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MULTI-OBJECTIVE DECISION-MAKING FOR ROAD DESIGN

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Abstract. Multi-objective analysis is a popular tool to solve many economic, managerial and construction problems. The objective of this research is to develop and implement a methodology for multi-objective optimization of multi-alternative decisions in road construction. After a rough overview of the articles dealing with the multi-objective decision and assessment of road design alternatives described by discrete values, Multi-Objective Optimization on the basis of the Ratio Analysis (MOORA) method was selected. This method focuses on a matrix of alternative responses on the objectives. A case study demonstrates the concept of multi-objective optimization of road design alternatives and the best road design alternative is determined.

Keywords: decision making, multi-objective optimization, MOORA method, road design, alternatives.

1. Introduction

Our values, beliefs and perceptions are forces behind almost any decision-making activity. They are responsible for the perceived discrepancy between the present and a desirable state. Especially in construction, the diversity of structures and processes, hardly commensurable variables, conflicting development objectives and constraints characterize contemporary decision problems. Different stakeholders with different interests and values make a decision-making process on different decision alternatives even much more complicated. In the Multi-Objective Decision-Making (MODM) context, the evaluation of each alternative on the set of objectives facilitates the selection.

MODM is also referred as:

- Multi-Criteria Decision Analysis (MCDA);
- Multi-Dimensions Decision-Making (MDDM);
- Multi-Attributes Decision-Making (MADM).

The objectives must be measurable, even if the measurement is performed only at the nominal scale (yes/no; present/absent) and their outcomes must be measured for every decision alternative. Objective outcomes provide the basis for a comparison of the alternatives and consequently facilitate the selection. Therefore, multi-objective techniques seem to be an appropriate tool for ranking or selecting one or more alternatives from a set of the available options based on the multiple, sometimes conflicting, objectives. A large number of methods have been developed for solving multi-objective problems. MODM frameworks vary from simple approaches, requiring very little information, to the methods based on mathematical programming techniques, requiring extensive information on each objective and the preferences of the stakeholders. Different publications present various classifications of the above-mentioned methods, but it is still a problem of choosing an appropriate method in a given situation.

Considering the nature of information available to decision makers, MODM can be divided into the following groups (Ustinovichius *et al.* 2007):

1. The method of rank correlation consisting of totalizing ranks is a first method to be considered. Rank correlation was first introduced by psychologist Spearman (1904, 1906 and 1910) and later taken over by statistician Kendall (1948). Bardauskiene (2007), Turskis *et al.* (2006), Zavadskas and Vilutiene (2006) applied this method for construction problems solution.
2. The methods based on quantitative measurements using a few criteria to compare the alternatives (comparison preference method). This group

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consists of the preference comparison methods like ELECTRE and PROMETHEE.

3. The methods based on initial qualitative assessment the results of which take a quantitative form at a later stage. This group consists of the analytic hierarchy process (AHP) methods as well as of the methods based on game theory and fuzzy sets. Peldschus and Zavadskas proposed (2005) fuzzy matrix games multi-criteria model for decision-making in engineering, Zavadskas and Turskis (2008) suggested and applied the logarithm normalization method in game theory for multi-criteria construction problems solution.
4. The methods based on a reference point or goal such as the Reference Point Method which is used in TOPSIS (Hwang and Yoon 1981; Zavadskas, Turskis 2006; Zavadskas *et al.* 2006; Jakimavičius and Burinskienė 2007; Kapliński, Janusz 2006; VIKOR (Opricovic and Tzeng 2004; Ginevicius and Podvezko 2006), COPRAS (Zavadskas *et al.* 1994, 2008) and Goal Programming.

These groups also deal with engineering problems including civil engineering (a list of references shows many different examples). Especially, the ELECTRE-3 method determines preferences when selecting a public transport expansion scenario (Thiel 2006). The PROMETHEE method was used to assess investment projects (Nowak 2005).

A rational variant of design documentation for a large transportation system (Su *et al.* 2006) was selected by the AHP method. Ugwu *et al.* (2006a, 2006b) offered an analytical decision model and a structured methodology for sustainability appraisal in the infrastructure projects. They used the 'weighted sum model' technique in multi-criteria decision analysis and 'additive utility model' in AHP for multi-criteria decision making to develop the model from first principles.

The method based on game theory was used to select a rational variant for road reconstruction (Peldschus 2005), to refurbishment selection of construction objects (Antucheviciene *et al.* 2006), to assess sustainable compactness of a city (Turskis *et al.* 2006), to select the best alternative of external wall finishing in cast-in-place buildings (Zavadskas and Turskis 2008). Finally, fuzzy sets methods were applied to deal with the problem related to the construction of a water supply pipeline (Peldschus and Zavadskas 2005).

