Measuring viscoelastic parameters in Magnetic Resonance Elastography: a comparison at high and low magnetic field intensity

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Abstract

Magnetic Resonance Elastography (MRE) is a non-invasive imaging technique which involves motion-encoding MRI for the estimation of the shear viscoelastic properties of soft tissues through the study of shear wave propagation. The technique has been found informative for disease diagnosis, as well as for monitoring of the effects of therapies. The development of MRE and its validation have been supported by the use of tissue-mimicking phantoms. In this paper we present our new MRE protocol using a low magnetic field tabletop MRI device at 0.5 T and sinusoidal uniaxial excitation in a geometrical focusing condition. Results obtained for gelatin are compared to those previously obtained using high magnetic field MRE at 11.7 T. A multi-frequency investigation is also provided via a comparison of commonly used rheological models: Maxwell, Springpot, Voigt, Zener, Jeffrey, fractional Voigt and fractional Zener. Complex shear modulus values were comparable when processed from images acquired with the tabletop low field scanner and the high field scanner. This study serves as a validation of the presented tabletop MRE protocol and paves the way for MRE experiments on ex-vivo

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tissue samples in both normal and pathological conditions.

*Keywords:* Magnetic Resonance Elastography (MRE), Complex shear modulus, Tabletop MRI, Rheological models
1. Introduction

Elastography is a non-invasive imaging technique for the estimation of shear viscoelastic properties in soft tissues [1, 2, 3, 4, 5, 6, 7] and is related to manual palpation, a fundamental step in a clinical physical evaluation. Elastography techniques can be classified into quasi-static and dynamic approaches [8]: while the former apply a time-invariant force to the tissue, the latter use a time-varying force that results in the propagation of mechanical waves, both of the compressional and shear kinds [9]. Dynamic elastography techniques observe shear wave propagation and produce quantitative stiffness maps compared to quasi static methods that only quantify contrast (relative stiffness values), without quantitative information about boundary conditions [10, 11]. Elastography measurements can be made using different imaging modalities, such as ultrasound [1, 2, 12, 13, 14], optics [15, 16, 17, 18], or magnetic resonance imaging (MRI) [7, 19]; in particular, the last approach is referred to as Magnetic Resonance Elastography (MRE), which involves motion-encoding MRI. MRE offers resolution up to hundreds of micrometers [20, 21], great penetration depth [22] and added value in a multiparametric MRI approach [23]. MRE has high diagnostic accuracy for the staging of hepatic fibrosis [24] and has diagnostic potential for the detection of breast [25], thyroid [26] and prostate cancers [27, 28]. Promising studies on kidney [29], brain [30], heart [31] and muscle [32] tissues have also been reported. The development of these MRE approaches and their validation have been supported by the use of tissue-mimicking phantoms.

Phantoms play an essential role in the development of elastography: given their accessibility and convenience, they have been used as a means of standardization and validation [33], and they have also been employed to improve the performance and reliability of inversion algorithms to obtain material properties maps, called elastograms [34, 35, 7], and to evaluate motion acquisition approaches [36, 37]. Thus, scientific literature contains many studies about phantom materials: the reviews by Cao et al. [35] and Culjat et al. [38] classify the different types of tissue mimicking materials and describe their fabrication, benefits and disadvantages. Gelatin is the most common tissue substitute used in investigations [39, 40, 41, 42]. Doyley et al. have used gelatin phantoms for assessing the quality of elastograms produced using an innovative imaging system for clinical breast elastography [41], while Hall proposed numerous mechanical tests of tissue-like gelatin materials for elastography experiments [42]. Other examples of materials used for tissue-like
phantoms are agar-agar gel [43, 44, 45], agar-gelatin [46, 47, 48, 49], ecoflex [50, 51], polyurethane gel [52], oil in gelatin [53, 34], aqueous polyvinyl alcohol solutions [54, 45], silicone [45], polyacrylamide gel [45], glycerol in oil-based gel [55], paraffin gel [56] and copolymer in oil from mixtures of styrene-ethylene/butylene-styrene [57].

