

# Migration probability as an incentive for human capital accumulation when information is asymmetric

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## Abstract

This paper demonstrates the significant role of a migration probability on the premigration level of human capital accumulation in the source country. The study suggests that the opportunity to migrate to another, superior technology country, may well lead to a "brain gain" for the prospective migrants. Adversely, since the individuals face an opportunity to migrate illegally when the legal status is refused, an information asymmetry arises. Because in many countries laws are not enforced against visibly present illegal immigrants, the information symmetry between the social planner in the source country and the rest of the world cannot be reinstated efficiently. This mismatch leads to a "brain drain" in the source country from the social point of view.

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# 1 Introduction

In a globalising world, the low developing countries (LDCs) and the developed world are competing for human capital. This mainly results in migration flows from low to high income countries because the prospective return to human capital is higher in the latter. A vast part of the literature on this flow of human capital has concentrated on the unfavourable repercussion that emigration from LDCs is associated with a brain drain<sup>1</sup>. The highly skilled leave the developing countries because they face better opportunities in the developed economies. This adverse consequence leaves the LDCs with a less skillful workforce and a persistent lower per capita output.

The emphasis of earlier research on the effect of migration on economic growth via the human capital channel has been on the motivating characteristics of migrants. At first the belief was that the key to migrants' performance is to be found in their ability, skills and innately higher productivity<sup>2</sup>. Yet by shifting the focus of analysis from the vector of migrants' characteristics to the structure of incentives, Galor & Stark (1990), Stark et al. (1998) and Mountford (1997) reached the conditional conclusion that migration prospects serve as positive inducement devices for pre-migration human capital formation. In this sense, the migration probabilities increase the human capital levels of all workers in the source country, whether being migrants or not, thereby avoiding the undevelopment trap as often emphasised in the brain drain literature.

However promising the latter results may seem, the conclusions are based on a highly stylised world, wherein all actors are very disciplined in abiding the law as laid down in the restrictive migration policies of the Western world. One only has to open a popular newspaper to become aware of the vast presence of non-disciplined actors in today's globalising world. Even though most individuals would opt for a legal immigrant status, they will also migrate in the case this is refused. On the other side of the spectrum there are the many countries not enforcing laws against visibly illegal immigrants causing ongoing illegal migrants to arrive, ostensibly unhindered, to join the legal streams.

The available stylised facts on illegal migration as to date so perspicuously set out by Entorf (2000), Zolberg (1990), Epstein et al. (1999), Zanfrini (2000), Hillman & Weiss (1999) and Davila & Pagan (1997) force us to further study the effects of migration prospects on human capital accumulation. The theoretical background for our investigation of the effect of migration on human capital accumulation is based on the incentive model by Stark et al. (1998). Our model is different in the way that it allows illegal migration besides legal migration. Also, a distinction is made between the specification of the optimisation problem on the individual and the social welfare level. We compare the effects of

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<sup>1</sup>Of which the most important are Todaro (1969), Bhagwati & Hamada (1974), Miyagiwa (1991), Sjaastad (1962)

<sup>2</sup>See for instance Burda (1993), Chiswick (1986), Blomqvist (1986), Markussen (1988) and Borjas (1991).

migration probabilities on human capital accumulation with and without illegal migration from the different points of view. This way we can clarify whether migration leads to a brain drain or a brain gain.

Our key idea is that compared to the sole possibility of legal migration, illegal migration alters the structure of the incentives individuals in the source country face; higher prospective returns to human capital in the foreign country even if the legal immigrant status is not obtained. The ostensibly permissible illegal migration sets the social planner in the source country on the wrong track. In the process, an ineluctable information asymmetry comes into being.

We use a simple model in which the individual's expected utility is set alongside the social planner's optimisation problem. We examine the optimal formation of human capital in the economy in the absence and presence of a probability of illegal migration. We calculate the effect of a probability of legal migration on the premigration human capital accumulation. We show that when the legal immigrant status is not obtained, the possibility of redeeming the migration intention illegally is conducive to the accumulation of human capital of individuals. However, from the social point of view we reach an ambivalent conclusion. Due to the information asymmetry arising from the 'permissible' illegal migration, the social level of human capital is negatively affected by the migration probabilities under consideration. Thus, we endorse the concern about the leakage as much as the euphoria for the expanding cerebellum in pursuance of migrating individuals.