The COPRAS method (Zavadskas *et al.* 1994, 2008a, 2008b) was used for real estate evaluation (Kaklauskas, Zavadskas 2007) to assess sustainability of Vilnius city (Viteikiene and Zavadskas 2007) and to make a sustainable revitalisation of derelict property (Zavadskas and Antucheviciene 2007).

The problem of utility with different independent objectives and alternative solutions has to be optimized. The notion of utility has always been a crucial point for researchers. For us, the notion of utility boils down to four problems: the choice of units per objective, normalization, optimization and importance which is given to an objective. In addition, subjectivity has to be avoided which is not the case for the methods using weights or scores. A newly proposed method for multi-objective optimization with discrete alternatives MOORA (Multi-Objective Optimization on basis of Ratio Analysis) tries to satisfy all these preliminary requirements.

An example of evaluating road design illustrates the application of the MOORA method. It is concluded that the MOORA method is ready for practical use and can be a full-fledged method for multiple objective optimization.

2. The MOORA method

The method starts with a matrix of responses of different alternatives on different objectives:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1i} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{j1} & \cdots & x_{ji} & \cdots & x_{jn} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mi} & \cdots & x_{mn} \end{bmatrix}, \tag{1}$$

where: x_{ij} – the response of alternative j on objective or attribute i; $i = 1, 2,\dots, n$ – is the number of objectives or attributes; $j = 1, 2,\dots, m$ – is the number of alternatives.

In order to define the objectives, we have to closer focus on the notion of *Attribute*. Keeney and Raiffa (1993) present the example of the objective "reduce sulfur dioxide emissions" to be measured by the attribute "tons of sulfur dioxide emitted per year". An objective and a correspondent attribute always go together. Consequently, when the text mentions *objective*, the correspondent attribute is also meant.

The MOORA Method consists of two components: (a) the ratio system and (b) the reference point approach.

2.1. The Ratio System as a Part of MOORA

MOORA was introduced by Brauers and Zavadskas for the first time in 2006 (Brauers and Zavadskas 2006). We go for a ratio system in which each response of an alternative on an objective is compared to a denominator which is a representative for all alternatives concerning that objective (Brauers *et al.* 2007; Kalibatas, Turskis 2008). Appendix A proves that for this denominator the best choice is the square root of the sum of squares of each alternative per objective:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}}, \tag{2}$$

where: x_{ij} – response of alternative j on objective i; $j = 1, 2, ..., m$; m – is the number of alternatives; $i = 1, 2, \dots n$; n – is the number of objectives; x_{ij}^* – is a dimensionless number representing the normalized response of alternative j on objective i.

Dimensionless Numbers, having no specific unit of measurement are obtained for instance by deduction, multiplication or division. The normalized responses of the alternatives on the objectives belong to the interval [0; 1]. However, sometimes the interval could be [–1; 1].

Indeed, for instance, in the case of productivity growth some sectors, regions or countries may show a decrease instead of increase in productivity i.e. a negative dimensionless number.

For example, instead of a normal increase in productivity growth a decrease remains possible. Thus, the interval becomes [–1, 1]. Let's consider an example of productivity which has to increase (positive). Consequently, we look after productivity maximization e.g. in European and American countries. What if the opposite does occur? For instance, let's analyse productivity changes in Russia (in the first half of decade 9). Contrary to the other European countries, its productivity decreased which means that in formula (2) the numerator for Russia would be negative with the whole ratio becoming negative. Consequently, we get the interval [–1; +1] instead of [0; 1]. For optimization, these responses are added in case of maximization and subtracted in case of minimization:

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^*, \tag{3}$$

where: $i = 1, 2, \ldots, g$ as the objectives to be maximized; $i = g + 1, g + 2, \ldots, n$ as the objectives to be minimized; y_j^* – the normalized assessment of alternative j with respect to all objectives.

An ordinal ranking of y_j^* shows the final preference. Indeed, cardinal scales can be compared in the ordinal ranking, according to Arrow (1974): 'Obviously, a cardinal utility implies an ordinal preference but not *vice versa*'.

2.2. The reference point approach as a part of MOORA

Reference Point Theory is based on the ratios found in formula (2) whereby a Maximal Objective Reference Point is also deduced. The Maximal Objective Reference Point approach is called as realistic and non-subjective when the coordinates (r_i) selected for the reference point are realized in one of the candidate alternatives. For example, we have three alternatives described as follows: *A* (10; 100), *B* (100; 20) and *C* (50; 50). In this case the maximal objective reference point $\boldsymbol{R_m}$ results in (100; 100). The Maximal Objective Vector is self-evident if the alternatives are well defined as for the projects in the area of Project Analysis and Planning.