Customarily, MRE on phantoms has been performed using MRI scanners at high ($B_0 > 1$ T [57, 43, 48]) and ultra-high ($B_0 > 7$ T [58, 59]) magnetic field, taking advantage of the high sensitivity resulting in increased resolution and Signal-to-Noise Ratio (SNR) and reduced scan time [60] compared to using scanners at low magnetic field. Nevertheless, both ultra-high and high magnetic field scanners come with high costs and are extremely bulky, thus requiring large magnet rooms. A higher field strength results in longer $T_1$ relaxation times and thus longer repetition times in most of the pulse sequences for accurate relaxometry mapping [61]. The chemical shift artifact is particularly evident as well, especially along the readout direction and this pitfall may necessitate either increasing the readout bandwidth or switching to fat suppression techniques, which can lead to Specific Absorption Rate (SAR) issues. Recently, a setup based on a 0.5 T tabletop MRI scanner was used for investigations of viscoelastic properties of small ex-vivo tissue samples and gel phantoms through MRE [62, 63]. With a permanent magnet in a tabletop device, the paradigm shift to the low field imaging comes with advantages: lower initial purchase price, operational and maintenance costs, lower fringe field effects – so a lower projectile risk and easier field shielding – as well as the decrease of chemical shift and susceptibility artifacts.

In this paper we present our new MRE protocol using a low magnetic field tabletop MRI device at 0.5 T and sinusoidal uniaxial excitation in a geometrical focusing condition. Results obtained for gelatin are compared to previously implemented ultra-high magnetic field MRE at 11.7 T. A multi-frequency investigation at 0.587 T is also provided via a comparison of commonly used rheological models. This study serves as a validation of the presented tabletop MRE protocol and paves the way for MRE experiments on ex-vivo tissue samples in both normal and pathological conditions.

2. Materials and Methods

2.1. Materials

A total of 5 samples for each of the 4 concentrations of gelatin solutions – 5%, 10%, 15% and 20% weight over volume (w/v) – were prepared with the
intent to span over the shear stiffness range of biological tissue by solubilization of Gelatin Powder (IS16003, Lab Grade, Innovating Science \textsuperscript{TM}, Aldon Corporation, Avon, NY) in distilled water and inserted in glass test tubes of 8mm inner diameter \cite{39}. Within the deformation regime of MRE, gelatin – as soft tissues – can be modeled as linear viscoelastic materials which are characterized by a storage modulus and a loss modulus ([64, 65, 66, 67]). The material is considered as nearly incompressible with a Poisson’s ratio approximable to 0.5 and with density of water ($\rho = 1000 \text{kg/m}^3$). Given these assumptions, the relationship between Young’s modulus $E$ and shear modulus $\mu$ is $E \sim 3\mu$.

2.2. Experimental setup

MRE experiments at low magnetic field were performed using a 10mm diameter vertical bore tabletop MR scanner with a 0.5 T permanent magnet (MagSpec, Pure Devices GmbH, Würzburg, Germany). The scanner is controlled by a driver console (drive L, Pure Devices GmbH, Würzburg, Germany), requiring a licensed version of MATLAB (Mathworks, Natick, MA, USA) not older than Version 2012a. An external gradient amplifier (DC 600, Pure Devices GmbH, Würzburg, Germany) and an integrated custom-built piezoelectric actuator system were added to the setup (see Table 1). A similar system was used in the studies of Ipek-Ugay et al. \cite{62} and Braun et al. \cite{63}.

