DEPARTMENT OF ENGINEERING MANAGEMENT

Analysis of different cost allocation methods in a collaborative transport setting

Christine Vanovermeire, Dries Vercruysse & Kenneth Sörensen

UNIVERSITY OF ANTWERP
Faculty of Applied Economics

Stadscampus
Prinsstraat 13, B.226
BE-2000 Antwerpen
Tel. +32 (0)3 265 40 32
Fax +32 (0)3 265 47 99
www.ua.ac.be/tew

FACULTY OF APPLIED ECONOMICS

DEPARTMENT OF ENGINEERING MANAGEMENT

Analysis of different cost allocation methods in a collaborative transport setting

Christine Vanovermeire, Dries Vercruysse & Kenneth Sörensen

RESEARCH PAPER 2013-009
APRIL 2013

University of Antwerp, City Campus, Prinsstraat 13, B-2000 Antwerp, Belgium
Research Administration – room B.226
phone: (32) 3 265 40 32
fax: (32) 3 265 47 99
e-mail: joeri.nys@ua.ac.be

The papers can be also found at our website:
www.ua.ac.be/tew (research > working papers) &
www.repec.org/ (Research papers in economics - REPEC)

D/2013/1169/009

Draft Paper

Analysis of different cost allocation methods in a collaborative transport setting*

Christine Vanovermeire Dries Vercruysse
Kenneth Sörensen

June 2012

Abstract

In collaborative transport, dividing the total cost of the coalition between its different partners is a key issue. However, as each coalition has its own set of preferences and has partners with different characteristics, a cost allocation method suitable in all situations does not exist.

In this paper, a set of cost allocation methods, some academic, some used in practice, are evaluated in different situations. We investigate how well these methods behave when partners have different characteristics. E.g., when one partner ships a much larger volume than the others, it is very likely that this partner does not agree to allocate costs according to the volume of each partner. We show which cost allocation methods suffice in which situations, showing that a right cost allocation is highly dependent on the characteristics of the coalition.

1. Introduction

According to Ireland and Bruce (2000), the measures that companies implement to increase the efficiency of a single functional entity in their supply chain are often detrimental to the companies or functions downstream and upstream in the supply chain. They argue that a total supply chain view is needed, and that collaboration is essential to generate benefits supply-chain wide. Barratt (2004) classifies such collaborative initiatives in two main categories. A first category is *vertical collaboration*, i.e., collaboration of a company with its customers or suppliers. *Horizontal collaboration*, the main topic of this paper, is defined as “identifying and exploiting win-win situations among

*Research funded by a Ph.D. grant of the Agency for Innovation by Science and Technology (IWT)

companies *at the same level of the supply chain* in order to increase their performance” (Cruijssen et al., 2007b).

Although vertical collaboration has been studied far more often (Barratt and Oliveira, 2001), case studies and pilot studies of horizontal collaboration initiatives are increasingly common. Verstrepen et al. (2009) report that — to their knowledge — fifty pilot studies have been performed in Western Europe alone. However, the specific way in which a horizontal collaboration initiative is implemented can be very different, as can be seen in the studies of Verstrepen et al. (2009) and Audy et al. (2010). Verstrepen et al. (2009) list a large number of different forms of horizontal logistics collaboration: a lobbying group, a maintenance group, a purchasing group, chartering, warehouse sharing, freight sharing, a knowledge center, road assistance, co-branding, a tender group, asset pooling, intermodal transport, and a shared cross-dock.

In this paper, we focus on the sharing of freight transport. By bundling orders that are delivered in the same region or to a mutual client in an intelligent manner, redundant trips can be eliminated, and better prices can be negotiated from a third party logistics provider. Audy et al. (2011), e.g., present a case study of four furniture manufacturers that collaboratively transport their furniture to the customers by truck. They achieve a cost reduction of 12.9%. However, dividing those gains between the four partners proves to be difficult in this case, as some of the companies imposes more restrictions on the operational plan than the others (i.e., prohibiting to use a regional carrier, or very strict delivery times). Such restrictions lower the coalition gain, while the more flexible attitude of the other three partners influences the gains in a positive way. As a general rule, it can be stated that a higher level of flexibility of the partners in a coalition leads to higher coalition gain, and should therefore be encouraged (Vanovermeire et al., 2012). For this purpose, Audy et al. (2011) propose to use a modified Alternative Cost Avoided Method (Tijs and Driessen, 1986) and a modified Equal Profit Method (Frisk et al., 2010). In Vanovermeire and Sörensen (2012), the Shapley value is proposed to incorporate those differences. However, for coalitions of two, the Shapley value is found to be inadequate and is also adapted.

The division of costs is clearly a key issue in collaboration (Cruijssen et al., 2007a; EyeforTransport, 2010-2011) and there is no clear answer as to which cost allocation method is best. Moreover, there is a large difference between the set of cost allocations that are used in practice — allocations based on stand-alone costs or volume are the most preferred models (Frisk et al., 2010) — and those that are recommended in theory (Krajewska et al., 2008; Liu et al., 2010).

