}{x_c} = \phi \qquad \text{(A1.7)}$$

Now denote the own compensated elasticity of demand for activity *j* with respect to its generalized price as:

$$\hat{\varepsilon}_{jj} = \frac{\partial \hat{Z}_j}{\partial Q_j}\frac{Q_j}{Z_j} \qquad \text{(A1.8)}$$

Furthermore, let the share of market goods and time in the generalized price of *j* be given by, respectively:

$$\alpha_{x_j} = \frac{p_j x_j}{Q_j}; \quad \alpha_{t_j} = \frac{wt_j}{Q_j} \qquad \text{(A1.9)}$$

Finally, following Kleven (2004), note that the price elasticity of input demand can be expressed in terms of the substitution elasticity between commodity and time input σ_j and the generalised cost share of time as follows:

$$\frac{\partial x_j}{\partial p_j}\frac{p_j}{x_j} = -\sigma_j \alpha_{t_j}. \qquad \text{(A1.10)}$$

Substituting these results into (A1.6)-(A1.7) and rearranging, we find the rules (9)-(10) discussed in the paper. Finally, differentiating $X_j = x_j Z_j$ with respect to p_j and using (A1.8), (A1.9) and (A1.10), it immediately follows that the compensated elasticity with respect to the market good can be written as:

$$\frac{\partial \hat{X}_j}{\partial p_j}\frac{p_j}{\hat{X}_j} = \alpha_{x_j}\hat{\varepsilon}_{jj} - \sigma_j \alpha_{t_j}$$

The next step is to derive the Corlett-Hague rule for this model. To do so, assume there are just two activities and leisure, and let cross-elasticities be nonzero. Assume that the two activities are activity 1 and car transport c. Using (A1.4) and (A1.5) for the case of these two goods, we can write the resulting two-equation system as follows:

$$\begin{aligned} &\left(\frac{p_1-q_1}{p_1}\right)\ \alpha_{x_1}\hat{\varepsilon}_{11}-\sigma_1\alpha_{t_1}\ +\left(\frac{p_c-q_c}{p_c}\right)\ \alpha_{x_c}\hat{\varepsilon}_{1c}\ =\phi+\frac{MECC}{Q_c}\hat{\varepsilon}_{1c} \\ &\left(\frac{p_1-q_1}{p_1}\right)\ \alpha_{x_1}\hat{\varepsilon}_{c1}\ +\left(\frac{p_c-q_c}{p_c}\right)\ \alpha_{x_c}\hat{\varepsilon}_{cc}-\sigma_c\alpha_{t_c}\ =\phi+\frac{MECC}{Q_c}\hat{\varepsilon}_{cc} \end{aligned} \qquad \text{(A1.11)}$$

Now solve the system by Cramer's rule, and use the homogeneity of compensated demands; this implies $\hat{\varepsilon}_{11}+\hat{\varepsilon}_{1c}+\hat{\varepsilon}_{10}=0;\ \ \hat{\varepsilon}_{c1}+\hat{\varepsilon}_{cc}+\hat{\varepsilon}_{c0}=0$, where $\hat{\varepsilon}_{10},\hat{\varepsilon}_{c0}$ are the compensated cross elasticities between the two goods and leisure, respectively. Finally, divide the first percentage tax by the second to find:

$$\frac{\dfrac{p_1-q_1}{p_1}}{\dfrac{p_c-q_c}{p_c}}=\frac{\phi\left\{\alpha_{x_c}\left[\hat{\varepsilon}_{cc}+\hat{\varepsilon}_{11}+\hat{\varepsilon}_{10}-\sigma_c\dfrac{\alpha_{t_c}}{\alpha_{x_c}}\right]\right\}-\dfrac{MECC}{Q_c}\left[\hat{\varepsilon}_{1c}\sigma_c\alpha_{t_c}\right]}{\phi\left\{\alpha_{x_1}\left[\hat{\varepsilon}_{cc}+\hat{\varepsilon}_{11}+\hat{\varepsilon}_{c0}-\sigma_1\dfrac{\alpha_{t_1}}{\alpha_{x_1}}\right]\right\}+\dfrac{MECC}{Q_c}\left[\hat{\varepsilon}_{cc}(\alpha_{x_1}\hat{\varepsilon}_{11}-\sigma_1\alpha_{t_1})-\alpha_{x_1}\hat{\varepsilon}_{1c}\hat{\varepsilon}_{c1}\right]}$$

(A1.12)

This is the Corlett-Hague equivalent for our model with time allocation, substitution between market inputs and time in activity production, and externalities. The standard Corlett-Hague rule immediately follows as a special case. Ignore time allocation issues (hence $\alpha_{t_1}=\alpha_{t_c}=0$, $\alpha_{x_1}=\alpha_{x_c}=1$), and assume there are no externalities. For further interpretation, we refer to the main body of the paper.