Our setup includes a custom-made piezoelectric actuator support, as shown in Figure 1, which is made of polycarbonate characterized by very low relative magnetic permeability, high density, high Poisson ratio and high Young’s modulus: it has been designed using SolidWorks (Dassault Systèmes, Vélizy-Villacoublay, France) with the aim of building a high-inertial, deformation-resistant and durable support at low cost. The components of the support were first fabricated separately and then chemically bonded; the support is provided with two flat sustains that touch the upper case of the scanner and with two holes on the lateral blocks hosting plastic screws, so that possible movements of the scanner are rigidly transferred to the support. The hollowed top block hosts the actuator in its cavity, with a H6 ISO tolerance to ensure the lowest clearance and, thus, an axial transmission of the displacement provided by the actuator through a threaded rod (hooking the sample tube extremity at one end and screwed in the actuator at the other end). The piezoelectric driver has an external diameter of 20 mm and a length of 72 mm. MRE at 0.5 T was followed by a comparison at
ultra-high magnetic field using the experimental setups of MRE described in previous publications [27, 68, 59, 58]. These experiments were performed on a 56mm vertical bore MR scanner (Bruker 11.7 T, Billerica, MA) and on a 310mm horizontal bore MR Scanner (Agilent 9.4 T 310/ASR, Santa Clara, CA) with the same setup as in Yasar et al. [59]. A total of 3 samples with 3 concentrations – 5%, 10% and 15% w/v – were prepared for the 11.7 T experiments, similarly as discussed for tabletop experiments. At 9.4 T we acquired data from a single sample with 10% w/v concentration only as a validation at that specific concentration.

2.3. MRE experiments

The piezoelectric driver was fed with a sinusoidal alternating current with a vibration-amplitude dependent voltage (maximum 90 V) and 4 driving frequencies \( f = 500, 1000, 1500 \) and 2000 Hz). The range of frequencies was chosen on the basis of the study of Guidetti et al. [58], by which the lower bound frequency is chosen relying on the ratio of the test tube diameter to the wavelength. This makes sure that at least half a wavelength can be detected for the reconstruction procedure. On the other hand, the higher frequency limit is selected based on the attenuation given by the damping effect. In order to encode vibrations into the MRI phase map values using a spin echo pulse sequence, the sinusoidal excitation provided by the actuator was synchronized to the application of a bipolar 8-lobes trapezoidal motion-encoding gradient (MEG). Data acquisition was performed during the echo formation after switching off both the MEG and the mechanical excitation. Complex phase subtraction was performed to correct for static phase offsets, requiring two acquisitions with inverse-polarity MEGs at each of the four time instances per frequency.

The vibrations inside the test tube were polarized along the main axis of the cylinder due to the constrained axial motion direction of the actuator (Figure 1a). Shear waves were introduced into the samples from the cylinder walls by producing concentric cylinder waves propagating from the outer sample boundaries towards the central axis of the test tube. This setup allowed the motion field acquisition to be limited by uniaxial z-component encoding and represented the geometric focusing technique [59], which compensates for the damping effect due to the viscoelastic properties of the analyzed materials. When a viscoelastic isotropic material is considered, the cylindrical coordinate wave equation provides the out-of-plane displacement.
$u_z$ as a function of the radial position $r$:

$$u_z(r, t, k_\beta) = u_{za} \frac{J_0(k_\beta r)}{J_0(k_\beta a)} e^{j\omega t}$$ (1)

with $u_{za}$ being the oscillation amplitude for $r = a$, $k_\beta = \omega \sqrt{\frac{\rho}{\mu_R + j\mu_I}}$ being the shear wave number, $J_0(z)$ the Bessel function of the first kind 0th order, with $j = \sqrt{-1}$, $\rho$ the density, $\omega$ the angular frequency, $\mu_I$ and $\mu_R$ referring to the imaginary and real part of the complex shear modulus $\mu$ respectively, $a$ being the radius of the test tube, and $u_{za}$ the amplitude of the harmonic excitation set on the boundary by the piezoelectric actuator.

For MRE at 9.4 T and at 11.7 T a similar kind of sample materials and similar actuation setups were used as for the low field MRE experiments, with different piezoelectric actuators and MRE sequences and adjusted container dimensions. When using the SLIM-MRE sequence [37], the motion was encoded by concentrating the gradient power in the slice direction and setting to zero the gradient amplitudes along the other two directions. A complete list of the acquisition parameters can be found in Table 1.

