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Green capacity investment under subsidy withdrawal risk☆

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ABSTRACT

Subsidies initially installed to stimulate green capacity investments tend to be withdrawn after some time. This paper analyzes the effect on investment of this phenomenon in a dynamic framework with demand uncertainty. We find that increasing the probability of subsidy withdrawal incentivizes the firm to accelerate investment at the expense of a smaller investment size. A similar effect is found when subsidy size as such is increased. When subsidy withdrawal risk is zero or very limited, installing a subsidy could increase welfare. In general we get that the larger the subsidy withdrawal probability, the smaller the welfare maximizing subsidy rate is. Therefore, a policy maker aiming to maximize welfare should try to reduce subsidy withdrawal risk.

1. Introduction

In an attempt to limit climate change, many countries have set ambitious targets to reduce greenhouse gas emissions during the past two decades. Increasing the share of renewable energy production to the overall energy mix is recognized as critical in reaching those targets [European Commission, 2017]. As of 2017, 179 countries had renewable energy targets, where, in particular, 90 countries had targets to generate more than 50% of their electricity from renewables no later than by 2050 [REN21, 2018b]. The European Commission, for example, has set a recent new target according to the "2030 framework for climate and energy policies", which is to achieve 32% of total energy consumption for the entire European Union in 2030 to be delivered by renewable energy sources. Another example is China that has just reached an accumulated wind capacity of 217 gigawatts (GW) in 2019 [World Wind Energy Association, 2019], and aims to increase total renewable power capacity to 680 GW by 2020 [REN21, 2018b].

Many countries have introduced support schemes aimed at accelerating investments in renewable energy over the past two decades, in order to reach these ambitious targets. Governments therewith, want to ensure competitiveness of renewable energy production and encourage investment. As of 2017, 128 countries had power regulatory incentives and mandates [REN21, 2018b]. China, for example, implemented the world's largest emissions trading scheme in 2017 [REN21, 2018b].

However, many support schemes have been retracted or revised suddenly and unexpectedly over the last years. For example, Ukraine removed a tax exemption on companies selling renewable energy [REN21, 2015]. Furthermore, the size of subsidy payments was retroactively adjusted in Belgium, Bulgaria, the Czech Republic, Greece and Spain [Boomsma and Linnerud, 2015], and the feed-in-tariffs were reduced in Bulgaria, Germany, Greece, Italy and Switzerland in 2014 [REN21, 2015]. China implemented sudden changes in their feed-in tariff in 2018, making new solar power projects less likely to be eligible for subsidy [The Economist, 2018].

One of the main reasons for subsidy policy change results from technological progress. Initially, a subsidy is implemented to ensure competitiveness of renewable energy production, but when technology advances such that the technique is profitable on itself, the subsidy is no longer needed and can be withdrawn. Another reason for subsidy withdrawal can be that the original renewable energy capacity target has been reached or that the budget has been depleted. Norway and Sweden created a joint electricity certificate market in 2012 to boost renewable electricity production in both countries. Norway will no longer provide electricity certificates to facilities that start operating after 31 December 2021, because the goal of having a green energy production of 28.4 TWh by 2020 has been reached [Energy Facts Norway, 2015]. Alternatively, a policy can be withdrawn or altered due to a depleted

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budget, as was the case in Italy for their solar photovoltaic (PV) support in 2013 [Karneyeva and Wüstenhagen, 2017]. However, in some locations green technologies are still unable to survive without subsidies [Institute for Energy Research, 2017]. For these countries, the question what consequences subsidy withdrawal has for renewable energy production and renewable energy investment, will be a relevant question in the near future.

In countries where policy changes already occurred, it had a severe impact on the profitability of renewable energy projects and investment behavior. In Spain an unforeseen subsidy retraction caused a 40% drop in profitability for investors [Del Rio and Mir-Artigues, 2012]. Spain's largest power group, Iberdrola, reported a 91% decline in net profits from wind after subsidies were reduced [Financial Times, 2014]. Similarly, subsidy cuts in the UK for solar PV damaged investor confidence and could also delay the point at which solar could be cost competitive [The Guardian, 2015]. Del Rio and Mir-Artigues [2012] mentions that when policy costs are high, the social acceptance of the policy decreases, increasing the pressure to implement (retroactive) changes to the policy. This increases policy instability, creating uncertainty and risks for investors, who, in return, want higher risk premiums. This all increases costs and reduces profitability.

This paper aims to determine how the optimal investment decisions related to renewable energy projects depend on the availability of a subsidy, the size of the subsidy and the withdrawal risk of the subsidy. A social planner wants to know how social welfare is affected by the subsidy, its size and the withdrawal risk. Studying the effect on social welfare is the standard approach in the public economics literature. However, it is not necessarily the standard approach in public decision-making, where it is of main importance to set the right goals and targets [Stern, 2018]. We, therefore, also look at the question how the ability to reach a capacity target within a certain time-frame is affected by the subsidy, subsidy size and subsidy withdrawal risk.