Finally, assume zero income effects, and assume the cost of funds equals the marginal utility of income. These assumptions imply $\phi=0$. Furthermore, assume zero substitution in rail trip production. Solving (A1.11) then yields the following optimal tax rules:

$$\frac{p_1-q_1}{p_1}=-\frac{1}{\Delta}\left\{\frac{MECC}{Q_c}\ \sigma_c\alpha_{t_c}\hat{\varepsilon}_{1c}\ \right\}$$

$$\frac{p_c-q_c}{p_c}=\frac{1}{\Delta}\left\{\frac{MECC}{Q_c}\left[\alpha_{x_1}\ \ \hat{\varepsilon}_{cc}\hat{\varepsilon}_{11}-\hat{\varepsilon}_{c1}\hat{\varepsilon}_{1c}\ \right]\right\}$$

where $\Delta = \alpha_{x_1}\alpha_{x_c} \quad \hat{\varepsilon}_{11}\hat{\varepsilon}_{cc} - \hat{\varepsilon}_{c1}\hat{\varepsilon}_{1c} \quad -\alpha_{x_1}\hat{\varepsilon}_{11}\sigma_c\alpha_{t_c} > 0$.

Appendix 2: Extension to multiple car inputs

Consider the case of multiple inputs in transport production. The problem determining optimal taxes can be written as:

$$\underset{p_1,\dots,p_n,p_c}{Max} \; v\left[Q_1(p_1,w),\dots,Q_n(p_n,w),Q_c(p_C^1,\dots,p_C^m,w,E),w,w\overline{T}+y\right]$$

$$s.t. \quad \left(\sum_{i=1}^{n}(p_i-q_i)*x_i(p_i,w)*Z_i(.)\right)+\left[\sum_{k=1}^{m}(p_C^k-q_C^k)*x_C^k(p_C^1,.,p_C^m,w,E)\right]*Z_c(.)=R$$

where

$$Z_i(.)=Z_i\left[Q_1(p_1,w),\dots,Q_n(p_n,w),Q_c(p_C^1,\dots,p_C^m,w,E),w,w\overline{T}+y\right],\quad i=1,2,\dots,n;c$$

The generalised price of a unit of transport activity can be written as:

$$Q_C(p_C^1,p_C^2,\dots,p_C^m,w,E)=\quad\sum_{k=1}^{m}p_C^k x_C^k(p_C^1,p_C^2,\dots,p_C^m,w,E)+wt_C(p_C^1,p_C^2,\dots,p_C^m,w,E)$$

Going through exactly the same derivations as in Appendix 1, it follows that the first-order conditions with respect to commodity input prices p_j (*j=1,2,….n*) and car input prices p_C^l (*l=1,2,…,m*) can be written as, respectively[15]:

$$\left[\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial\hat{Z}_i}{\partial Q_j}\frac{1}{Z_j}\right]+(p_j-q_j)\frac{\partial x_j}{\partial p_j}\frac{1}{x_j}+\left[\sum_{k=1}^{m}(p_C^k-q_C^k)x_C^k-MECC\right]\frac{\partial\hat{Z}_c}{\partial Q_j}\frac{1}{Z_j}=\phi$$

for all j

$$\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial\hat{Z}_i}{\partial Q_c}\frac{1}{Z_c}+\left[\sum_{k=1}^{m}(p_C^k-q_C^k)\frac{\partial x_C^k}{\partial p_C^l}\frac{1}{x_C^l}\right]+\left[\sum_{k=1}^{m}(p_C^k-q_C^k)x_C^k-MECC\right]\frac{\partial\hat{Z}_c}{\partial Q_c}\frac{1}{Z_c}=\phi$$

for all l

Assuming zero cross-elasticities between activities and slightly rearranging yields:

$$(\frac{p_j-q_j}{p_j})\frac{\partial x_j}{\partial p_j}\frac{p_j}{x_j}+(\frac{p_j-q_j}{p_j})p_jx_j\frac{\partial\hat{Z}_j}{\partial Q_j}\frac{1}{Z_j}=\phi\qquad\forall j$$