Having given the dimensionless number representing the normalized response of alternative j on objective i, i.e. x_{ij}^* in formula (2), we come to:

$$\left(r_i - x_{ij}^*\right), \tag{4}$$

where: $i = 1, 2, \ldots, n$ as the attributes; $j = 1, 2, \ldots, m$ as the alternatives; r_i – the i^{th} coordinate of the reference point; x_{ij}^* – the normalized attribute i of alternative j;

This matrix is subject to the *Min-Max Metric of Tchebycheff* (Karlin, Studden, 1966):

$$\underset{(j)}{\text{Min}} \left\{ \underset{(i)}{\max} \left(r_i - x_{ij}{}^* \right) \right\}. \tag{5}$$

Appendix B shows that the Min-Max metric is the best choice between all the possible metrics of reference point theory.

2.3. Importance given to an objective in MOORA

One objective i of x_{ij}^* cannot be more important than the smaller ones (see formula 3). Nevertheless, it may turn out to be necessary to stress that some objectives are more important than the others. In order to give more importance to an objective, it could be multiplied with a *Significance Coefficient*.

The *Attribution of Sub-Objectives* represents still another solution. Let us consider an example of purchasing fighter planes (Brauers 2002). From an economic point of view, apart from military effectiveness the objectives concerning the fighter planes are threefold – price, employment and balance of payments. In order to give more importance to military defense, effectiveness is broken down in, for instance, the maximum speed, the power of the engines and the maximum range of the plane. Anyway, the Attribution Method is more refined than that of significance coefficient. The attribution method better succeeds in characterizing an objective.

3. Application of the proposed method for evaluating road design alternatives

Roads and bridges have a special role in the infrastructure of cities and residential areas as these places are complex engineering facilities and their construction and use require much special scientific knowledge.

The harmony in the residential environment depends much of the density of road network and the number and capacity of bridges. Lately, research of general plans (Zagorskas and Turskis 2006) and sustainable development and transport flows has received increasing attention (Zagorskas and Turskis 2006; Šaparauskas and Turskis 2006; Turskis *et al.* 2006). Methods to evaluate citizen opinions (Thiel 2006; Su *et al.* 2006; Jakimavičius and Mačerinskienė 2006), special forecasting methods and decision support systems (Kaklauskas and Zavadskas 2007) are being developed for an integrated assessment of the variants of sustainable urban development.

The constant growth of the number of traffic participants demands the expansion of district, national and arterial roads and especially the highway network. Alongside with the construction of new roads, adding more lanes to the existing highways plays an exclusive role.

In the expansion of the highway network, preparation for a good design has an important role to play. Considering the large costs for road construction and widening, producing highly rational solutions is a very important task. Therefore, it is necessary to assess accumulated previous experiences in order to improve the quality and longevity of roads. In addition, special attention must be paid to road safety. Moreover, some scientific research in these fields is performed.

The variations of the Northern climate affect the roads in the Baltic States with the additional problems of maintenance. Certainly, in winter, they damage road surface and have negative effects on the environment. Currently, the road safety of Lithuanian roads is the worst compared to the other EU countries. Lithuania is the last among all EU countries according to the number of people killed in fatal traffic accidents per one million residents. Ratkevičiūtė *et al.* (2007) provide an exhaustive analysis of the causes behind the accident rates and offer means to increase the road safety. Kashevskaya (2007), Leonovich and Kashevskaya (2007) analyze the problems related to the quality of road infrastructure and on the basis of the main statements of the road quality management theory offer a few methods to guarantee high quality road maintenance.

The quality of roads and bridges and flyovers as a part of roads depends on the quality of design solutions. The quality of the last ones is determined by knowledge of the designers and their ability to apply the newest and most advanced constructional technologies. Road surfacing was analyzed by Ziari and Khabiri (2007), Laurinavičius and Oginskas (2006), Petkevičius *et al.* (2006), Chang *et al.* (2005), Ziari *et al.* (2007) in order to improve its longevity and maintenance qualities. Frangopol and Liu (2007) analyzed Bridge constructions.

While planning the construction of the roads or determining which road sections need repair of surfacing, the actual condition of road surface must be assessed. However, the methods for an assessment of road surface and its construction have their limitations. It is easy to notice that some of them are insufficiently precise while the others remain too complex. When assessments are made on one of the objectives – as is usually the case – the best solutions are not always selected.

When constructing new or renovating old highways, the cost and duration of construction, longevity, environmental issues and economic validity are the most important objectives for assessing design solutions. The best solution is sought to achieve the best values of these objectives. However, it is impossible to come to this question at the time. Consequently, to deal with similar tasks, Multi Objective Decision Making methods are applied.

Tille and Dumont (2003) described how the problem of choice between various alternatives was permanent and crucial in the projects for road infrastructure. The designer must use objective and global methods in order to propose an optimal alternative to the decision maker. Only multi-objective decision-making is advisable for application by the designer, the more as it can also consider the complexity of the problem. In addition, the use of such methods makes it possible to stress the non-subjective elements of the choice, based in particular on the technical evaluation of the performance indicators.

This article tries to select a variant for expanding a highway in Thuringia, Germany, from four to six lanes. The rationality of using MODM methods for road and bridge construction is attempted.

