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Taming singularities in transverse-momentum-dependent parton densities

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Abstract. We propose a consistent treatment of divergences emerging in the computation of transverse-momentum-dependent parton densities in leading α_s-order of QCD perturbation theory.

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Transverse-momentum-dependent (TMD) parton distribution functions (PDF)s (abbreviated in what follows by the term "TMD") accumulate information about the intrinsic 3-dimensional motion of partons in a hadron [1]. They depend, therefore, on the longitudinal ($x = k^+/p^+$), as well as on the transverse ($\boldsymbol{k}_\perp$) momentum fractions of a given parton. Trying to work out a consistent operator definition of TMDs, one encounters the puzzle of emergent divergences [2, 3]. These, being hidden in the case of collinear PDFs, become visible in the TMDs and jeopardize, in particular, their renormalizability [4, 5, 6, 7, 8]. In the present work, we explore the issue of extra rapidity divergences in the TMDs in leading α_s-order and describe a consistent method to take care of them.

We start from the definition of a TMD (of a quark with flavor i in a hadron h) that respects gauge invariance and collinear factorization on the tree-level [9, 10, 11, 12, 13, 14], but has no concern with any singularities — as these arise only in the one-loop corrections:

$$\mathscr{F}^{\text{tree}}_{i/h}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} \Big\langle h|\bar{\psi}_i(\xi^-,\boldsymbol{\xi}_\perp)[\xi^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\xi}_\perp]^\dagger$$
$$\times[\infty^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\infty}_\perp]^\dagger\gamma^+[\infty^-,\boldsymbol{\infty}_\perp;\infty^-,\boldsymbol{0}_\perp][\infty^-,\boldsymbol{0}_\perp;0^-,\boldsymbol{0}_\perp]\psi_i(0^-,\boldsymbol{0}_\perp)|h\Big\rangle \ . \qquad (1)$$

Tree-level gauge invariance is ensured by the inserted gauge links (path-ordered Wilson-line operators) having the generic form

$$[y,x|\Gamma] = \mathscr{P}\exp\left[-ig\int_{x[\Gamma]}^{y} dz_\mu \mathscr{A}^\mu(z)\right] , \qquad (2)$$

where $\mathscr{A} \equiv t^a A^a$. The transverse gauge links, extending to light-cone infinity [11, 12, 13], are also included in (1). Beyond the tree level, the function (1) will be shown to be

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dependent on the renormalization scale μ and the rapidity cutoff η. We assume that any soft and collinear singularities can be properly factorized out and be treated by means of the standard procedure so that we don't have to consider them anymore. Thus, we only concentrate on the unusual divergences which are specific to the TMD case.

It was shown in [5] that in the light-cone gauge, the anomalous divergent term containing overlapping (UV $\otimes$ rapidity singularity) stems from the virtual-gluon contribution

$$\Sigma^{\rm LC}_{\rm virt.} = -\frac{\alpha_s}{\pi} C_{\rm F}\, \Gamma(\varepsilon)\, \left[4\pi \frac{\mu^2}{-p^2}\right]^{\varepsilon}\, \delta(1-x)\delta^{(2)}(k_\perp)\, \int_0^1 dx \frac{(1-x)^{1-\varepsilon}}{x^{\varepsilon}[x]_\eta}\,, \tag{3}$$

where the UV divergence is treated within the dimensional-regularization $\omega = 4-2\varepsilon$ approach, while the rapidity divergence in the gluon propagator in the light-cone gauge is regularized by the parameter η, entailing the following regularization of the last integral in Eq. (3):

$$\frac{1}{[x]_\eta^{\rm Ret./Adv./P.V.}} = \left[\frac{1}{x+i\eta}\,, \qquad \frac{1}{x-i\eta}\,, \qquad \frac{1}{2}\left(\frac{1}{x+i\eta}+\frac{1}{x-i\eta}\right)\right]\,. \tag{4}$$

Within this approach, one can extract from Eq. (3) the UV-divergent part and obtain the overlapping singularity in the logarithmic form

$$\Sigma^{\rm LC}_{\rm virt.} = -\frac{\alpha_s}{\pi} C_{\rm F}\, \frac{1}{\varepsilon}\left[-\frac{3}{4} - \ln\frac{\eta}{p^+} + \frac{i\pi}{2}\right] + [\text{UV finite part}]\,, \tag{5}$$

where the contribution of the transverse link is taken into account, while the mirror diagram is omitted (see for technical details in [5]). The exact form of the overlapping singularity drops us a hint at the form of the additional soft factor which must be introduced into the definition of TMD (1), if one wants to extend it beyond the tree level in order to render it renormalizable and free of undesirable divergences — at least at one-loop [4, 5]. Hence, a generalized renormalization procedure has been formulated [15] in terms of a soft factor supplementing the tree-level TMD, i.e.,