Game-theoretical cost allocation methods are found to be perfectly applicable in supply chains (Leng and Parlar, 2005). However, these methods are in general significantly more difficult to calculate (see Section 2) and may require data that is difficult to obtain. Moreover, the need to implement a complicated method can be questioned. For example, if partners are quite similar, the question can be asked whether the additional effort of using a complicated cost allocation method, the results of which are not always easy to interpret, is justified.

The aim of this paper is to evaluate different cost allocation methods. More specifically, we provide insight in how various cost allocation methods behave when partners have different characteristics or put different restrictions on the operational planning.

The paper is organized as follows. In Section 2, we list a set of cost allocation methods and show how to calculate them. In Section 3, we choose a number of situations (different partner characteristics and requirements) on which to test these cost allocation methods. Given these situations, the total coalition cost needs to be determined in order to be able to divide it between the partners in the coalition. This is also explained in Section 3. The outcome of the different cost allocation mechanisms and an analysis of their behaviour is shown in Section 4. We present some final conclusions in Section 5, as well as a decision tree that can be used to determine the most appropriate cost allocation method for a given situation.

2. Cost allocation methods in logistics collaboration

In horizontal logistics collaboration a set of companies form a group or *coalition* and thus become *partners* who jointly organize their distribution of goods. In game-theoretical terms, these partners are called *players*. Each partner issues a number of order requests, with a varying number of pallets that needs to be delivered. To deliver the pallets of all partners in the coalition, a total *coalition cost* is incurred, which needs to be divided between the different partners. The method used to determine the share of the coalition cost that is paid by each partner is called the *cost allocation method*. Table 1 shows the notation used to formulate the different cost allocation methods.

Table 1: Notation

i, j	An individual partner
N	The grand coalition, or the coalition with all partners included
S, T	A (sub)coalition
$ N , S $	The number of participants in a (sub)coalition
$c(S), c(N)$	The cost of a (sub)coalition
$c(i)$	The stand-alone cost of partner i
$v(S), v(N)$	The benefits created by a coalition, which we assume to be $\sum_{i \in S} c(i) - c(S)$
c_i	The cost allocated to partner i
v_i	The profit allocated to partner i (equals $c(i) - c_i$)
x_i, y_i	The cost allocated to partner i by the cost allocation method
z_i	The volume of partner i , expressed in number of pallets
m_i	Marginal cost when partner i enters coalition $N \setminus i$ or $c(N) - c(N \setminus i)$
w_i	The weight or importance of player i

In game theory, cost allocation methods are often evaluated against the properties they demonstrate. The two most important properties that a cost allocation method should

preferably have are *individual rationality* and *stability* (Moulin, 1988). The first states that an allocation should be so that each company does not pay a larger share of the coalition cost than its stand-alone cost. Assuming a company is *rational*, it would under such circumstances rather work alone than co-operate and suffer a higher cost.

A *stable* cost allocation on the other hand is such that none of the partners can improve by forming a subcoalition.

2.1. The Shapley value

The Shapley value (Shapley, 1953) is a well known game-theoretical cost allocation method based solely on the marginal effect of a partner on *all* (sub)coalitions containing that partner. It is therefore said to be based only on a partners co-operative productivity (Loehman and Whinston, 1974). The formula takes a weighted average of the marginal costs of a partner in each possible coalition:

$$x_i = \sum_{S \subseteq N \setminus i} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \times (c(S \cup i) - c(S))$$

One of the most important properties of the Shapley value is the fact that individual rationality is guaranteed (under the condition that the cost decreases when more players enter the coalition). Stability, however, is not guaranteed. Still, the Shapley value has other appealing characteristics, such as the fact that if a partner neither harms nor creates benefits for the coalition, it is allocated its stand-alone cost (i.e., the dummy property) (Shapley, 1953).

2.2. Nucleolus

The Nucleolus (Schmeidler, 1969) is another well known cost allocation method in game theory. Its approach is different from the Shapley value, as its main assumption is that not the co-operative productivity is the most important aspect, but stability and equality. The equality principle states that the best cost allocation is situated in the centre of the *core*, i.e. the union of all stable allocations.

The Nucleolus checks for each allocation x the *excess* in all coalitions. The excess for a coalition S given an allocation x is denoted as $e(x; S) = \sum_{i \in S} v_i - v(S)$, and is equal to the extra benefit to coalition S above the sum of benefits that are allocated to this subset of partners in the grand coalition N . The nucleolus tries to maximize the smallest excess, then the second smallest and so on until a single allocation is reached. The calculation of the Nucleolus requires solving a series of linear programs, and is therefore quite complicated. However, for coalitions of two or three partners ($|N| = 2$ or $|N| = 3$), simpler rules can be derived.