3.1. Assessment of alternatives

A case study considers six possible alternatives of highway design (Peldschus 2005; Zavadskas *et al.* 2007):

Variant 1. Construction of a new road by changing the axis and gradients of the highway and using concrete surfacing. A change of gradients requires deep excavations and embankments i.e. a large amount of earthwork and makes up 70 m^3/m in the average.

Variant 2. Construction of a new road by changing the axis and gradients of the highway and using asphalt concrete surfacing. Differences in surfacing compared to Variant 1.

Variant 3. Construction of a new road by changing the axis and retaining the gradients of the highway with concrete surfacing. While retaining the gradients, Variant 3 reduces the amount of earthwork and makes up 36.2 m^3/m.

Variant 4. Construction of a new road by changing the axis and retaining the gradients of the highway using asphalt concrete surfacing. With differences in surfacing, this variant corresponds to Variant 3.

Variant 5. Construction of a new road retaining the axis and the gradient of the highway with concrete surfacing. With differences in duration, the amount of earthworks in this variant is similar to that of Variant 3.

Variant 6. Construction of a new road retaining the axis and the gradient of the highway with asphalt concrete surfacing. With differences in road surfacing, Variant 6 corresponds to Variant 5.

Each of the alternatives, taken from the article by Peldschus (2005) and provided in Table 1, is described by five objectives calculated to assess the listed variants: price, duration of construction, distance of transportation, noise level and longevity.

Longevity **–** x_1 [*years*]. Longevity is one of the most important objectives in assessing highway design. The total price depends much on the longevity needed. Consequently, we have a road whereas it can be used without expenditures on renovation. Cheap solutions determine large renovation costs and usually become more expensive in the end. Asphalt and concrete surfacing are compared.

Construction price **–** x_2 [10^6 *€*]. One of the main requirements for designing is to strive for the lowest construction price and to simultaneously guarantee good quality and hardness, to achieve the shortest duration of construction, to guarantee a smaller number of detours or changes of direction and to reduce the number of accidents in the stages of construction and maintenance. It is also important to consider the interests of the people living in or the owners of neighboring land plots.

Environment protection **–** x_3 [*10 db(A)*]**.** The construction of new or renovation of old highways has a negative effect on nature. Damage to environment must be minimized during construction. The roads are part of the landscape. They cannot deface the terrain. The amount of earthworks and duration of construction must be minimized during construction. For this purpose, special methods are developed using the theory of mass service and neural mathematical models (Schabovicz and Hola 2007). Strategic environment studies are prescribed in the EU. Therefore, special studies are performed. The effect on flora, fauna, soil, water, air, climate, landscape, existing situations and environment quality must be determined considering growing demand. Besides, consumption of natural resources, CO_2 emissions and increased noise must be assessed.

Economic validity **–** $\boldsymbol{x_4}$ [*100 m*]**.** The economic validity of construction much depends on average distance for soil transportation. Consequently, when preparing the profiles of a road, the distance between the embankment and the excavations must be considered. The transportation distances and volumes of transported soil have a strong impact on construction costs and duration.

Construction duration **–** $\boldsymbol{x_5}$ [*100 days*]**.** The reconstruction of highways impedes communication, and therefore efforts are made to substantially reduce the duration of work. Speed up requires additional costs and foreseeing more capacity (labor force and machinery). Consequently, it must be considered whether it is really necessary. The most rational way is to find such construction variants that could help with reducing construction duration.

Asphalt surfacing is not rigid. It usually consists of an upper layer, lower layer and the road base. Weather and temperature variations during construction can affect the quality and longevity of the different variants of asphalt surfacing.

Table 1. MOORA road design: highways reconstruction projects in Thuringia (Germany)
The ratio system as a part of MOORA (1a until 1c) and reference point approach as a part of MOORA (1d-1e)

1a. Matrix of responses of alternatives on objectives: (x_{ij})

	1	2	3	4	5
	max	min	max	min	min
A_1	30	12.49	6.26	10.88	7.61
A_2	20	12.37	5.96	10.88	7.46
A_3	27	11.1	6.26	9.92	6.69
A_4	18	10.98	5.96	9.92	6.54
A_5	24	11.02	6.28	9.98	7
A_6	16	10.9	5.98	9.98	6.85

1b. Sum of squares and their square roots

A_1	900	156	39.188	118.37	57.912
A_2	400	153.02	35.522	118.37	55.652
A_3	729	123.21	39.188	98.406	44.756
A_4	324	120.56	35.522	98.406	42.772
A_5	576	121.44	39.438	99.6	49
A_6	256	118.81	35.76	99.6	46.923
sum of squares	3185	793.04	224.62	632.76	297.01
square roots	56.436	28.161	14.987	25.155	17.234