We consider a firm that has the option to invest in a renewable energy project. It has to decide on both the time to invest, as well as the size of the capacity it wants to install. We consider a dynamic framework with demand uncertainty. The cost of installing capacity of a certain size depends on the size of the capacity as well as the availability of support. Support is provided in the form of a lump-sum investment subsidy, which represents a general class of investment subsidies including investment tax credits and capital subsidies. Investment tax credits constitute the most widespread policy instrument for renewable energy globally,[1] and is often implemented with the aim to increase the affordability and profitability of renewable energy production [REN21, 2018a, page 70]. We study the effect of policy uncertainty in the form of retraction of a currently provided subsidy.

We first derive the optimal investment decisions of a profit-maximizing firm facing subsidy retraction risk. We find that increasing the subsidy size speeds up investment but this goes at the expense of a decreased optimal investment size. Increasing subsidy retraction risk for a given subsidy size has the same effect. Surprisingly, the firm's optimal investment size when there is no subsidy provided is larger than the optimal investment size when subsidy is provided but there is risk of future retraction.[2]

We then take the viewpoint of a policy maker, where we analyze the effect of the subsidy on the resulting investment decision of the firm. We find that a subsidy could increase welfare. Numerical experiments suggest that a subsidy increases welfare when subsidy withdrawal risk is sufficiently small. A policy maker aiming to maximize welfare should minimize subsidy withdrawal risk, since welfare decreases with a larger subsidy withdrawal risk. We also derive the impact of a subsidy on the ability to reach certain policy targets. When a proposed capacity target is smaller than the firm's optimal investment size, a subsidy can be used to speed up investment, thereby raising the probability that the target is reached in time.

Our paper contributes to different strands of literature. First, we contribute to the literature on incentive regulation of a firm within an uncertain dynamic framework (see, e.g., Brennan and Schwartz, 1982, Dobbs, 2004, Evans and Guthrie, 2005, 2012, Guthrie, 2006, 2020, Willems and Zwart, 2018 and Azevedo et al., 2020). Azevedo et al. [2020] consider revenue neutral tax-subsidy package on the firm's timing and capacity decision under demand uncertainty without regulatory uncertainty. Within the aforementioned strand of literature, regulatory uncertainty is considered by Teisberg [1993], Dixit and Pindyck [1994, Chapter 9], and Hassett and Metcalf [1999], where the latter two also consider the effect of subsidy size on investment timing. Motivated by recent frequent occurrences of changes in regulatory policies in the green energy industry, we contribute to this literature by focusing on the effect of policy risk in the form of potential subsidy withdrawal. In addition we determine the optimal subsidy size looking at different aims, such as welfare maximization and capacity targets, and we study the role of policy risk in determining the optimal subsidy size.

Our paper also contributes to an increasing strand of literature that studies the effect of subsidies on green investment (e.g., Pizer, 2002; Eichner and Runkel, 2014; Nesta et al., 2014; Abrell et al., 2019; Bigerna et al., 2019). Pennings [2000] and Danielova and Sarkar [2011] focus on the combination of subsidy and tax rate reduction. Unlike the aforementioned papers, we also analyze how the risk of policy change intervenes with the effect of policy measures. Some of the literature focuses on carbon pricing and studies how policy uncertainty affects the volatility of the prices (see, e.g., Blyth et al., 2007, Fuss et al., 2008, Yang et al., 2008, and Kang and Létourneau, 2016). The carbon pricing literature generally concludes that more policy uncertainty results in larger volatility in prices and, therefore, delays investment.

Some recent literature related to renewable energy accounts for policy uncertainty related to random provision, revision or retraction of a subsidy, such as, for example, Boomsma et al. [2012], Boomsma and Linnerud [2015], Adkins and Paxson [2016], Eryilmaz and Homans [2016], Ritzenhofen and Spinler [2016] and Chronopoulos et al. [2016]. These papers focus on how uncertainty in the availability of a certain type of subsidy affects investment behavior. The effect of uncertainty in availability of a subsidy on investment behavior strongly depends on the type of subsidy in place as well as the level of uncertainty. We contribute to this literature by studying a lump-sum investment subsidy, the most widespread policy instrument for renewable energy globally [REN21, 2018a, page 70], and study the role of subsidy size and the risk of potential subsidy withdrawal on investment. Furthermore, we do not solely focus on the firm's investment behavior, but also study the effect of policy risk on the goals of the social planner and welfare. To our knowledge, we are the first to conclude that a larger likelihood of an investment subsidy withdrawal damages both welfare and the policy maker's ability to increase renewable energy capacity.