---

[15] Note that the definition of *MECC* as well as the definition of ϕ slightly differs from the definitions in Appendix 1, due to the different tax instruments. In order not to overburden the text with different notation, we keep notation the same, despite the slightly different formulation of these concepts. The same remark holds for the following appendices as well.

$$\left[\sum_{k=1}^{m}(p_C^k - q_C^k)\frac{\partial x_C^k}{\partial p_C^l}\frac{1}{x_C^l}\right] + \left[\sum_{k=1}^{m}(p_C^k - q_C^k)x_C^k - MECC\right]\frac{\partial \hat{Z}_c}{\partial Q_c}\frac{1}{Z_c} = \phi \qquad \forall l$$

Obviously, the condition for taxes on market inputs into regular activities j is identical to (A1.6) above. To interpret the rule for car transport inputs, assume for simplicity that there are just two commodity inputs. Solving the first-order condition for the two car input taxes by Cramer's rule and rearranging yields, after simple algebra:

$$\frac{\dfrac{p_c^1 - q_c^1}{p_c^1}}{\dfrac{p_c^2 - q_c^2}{p_c^2}} = \frac{\left[\eta_{22}^c - \eta_{21}^c \dfrac{p_c^2 x_c^2}{p_c^1 x_c^1}\right]}{\left[\eta_{11}^c - \eta_{12}^c \dfrac{p_c^1 x_c^1}{p_c^2 x_c^2}\right]}$$

In these expressions,

$$\eta_{ij}^c = \frac{\partial x_c^i}{\partial p_c^j}\frac{p_c^j}{x_c^i}$$

Now use the homogeneity of the input demand functions into car trip production; this implies:

$$p_c^1 \frac{\partial x_c^1}{\partial p_c^1} + p_c^2 \frac{\partial x_c^2}{\partial p_c^1} = -w\frac{\partial t_c}{\partial p_c^1}$$

$$p_c^1 \frac{\partial x_c^1}{\partial p_c^2} + p_c^2 \frac{\partial x_c^2}{\partial p_c^2} = -w\frac{\partial t_c}{\partial p_c^2}$$

Substituting and rearranging finally yields (13):

$$\frac{\dfrac{p_c^1 - q_c^1}{p_c^1}}{\dfrac{p_c^2 - q_c^2}{p_c^2}} = \frac{\left[\eta_{11}^c + \eta_{22}^c\right] + w\dfrac{\partial t_c}{\partial p_c^1}\dfrac{1}{x_c^1}}{\left[\eta_{11}^c + \eta_{22}^c\right] + w\dfrac{\partial t_c}{\partial p_c^2}\dfrac{1}{x_c^2}}$$

Appendix 3: The case of road pricing

Denote the per kilometre tax by θ_c. Budget and time restrictions now read

$$\sum_{i=1}^{n} p_i X_i + p_c X_c + \theta_c Z_c = wT_W + y$$

$$Z_0 + \sum_{i=1}^{n} T_i + T_c + T_W = \bar{T}$$

For non-transport goods, the analysis of the previous case (subsection 2.2) still applies; nothing changes. For car transport, the generalised price now equals:

$$Q_c(p_c,\theta_c,w,E) = p_c x_c(p_c,w,E) + wt_c(p_c,w,E) + \theta_c$$

The consumer's problem can be reformulated as

$$\underset{Z_0,Z_1...,Z_n,Z_c}{Max} \quad u(Z_0,Z_1,...,Z_n,Z_c)$$

$$s.t. \sum_{i=1}^{n} Q_i(p_i,w)Z_i + Q_c(p_c,\theta_c,w,E)Z_c + wZ_0 = w\overline{T} + y$$