1c. Objectives divided by their square roots and MOORA

						total	+ 2.116	rank
A_1	0.5316	0.4435	0.4177	0.4325	0.4416	−1.204	0.912	3
A_2	0.3544	0.4393	0.3977	0.4325	0.4329	−1.348	0.768	6
A_3	0.4784	0.3942	0.4177	0.3944	0.3882	−1.116	1.000	1
A_4	0.3189	0.3899	0.3977	0.3944	0.3795	−1.242	0.874	4
A_5	0.4253	0.3913	0.419	0.3967	0.4062	−1.188	0.928	2
A_6	0.2835	0.3871	0.399	0.3967	0.3975	−1.297	0.819	5

1d. Reference point theory with ratios: coordinates of the reference point equal to the maximal objective values

r_i	0.5316	0.3871	0.3977	0.3944	0.3795

1e. Reference point theory: deviations from the reference point

						max	rank min
A_1	0	0.0565	0.02	0.0382	0.0621	0.06209	2
A_2	0.1772	0.0522	0	0.0382	0.0534	0.17719	4
A_3	0.0532	0.0071	0.02	0	0.0087	0.05316	1
A_4	0.2126	0.0028	0	0	0	0.21263	5
A_5	0.1063	0.0043	0.0214	0.0024	0.0267	0.10632	3
A_6	0.2481	0	0.0013	0.0024	0.018	0.24807	6

Highways with concrete surfacing have an advantage. This surfacing is rigid, rhea logy depends less on loads and temperature, thus it is stable and does not deform. No tracks appear and elevations emerge due to stopping of heavy vehicles. Besides, concrete roads are less sensitive to the effect of water. The older and more porous is the asphalt, the more serious the effect of water on asphalt concrete solidity. Therefore, the lifetime of concrete roads is from 20 to 30 years whereas asphalt concrete road surfacing makes from 15 to 20 years. Upon expiration of this term, the road must be renovated.

The data presented in Table 1 shows that there is no alternative dominating in all objectives.

3.2. Ranking of alternatives

MOORA optimization technique with discrete alternatives was used for ranking alternatives in the case study. Table 1 represents the results of the multi-objective analysis.

The negative figures shown in Table 1c, due to many objectives to be minimized, quite common for construction, do not look very 'elegant'. Therefore, the highest ranked is increased to one and the other totals are increased in the same way

The Ratio System as a part of MOORA and the Reference Point approach as a part of MOORA mutually control each other. The ultimate preference for variant A_3 (construction of a new road by changing the axis of the highway with concrete surfacing; when the gradients of the road are retained, the amount of earthwork is reduced) is well pronounced.

The study proposes second best cores under the form of either variant 1 or variant 5 (Spulber 1989).

- Variant 1 (A_1): Construction of a new road by changing the axis and gradients of the highway and using concrete surfacing. A change of gradients requires deep excavations and embankments, i.e. a large amount of earthwork and makes up 70 m^3/m in the average.
- Variant 5 (A_5): Construction of a new road retaining the axis and the gradient of the highway with concrete surfacing. With differences in duration, the amount of earthworks in this variant is similar to that of Variant 3.

Combining both approaches of MOORA, variants 2, 4 and 6 are rejected. It is also clear that concrete surfacing is preferred to asphalt concrete surfacing.

4. Conclusions

The problem of utility with different independent objectives and alternative solutions has to be optimized. The notion of utility has always been a crucial point for researchers in decision-making. MOORA (Multiple Objectives Optimization by Ratio Analysis) consists of two components. The first component is a ratio system in which per objective each response of an alternative is compared to the square root of the sum of the squares of the responses of each alternative.

For us, the notion of utility boils down to four problems: the choice of units per objective, normalization, optimization and importance which is given to an objective. MOORA tries to satisfy all these preliminary conditions. In this way, this ratio development can be a full-fledged method for multiple objective optimization.

The second part of MOORA consists of the Reference Point Method with a Maximal Objective Reference Point. As well for the ratio system as for the Reference Point Method, the square roots ratios are used.

With the square roots ratios of MOORA one objective cannot be much more important than the other as all their ratios are all smaller than one. Nevertheless, it may be necessary that some objectives are considered as more important than the others are. The use of Significance Coefficients of importance is a traditional answer. The breakdown of an important objective in sub-objectives represents another solution.

In Reference Point Theory, preference is given to the Tchebycheff Min-Max Metric with the maximum objective reference point. This reference point per objective possesses as coordinates the dominating coordinates of the candidate alternatives. For minimization, the lowest coordinates are chosen.

In conclusion, the following steps are foreseen in MOORA.

1) Square Roots Ratios for MOORA (Multi Objective Optimization on basis of Ratio Analysis) are accepted as the best choice.
2) Eventually more importance is introduced for an objective replacing it with different sub-objectives.
3) The ratios per alternative are added for the objectives to be maximized. The ratios per alternative for the objectives to be minimized are subtracted. The general total per alternative will compete in a ranking of all alternatives.
4) The ranking is set up.
5) Reference Point Theory with the Min-Max Metric is used as the second part of MOORA.