$$\mathscr{F}^{\rm tree}(x,\boldsymbol{k}_\perp) \to \mathscr{F}(x,\boldsymbol{k}_\perp;\mu,\eta,\varepsilon) \times R^{-1}(\mu,\eta,\varepsilon)\,, \tag{6}$$

so that the above expression is free of overlapping divergences and can be renormalized by means of the standard $R-$operation. Within this framework, the introduction of the small parameter η allows one to keep the overlapping singularities under control and treat the extra term in the UV-divergent part via the cusp anomalous dimension, which in turn determines the specific form of the gauge contour in the soft factor R.

It is worth comparing the result obtained in the light-cone gauge with the calculation in covariant gauges. In Ref. [2], it was shown that the virtual-gluon exchange between the quark line and the light-like gauge link (this graph is obviously absent in the light-cone gauge) yields (in the dimensional regularization)

$$\Sigma^{\rm cov.}_{\rm virt.} = -\frac{\alpha_s}{\pi} C_{\rm F}\Gamma(\varepsilon)\left[4\pi\frac{\mu^2}{-p^2}\right]^{\varepsilon}\, \delta(1-x)\delta^{(2)}(k_\perp)\, \int_0^1 dx\, \frac{x^{1-\varepsilon}}{(1-x)^{1+\varepsilon}}\,. \tag{7}$$

This expression contains the double pole $1/\varepsilon^2$, which is not compensated by the real counter part in the TMD case, while in collinear PDFs, such a compensation does indeed take place. Going back to our Eq. (3), we observe that without the η-regularization of the last integral and after a trivial change of variables it is reduced to Eq. (7) and reads

$$\Sigma^{\text{cov.}}_{\text{virt.}}(\varepsilon) = \Sigma^{\text{LC}}_{\text{virt.}}(\varepsilon, \eta = 0) \ . \tag{8}$$

The latter result allows us to conclude that the generalized renormalization procedure, described above, is gauge invariant and regularization-independent: in principle, one can use dimensional regularization to take care of the overlapping singularities as well. However, in the latter case the structure of extra divergences is much less transparent and one is not able to conclude about the specific form of the soft factor. Let us note that the applicability of dimensional regularization to for a consistent treatment of the divergences arising in the path-dependent gauge invariant two-quark correlation function had been studied in Ref. [16].

Another obstacle still arises in the soft factor. Evaluating in the light-cone gauge one-loop graphs, one finds the expression

$$\Sigma^{\text{LC}}_{\text{soft}}(\varepsilon, \eta) = ig^2\mu^{2\varepsilon} C_{\text{F}} 2p^+ \int \frac{d^\omega q}{(2\pi)^\omega} \frac{1}{q^2(q^- \cdot p^+ - i0)[q^+]_\eta} \ , \tag{9}$$

which contains a new singularity, that can not be circumvented by dimensional regularization or by the η-cutoff, leading to

$$\Sigma^{\text{LC}}_{\text{soft}}(\varepsilon, \eta) = -\frac{\alpha_s}{\pi} C_{\text{F}} \left[\frac{4\pi\mu^2}{\lambda^2}\right]^\varepsilon \Gamma(\varepsilon) \int_0^1 dx \frac{x}{x^2[x-1]_\eta} \ , \tag{10}$$

where λ is the IR regulator. In our previous paper [5], we have argued that this divergence is irrelevant, since it doesn't affect the rapidity evolution. However, for the sake of completeness, we propose here a procedure, which allows one to remove this divergence in a proper way. Taking into account that the extra singularity is cusp-independent, we conclude that it represents the self-energy of the Wilson line, evaluated along a "straightened" path, i.e., assuming that the cusp angle becomes very small: $p^+ \to \eta$. Subtraction of this self-energy part is presented graphically in Fig. 1. Note that there is no need to introduce additional parameters in this subtraction. Moreover, it has a clear physical interpretation: only an irrelevant contribution due to the self-energy of the light-like gauge links is removed, which is merely part of the unobservable background.