The remainder of this paper is organized as follows. Section 2 presents the model and characterizes the optimal investment decisions both from a profit-maximizing firm and social welfare point of view. In Section 3, we study the optimal investment decision of a firm in more detail by providing comparative statics. Numerical experiments are performed in Section 4. Section 5 focuses on the effect of both the subsidy size and the likelihood of subsidy withdrawal on reaching certain environmental targets as well as welfare. In Section 6 we discuss the role of the type of subsidy we study on our results. Section 7 concludes.

[1] Worldwide, an estimated amount of 30 to 40 countries used investment or production tax credits to support renewable energy installations over the past decade [REN21, 2018a, page 69].

[2] On the macro level it could still be the case that more firms invest when a subsidy is provided, and that - despite that the average installed capacity per firm is smaller - the total renewable energy capacity on the market increases. See, for example, Hassett and Metcalf [1999], in which it is obtained that providing a lump-sum subsidy increases the total market capacity when many firms are faced with the option to invest in a project of fixed size.

2. Model

We propose a theoretical framework that studies a firm's optimal investment decision under uncertain subsidy support. We consider a risk-neutral, profit-maximizing firm that holds the option to invest in a renewable energy project with an uncertain future revenue stream. The firm has to determine the optimal timing of the investment and the size of the capacity to be acquired. We assume that the firm produces up to capacity, and cannot scale up capacity in the future. Renewable energy projects, such as wind parks, are location- and firm-specific due to governmental concessions needed to obtain the investment option. In most concession-based contracts for renewable energy generation capacity, the investment is a one-time lumpy decision.

We assume the firm to be sufficiently large so that it exerts market power. This is supported by the fact that a series of studies has indicated that the electricity market is highly concentrated. In the United States, a government report by the United States General Accounting Office [2005] states that the four federal Power Marketing Administrations (PMAs) exert market power from the federal hydroelectric dams and projects. Signs of market power on the US electricity market are also reported on a state level.[3] In Europe, signs of market power are reported on a national level, for example in Italy,[4] England and Wales,[5] and the Nordic countries.[6] We refer to Karthikeyan et al. [2013] for a thorough review on market power in the electricity market in different countries. The output price at time t, $P(t)$, is given by:

$$P(t) = X(t)(1-\eta K), X(0) = x \tag{2.1}$$

where K is the firm's production capacity, and $\eta > 0$ is a constant.[7]

The output price $P(t)$ depends on an exogenous shock $X(t)$, which is assumed to follow a geometric Brownian motion process given by:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t) \tag{2.2}$$

where μ is the drift rate, σ the uncertainty parameter and $dW(t)$ the increment of a Wiener process. The inverse demand function (2.1) is a special case of the one used by Dixit and Pindyck [1994, Chapter 9], which assumes $P = XD(K)$ with an unspecified demand function $D(K)$, and is frequently used in the literature (see, e.g., Pindyck, 1988, He and Pindyck, 1992, and Huisman and Kort, 2015).

The cost of one unit of investment is set equal to δ. Hence, installing a production capacity of size K yields an investment cost of δK when no subsidy is in effect. Subsidy provides a one time discount at rate θ on the investment cost, so that the investment costs are then equal to $(1-\theta)\delta K$.

Initially, the lump-sum subsidy[8] is assumed to be available, but due to technological development (or a restriction from the budget constraint or a change in government), the firm expects the subsidy to be withdrawn. We model the firm's perceived risk of subsidy retraction by an exponential jump with parameter λ. This implies that the firm's perceived probability that the subsidy will be retracted in the next time interval dt is equal to λdt.

The optimization problem for the profit-maximizing firm is then given by an optimal stopping problem in which it aims to find the optimal time τ to invest in a capacity of optimal size K:

$$F(x, \theta) = \sup_{\{\tau, K\}} \mathbb{E}\left[\int_{\tau}^{\infty} P(t)Ke^{-rt}dt - \left(1-\theta\cdot 1_{\xi(\tau)}\right)\delta K e^{-r\tau} \,|\, X(0) = x, \xi(0) = 1\right] \tag{2.3}$$

with

$$\xi(t) = \begin{cases} 0 & \text{if subsidy retraction has occurred at time } t \text{ or earlier} \\ 1 & \text{otherwise} \end{cases} \tag{2.4}$$

When investing, the firm pays a lump-sum investment cost and obtains the revenue stream $P(t)K$ from time τ on. r is the risk-free rate, where we assume $r > \mu$. In case $r \leq \mu$, the problem is trivial as it would always be optimal to wait with investment.