The case study concerned highway reconstruction projects in Eastern Germany. The results proved that the best alternative was the construction of a new road by changing the axis and retaining the gradients of the highway with concrete surfacing. The worst solution is constructing a new road by changing the axis and gradients of the highway and using asphalt concrete surfacing.

The case study shows that multi-objective analysis in construction is necessary. The selection of the best alternative cannot be based on a single objective. The case study proved that the proposed theoretical model was effective in a real life situation and could be successfully applied to solving similar utility problems.

Appendix A

Is the square root of the sum of squares of each alternative per objective the best choice for the denominator in the ratio system?

In the ratio system, each response of an alternative on an objective is compared to a denominator which is a representative for all alternatives concerning that objective. Until now, the square root of the sum of squares of each alternative per objective was used for this denominator, namely $\sqrt{\sum_{j=1}^{m} x_{ij}^2}$ was chosen in the MOORA formula (2):

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}}.$$

Was it the best choice? Other approaches are discussed in this appendix which does not mean that the following description is exhaustive.

1. Total ratios

The formula of total ratios replaces formula (2):

$$x_{ij}'^{*} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}}, \quad (6)$$

with: x_{ij} – response of alternative j on objective i; $j = 1, 2, \ldots, m$; m the number of alternatives; $i = 1, 2, \ldots, n$; n the number of objectives; $x_{ij}'^{*}$ – a dimensionless number representing the normalized response of alternative j on objective i.

Allen (1951) used this formula and Voogd (1983) applied it for multi-objective evaluation. For optimization purposes, these responses are added in case of maximization and subtracted in case of minimization (formula 8).

The total ratios are smaller than those in the square roots method are, but their calculation is less complicated than that used in the square roots method. However, they will not necessarily lead to the same outcome (e.g. the simulation for marketing in a department store showed different results (Brauers 2004)). Moreover, if many similar situations e.g. an example of productivity occur, see Footnote 3, the denominator of the ratio could become positive, negative or even equal to zero. In that case, the ratio itself could obtain all positive or negative values or even be undefined. Consequently, the intervals [0; 1] or [–1; 1] are not maintained for the formula of total ratios.

2. Schärlig (1985) Ratios

When dealing with Schärlig Ratios, one of the alternatives is taken as a reference. This mechanical approach is comparable with the formula of Schärlig multiplying all these ratios (Schärlig 1985).

A problem arises if one of the objectives is missing in an alternative and the alternative is used as a base. The result is that some obtained ratios are undefined because the denominator is zero. Therefore, an alternative has to be chosen as a base with none of its objectives equal to zero.

If another alternative is chosen as a base, other results are obtained. Consequently, ratio analysis, with one of the alternatives taken as a reference, produces no univocal outcome (Brauers 2004).

3. Weitendorf (1976) ratios

Weitendorf compares the responses with the interval Maximum-Minimum in the following way:

- if x_{ij}* should be maximized:

$$x_{ij}^{*} = \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-}; \quad (7)$$

- if x_{ij}* should be minimized:

$$x_{ij}^{*} = \frac{x_i^+ - x_{ij}}{x_i^+ - x_i^-}, \quad (8)$$

with: x_i^+ representing the maximum value and x_i^- the minimum value of objective i.

The normalized responses belong to the interval [0; 1].

Although at the first glance this method seems to be interesting, it has to be rejected on the following grounds:

1) The Min-Max metric cannot be applied as all coordinates of the reference point are equal to one which makes ranking impossible. Brauers and Zavadskas (2006) show an example of privatization in a transition economy.
2) With only the maximum and minimum per objective of all alternatives the composition of the whole series of objectives is not taken into consideration i.e. are not considered:
   - the spread as measured by the standard deviation as it can be different for several series though with the same maxima and minima;
   - the median and quartiles can be different for several series though with the same maxima and minima.

Given these remarks, a simulation was made with Weitendorf ratios in Brauers and Zavadskas (2006) showing other results compared to the square roots ratios.

Moreover, thousands and thousands other matrices of the responses of the alternatives on objectives with the same outcomes of formulae (10) and (11) will lead to the same ranking even if the same results could be obtained. For example, Brauers and Zavadskas (2006) show the same ranking and even the same results though with another matrix of responses as a starting point and having the same relations to their maxima and minima. If MOORA uses this matrix of responses instead of Weitendorf approach, the outcome is entirely different.

4. Van Delft and Nijkamp (1977) ratios of maximum value

Referring to the method of maximum value, the objectives per alternative are divided by the maximum or minimum value of that objective found in one of the alternatives.

$$x_{ij}^{*} = \frac{x_{ij}}{x_i^+}, \quad (9)$$

with: x_i^+ as the maximum or minimum x_{ij} depending if a maximum or a minimum of an objective is strived for. As only maxima, minima and the responses are involved, the same comments on the spread, median and quartiles mentioned for the Weitendorf ratios are also applicable here.