This yields activity demand functions:

$$Z_i\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right] \qquad i=0,....n$$

$$Z_c\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right]$$

The indirect utility function reads:

$$v\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right].$$

The government's problem is to solve

$$\underset{p_1,...,p_n,p_c,\theta_c}{Max} \; v\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right]$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{n}(p_i-q_i)*x_i(p_i,w)*Z_i\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right] \\ & +(p_c-q_c)*x_c(p_c,w)*Z_c\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right] \\ & +\theta_c*Z_c\left[Q_1(p_1,w),....,Q_n(p_n,w),Q_c(p_c,\theta_c,w,E),w,w\overline{T}+y\right] = R \end{aligned}$$

Using the same simple derivations as in Appendix 1, we find that the first-order conditions with respect to commodity inputs p_j $(j=1,...,n)$ in regular non-transport activities, the price of transport commodity input p_c, and the road toll θ_c are given by, respectively:

$$\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial\hat{Z}_i}{\partial Q_j}\frac{1}{Z_j} + (p_j-q_j)\frac{\partial x_j}{\partial p_j}\frac{1}{x_j} + \;(p_c-q_c)x_c+\theta_c-MECC\;\frac{\partial\hat{Z}_c}{\partial Q_j}\frac{1}{Z_j} = \phi$$

$$\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial\hat{Z}_i}{\partial Q_c}\frac{1}{Z_c} + (p_c-q_c)\frac{\partial x_c}{\partial p_c}\frac{1}{x_c} + \;(p_c-q_c)x_c+\theta_c-MECC\;\frac{\partial\hat{Z}_c}{\partial Q_c}\frac{1}{Z_c} = \phi$$

$$\sum_{i=1}^{n}(p_i-q_i)x_i\frac{\partial\hat{Z}_i}{\partial Q_c}\frac{1}{Z_c} + \;(p_c-q_c)x_c+\theta_c-MECC\;\frac{\partial\hat{Z}_c}{\partial Q_c}\frac{1}{Z_c} = \phi$$

For zero cross elasticities, this produces the results in the main body of the paper.

Appendix 4: Transport as derived from other activities

1. Optimal uniform road toll

As argued in the main body of the paper, we restrict the analysis to two transport-using activities. The optimal tax problem can be formulated as:

$$\underset{p_1,p_2,\theta_c}{Max}\; v\left[Q_1(p_1,w,P_c^1),Q_2(p_2,w,P_c^2),w\overline{T}+y\right]$$

$$s.t. \quad (p_1-q_1)*x_1(.)*Z_1 \;.\; +(p_2-q_2)*x_2(.)*Z_2 \;.$$
$$+\theta_c * \; c_1(.)Z_1(.)+c_2(.)Z_2(.) \; = R$$

where

$$P_c^i(p_c,\theta_c,w,E_i)=p_c x_c(E_i)+wt_c(E_i)+\theta_c$$

are the generalized prices per kilometre of transport associated with activity *i (i=1,2).* Demands for activities are:

$$Z_i\left[Q_1(p_1,w,P_c^1),Q_2(p_2,w,P_c^2),w\overline{T}+y\right] \text{ for } i=1,2$$

The first order condition with respect to p_1 can be written as, using Roy's identity and Shephard's lemma:

$$\begin{aligned}&-(\lambda-\gamma)Z_1x_1+\gamma\left\{(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}x_1+Z_1\frac{\partial x_1}{\partial p_1}\right)+\theta_c\left(c_1x_1\frac{\partial Z_1}{\partial Q_1}+Z_1\frac{\partial c_1}{\partial p_1}\right)\right\}\\&+\gamma\left\{\frac{1}{\gamma}\frac{\partial v}{\partial Q_1}c_1+(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial x_1}{\partial P_c^1}\right)+\theta_c\left(c_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial c_1}{\partial P_c^1}\right)\right\}\frac{\partial P_c^1}{\partial E_1}\frac{dE_1}{dp_1}=0\end{aligned} \qquad \text{(A4.1)}$$

We have immediately assumed zero cross elasticities between activities for simplicity. As always, γ is the shadow price of the government's budget constraint.