## 2 The model

### 2.1 The basic model

We begin by posing initial conditions where there is a perfectly competitive world in which economic activity is extended over discrete time. The economy is characterised by generations that live for two periods. Members of the workforce derive periodic income from per period utility,  $U(X)$ . The properties of  $U(x)$  are:

$$U(x) > 0 \quad ; \quad U'(x) > 0 \quad ; \quad U''(x) < 0 \quad \forall \quad x \geq 0$$

The utility function is thus strictly concave and satisfies the expected utility properties.

### 2.2 The model with premigration HC accumulation

In the first period individuals may either work or engage in education. In the second period individuals may work in the source country or abroad, either

legally or illegally. If the migrant is caught working on the illegal market, she is sent back to work in the source country. Only when working legally in the host country does the migrant make use of her latent human capital. This amount of human capital which is available to agents in their productive period of life, is given by the continuously differentiable function  $\psi(e_1)$  with  $\psi'(e_1) > 0$  and  $\psi''(e_1) < 0 \forall e_1$ .

Assume that each individual is endowed with one unit of resources, for instance time, and denote this amount of effort a representative individual chooses to spend on education by  $e_1 \in (0, 1)$ . The amount of effort that the individual has for working is therefore  $(1 - e_1)$ , which can provide an income of  $(1 - e_1)\underline{\omega}$ . Also assume that it is known to individuals that there exists a probability  $\alpha$  in their productive period of life to emigrate to a country with a higher competitive wage per efficiency unit of labour,  $\bar{\omega}$ . Then, when there is no possibility for illegal migration, and given the properties of the production function of human capital,  $\psi(e_1)$ , the second period earnings of the worker are:

$$\begin{aligned} \psi(e_1)\bar{\omega} & \quad \text{with probability } \alpha \\ \psi(e_1)\underline{\omega} & \quad \text{with probability } (1 - \alpha) \end{aligned}$$

We would like to compare this situation to the one where there is a possibility for illegal migration. In this instance the second period earnings of the workers are:

$$\begin{aligned} \psi(e_1)\bar{\omega} & \quad \text{with probability } \alpha \\ \omega & \quad \text{with probability } (1 - \gamma)(1 - \alpha) \\ \underline{\omega} & \quad \text{with probability } \gamma(1 - \alpha) \end{aligned}$$

where  $\bar{\omega} > \omega > \underline{\omega}$ .  $\bar{\omega}$  is the higher legal competitive wage per efficiency unit of labour and  $\omega$  is the higher illegal competitive wage that can be earned in the host country after emigration materialises. The risk of being caught working on the illegal market in the host country is represented by  $\gamma$ .

Given the second period migration opportunity, with a possibility if migrating illegally when a legal move is denied, and including the risk of being caught and sent back to the source country, the individual's optimisation problem involves choosing the level of human capital accumulation,  $e_1$ , such as to maximise her intertemporal utility:

$$Max_{e_1} \{U((1 - e_1)\underline{\omega}) + \phi[\alpha U(\psi(e_1)\bar{\omega}) + (1 - \alpha)\{(1 - \gamma)U(\omega) + \gamma U(\underline{\omega})\}]\} \quad (1)$$

In this instance,  $\phi$  is the subjective time rate of discount. Optimising the intertemporal utility function with regard to  $e_1$  and rewriting this equation gives the optimal level of human capital accumulation in the source country:

$$\psi(e_1, \alpha) = \psi'(e_1) = \frac{U'((1 - e_1)\underline{\omega})}{\phi[\alpha U'(\psi(e_1)\bar{\omega})\left(\frac{\bar{\omega}}{\underline{\omega}}\right)]} \quad (2)$$

Now it can be shown that a legal migration probability affects the investment in human capital in the home country prior to migration. Using the implicit function theorem, the effect on human capital formation in the source country of a probability  $\alpha$  of migrating legally to a higher wage country becomes:

$$\frac{\partial \eta \psi(\bar{\alpha})}{\partial \alpha} = - \frac{\partial \psi(\bar{e}_1, \bar{\alpha}) / \partial \alpha}{\partial \psi(\bar{e}_1, \bar{\alpha}) / \partial e_1} \quad (3)$$

Our interest is in providing conclusions about the resulting sign of this expression under the assumption of legal or illegal migration materialising.