Aumann and Maschler (1985) show that in a coalition of two partners, the Nucleolus (as well as the Shapley value) will always give the standard solution, in which each player i receives the amount $v(i)$ or the reduction in stand-alone cost that can be achieved by becoming flexible, and the remainder of the gains are divided equally between the two players. In a coalition of three partners for superadditive¹ games, Moulin (1988) gives five conditions, for which a simple rule to allocate costs can be derived:

1. if $v(1, 2) \leq \frac{1}{3}v(1, 2, 3)$
 - $v_1 = v_2 = v_3 = \frac{1}{3}v(1, 2, 3)$
2. if $\frac{2}{3}v(1, 3) + \frac{1}{3}v(1, 2) \leq \frac{1}{3}v(1, 2, 3) \leq v(1, 2)$
 - $v_1 = v_2 = \frac{1}{4}(v(1, 2, 3) + v(1, 2))$
 - $v_3 = \frac{1}{2}(v(1, 2, 3) - v(1, 2))$
3. if $\frac{2}{3}v(2, 3) + \frac{1}{3}v(1, 2) \leq \frac{1}{3}v(1, 2, 3) \leq \frac{2}{3}v(1, 3) + \frac{1}{3}v(1, 2)$
 - $v_1 = \frac{1}{2}(v(1, 2) + v(1, 3))$
 - $v_2 = \frac{1}{2}(v(1, 2, 3) - v(1, 3))$
 - $v_3 = \frac{1}{2}(v(1, 2, 3) - v(1, 2))$
4. if $\frac{2}{3}(v(1, 3) + v(2, 3)) - \frac{1}{3}v(1, 2) \leq \frac{1}{3}v(1, 2, 3) \leq \frac{2}{3}v(2, 3) + \frac{1}{3}v(1, 2)$
 - $v_1 = \frac{1}{4}(v(1, 2, 3) + v(1, 2)) + \frac{1}{2}(v(1, 3) - v(2, 3))$
 - $v_2 = \frac{1}{4}(v(1, 2, 3) + v(1, 2)) + \frac{1}{2}(v(1, 3) - v(2, 3))$
 - $v_3 = \frac{1}{2}(v(1, 2, 3) - v(1, 2))$
5. if $\frac{1}{3}(v(1, 2, 3) \leq \frac{2}{3}(v(1, 3) + v(2, 3)) + \frac{1}{3}v(1, 2))$
 - $v_1 = \frac{1}{3}(v(1, 2, 3) + v(1, 2) + v(1, 3) - 2v(2, 3))$
 - $v_2 = \frac{1}{3}(v(1, 2, 3) + v(1, 2) + v(2, 3) - 2v(1, 3))$
 - $v_3 = \frac{1}{3}(v(1, 2, 3) + v(1, 3) + v(2, 3) - 2v(1, 2))$

¹ $v(i, j) > v(i) + v(j)$

2.3. Allocation based on volume or stand-alone cost

These methods, described as *proportional* cost allocation methods (Stevens, 1988), are the ones most commonly used in practice. Their calculation is straightforward: a weight is determined for each partner based on the volume (i.e., the number of pallets) that it ships or on its stand-alone cost. The cost assigned to each partner is then based on these weights.

A weight $w_i = z_i / (\sum_{j \in N} z_j)$ is volume-based, while $w_i = c(i) / (\sum_{j \in N} c(j))$ is based on the stand-alone cost. The allocated cost is then $x_i = w_i c(N)$. When referring to the cost allocation using the total volume as the weight to divide the total cost, we refer to the method "Volume". When allocating according to the stand-alone cost, we refer to the method "SA".

Although commonly used, both are not guaranteed to be individually rational, nor stable.

2.4. Separable and non-separable costs

Tijs and Driessen (1986) discuss allocation methods that divide the total cost in a separable and a non-separable part. The part that can be separated, i.e. linked to one specific partner, is the marginal cost when that player enters the coalition consisting of all other partners ($m_i = c(N) - c(N \setminus i)$).

The remaining part of the coalition cost is called the *non-separable* cost and can be divided in various ways.

First, the Equal Charge Method (ECM) divides the non-separable cost equally among all the partners. The total amount allocated to a player i is thus:

$$x_i = m_i + \frac{c(N) - \sum_j m_j}{|N|}.$$

Second, the Alternative Cost Avoided Method (ACAM) will take into account the individual contributions of each player. It defines weights based on the difference between the individual cost and the marginal cost of that player, or

$$x_i = m_i + (c(N) - \sum_j m_j) \frac{c(i) - m_i}{\sum_j c(j) - m_j}$$

A third allocation method, developed by Tijs and Driessen (1986) themselves, is the Cost Gap Method (CGM). Similar to the Shapley value and the Nucleolus, this method is based on the costs of all (sub)coalitions. It assumes that each partner needs to pay at least its own marginal cost m_i and thus can maximally only pay its own marginal cost plus the entire non-separable cost. However, if a subcoalition S exists in which the

non-separable cost to be paid $c(S) - \sum_{j \in S} m_j$ is smaller than $c(N) - \sum_j m_j$, this will become the maximal contribution to the non-separable cost in coalition N .