Since congestion is activity-specific by assumption, let us reconsider the effect of an increase in p_1 on the traffic level E_1. Differentiating

$$E_1=NC_1=Nc_1Z_1=N\;c_1(p_1,w,P_c^1)\left[Z_1\;Q_1(p_1,w,P_c^1),Q_2(p_2,w,P_c^2),w\overline{T}+y\;\right]$$

we easily derive:

$$\frac{dE_1}{dp_1}=\frac{N\left[c_1x_1\frac{\partial Z_1}{\partial Q_1}+Z_1\frac{\partial c_1}{\partial p_1}\right]}{1-\beta_1}, \quad \beta_1=N\frac{\partial P_c^1}{\partial E_1}\left[c_1\frac{\partial Z_1}{\partial Q_1}\frac{\partial Q_1}{\partial P_c^1}+Z_1\frac{\partial c_1}{\partial P_c^1}\right]$$

where β_1 is the feedback effect of congestion on demand for transport associated with activity one. Using this result, (A4.1) can be rewritten as:

$$-(\lambda-\gamma)Z_1x_1+\gamma\left\{(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}\frac{\partial Q_1}{\partial p_1}+Z_1\frac{\partial x_1}{\partial p_1}\right)+(\theta_c-MECC_1)\left(c_1x_1\frac{\partial Z_1}{\partial Q_1}+Z_1\frac{\partial c_1}{\partial p_1}\right)\right\}=0 \tag{A4.2}$$

where $MECC_1$ is the full marginal external cost of an increase in transport demand C_1 associated with activity Z_1. It is defined as:

$$MECC_1 = -\frac{1}{\gamma}\left\{\frac{\partial v}{\partial Q_1}\frac{\partial Q_1}{\partial P_c^1}+\gamma\left[(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}\frac{\partial Q_1}{\partial P_c^1}+Z_1\frac{\partial x_1}{\partial P_c^1}\right)+\theta_c\left(c_1\frac{\partial Z_1}{\partial Q_1}\frac{\partial Q_1}{\partial P_c^1}+Z_1\frac{\partial c_1}{\partial P_c^1}\right)\right]\right\}\frac{\partial P_c^1}{\partial E_1}\frac{1}{1-\beta_1} \tag{A4.3}$$

Assuming zero income effects, rearranging (A4.2) and dividing by $\gamma Z_1 x_1$, yields after similar transformations as in Appendix 1:

$$\left(\frac{p_1-q_1}{p_1}\right)\varepsilon_{11}\alpha_{x_1}+\eta_{xx}^1+\frac{\theta_c}{P_c^1}\left(\varepsilon_{11}\alpha_{c_1}+\eta_{cx}^1\frac{c_1P_c^1}{p_1x_1}\right)=\phi+\frac{MECC_1}{P_c^1}\left(\varepsilon_{11}\alpha_{c_1}+\eta_{cx}^1\frac{c_1P_c^1}{p_1x_1}\right) \tag{A4.4}$$

Here $\alpha_{x_1}=\frac{p_1x_1}{Q_1}, \alpha_{c_1}=\frac{P_c^1c_1}{Q_1}$ are the cost shares of market goods and transport in the generalized price of the first activity, respectively, and

$$\eta_{xx}^1=\frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}, \eta_{cx}^1=\frac{\partial c_1}{\partial p_1}\frac{p_1}{c_1}$$

are the own and cross price elasticities of input demands in the first activity.

By complete analogy, we have the first order conditions with respect to p_2:

$$\left(\frac{p_2-q_2}{p_2}\right)\varepsilon_{22}\alpha_{x_2}+\eta_{xx}^2+\frac{\theta_c}{P_c^2}\left(\varepsilon_{22}\alpha_{c_2}+\eta_{cx}^2\frac{c_2P_c^2}{p_2x_2}\right)=\phi+\frac{MECC_2}{P_c^2}\left(\varepsilon_{22}\alpha_{c_2}+\eta_{cx}^2\frac{c_2P_c^2}{p_2x_2}\right) \tag{A4.5}$$

where the definition of the marginal external cost of congestion associated with activity two $MECC_2$ is defined analogous to (A4.3); similarly, the feedback for transport associated with activity two β_2 is defined in a similar way as for β_1.