**Proposition 1** *A positive probability of legal emigration to a higher income country will lead to a higher pre-migration per worker investment in human capital if the individual has the opportunity to migrate illegally when the legal immigrant status is withheld, than had the probability of legal migration been zero.*

**Proof.** Applying expression (3) to (2), and simplifying, gives:

$$\frac{(U''(1 - e_1)\underline{\omega})(\phi U'(\psi(e_1)\bar{\omega}))}{U''((1 - e_1)\underline{\omega})(-\underline{\omega})\{\phi \alpha U'(\psi(e_1)\bar{\omega}) - U'((1 - e_1)\underline{\omega})\phi \alpha U''(\psi(e_1)\bar{\omega})(\psi'(e_1)\bar{\omega})\}} \quad (4)$$

The numerator of this expression is negative, while the denominator is positive, therefore  $\frac{\partial \eta \psi(\bar{\alpha})}{\partial \alpha}$  is unambiguously positive. ■

According to the proposition above the positive probability  $\alpha$  of working on the legal market abroad has a positive effect on the premigration level of human capital accumulation in the source country irrespective of the actual move being legal or illegal<sup>3</sup>.

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<sup>3</sup>This result coincides with the conclusion by Stark et al. (1998).

## 2.3 The social welfare model

### 2.3.1 Human capital formation without migration

So far we have studied the private returns of migration for the individual worker who does not internalise the social returns of human capital. We optimised the individual intertemporal utility function without taking the externalities from an economy-wide average level of human capital into account. To be able to compare the social returns of human capital to the individual return, we need to introduce externalities accruing from the average level of human capital and a social welfare function. The individual's intertemporal utility function becomes:

$$U((1 - e_1)\underline{\omega}) + \phi[U((\psi(e_1) + \xi\psi(\epsilon_1))\underline{\omega})] \quad (5a)$$

where  $\epsilon_1$  is the economy-wide average level of human capital, and  $\xi$  represents the externality accruing from this average level of human capital in the economy. The social planner is faced with another, social intertemporal utility function, namely

$$U((1 - \epsilon_1)\underline{\omega}) + \phi[U((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})] \quad (5b)$$

Clearly, the solution to the individual's optimisation problem does not lead to the socially optimal level of human capital accumulation. The individual optimal level of human capital is:

$$\psi(e_1, \alpha) = \psi'(e_1) = \frac{U'((1 - e_1)\underline{\omega})}{\phi[U'((\psi(e_1) + \xi\psi(\epsilon_1))\underline{\omega})]} \quad (6a)$$

while the socially optimal level is equal to:

$$\psi(\epsilon_1, \alpha) = \psi'(\epsilon_1) = \frac{U'((1 - \epsilon_1)\underline{\omega})}{\phi[U'((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})(1 + \xi)]} \quad (6b)$$

When comparing (6a) to (6b), it becomes apparent that the social planner's optimal level of human capital accumulation is lower the higher the externality parameter.

### 2.3.2 Human capital formation with migration

Assume that an opportunity,  $\alpha$ , to migrate to another, superior technology country presents itself. Assume further that human capital neither depreciates nor appreciates across countries. The return to human capital is higher in the host country than at home. This is represented by a higher wage rate,  $\bar{\omega}$ . Furthermore,  $\alpha$  and  $\xi$  are exogenous to the model. The workers in the source country, thus, face an intertemporal utility function:

$$U((1 - e_1)\underline{\omega}) + \phi[\alpha U(\psi(e_1)\bar{\omega}) + (1 - \alpha)U((\psi(e_1) + \xi\psi(e_1))\underline{\omega})] \quad (7a)$$

In this instance, the social intertemporal utility function becomes:

$$U((1 - \epsilon_1)\underline{\omega}) + \phi[\alpha U(\psi(\epsilon_1)\bar{\omega}) + (1 - \alpha)U((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})] \quad (7b)$$

The solution to the intertemporal optimisation problem for the individual then is:

$$\psi(e_1, \alpha) = \psi'(e_1) = \frac{U'((1 - e_1)\underline{\omega})}{\phi[\alpha U'(\psi(e_1)\bar{\omega})\left(\frac{\bar{\omega}}{\underline{\omega}}\right) + (1 - \alpha)\{U'((\psi(e_1) + \xi\psi(e_1))\underline{\omega})\}]} \quad (8a)$$

while the socially optimal level is equal to:

$$\psi(\epsilon_1, \alpha) = \psi'(\epsilon_1) = \frac{U'((1 - \epsilon_1)\underline{\omega})}{\phi[\alpha U'(\psi(\epsilon_1)\bar{\omega})\left(\frac{\bar{\omega}}{\underline{\omega}}\right) + (1 - \alpha)\{U'((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})\}((1 + \xi))]} \quad (8b)$$

Now it can be shown that in the presence of a migration opportunity the investment in human capital in the home country prior to migration is affected. Using the same method as before it can be shown that the effect on human capital formation in the source country of a probability  $\alpha$  of migrating legally to a higher wage country for the individual is equal to (3).