Therefore, the "completely subtracted" generalized definition of the TMD reads

$$\mathscr{F}(x, \boldsymbol{k}_\perp; \mu, \eta) = \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} \Big\langle h | \bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger$$
$$\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp][\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp] \psi_i(0^-, \boldsymbol{0}_\perp) | h \Big\rangle R^{-1} \ , \tag{11}$$

$$R^{-1}(\mu, \eta) = \frac{\langle 0 | \mathscr{P} \exp\left[ig \int_{\mathscr{C}_{\text{cusp}}} d\zeta^\mu \mathscr{A}^\mu(\zeta)\right] \cdot \mathscr{P}^{-1} \exp\left[-ig \int_{\mathscr{C}'_{\text{cusp}}} d\zeta^\mu \mathscr{A}^\mu(\xi + \zeta)\right] | 0 \rangle}{\langle 0 | \mathscr{P} \exp\left[ig \int_{\mathscr{C}_{\text{smooth}}} d\zeta^\mu \mathscr{A}^\mu(\zeta)\right] \cdot \mathscr{P}^{-1} \exp\left[-ig \int_{\mathscr{C}'_{\text{smooth}}} d\zeta^\mu \mathscr{A}^\mu(\xi + \zeta)\right] | 0 \rangle} \ ,$$

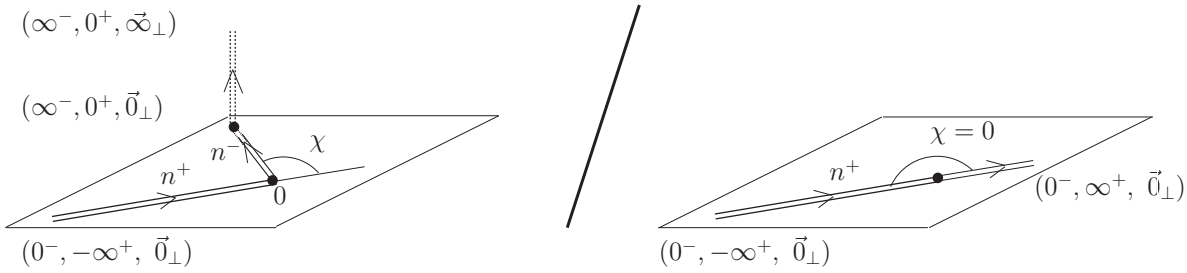

FIGURE 1. Subtraction of the Wilson-line self-energy contribution in the soft factor.

where the cusped and smooth contours are presented in Fig. 1.

To conclude, we have demonstrated that the generalized definition of the TMD (11) is completely gauge- and regularization-invariant, renormalizable and free of any kind of emergent overlapping divergences, including those produced by the artifacts of the soft factor — at least in leading α_s-order. For completeness, one has yet to prove that this definition is part of a TMD factorization theorem (see for an example of such an explicit proof in covariant gauges with gauge links shifted from the light-cone in [17] and the discussion in Ref. [18]), and clarify the relationship of our approach (in particular, the precise form of the soft factors, which might vary within different schemes) to other approaches for the operator definitions of TMDs (e.g., Refs. [19, 20]). This issue is left for future work.

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REFERENCES

1. D. E. Soper, Phys. Rev. Lett. **43** (1979) 1847.
2. J.C. Collins, D.E. Soper, Nucl. Phys. B **194** (1982) 445.
3. J. Collins, PoS **LC2008** (2008) 028 [arXiv:0808.2665 [hep-ph]].
4. I. O. Cherednikov, N. G. Stefanis, Phys. Rev. D **77** (2008) 094001;
5. I. O. Cherednikov, N. G. Stefanis, Nucl. Phys. B **802** (2008) 146.
6. I. O. Cherednikov, N. G. Stefanis, Phys. Rev. D **80** (2009) 054008.
7. N. G. Stefanis, I. O. Cherednikov, Mod. Phys. Lett. A **24** (2009) 2913.
8. I. O. Cherednikov, A. I. Karanikas and N. G. Stefanis, Nucl. Phys. B **840** (2010) 379.
9. J. C. Collins, D. E. Soper, and G. Sterman, Adv. Ser. Direct. High Energy Phys. **5** (1988) 1.
10. J. C. Collins, Acta Phys. Pol. B **34** (2003) 3103.
11. X. Ji, F. Yuan, Phys. Lett. B **543** (2002) 66.
12. A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B **656** (2003) 165.
13. D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. **B667** (2003) 201.
14. A. V. Belitsky, A. V. Radyushkin, Phys. Rept. **418** (2005) 1.
15. G. P. Korchemsky, A. V. Radyushkin, Nucl. Phys. B **283** (1987) 342.
16. N. G. Stefanis, Nuovo Cim. A **83** (1984) 205.
17. X. Ji, J. Ma, F. Yuan, Phys. Rev. D **71** (2005) 034005.
18. J. C. Collins, T. C. Rogers, A. M. Stasto, Phys. Rev. D **77** (2008) 085009.
19. J. C. Collins, A. Metz, Phys. Rev. Lett. **93** (2004) 252001.
20. J. C. Collins, F. Hautmann, Phys. Lett. B **472** (2000) 129.