CGM uses this maximal contribution to calculate the weights to divide the non-separable cost:

$$w_i = \min_{S|i \in S} (c(S) - \sum_{j \in S} m_j); \quad x_i = m_i + \frac{w_i}{\sum_{j \in N} w_j} (c(N) - \sum_j m_j)$$

2.5. Equal Profit Method

Frisk et al. (2010) propose a method to allocate the coalition cost in such a way that relative savings of each partner, i.e., $(c(i) - x_i)/c(i)$, are as equal as possible. The following linear program needs to be solved:

$$\begin{aligned} \min \quad & f \\ \text{s.t.} \quad & f \geq \frac{x_i}{c(i)} - \frac{x_j}{c(j)} & \forall i, j \sum_{j \in S} x_j \leq c(S) & \forall S \subseteq N \\ & \sum_{j \in N} x_j = c(N) - \sum_{i \in N} c(i) & \forall x_i \geq 0, \quad \forall i \\ & x_i \geq 0, x_i \in \mathbb{R} & \forall i \in N \\ & i, j \in N \end{aligned}$$

The objective function tries to minimize a variable f . This variable, according to the first constraint set, is the largest relative savings difference between any pair of partners. The next constraint sets ensures that the allocation is stable and that the total coalition cost is allocated. Finally, an allocated cost should always be positive.

2.6. Weighted relative savings model

The weighted relative savings model of Liu et al. (2010) is an adaptation of the Equal Profit Method. Liu et al. (2010) argue that the relative savings should not be as equal as possible, but should reflect the contributions of each partner. To this end, they define the following ratio of contribution.

$$w_i = 1 - \frac{\sum_{i \in S} v(S) - v(S \setminus i)}{\sum_{i \in N} \sum_{i \in S} v(S) - v(S \setminus i)}$$

The difference in *weighted* relative savings between a player i and a player j is then given by

$$\frac{w_i v(i)}{c(i)} - \frac{w_j v(j)}{c(j)}$$

The linear program to be calculated is then easily derived from EPM:

$$\begin{aligned} \min \quad & f \\ \text{s.t.} \quad & f \geq \frac{w_i v_i}{c(i)} - \frac{w_j v_j}{c(j)} && \forall i, j \\ & \sum_{j \in S} v_j \geq v(S) && \forall S \subseteq N \\ & \sum_{j \in N} v_j = v(N) \\ & v_j \in \mathbb{R}, i \in \mathbb{N}, j \in \mathbb{N} \end{aligned}$$

2.7. Activity Based Costing

Activity Based Costing had been used in the private sector for several years before it was picked up in the academic literature by Cooper and Kaplan (1991). ABC is a system that links the activities to the resources of a firm. It is clear that the activities of a firm to its resources and their costs differ from firm to firm and from product to product. A good analysis is therefore needed beforehand to calculate the actual cost of an activity correctly.

For the purpose of this paper, we consider each trip as an activity, and the number of pallets as the cost driver in each trip. This means that we allocate the cost of a trip proportionally to the number of pallets each partner has in that trip. It differs from the Volume-method, as it does not use the *total* volume as a weight to divide the *total* cost, but divides the cost of each trip according to the volume in that particular trip.

This method of calculating the individual cost is based on our specific situation. Some of the conclusions regarding ABC are thus dependent on the cost drivers and activities used.

3. Methodology

3.1. General method

In this section, the various cost allocation methods discussed in the previous section are applied to a set of simulated coalitions to determine their suitability in a different collaborative transport settings.

The coalitions differ in the number of collaborating partners, the average order size per partner, the total number of orders per partner and the number of days each partner allows its delivery dates to change. Hence, the costs that are linked to transporting the orders of each partner in each coalition, or, the characteristic functions of these games, will also become very different.

By analysing the formulas and the properties of each of the different cost allocation methods, as well as by solving test cases (see Section 3.2) and analysing the results, we determine how these differences can affect the behaviour of the cost allocation methods. We give an overview of these differences and create a guideline in the form of a decision tree of which cost allocation methods to use in which situations.

3.2. Creating test cases

To test certain concepts, we develop a set of test instances. We assume that the capacity of a truck (33 pallets) is constant, that there are no additional fixed costs, that no extra cost is associated with transportation of goods between the partners' warehouses. In the first set of (simple) instances, each trip has a cost of one unit. Additionally, several realistic instances are generated, based on real-life case studies. In those instances the total cost of a trip is determined by the (decreasing) transport prices per pallet (see Table 3 in Appendix). These instances can be found on-line at <http://antor.ua.ac.be/downloads>.

Each test instance consists of the orders of a coalition with two up to five partners. An order i is sent from a forwarder (partner) m to a client n . It has a size q_i (in number of pallets) and is sent on date f_i , or on a date that does not exceed the maximum number of days A_i that the forwarder of this order allows it to be moved in time. The orders are bundled in trips j with a maximum capacity of $C = 33$ pallets. A trip will have a cost $c(p)$, with p the number of pallets shipped in that trip.