2.4. Data processing and analysis

Complex wave images were taken from the first harmonic after applying a discrete Fourier transform of the MR phase-difference images along the four time instances. For isotropic and homogeneous materials, the diameter profiles – lines representing the out-of-plane complex displacement crossing the center of the sample – can be used to estimate $\mu(\omega)$ at different frequencies by matching the analytical closed form solution in Equation (1). Subsequently, rheological model parameters can be fit to the complex shear modulus when MRE is performed at multiple frequencies. With respect to propagation of shear waves over a broad frequency range, linear viscoelastic materials such as tissues and gelatin phantoms generally show an increase of shear storage and loss moduli with excitation frequency [59, 58, 10, 69]. The displacement profiles were acquired along 4 arbitrary directions with equispaced angles. Standard deviation was defined over the four fitted profiles at each excitation frequency and median-averaged over the samples with the same concentration. The complex modulus was computed by fitting the analytical closed solution to the real and imaginary parts of the complex displacement profiles through a constrained least-square optimization for a minimum search performed using the fmincon MATLAB function. To compare
the SNR among different scanners, we computed the Coefficient of Variation (CV) as the standard deviation over the mean of the values of the magnitude of the displacement along each visible circular wave crest. Wave crests were automatically detected as displacement local maxima along a diameter after using a 3 pixel moving-average.

The mechanical behavior of soft tissues and their mimicking phantoms can be modeled through linear viscoelasticity when assuming the small deformation regime of MRE; the constitutive equations of rheological models are obtained by the combination of basic elements in serial or parallel arrangements. It is preferable to minimize the number of independent parameters in the constitutive model that is used [70]. Constitutive models incorporating fractional derivative elements in them have been shown to optimally describe dynamic behavior of such materials using a minimal number of independent parameter [71, 72, 73, 74, 75]. Essentially, a generalization of conventional viscoelastic models is obtained by using a fractional basic rheological element, commonly referred to as the springpot, in addition to the spring and the dashpot. The springpot is described by two independent parameters. The first is $\mu_\alpha$, and the second is $\alpha$ ($0 \leq \alpha \leq 1$), which is an interpolation parameter that represents the matrix geometry and varies between the pure elastic ($\alpha = 0$) and viscous ($\alpha = 1$) cases. The complex modulus of the springpot in the frequency domain is simply given as

$$\mu_\alpha^{1-\alpha}(j\omega\mu_0)^\alpha,$$

where $j = \sqrt{-1}$, $\omega$ is the circular frequency in radians/second and $\mu_0$ is support variable typically set to 1 Pa·s [59].

The storage and loss moduli of the springpot by itself are the real and imaginary parts, respectively, of this expression. Combining the springpot with other basic elements, such as springs, dashpots or additional springpots leads to a variety of rheological models with anywhere from 2 (springpot by itself) to more independent parameters. Some commonly used rheological models, with and without the springpot, include the following: springpot by itself, Maxwell, Voigt, fractional Voigt, Jeffrey, Zener, and fractional Zener [10, 69, 76, 77, 78, 79]. An overview of the scheme, and the complex modulus function of these rheological models are shown in Table 2.

The merit function for the optimization algorithm was chosen to be the
square root of the Residual Sum of Squares, RSS, computed as

\[ RSS = \sum_{k=1}^{n} (\mu_{k,fit} - \mu_{k,exp})^2 \]  

thus representing the sum of the square difference between the theoretical and experimental values of the complex modulus for all the n sampled frequencies.

2.5. Statistical analysis

The storage and loss moduli were computed for each gelatin sample as the average optimized value along 4 equispaced angles: Lilliefors tests rejected data normality for the data populations at each frequency, concentration and magnetic field intensities, so non-parametric tests were adopted and performed with MATLAB version 2016b. The level of significance was set to \( p = 0.05 \) for all the statistical tests performed, which included a one-way ANOVA for different gelatin concentrations, same excitation frequency and magnetic field, and also another one-way ANOVA for different excitation frequency but same gelatin concentration and magnetic field. A Wilcoxon rank-sum (Mann-Whitney) test was performed to compare data from 0.5 T and 11.7 T scanners.