Finally, consider the first-order condition for θ_c. It can be written as, using Roy's identity and Shephard's lemma:

$$
\begin{aligned}
&-(\lambda-\gamma)\ \ Z_1c_1+Z_2c_2\ \ -\lambda Z_1c_1\frac{\partial P_c^1}{\partial E_1}\frac{dE_1}{d\theta_c}-\lambda Z_2c_2\frac{\partial P_c^2}{\partial E_2}\frac{dE_2}{d\theta_c}\\
&+\gamma\left\{(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial x_1}{\partial P_c^1}\right)+(p_2-q_2)\left(x_2\frac{\partial Z_2}{\partial Q_2}c_2+Z_2\frac{\partial x_2}{\partial P_c^2}\right)\right\}\\
&+\gamma\left\{\theta_c\left(c_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial c_1}{\partial P_c^1}+c_2\frac{\partial Z_2}{\partial Q_2}c_2+Z_2\frac{\partial c_2}{\partial P_c^2}\right)\right\}\\
&+\gamma\left\{(p_1-q_1)\left(x_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial x_1}{\partial P_c^1}\right)+\theta_c\left(c_1\frac{\partial Z_1}{\partial Q_1}c_1+Z_1\frac{\partial c_1}{\partial P_c^1}\right)\right\}\frac{\partial P_c^1}{\partial E_1}\frac{dE_1}{d\theta_c}\\
&+\gamma\left\{(p_2-q_2)\left(x_2\frac{\partial Z_2}{\partial Q_2}c_2+Z_2\frac{\partial x_2}{\partial P_c^2}\right)+\theta_c\left(c_2\frac{\partial Z_2}{\partial Q_2}c_2+Z_2\frac{\partial c_2}{\partial P_c^2}\right)\right\}\frac{\partial P_c^2}{\partial E_2}\frac{dE_2}{d\theta_c}=0
\end{aligned}
$$

Working out the effect of tolls on congestion, we have:

$$
\frac{dE_1}{d\theta_c}=\frac{c_1\dfrac{\partial Z_1}{\partial Q_1}c_1+Z_1\dfrac{\partial c_1}{\partial P_c^1}}{1-\beta_1};\quad \frac{dE_2}{d\theta_c}=\frac{c_2\dfrac{\partial Z_2}{\partial Q_2}c_2+Z_2\dfrac{\partial c_2}{\partial P_c^2}}{1-\beta_2}
$$

Using these results and the definition of the activity specific marginal external congestion costs ((A4.3) and its equivalent for $MECC_2$) we have, after simple algebra:

$$
\begin{aligned}
&\left(\frac{p_1-q_1}{p_1}\right)\left(\varepsilon_{11}\alpha_{x_1}+\eta_{xc}^1\frac{p_1x_1}{P_c^1c_1}\right)s_1+\left(\frac{p_2-q_2}{p_2}\right)\left(\varepsilon_{22}\alpha_{x_2}+\eta_{xc}^2\frac{p_2x_2}{P_c^2c_2}\right)(1-s_1)\\
&\quad+\frac{\theta_c}{P_c^1}\ \ \varepsilon_{11}\alpha_{c_1}+\eta_{cc}^1\ \ s_1+\frac{\theta_c}{P_c^2}\ \ \varepsilon_{22}\alpha_{c_2}+\eta_{cc}^2\ \ (1-s_1) \qquad \text{(A4.6)}\\
&\quad=\phi+\frac{MECC_1}{P_c^1}\ \ \varepsilon_{11}\alpha_{c_1}+\eta_{cc}^1\ \ s_1+\frac{MECC_2}{P_c^2}\ \ \varepsilon_{22}\alpha_{c_2}+\eta_{cc}^2\ \ (1-s_1)
\end{aligned}
$$

where $s_1=\dfrac{c_1Z_1}{c_1Z_1+c_2Z_2}$ is the share of activity one in total transport demand.