To determine the cost of a (sub)coalition, all orders of all partners in that coalition are scheduled in such a way that the total cost is minimized. This requires solving, for each destination n , an optimization problem with the following decision variables:

$d_j \in \mathbb{N}$	The shipping date of trip j
$x_{ij} \in \{0, 1\}$	1 if order i is in trip j , 0 if it is not
$y_{jp} \in \{0, 1\}$	1 if trip j contains p pallets, 0 if it does not

For all orders with destination n , the following linear program is solved:

$$\min \sum_j \sum_p c(p) y_{jp} \quad (1)$$

s.t.

$$\sum_j x_{ij} = 1 \quad \forall i \quad (2)$$

$$\sum_p y_{jp} = 1 \quad \forall j \quad (3)$$

$$C(1 - y_{jp}) \geq \sum_i x_{ij} q_i - p \quad \forall j, \forall p \quad (4)$$

$$\sum_i q_i x_{ij} \leq C \quad \forall j \quad (5)$$

$$d_j + M(1 - x_{ij}) \geq f_i - A_i \quad \forall i, \forall j \quad (6)$$

$$d_j - M(1 - x_{ij}) \leq f_i + A_i \quad \forall i, \forall j, i, j, p \in \mathbb{N} \quad (7)$$

$$(8)$$

The objective function (1) minimizes the total cost. Constraints (2) ensure that each order is transported exactly once. Constraints (3) impose that each trip needs to transport exactly one quantity of pallets p . According to constraints (4), this quantity cannot be lower than the sum of the number of pallets that are in the trip. The number of pallets in a trip cannot exceed the capacity C (constraints (5)). Finally, constraints (6–7) ensure that an order is delivered within its time window $[f_i - A_i, f_i + A_i]$. The total cost of the coalition is determined by summing the costs for all destinations n .

4. Analysis

Unsurprisingly, when companies have similar characteristics, we find that all cost allocation methods allocate a similar share to all the partners. This is illustrated in Figure 1, in which the result of various cost allocation methods on a coalition of three very similar partners, are shown.

However, when companies have asymmetric properties, the cost allocations of the different methods diverge. The question remains which cost allocation is then the best. As this is highly dependent on the situation and the preferences of the partners, we give an analysis of various situations.

4.1. Number of partners

When there are only two partners in the coalition, many of the cost allocation methods propose the same cost allocation. When there is an asymmetry in the number of days

Figure 1: Cost allocation for three similar companies: Company A (276 pallets), Company B (305 pallets) and Company C (285 pallets)

Figure 2: Cost allocation to company A (130 pallets) and company B (661 pallets)

that orders can be moved, in the order size, or in the number of orders, the cost allocation methods can be organized as follows. A first group (Shapley, Nucleolus, ECM, ACAM and CGM) allocates the total profit equally among the two partners. A second group (SA, WRSM and EPM) allocates the cost in such a way that the relative profit is equal. The volume-method and ABC are different from the other methods. They are discussed in Section 4.2 and Section 4.3.

The larger the coalition, the more apparent the differences between cost allocation methods become. For several of them, it also becomes more difficult to collect all the information needed. The Nucleolus, the Shapley value, CGM, EPM and WRSM require knowledge of the cost of all possible (sub)coalitions. The number of possible subcoalitions for a grand coalition N is equal to $2^{|N|}$, including both the grand coalition itself and the empty coalition. For large coalitions, it can be difficult or time-consuming to gather all required data. This is especially the case if, like in the example presented in this paper, determining the cost of each coalition requires solving an optimization problem, like the linear programming problem mentioned before. Finally, the calculation of the cost allocation itself can become difficult or even impossible. To solve the Nucleolus, a series of linear programs need to be solved. The degree of difficulty thus increases significantly when more than three partners are involved. We therefore prefer not to choose the Nucleolus when the number of partners exceeds three. EPM and WRSM on the other hand, although they also require the solution of a linear programming problem, are less difficult to determine. However, these methods have the drawback that, for large coalitions, it is increasingly likely that no stable allocation exists. In this case, EPM and

Figure 3: Allocated cost to company B (4073 pallets), collaborating with partners A (967 pallets) and C (1375 pallets), based on Table 4

WRSM will offer no solution at all.

4.2. Total number of pallets

An asymmetry in the total number of pallets — whether the average order size of one company is significantly higher or lower, or the total number of pallets differs — has a large impact on the cost allocations. The pace list (Appendix A) has a decreasing cost per pallet, which implies that the more pallets in a trip, the more profit can be made (decreasing marginal costs). However, the volume method regards the number of pallets as the only cost driver. Therefore, we see in Figure 3 and in Table 4 that this method disfavors companies with a large number of pallets to such an extent that it very easily allocates such companies a share of the total cost that is higher than their own stand-alone cost.

As the marginal cost per pallet decreases in a trip² companies will receive the least benefits from cost allocation methods that only consider stand-alone cost as a cost driver (SA). The opposite is true for EPM and WRSM, which base the main calculation on the stand-alone cost, but use it to divide the *profits*. A large company will thus prefer EPM and WRSM. Another cost allocation method that favours large companies is ECM. It only considers the marginal cost in the final coalition and divides the rest equally.

Comparing the other methods, the Shapley value and ACAM behave very similarly. ACAM is, however, easier to calculate. In general, the Nucleolus and CGM allocate slightly lower costs to the large company than ACAM and the Shapley value. However, exceptions can be found in Section 4.4. ABC on the other hand (slightly) benefits small companies as it divides the cost of a trip according to the number of pallets each partner ships in that trip, even though pallet prices are decreasing and thus, partners with a higher number of pallets in that trip contributed more to the lower price of that trip. This could be a sign that another cost driver should be included. However, the final outcome of the cost allocation is always very dependent on the operational plan, which is demonstrated in Section 4.3. Possible differences between the Nucleolus and the Shapley value are discussed in Section 4.4.