3. Results and Discussion

More than 90% of the data did not show any significant difference among magnetic field strengths (see Table 3): based on this observation the storage modulus values for all frequencies and gelatin concentrations at 0.5 T were comparable with the ones at 11.7 T, and same applied to loss modulus data except for the lowest concentration (\( p\)-value<0.05 at 5% w/v). However, for low gelatin concentrations the number of visible wavelength is reduced due to high attenuation.

MRE using the tabletop system was able to detect differences of complex shear moduli for different gelatin concentrations considered at all excitation frequencies: indeed, the real part and magnitude were different in all the cases, specifically denoting an increase in stiffness corresponding to increasing gelatin concentrations. On the other hand, 75% of the imaginary part values did not exhibit differences, which suggests that the damping characteristics are less sensitive to gel concentrations. Values for real and imaginary parts of the complex shear modulus are reported in Figure 2 for different excitation
frequencies and gelatin concentrations at 0.5, 9.4 and 11.7 T, respectively. A slight but not significant increasing trend in complex shear moduli was detected for increasing frequencies.

An example of Coefficient of Variation (CV) computation is reported for the same gelatin concentration (10%) and excitation frequency (2000 Hz). Examples of displacement magnitude image used for CV calculation are represented in Figure 3 (with both 0.5 T and 11.7 T scanner) where blue and yellow colors refer to in- and out-of-plane displacement, respectively. CV values at 2000 Hz for a 10% gelatin were 0.5970 for the tabletop scanner and 0.2302 for the high-field scanner. Although in the same order of magnitude, CV values reveal higher SNR in the images from high-field scanner as expected from the monotonic increasing dependency of MR signal on the static magnetic field.

Figure 4 illustrates both the experimental data at 0.5 T and the fitting results for the models reported in Table 2. Values of the fitted parameters and computed errors are found in Table 4, subdivided for gelatin concentration.

The best performing models are Zener and its fractional version, which display identical errors except for at 10% concentration, and the Jeffrey model, which outperforms the others for the 10% gelatin. In three out of four analyzed gelatin concentrations, the fractional Zener is equivalent to the Zener model as can be deduced by the value of the fractional coefficient ($\alpha = 1$) (IV parameter in the Table 4).

This is in agreement with previous studies performed in brain and liver tissues [76], where the Zener model has shown to provide the best agreement between fit and experimental data among the 3-parameter models in a lower frequency range than the one used in this study. The Zener model parameters were also highly reproducible, while the fractional Zener model fit parameters scattered in a wide range in follow-up studies and provided only an equal or marginally better fit quality in individual experiments.

The imaginary part of the Maxwell model fitting always showed a broad peak for low frequency values, resulting in generally high errors, while the higher order Jeffrey model fitting had a narrower peak in a lower frequency regime, thus generally impacting less the optimization error.

Maxwell and Voigt models generally show higher fitting errors since the lower number of parameters is unable to follow data dynamics.

Indeed, Maxwell and Voigt models can be seen as subsets of more complex models such as the Zener model [80]: while the Voigt model can be interpreted as a low-frequency approximation to the Zener model, the Maxwell model
can be interpreted as its high-frequency approximation, which could make them appropriate models for specific frequency ranges.

The Fractional Voigt model provided lower errors with respect to the non-fractional equivalent, benefitting from the additional fractional parameter.

Springpot and fractional Voigt models follow data variability at all concentrations with average error values.

Table 4 also shows an increasing trend for the fitted parameters with concentration, in particular for the parameters corresponding to $\mu$ and $\eta$ coherently with their physical meaning.

As can be seen in Figure 4, the gelatin samples show a large variability in the estimation of the shear moduli. We hypothesize that this could be explained by the lack of a standardization process in the production of the samples, by the temperature variations in the scanner room and by the estimation errors provided by the fitting algorithm (approximately 5-10%).