A fundamental problem arises for minimization. The ideal situation for minimization occurs when zero is attained. This could mean dividing by zero. If the numerator is not zero at that point, the fraction is undefined. Even if in that case a symbolic number, for instance 0.001, was given as an alternative, the result would be negatively biased for other alternatives. It could even solely determine the final ranking of the alternatives which is not correct (Brauers 2004). Anyway, in the case of minimization, the ratios can deviate largely from the interval [*0; 1*]. Consequently, one of the advantages of the ratio system is dropped, viz. that the normalized responses belong to the interval [*0; 1*] which makes them comparable.

With the application of the reference point theory, all coordinates of the maximal objective reference point are equal to one. Indeed, the maximal criterion values are either the maximum or minimum value divided by itself.

5. Jüttler (1966) ratios

For normalization purposes, the use of Jüttler's ratios is also possible:

$$x_{ij}{}^* = \frac{x_j^+ - x_{ij}}{x_i^+}. \tag{10}$$

As only maxima, minima and the responses are involved, the above mentioned comments on the spread, median and quartiles are also applicable here.

If x_i^+ represents a minimum, it can have a zero value in the denominator. Thus, the same objections can be made as against the van Delft and Nijkamp Method of Maximum Value.

6. Stopp (1975) ratios

If max x_{ij} is desirable:

$$x_{ij}{}^* = \frac{100 x_{ij}}{x_i^+}. \tag{11}$$

If min x_{ij} is desirable:

$$x_{ij}{}^* = \frac{100 x_i^-}{x_{ij}}. \tag{12}$$

These normalized values are expressed in percentages. As maxima and minima are used, the same objections as against Weitendorf Ratios are valuable here. Hwang and Yoon (1981) mention the same formulae with no percentages.

7. Körth (1969 a and b) ratios

$$x_{ij}{}^* = 1 - \left| \frac{x_i{}^+ - x_{ij}}{x_i{}^+} \right|. \tag{13}$$

The same objections against van Delft and Nijkamp Method of Maximum Value and against Weitendorf Ratios are also valuable here as the maximum value is used.

8. Peldschus *et al.* (1983) and Peldschus (1986) ratios for nonlinear normalization

If Minimum x_{ij} is desirable:

$$x_{ij}{}^* = \left(\frac{x_i^-}{x_{ij}} \right)^3. \tag{14}$$

If Maximum x_{ij} is desirable:

$$x_{ij}{}^* = \left(\frac{x_{ij}}{x_i^+} \right)^2. \tag{15}$$

Only maxima and minima are used, and therefore the same objections as against Weitendorf Ratios are valuable here.

Concerning an effective ratio system, the choice is not difficult to make. The ratio system in which each response of an alternative on an objective is divided by the square root of the sum of squares of each alternative per objective is representative for the comparison between alternatives and objectives. On basis of these ratios, MOORA results in ranking between the alternatives.

As a second method in MOORA, Reference Point Theory considered the Min-Max metric as the most representative choice. Was it a right choice?

Appendix B

Is the Min-Max metric the First Choice for Reference Point Theory?

1. The choice of the reference point

Reference point theory is a very respectable theory going back to such forerunners as Tchebycheff (1821–1894) and Minkowski (1864–1909); (see Karlin and Studden 1966 and Minkowski 1896, 1911). The choice of a reference point and the distance to the reference point is essential for reference point theory.

Preference is given to a reference point possessing as coordinates the dominating coordinates per objective of the candidate alternatives, which is designated as the *Maximal Objective Reference Point*. On the contrary, the Utopian Objective Reference Point gives higher values to the coordinates of the reference point rather than the maximal objective one. The Aspiration Objective Reference Point represents the other extreme, namely moderating the aspirations of the stakeholders by choosing smaller coordinates than in the maximal objective reference point.

2. How to measure the distance between the discrete points of the alternatives and the reference point?

The Minkowski Metric as a discrepancy measure brings the most general synthesis (Minkowski 1896, 1911; Pogorelov 1978):

$$\text{Min } M_j = \left\{ \sum_{i=1}^{i=n} \left(r_i - x_{ij}^* \right)^{\alpha} \right\}^{1/\alpha}, \quad (16)$$

where: M_j – Minkowski metric for alternative j; r_i – the i^{th} coordinate of the reference point; x_{ij}^* – the normalized attribute i of alternative j; $j = 1, 2, ..., m$, with m as the number of alternatives; $i = 1, 2, ..., n$, with n as the number of attributes.

The Minkowski metric represents the basis of what is designated in literature as *Goal Programming*. From the Minkowski formula, the different forms of goal programming are deduced. The metric shows these forms depending on the values given to α.