4.3. Flexibility

When there is a clear asymmetry in flexibility — either because one company has more rigid constraints than the others (in this case, allows its orders to be moved less) or because one company is more flexible than the others — we find that some cost allocations are more adequate than others.

As mentioned in Section 4.1, for a two-partner-coalition, the share of the profit is fixed for the Shapley value, the Nucleolus, ACAM, ECM and CGM. The surplus of becoming flexible will thus always be equally shared by these methods. The flexible partner will benefit from becoming flexible, however, the non-flexible partner will receive an increase in profit that is exactly the same. EPM, WRSM and SA will give a larger share to the flexible partner, but only when this flexibility also enables it to lower its own stand-alone cost. As shown later in this section, the Volume method and ABC cannot reward flexibility in an adequate manner, in a two-partner-coalition nor in a larger coalition.

The best options among these cost allocation methods to reward flexibility in a two-partner-coalition are thus SA, EPM and WRSM. However, in this section, we show that these methods still reward flexibility inadequately. This issue is addressed in Vanovermeire and Sörensen (2012).

In Figure 4, we show the stand-alone costs and the attributed costs for each cost allocation method, and for a specific partner in a three-partner-coalition. In a first scenario, all companies are rigid (do not allow their orders to be moved to a different delivery date). In the second scenario, partner A allows its orders to be delivered on any day of the same week.

Dividing the coalition costs according to the volume shipped by each company can never incorporate the asymmetry in flexibility. A company will receive the best incentive to be flexible when the difference between the attributed cost when rigid and when flexible is the highest. As can be derived from Figure 4, the Volume method attributes a cost to company A that is much lower than the others in the rigid situation, while attributing the highest cost in the flexible situation.

Figure 4: Allocated cost to company A (967 pallets), collaborating with partners B (4073 pallets) and C (1375 pallets), based on Table 4.

The method that performs second-to-worst, is ABC. Although basing the attributed cost on the actual operational plan might give incentives to be more flexible (by being more flexible, its orders can be easily bundled with other trips and for each order bundled, a smaller part of a trip should be paid), we show in Figure 5 that a cost allocation that exceeds the stand-alone cost is possible. The cost allocation can thus be influenced by the operational planner, which is not desirable.

In Figure 5, three partners, A, B and C, deliver to a mutual client. Both B and C allow to move their orders for two days. As a trip has a maximum capacity of five pallets, it is not possible to bundle all the orders in one trip. This implies that there are multiple operational plans possible. In the first scenario, none of the orders are moved, and dividing according to this operational plan, the costs are allocated respectively 1 , $\frac{1}{2}$, $\frac{1}{2}$. However, when the operational planner moves the order of company B, as he has done in the second operational plan, the allocation changes to $\frac{3}{5}$, $\frac{2}{5}$ and 1 . This can easily be perceived as unfair, as the flexible partner C has to pay an additional cost, and the rigid partner A receives more gains than C. In scenario 3, partner A pays $\frac{3}{5}$ of the trip, while B pays an entire trip and C $\frac{2}{5}$. The (arbitrary) choice of the operational plan can thus influence the cost allocation significantly. Note that — although we have used our own specific formula for ABC — the fact that ABC is based on the operational plan at hand will always allow the planner to influence the cost allocation when using this method. This implies that the optimal plan can be rejected by one of the partners, knowing that there exist alternatives with a similar cost (or even a higher total cost),

(a) Operational plan 1 (nobody moves its orders)

(b) Operational plan 2 (B moves its orders)

(c) Operational plan 3 (C moves its orders)

Figure 5: Different options to plan shipments of A, B and C

that can generate a much lower allocated cost for this partner.

Although flexibility should be stimulated as much as possible (the more flexible the constraints, the lower the total transportation cost can be), we argue that in this case, the methods that give the highest incentives are not necessarily the best (SA and EPM). As can be derived from Table 4, when allocating costs using these methods, partner B would be allocated a higher cost than when partner A was rigid. It is clear from Table 4 that no cost allocation method guarantees that all partners profit when one partner becomes flexible. However, as only EPM and SA attribute a higher cost to company B, one might reconsider the fairness of the allocation.

Indeed, flexibility on the delivery dates reduces the stand-alone cost of the flexible partner to a greater extent than the reduction when becoming flexible in the collaborative setting (Some of the bundling that can be achieved when working alone and flexible, can also be achieved when working collaboratively and rigidly). Therefore, cost allocation methods that use the stand-alone cost as a measure to divide the costs (EPM, SA and to a lesser extent WRSM) allocate a cost that is disproportionate to the actual contribution. The reverse can be true as well: flexibility that has a large effect on the final coalition cost, but not on the stand-alone cost of a partner will not earn this specific partners any rewards. It is thus better to use a method that incorporates the marginal cost of the flexible partner *in the final coalition*, such as the Shapley value, CGM, ACAM, ECM or the Nucleolus.

The method that should be chosen depends on the preference of the coalition. If the partners prefer to give the largest possible incentives for flexibility, because they feel this is an important factor to lower the total coalition cost, CGM, ACAM and the Shapley value are the best methods. If they prefer a more equal division of profit, the Nucleolus and ECM are the best option. ECM should, however, be used with care, as it possibly allocates a cost that is higher than the stand-alone cost.