Next we turn to the solution of system (A4.4-A4.5-A4.6) [16]. To solve, it will be convenient to replace the third equation (A4.6) by ((A4.6))- s_1*(A4.4)- s_2*(A4.5)). The resulting three-equation system can equivalently be written in matrix notation as follows:

$$\begin{bmatrix} \varepsilon_{11}\alpha_{x_1} + \eta_{xx}^1 & 0 & \frac{1}{P_c^1}\left(\varepsilon_{11}\alpha_{c_1} + \eta_{cx}^1 \frac{c_1 P_c^1}{p_1 x_1}\right) \\ 0 & \varepsilon_{22}\alpha_{x_2} + \eta_{xx}^2 & \frac{1}{P_c^2}\left(\varepsilon_{22}\alpha_{c_2} + \eta_{cx}^2 \frac{c_2 P_c^2}{p_2 x_2}\right) \\ \left(\eta_{xc}^1 \frac{\alpha_{x_1}}{\alpha_{c_1}} - \eta_{xx}^1\right) s_1 & \left(\eta_{xc}^2 \frac{\alpha_{x_2}}{\alpha_{c_2}} - \eta_{xx}^2\right)(1-s_1) & \left(\eta_{cc}^1 - \eta_{cx}^1 \frac{\alpha_{c_1}}{\alpha_{x_1}}\right)\frac{s_1}{P_c^1} + \left(\eta_{cc}^2 - \eta_{cx}^2 \frac{\alpha_{c_2}}{\alpha_{x_2}}\right)\frac{(1-s_1)}{P_c^2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{p_1 - q_1}{p_1} \\ \frac{p_2 - q_2}{p_2} \\ \phi_c \end{bmatrix} = \begin{bmatrix} \phi + \frac{MECC_1}{P_c^1}\left(\varepsilon_{11}\alpha_{c_1} + \eta_{cx}^1 \frac{c_1 P_c^1}{p_1 x_1}\right) \\ \phi + \frac{MECC_2}{P_c^2}\left(\varepsilon_{22}\alpha_{c_2} + \eta_{cx}^2 \frac{c_2 P_c^2}{p_2 x_2}\right) \\ MECC_1\left(\eta_{cc}^1 - \eta_{cx}^1 \frac{\alpha_{c_1}}{\alpha_{x_1}}\right)\frac{s_1}{P_c^1} + MECC_2\left(\eta_{cc}^2 - \eta_{cx}^2 \frac{\alpha_{c_2}}{\alpha_{x_2}}\right)\frac{(1-s_1)}{P_c^2} \end{bmatrix}$$

The system has, therefore, the following structure:

$$\begin{bmatrix} d_1 & 0 & a_1 \\ 0 & d_2 & a_2 \\ e_1 & e_2 & b_1 + b_2 \end{bmatrix} \begin{bmatrix} \frac{p_1 - q_1}{p_1} \\ \frac{p_2 - q_2}{p_2} \\ \phi_c \end{bmatrix} = \begin{bmatrix} \phi + a_1 MECC_1 \\ \phi + a_2 MECC_2 \\ b_1 MECC_1 + b_2 MECC_2 \end{bmatrix}$$

where the meaning of the notation is obvious. The solution is given by:

[16] A few preliminary results immediately follow from this system of equations. First, in the presence of lump-sum taxes and activity-specific congestion tolls, setting all commodity taxes equal to zero and having the transport taxes equal to the respective marginal external costs solves the first-order conditions. Intuitively, since the transport tax directly taxes transport activity and not transport market inputs, there is no need to deviate from marginal cost pricing. Second, the tax rules for transport and the transport requiring goods are linearly dependent if there were no substitution possibilities between inputs into the production of the different transport requiring goods. The intuition is simple: if there is no input substitution then the externality can be taxed directly via the transport tax, or it can be taxed via commodity input taxes.

$$\frac{p_1 - q_1}{p_1} = \frac{1}{\Omega} \ \phi \ \ d_2(b_1 + b_2) + e_2(a_2 - a_1) \ \ + (MECC_1 - MECC_2) \ \ a_1(d_2 b_2 - a_2 e_2)$$

$$\frac{p_2 - q_2}{p_2} = \frac{1}{\Omega} \ \phi \ \ d_1(b_1 + b_2) + e_1(a_2 - a_1) \ \ + (MECC_2 - MECC_1) \ \ a_2(d_1 b_1 - a_1 e_1)$$

$$\theta_c = \frac{1}{\Omega} \ \phi \ \ e_1 d_2 - e_2 d_1 \ \ + (MECC_1) \ \ d_2(d_1 b_1 - a_1 e_1) \ \ + (MECC_2) \ \ d_1(d_2 b_2 - a_2 e_2)$$

where $\Omega = d_2(d_1 b_1 - a_1 e_1) + d_1(d_2 b_2 - e_2 a_2)$.