**Proposition 2** *A positive probability of legal emigration to a higher income country will lead to a lower pre-migration per worker investment in human capital if the individual has the opportunity to migrate only legally, than had the probability of legal migration been zero.*

**Proof.**

Applying expression (3) to (8a), and simplifying, gives the numerator of (3)<sup>4</sup>:

$$-U'((1 - e_1)\underline{\omega})\{\phi[U'(\psi(e_1)\bar{\omega})\left(\frac{\bar{\omega}}{\underline{\omega}}\right) - (U'((\psi(e_1) + \xi\psi(e_1))\underline{\omega}))]\} \quad (9)$$

which is positive. The denominator is also positive, therefore  $\frac{\partial \eta\psi(\bar{\alpha})}{\partial \alpha}$  is negative. ■

For the social planner we have to go through the same procedure:

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<sup>4</sup>Refer to the appendix for the full expression of (9)

**Proposition 3** *A positive probability of legal emigration to a higher income country will lead to a lower average investment in human capital if a proportion of the population of the source country has the opportunity to migrate only legally, than had the probability of legal migration been zero.*

**Proof.**

Applying expression (3) to (8b), and simplifying, gives the numerator of (3)<sup>5</sup>:

$$-U'((1 - \epsilon_1)\underline{\omega})\{\phi[U'(\psi(\epsilon_1)\bar{\omega})(\frac{\bar{\omega}}{\underline{\omega}}) - (U'((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})(1 + \xi))]\} \quad (10)$$

which is positive. The denominator is also positive, therefore  $\frac{\partial \eta \psi(\alpha)}{\partial \alpha}$  is negative. ■

Studying the latter two results we may conclude that including externalities accruing from the average level of human capital leads to a negative effect of legal migration on the premigration level of human capital accumulation. For the individual as for the social planner, the negative effect of a probability of migrating legally is larger when the wage difference between the host and the source country is larger. The same goes for the externality effect of  $\xi$  being smaller.

### 2.3.3 Human capital formation with illegal migration

In this section there is a possibility for the individual to migrate illegally when the possibility for a legal move is denied. Given the second period migration opportunities and the risk of being caught working on the illegal market, the individual's intertemporal optimisation problem becomes:

$$U((1 - e_1)\underline{\omega}) + \phi[\alpha U(\psi(e_1)\bar{\omega}) + (1 - \alpha)\{(1 - \gamma)U(\omega) + \gamma U(\xi\psi(\epsilon_1))\}] \quad (11)$$

with its solution:

$$\psi(e_1) = \frac{U'((1 - e_1)\underline{\omega})}{\phi[\alpha U'(\psi(e_1)\bar{\omega})(\frac{\bar{\omega}}{\underline{\omega}})]} \quad (12)$$

Due to an information asymmetry, the social planner does not have any reliable information about the magnitude of illegal migration. Therefore, the social intertemporal utility function remains as in (7b).

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<sup>5</sup>Refer to the appendix for the full expression of (10)

Now we can study the effect on human capital formation in the source country of a probability of migrating legally or illegally to a superior technology country for the individual.

**Proposition 4** *With externalities accruing from the average level of human capital, a positive probability of legal emigration to a higher income country will lead to a higher premigration per worker investment in human capital if the individual has the opportunity to migrate illegally when legal migration is denied than had the probability of legal migration been zero.*

**Proof.**

Applying expression (3) to (12), and simplifying gives:

$$\frac{-U'((1 - e_1)\underline{\omega})\{\phi[U'(\psi(e_1)\bar{\omega})(\frac{\bar{\omega}}{\underline{\omega}})]\}}{U''((1 - e_1)\underline{\omega})(-\underline{\omega})\{den\} - U'((1 - e_1)\underline{\omega})\{\phi[\alpha\{U''(\psi(e_1)\bar{\omega})(\frac{\bar{\omega}^2}{\underline{\omega}})(\psi(e_1))\}]\}} \quad (13)$$

where the numerator is positive. Since the denominator is also positive,  $\frac{\partial \eta \psi(\bar{\omega})}{\partial \alpha}$  is unambiguously positive. ■

Also, we can study the effect on average human capital formation in the source country of a probability of migrating legally or illegally to a superior technology country from the social planner's point of view.