With the *Rectangular Distance Metric* ($\alpha = 1$) the results are very unsatisfactory. In the case of two attributes, suppose e.g. reference point (100;100) and the following points (100; 0), (0;100), (50; 50), (60; 40), (40; 60), (30; 70) and (70; 30) show the same rectangular distance and belong to the same line $x + y = 100$. Ipso facto, a midway solution like (50; 50) takes the same ranking as the extreme positions (100; 0) and (0; 100). In addition, the points (30; 30), (20; 40), (40; 20), (50; 10), (25; 35), (0; 60) and (60; 0), belonging to line $x, + y = 60$, show the same rectangular distance to reference point (50; 40) which is not defendable. Even worse, theoretically, for each line an infinite number of points will result in the same ranking.

With weights the negative remarks can be repeated when for normalization an Additive Method with Weights is used.

In the *Rectangular Distance Metric,* weights can be applied on the distance to the reference point or on the attributes.

The comparison with the Additive Method with Weights is completed on the distance to the reference point. Indeed, the formula runs as follows (Tamiz and Jones 1995, 1996; Tamiz *et al.* 1996):

$$\text{Min } M_j = \sum_{i=1}^{i=n} w_i (r_i - x_{ij}), \quad (17)$$

with: w_i as the weight for the distance of attribute i to the coordinate i of the reference point.

With weights joined to the attributes the link with the Additive Method with Weights is less direct:

$$\text{Min } M_j = \sum_{i=1}^{i=n} (r_i - w_i x_{ij}), \quad (18)$$

where: w_i as the weight for attribute i.

With $\alpha = 2$, radii of concentric circles, with the reference point as a central point, will represent the *Euclidean Distance Metric*. This distance metric applied for two attributes is similar to linear distances. Applying the Euclidean distance metric to the example above, the outcome is very unusual. The midway solution (50; 50) is ranked first with symmetry in ranking for the extreme positions (100; 0) and (0; 100); the same is for (60; 40) and (40; 60) and for (30; 70) and (70; 30) positions. Once again, numerous solutions are available.

Considering consumer sovereignty, a danger that the reference point is situated in the inadmissible non-convex zone above the highest possible indifference curve exists. Brauers and Zavadskas have proven that the Rectangular Distance Metric, the Rectangular Distance Metric with weights and the Euclidean Distance Metric rank non-convex points on the first places whereas the ranking of their convex points is also wrong (Brauers 2008; Brauers and Zavadskas 2006).

With three attributes, radii of concentric spheres with the reference point as a centre represent the Euclidean Distance Metric resulting also in numerous solutions. Automatically similar as for two attributes, the ranking of non-convex and even convex points will be incorrect. One could imagine that for more than three attributes corresponding conclusions can be drawn.

With $\alpha = 3$, negative results are possible if some coordinates of the alternatives exceed the corresponding coordinate of the reference point.

The same remarks made above would play with $\alpha > 3$, with the exception of $\alpha \to \infty$. In this special case of the Minkowski metric only one distance per point, viz. the largest one, is kept in the running. The Minkowski metric becomes the *Tchebicheff Min-Max Metric* with the formula already given above under (6).

Also here it is possible that exceptionally more than one solution is obtained, but not as general as in the previous cases whereas consumer sovereignty is fully respected (Brauers 2008; Brauers and Zavadskas 2006).

In the field of reference point theory, a method under the name of TOPSIS excites much interest by practitioners.

3. Is TOPSIS a better choice for reference point theory?

In fact, TOPSIS is a Reference Point Theory launched later than the traditional reference point theories (Hwang and Yoon 1981).

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is *based upon the concept that the chosen alternative should have the shortest distance from the ideal solution* (Hwang and Yoon 1981) which is in fact the aim of every Reference Point Theory. The distinction lies in the definition of the distance and how the coordinates of the reference point are defined. Moreover, an objective can ask for a maximum or minimum attainment. The choices of the distance function and of how to handle maxima and minima make TOPSIS a target for comments.

In TOPSIS, the Euclidean distance is chosen to define the shortest distance. The Euclidean distance was criticized above.

After normalization and eventually attributing weights, TOPSIS proposes two kinds of reference points – positive and negative. The positive reference point has as coordinates the highest corresponding coordinates of the alternatives (the lowest in the case of a minimum). The negative reference point has as coordinates the lowest corresponding coordinates of the alternatives (the highest in the case of a minimum). With regard to these two kinds of reference points, Euclidean distances are calculated. Consequently, each alternative will have two outcomes. Let us call them: y^*_{j+} and y^*_{j-}.

In order to come to one solution, TOPSIS proposes the following formula, which is rather arbitrarily chosen (Hwang and Yoon 1981):

$$y_j{}^* = \frac{y^*_{j-}}{y^*_{j+} + y^*_{j-}}, \tag{19}$$

where: $j = 1, 2, ..., m$; m the number of alternatives.

In addition, Opricovic and Tzeng (2004) conclude that the relative importance of the two outcomes is not considered, although it could be a major concern in decision-making.

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