In all other situations simulated with asymmetric flexibility, the same conclusions can be derived.

4.4. Strong subcoalitions

We use the term *strong subcoalitions* to denote a situation in which a subcoalition that can already generate a large part of the total benefit. In general, if there exists $S \in N$ and $T \in N$ for which $v(S) + v(T) \geq 1$. There can be many reasons for such strong subcoalitions to exist.

Imagine three partners, each delivering five pallets to a client each Monday. Given that a truck has a capacity of ten pallets, the number of trips needed can be reduced from three to two trips. However, this reduction in trips can already be achieved when two partners in that coalition collaborate.

Table 2: Outcomes for the example in Section 4.4 given different methods

Method	Allocation to A	Allocation to B	Allocation to C
Shapley	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{6}$
Nucleolus	0	1	1
Volume	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
SA	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
ECM	0	1	1
ACAM	0	1	1
GCM	0	1	1
EPM	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

Another example of strong subcoalitions is the following: given are three partners A, B and C, delivering an order of three pallets on respectively Monday, Tuesday, and Wednesday. Although all orders would fit together in a trip, only company A allows its orders to be moved. This means that $v(AB) = 1$, $v(AC) = 1$, $v(BC) = 0$ and $v(ABC) = 1$. It is clear that the coalitions with company A in it are very strong.

In case of strong subcoalitions, the cost allocation methods that incorporate the costs of these subcoalitions will evidently start to differ from the other methods (as can be derived from Table 2). The partner that is able to generate more profits in the subcoalitions will generally benefit in comparison to the other methods. The degree to which it can differ however is different.

Nucleolus and EPM search for a stable allocation. It is possible that when strong subcoalitions occur, there are not many stable allocations possible. In this case, both methods will greatly favour the partners in the strong subcoalition. Note that, when there are still many stable allocations possible, EPM will behave in an opposite way, as the goal of relative equality will be of disadvantage for the companies in the strong subcoalitions. In any case, the Nucleolus will result in much larger inequality than the Shapley value or ACAM.

5. Conclusion and recommendations

5.1. Conclusion

The choice of cost allocation method can vary much, depending on the characteristics of the partners as well as their preferences. The cost allocation methods that are preferred in the literature because of their properties (Shapley or the Nucleolus) will not always be best in practice. Moreover, depending on the situation, it might be possible to use a cost allocation methods that will behave similarly in the same situation, but is easier to calculate. When there is a clear asymmetry in flexibility, cost allocation methods based

on the marginal cost (or the costs of all coalitions) are needed. When the volume or stand-alone cost clearly differs, cost allocation methods that ensure individual rationality need to be used. Finally, strong subcoalitions can result in large differences between various methods.

Figure 6 gives an overview of the cost allocation methods that are best suited for every given situation. In bold, the methods that are the easiest to calculate are indicated.

5.2. Future work

The conclusions in this paper have been based on one specific situation, in which orders can only be bundled when they have the same destination, and each partner has the same client base. Moreover, we have assumed the transport is organized by a logistics service provider and therefore, there is no restriction on fleet capacity and prices are set using a pace list. All other assumptions can be found in Section 3.

Evidently, much more possible situations exist, and a lot more characteristics and the resulting behaviour of even more cost allocation methods. However, this paper already indicates for a number of common situations which cost allocation methods can be applied.

The methodology of this paper can be applied to many other situations. It is important to define beforehand the characteristics and preferences of the coalition. By considering the data needed and the manner of calculating, many of the behaviours of the different methods can already be predicted in advance. Finally, testing the cost allocation methods can reveal additional behaviour.

This work has focused on the bundling of orders from one origin to one destination point. However, the behaviour of the cost allocation methods when round trips are created has to be investigated. Moreover, the incorporation of additional fixed cost can also influence the performance of the cost allocation methods.

References

- J.-F. Audy, N. Lehoux, S. D'Amours, and M. Rönnqvist. A framework for an efficient implementation of logistics collaborations. *International transactions in operational research*, pages 1–25, October 2010.
- J.-F. Audy, S. D'Amours, and L.-M. Rousseau. Cost allocation in the establishment of a collaborative transportation agreementan application in the furniture industry. *Journal of the Operational Research Society*, (62):960970, 2011.
- R.J. Aumann and M. Maschler. Game theoretic analysis of a bankruptcy problem from the talmud. *Journal of Economic Theory*, 36(2):195 – 213, 1985.

Figure 6: Decision tree for choosing a cost allocation method. Bold cost allocation methods are the easiest to calculate.

¹ Yes, but give a slight advantage to *small* companies

² Yes, but give a slight advantage to *large* companies.