To simplify, define δ as follows:

$$\delta = \frac{d_2(d_1 b_1 - a_1 e_1)}{\Psi}, \text{ so } 1 - \delta = \frac{d_1(d_2 b_2 - a_2 e_2)}{\Psi}$$

It follows:

$$\frac{p_1 - q_1}{p_1} = \frac{1}{\Omega} \ \phi \ \ d_2(b_1 + b_2) + e_2(a_2 - a_1) \ \ \ + \frac{a_1}{d_1}(1 - \delta)(MECC_1 - MECC_2)$$

$$\frac{p_2 - q_2}{p_2} = \frac{1}{\Omega} \ \phi \ \ d_1(b_1 + b_2) + e_1(a_2 - a_1) \ \ \ + \frac{a_2}{d_2}\delta(MECC_2 - MECC_1)$$

$$\theta_c = \frac{1}{\Omega} \ \phi \ \ e_1 d_2 - e_2 d_1 \ \ \ + \delta(MECC_1) + (1 - \delta)(MECC_2)$$

Finally, let there be a lump sum instrument and zero income effects so that $\phi = 0$, and use the definition of the terms a_1, a_2, d_1, d_2. Then we can write the tax rules as:

$$\frac{p_1 - q_1}{p_1} = \frac{\left(\varepsilon_{11}\alpha_{c_1} + \eta_{cx}^1 \frac{c_1 P_c^1}{p_1 x_1} \right)}{P_c^1 \ \ \varepsilon_{11}\alpha_{x_1} + \eta_{xx}^1} (1 - \delta)(MECC_1 - MECC_2)$$

$$\frac{p_2 - q_2}{p_2} = \frac{\left(\varepsilon_{22}\alpha_{c_2} + \eta_{cx}^2 \frac{c_2 P_c^2}{p_2 x_2} \right)}{P_c^2 \ \ \varepsilon_{22}\alpha_{x_2} + \eta_{xx}^2} \delta(MECC_2 - MECC_1)$$

$$\theta_c = \delta(MECC_1) + (1 - \delta)(MECC_2)$$

These are the expressions discussed in the paper.

2. Suboptimal uniform road toll

Finally, assume the transport tax is fixed at a suboptimal level $\bar{\theta}_c$. As before, let cross-elasticities be zero to facilitate the interpretation. The first order conditions with respect to p_1, p_2 are given by (A4.4)-(A4.5), but adjusted for the exogenous toll level:

$$\left(\frac{p_1 - q_1}{p_1}\right) \varepsilon_{11}\alpha_{x_1} + \eta_{xx}^1 = \phi - \left(\frac{\bar{\theta}_c - MECC_1}{P_c^1}\right)\left(\varepsilon_{11}\alpha_{c_1} + \eta_{cx}^1 \frac{c_1 P_c^1}{p_1 x_1}\right)$$

$$\left(\frac{p_2 - q_2}{p_2}\right) \varepsilon_{22}\alpha_{x_2} + \eta_{xx}^2 = \phi - \left(\frac{\bar{\theta}_c - MECC_2}{P_c^2}\right)\left(\varepsilon_{22}\alpha_{c_2} + \eta_{cx}^2 \frac{c_2 P_c^2}{p_2 x_2}\right)$$

We immediately have:

$$\frac{p_1 - q_1}{p_1} = \frac{\phi - \left(\frac{\bar{\theta}_c - MECC_1}{P_c^1}\right)\left[\alpha_{c_1}\varepsilon_{11} + \frac{c_1 P_c^1}{p_1 x_1}\eta_{cx}^1\right]}{\alpha_{x_1}\varepsilon_{11} + \eta_{xx}^1}$$

$$\frac{p_2 - q_2}{p_2} = \frac{\phi - \left(\frac{\bar{\theta}_c - MECC_2}{P_c^2}\right)\left[\alpha_{c_2}\varepsilon_{22} + \frac{c_2 P_c^2}{p_2 x_2}\eta_{cx}^2\right]}{\alpha_{x_2}\varepsilon_{22} + \eta_{xx}^2}$$