In our setup, samples with diameters up to 10 mm can be scanned, which thus require higher frequencies of the piezoelectric actuator to be visualized [81]. Although the excitation frequencies applied do not match those for in vivo tissue analysis, which are found in the 50-400 Hz range [82, 83], these have been previously used for the characterization and study of the diagnostic potential of neurodegeneration in mouse models [84]. Also, this study could be extended for small-scale MRI scanners for other preclinical analysis and imaging of ex vivo human tissue samples such as biopsy samples. Therefore, this study represents a validation study of the new low-cost MRE system, in line with prior studies that compared tissue mechanical properties across MRE systems [51, 59, 69, 85]. Further studies are needed to identify the best suited model for tissue and phantom modeling. Indeed, the use of a wide range of frequencies densely sampled especially in the low frequency range is suggested to improve rheological model fitting and, consequently, parameter estimation. It is a limitation of the presented study that the analysis of earlier collected data using the high field scanners was performed retrospectively and with a limited sample size. Therefore, there is a limited overlapping in the excitation frequency ranges. A thorough study with a greater match between frequencies used at low and high field scanners and simulations replicating the experimental conditions should be performed to better validate the results presented in this study.
4. Conclusion

An MRE characterization of gelatin phantoms dynamic properties using a compact low field scanner was presented. Complex shear stiffness values were comparable when processed from images acquired with the tabletop low field scanner and an 11.7 T scanner, while the Coefficient of Variation of the former was double. Nevertheless, MRE on the tabletop system is capable of detecting differences of complex shear moduli for increasing gelatin concentrations. This indicates the feasibility of future low-cost MRE experiments on ex-vivo samples for the characterization of tissue in both normal and pathological states. The Springpot model provides the best fit among the 2-parameter models while this is the case for the Zener model among the 3-parameter models. The addition of a further parameter in the fractional Zener model improved the fit quality only at one of the four concentrations.

Acknowledgments

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Tables

Table legend

Table 1: Experimental parameters for the three scanners used in this study. For the sake of simplicity, we report the magnetic field intensities of the scanners truncated at the first decimal digit throughout the text.
(*) Piezosystem Jena, Jena, Germany. (**) Physik Instrumente GmbH & Co., Karlsruhe, Germany.
SLIM = SampLe Interval Modulation, SE = Spin-Echo.

Table 2: Rheological models: graphical depiction and complex modulus as a function of excitation frequency.

Table 3: Comparison of the complex modulus values between 0.5T and 11.7 T at specific concentrations and excitation frequencies by means of the p-value.

Table 4: Values of model parameters fit to experimental data at 0.5T with units as follows:
I: $\eta$ [Pa·s] II: $\mu$ [Pa] (Maxwell);
I: $\alpha$ [-] II: $\mu_\alpha$ [Pa] (Springpot*);
I: $\eta$ [Pa·s] II: $\mu$ [Pa] (Voigt);
I: $\eta$ [Pa·s] II: $\mu_1$ [Pa] III: $\mu_2$ [Pa] (Zener);
I: $\eta_1$ [Pa·s] II: $\eta_2$ [Pa·s] III: $\mu$ [Pa] (Jeffrey);
I: $\mu$ [Pa] II: $\mu_\alpha$ [Pa·s] III: $\alpha$ [-] (Fractional Voigt);
I: $\mu_\alpha$ [Pa·s] II: $\mu_1$ [Pa] III: $\mu_2$ [Pa] IV: $\alpha$ [-] (Fractional Zener).

(*) In Springpot model an additional parameter $\mu_0$ was considered and kept constant to the value of 1Pa·s.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MagSpec (Pure Devices)</th>
<th>310/ASR (Agilent)</th>
<th>Advance III HD (Bruker)</th>
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<tbody>
<tr>
<td>Magnetic Field [T]</td>
<td>0.587 (25 MHz)</td>
<td>9.4 (400 MHz)</td>
<td>11.74 (500 MHz)</td>
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<td>Piezoelectric actuator</td>
<td>PAHL 60/20 (*)</td>
<td>P-840.1 (**)</td>
<td>1000/50 N (**)</td>
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<tr>
<td>Oscillation amplitude [(\mu)m]</td>
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<td>6</td>
<td>11</td>
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<tr>
<td>Excitation and MEG frequencies [kHz]</td>
<td>[0.5, 1.0, 1.5, 2.0]</td>
<td>[0.5, 1.0, 1.5, 2.0]</td>
<td>[1.0, 2.0, 3.0, 4.0]</td>
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<td>68mm x 68mm</td>
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<td>78 x 78</td>
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<td>Scan time / frequency</td>
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<td>MRE sequence</td>
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Table 1:
Table 2:

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<th>Model</th>
<th>Scheme</th>
<th>Complex modulus</th>
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<tbody>
<tr>
<td>Maxwell [86]</td>
<td><img src="image1" alt="Diagram" /></td>
<td>$\frac{\mu j\omega \eta}{j\omega \eta + \mu}$</td>
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<tr>
<td>Springpot [59, 69, 87]</td>
<td><img src="image2" alt="Diagram" /></td>
<td>$\mu_1^{1-\alpha} (j\omega \mu_0)^\alpha$</td>
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<tr>
<td>Voigt [69]</td>
<td><img src="image3" alt="Diagram" /></td>
<td>$\mu + j\omega \eta$</td>
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<tr>
<td>Zener [88]</td>
<td><img src="image4" alt="Diagram" /></td>
<td>$\mu_2 \frac{1 + d(j\omega \tau)}{1 + j\omega \tau} \left[ d = \frac{\mu_1 + \mu_2}{\mu_2}, \quad \tau = \frac{\eta_2}{\mu_1} \right]$</td>
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<tr>
<td>Jeffrey [89]</td>
<td><img src="image5" alt="Diagram" /></td>
<td>$-\omega \eta_1 \frac{\omega \eta_2 - j\mu}{\mu_1 + j\omega (\eta_1 + \eta_2)}$</td>
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<td>Frac Voigt [59, 69]</td>
<td><img src="image6" alt="Diagram" /></td>
<td>$\mu + \mu \left( \frac{j\omega \eta_0}{\mu} \right)^\alpha$</td>
</tr>
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<td>Frac Zener [78]</td>
<td><img src="image7" alt="Diagram" /></td>
<td>$\mu_2 \frac{1 + d(j\omega \tau)^\alpha}{1 + (j\omega \tau)^\alpha} \left[ d = \frac{\mu_1 + \mu_2}{\mu_2}, \quad \tau = \frac{\eta_2}{\mu_1} \right]$</td>
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Table 3:

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<td>15%</td>
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<td>0.10</td>
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<td>5% I</td>
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<td>Maxwell</td>
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<td>Springpot</td>
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Table 4:
Figures

Figure 1: (a) Picture of the MRE setup for piezoelectric actuator support mounted on the tabletop MRI machine by Pure Devices GmbH, Würzburg, Germany. (b) SolidWorks model of the piezoelectric actuator support.

Figure 2: Real (solid line) and imaginary (dashed line) parts of the complex shear modulus estimated at 0.5 T (red and orange), 9.4 T (green and chartreuse) and 11.7 T (blue and light blue) from the gelatin samples at different concentrations: 5% w/v (a), 10% w/v (b), 15% w/v (c) and 20% w/v (d). Points represent the median values while error bars define population minimum and maximum values at each frequency. Outliers (represented as crosses) were chosen as the data points having an imaginary part of the complex shear modulus equal to zero or with a difference of more than an order of magnitude with respect to the median value.

Figure 3: Real and imaginary part of the complex displacement at 2000 Hz for a 10% gelatin concentration imaged at 0.5 T (a) and at 11.7 T (b).

Figure 4: Experimental values for complex moduli (storage modulus: circles - loss modulus: squares) and fitted curves based on the rheological modeling results in Table 4. Some of the models perform similarly and appear overlapped for the following cases and gelatin concentrations: Zener and Maxwell at 10%, Zener and its fractional version at all concentrations where $\alpha$ was estimated at 1, the imaginary parts for Zener and Voigt at 5%, Springpot and fractional Voigt at 10% and 20%.
Figure 3:
Bibliography

References


[43] U. Hamhaber, F. Grieshaber, J. Nagel, U. Klose, Comparison of quantitative shear wave mr-elastography with mechanical compression tests,