**Proposition 5** *With externalities accruing from the average level of human capital, a positive probability of legal emigration to a higher income country will lead to a lower premigration average investment in human capital if a proportion of the population has the opportunity to migrate illegally when legal migration is denied than had the probability of legal migration been zero.*

**Proof.**

Applying expression (3) to (8b), and simplifying, gives the numerator of (3)<sup>6</sup>:

$$-U'((1 - \epsilon_1)\underline{\omega})\{\phi[U'(\psi(\epsilon_1)\bar{\omega})(\frac{\bar{\omega}}{\underline{\omega}}) - (U'((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})(1 + \xi))]\} \quad (10)$$

which is positive. The denominator is also positive, therefore  $\frac{\partial \eta \psi(\bar{\omega})}{\partial \alpha}$  is negative. ■

We find that in the model with illegal migration a probability of migrating legally has a positive effect on the premigration human capital accumulation for

<sup>6</sup>Refer to the appendix for the full expression of (10)

the individual. As was shown in (10), this probability has a negative effect on the premigration average human capital accumulation level as faced by the social planner. From this we may conclude that there exists an information asymmetry between the social planner and the prospective migrants. This asymmetric situation may well lead to a brain gain for the individuals. From the social point of view, the investment in human capital will drop as a response to a migration prospect of part of the population. In the latter case, those who cannot gain from migration by participating in it stand to lose due to resulting brain drain.

### 3 Conclusion

The possibility to obtain a legal immigrant status carries both a "brain drain" and a "brain gain" with it. The inducement effect of legal migration has a positive ascendancy over the attainment of necessary skills by individuals. On the other hand, the propensity to acquire skills is not impervious to the waning externalities accruing from the reduced average level of human capital in the economy when migration materialises illegally.

In the former case, individuals who optimally choose how much to invest in costly human capital accumulation will over-invest from the social point of view in order to be well in the running for legal employment abroad. This result offers a positive implication: in the presence of the possibility of migration, workers who stay in the source country will have chosen to form more human capital than in the absence of the possibility of migration. Thus, even those who cannot gain directly from migration by participating in it are apt to be better off.

Quite a different kettle of fish is the effect of permissible illegal migration on the average level of human capital in the source country. Since the illegal migrants elude the policy sphere, the social planner's choice of the optimal level of human capital accumulation falls short of the socially desired optimum. This inefficiency is inextricably bound up with the information asymmetry arising from individuals leaving and entering countries unnoticed. This mismatch deprives the externalities accruing from the average level of human capital in the source country. Consequently, aggrieving the non-migrants.

Finally, our analysis has been based on a simple model with generalised expected utility and social welfare functions. These have availed considerable transparency and straightforward pliancy. We note that our model is quite robust to alternative functional specifications. Though in related papers intertemporal utility maximising individuals respond to migration probabilities in a manner akin to that of workers in the present paper, the social planners' choice is dissentient. Nonetheless, the procured results are commensurable and may be extended to attune future prospects on the topic.

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## APPENDIX

Proposition 2, expression (9):

Numerator:

$$-U'((1 - e_1)\underline{\omega})\{\phi[U'(\psi(e_1)\overline{\omega})\left(\frac{\overline{\omega}}{\underline{\omega}}\right) - (U'((\psi(e_1) + \xi\psi(\epsilon_1))\underline{\omega}))]\} \quad (9a)$$

Denominator:

$$\begin{aligned} &U''((1 - e_1)\underline{\omega})(-\underline{\omega})\{den\} - U'((1 - e_1)\underline{\omega})\{\phi[\alpha\{U''(\psi(e_1)\overline{\omega})\left(\frac{\overline{\omega}}{\underline{\omega}}\right)\psi'(e_1)\overline{\omega})\} \\ &\quad + (1 - \alpha)\{U''((\psi(e_1) + \xi\psi(\epsilon_1))\underline{\omega})(\psi'(e_1)\underline{\omega})\}] \end{aligned} \quad (9b)$$

Propositions 3 and 5, expression (10):

Numerator:

$$-U'((1 - \epsilon_1)\underline{\omega})\{\phi[U'(\psi(\epsilon_1)\overline{\omega})\left(\frac{\overline{\omega}}{\underline{\omega}}\right) - (U'((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})(1 + \xi))]\} \quad (10a)$$

Denominator:

$$\begin{aligned} &U''((1 - \epsilon_1)\underline{\omega})(-\underline{\omega})\{den\} - U'((1 - \epsilon_1)\underline{\omega})\{\phi[\alpha\{U''(\psi(\epsilon_1)\overline{\omega})\left(\frac{\overline{\omega}}{\underline{\omega}}\right)\psi'(\epsilon_1)\overline{\omega})\} \\ &\quad + (1 - \alpha)\{U''((\psi(\epsilon_1) + \xi\psi(\epsilon_1))\underline{\omega})(1 + \xi)^2(\psi'(\epsilon_1)\underline{\omega})\}] \end{aligned} \quad (10b)$$