- M. Barratt. Understanding the meaning of collaboration in the supply chain. *Supply Chain Management: An International Journal*, 9(1):30–42, 2004.
- M. Barratt and A. Oliveira. Exploring the experiences of collaborative planning initiatives. *International Journal of Physical Distribution & Logistics Management*, 31(4): 266–289, 2001.
- T. Cooper and R.S. Kaplan. Profit priorities from activity based costing. *Harvard Business review*, (May/June):130–135, 1991.
- F. Cruijssen, M. Cools, and W. Dullaert. Horizontal cooperation in logistics: Opportunities and impediments. *Transportation Research Part E: Logistics and Transportation Review*, 46(3):22–39, 2007a.
- F. Cruijssen, W. Dullaert, and H. Fleuren. Horizontal cooperation in transport and logistics: a literature review. *Transportation Journal*, 43(2):129–142, 2007b.
- EyeforTransport. European horizontal collaboration in the supply chain: a brief analysis of eyefortransport’s recent survey. Technical report, EyeforTransport, 2010-2011.
- M. Frisk, M. Göthe-Lundgren, K. Jörnsten, and M. Rönnqvist. Cost allocation in collaborative forest transportation. *European Journal of Operational Research*, 205(2): 448–458, 2010.
- R. Ireland and R. Bruce. Cpfr-only the beginning of collaboration. *Supply Chain Management Review*, 4(September/October):80–88, 2000.
- M.A. Krajewska, H. Kopfer, G. Laporte, S. Ropke, and G. Zaccour. Horizontal cooperation among freight carriers: request allocation and profit sharing. *Journal of the Operational Research Society*, 59(11):1483–1491, 2008.
- M. Leng and M. Parlar. Game theoretic applications in supply chain management. *INFOR*, 43(3):187–220, August 2005.
- P. Liu, Yaohua Wu, and N. Xu. Allocating collaborative profit in less-than-truckload carrier alliance. *Journal of Service Science and Management*, 3:143–149, 2010.
- E. Loehman and A. Whinston. An axiomatic approach to cost allocation for public investment. *Public Finance Quarterly*, pages 236–251, April 1974.
- H. Moulin. *Axioms of cooperative decision making*. Econometric Society Monographs. Press Syndicate of the University of Cambridge, 1988.
- D. Schmeidler. The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics*, 17(6):1163–1170, 1969.
- L.S. Shapley. A value for n-person games. *Annals of Mathematics Studies*, 28:307–317, 1953.

- R. Stevens. *Coperatieve speltheorie*. Eclectica. Economische Hogeschool Sint-Aloysius, 1988.
- S.H. Tijs and T.S.H. Driessen. Game theory and cost allocation problems. *Management Science*, pages 1015–1028, 1986.
- C. Vanovermeire and K. Sörensen. Measuring and rewarding flexibility in collaborative distribution using the shapley value. *Submitted for publication*, 2012.
- C. Vanovermeire, K. Sörensen, A. van Breedam, B. Vannieuwenhuyse, and S. Verstrepen. Efficient supply chains through flexible horizontal collaboration. Antwerp, 2012.
- S. Verstrepen, M. Cools, F. Cruijssen, and W. Dullaert. A dynamic framework for managing horizontal cooperation in logistics. *International Journal of Logistics Systems and Management*, 5(3):228–248, 2009.

A. Pace list

Pallets	Price	Pallets	Price	Pallets	Price
1	25.16	13	11.81	24	8.36
2	21.51	14	11.38	25	8.13
3	19.43	15	11	26	7.92
5	17.9	16	10.63	27	7.71
6	16.72	17	10.28	28	7.52
7	16.75	18	9.96	29	7.32
8	15.75	19	9.66	30	7.14
9	14.89	20	9.38	31	6.96
10	13.54	21	9.11	32	6.79
11	12.77	22	8.85	33	6.45
12	12.27	23	8.59		

Table 3: Price per pallet per order of x pallets

B. Case with 3 partners, A (967 pallets) - B (4073 pallets) - C (1375 pallets)

Method	Working together (1)			$\Delta_{\text{Stand-alone}} - (1)$			A is flexible (2)			$\Delta(1) - (2)$		
	A	B	C	A	B	C	A	B	C	A	B	C
Stand-alone cost $c(i)$	14489	40884	21710	0	0	0	9525	40884	21710	4964	0	0
Shapley	11579	37615	18534	2910	3269	3176	4140	37306	19078	4140	309	-545
Nucleolus	11787	37464	18476	2910	3420	3234	3670	36505	19201	3670	959	-725
Volume	10209	43001	14517	4280	-2117	7193	588	40522	13680	588	2479	837
SA	12730	35922	19075	1759	4962	2635	4306	36177	19221	4306	-256	-147
ECM	11787	37464	18476	2702	3420	3234	3670	36505	19201	3670	959	-725
ACAM	11640	37570	18517	2849	3314	3193	4068	37149	19102	4068	421	-586
CGM	11644	37562	18521	2845	3322	3189	4007	36996	19190	4007	566	-669
EPM	12730	35922	19075	1759	4962	2635	4306	36177	19221	4306	-256	-147
ABC	11116	38626	17985	3373	2258	3725	2665	37592	17593	2665	1034	392
WRSM	12796	35855	19076	1693	5029	2634	4198	35797	19428	4198	58	-352
Total cost	67727			9356			63823			3904		

Table 4: Allocated costs and profits in scenario "Working together" where all work together but rigidly, and a scenario "A is flexible" where A is flexible.