Obviously, it is optimal for the firm to invest when the output price $P(t)$ is large enough, where (2.1) learns that $P(t)$ is proportional to $X(t)$. It follows that the investment rule is of a threshold type. In particular, there exists a threshold value of $X(t)$ at which the firm is indifferent between investing and waiting with investment.[9] It is intuitively clear that when the price is below a certain threshold level, denoted by X_1, the firm will not invest, independently of whether the subsidy is available or not. Furthermore, when the price is high enough, i.e. above a threshold $X_0 > X_1$, the firm will always invest, independent of the availability of the subsidy. For $X(t)$ in the interval $[X_1, X_0]$, the firm will only invest when the subsidy is active, and it will not do so when the subsidy has been withdrawn. Therefore, X_1 (X_0) is the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing, while the policy is (not) in effect. Fig. 1 summarizes the above.

The thresholds X_0 and X_1 are directly linked to the investment timing. When there is (no) subsidy available, investment is done when the geometric Brownian motion defined in eq. (2.2) hits the value X_1 (X_0) for the first time from below. As a result, there exists a one-to-one mapping between the investment threshold and the investment time. Throughout this paper, we will refer to X_0 and X_1 both as the investment thresholds and the timing of investment.

Assuming the initial value of the geometric Brownian motion process, x, meets the requirement[10] $x < X_1$, then there are two cases that can occur regarding the timing of the investment. In the first case, the firm invests when the geometric Brownian motion hits the threshold X_1 for the first time while the subsidy has not been retracted. Alternatively, the subsidy is retracted before the GBM hits the threshold X_1 and the firm invests when the process hits the threshold X_0 for the first time. Let s denote the time at which the policy maker withdraws the subsidy. The firm's expected investment time follows from the investment thresholds and the withdrawal time of the subsidy, and is equal to:

$$\text{Expected time to investment} = \mathrm{P}[s>\tau_1]\cdot\mathbb{E}[\tau_1] + (1-\mathrm{P}[s>\tau_1])\cdot\mathbb{E}[\tau_0] \tag{2.5}$$

[3] A government report by the United States General Accounting Office [2002] on the California power market concluded prices did not follow patterns consistent with prices under competitive conditions. Furthermore, Woerman [2019] estimates the impact of market power on the Texas electricity market, and finds that a 10% increase in demand causes markups to more than double, showing that producers do have market power.

[4] European Commission [2011] reports that the Italian energy market is highly concentrated, and also Bosco et al. [2010], Bigerna et al. [2016] and Sapio and Spagnolo [2016] find empirical evidence of market power on the Italian energy market.

[5] David and Wen [2001] found that two dominant suppliers in the England and Wales pool, which is a highly concentrated market, decrease capacity to increase profits during peak periods.

[6] Lundin and Tangerås [2020] empirically reject the hypothesis of perfect competition on Nord Pool, the day-ahead market of the Nordic power exchange, during the period 2011–2013. Tangerås and Mauritzen [2018] test the hypothesis of perfect competition in some areas in Sweden in the period 2010–2013 and reject this hypothesis. Fleten and Lie [2013] conclude that Norway's largest hydro power producer has an incentive to reduce thermal production in order to increase the market spot price.

[7] Note that output price is always positive, as the production capacity K is endogenous. Therefore, the firm will choose the production capacity such that it will be less than $\frac{1}{\eta}$. By applying Eq. (2.1), we implicitly assume that the production quantity is constant. In the short and medium term, renewable energy generation is highly variable due to a large dependency on, among others, weather conditions. However, in the long run production is more predictable and less variable. As the decision to install a renewable energy project, as well as policy decisions have a long term focus, we refrain from focusing on fluctuations in productions on the short and medium term. See, for example, Boomsma et al. [2012], Dalby et al. [2018] and Bigerna et al. [2019] for similar assumptions. A reader interested in how production flexibility affects a firm's investment timing and size can for example look at Hagspiel et al. [2016].

[8] We also use subsidy to refer to the lump-sum subsidy.

[9] See, for example, Dixit and Pindyck [1994] or Huisman and Kort [2015].

[10] If $x \geq X_1$, it is optimal for the firm to invest immediately, and the problem is trivial.

in which $\mathbb{P}[s > \tau_1]$ is the probability that the subsidy withdrawal occurs after threshold X_1 is hit, and $\mathbb{E}[\tau_1]$ ($\mathbb{E}[\tau_0]$) is the expected first hitting time of threshold X_1 (X_0).[11]

To determine the optimal investment decision, the first step is to derive the value the firm obtains by investing. Denoting the value of the firm at the moment of investment by V_0 if the subsidy has already been retracted, and by V_1 in case the subsidy is still in effect, we get[12]

$$V_0(X,K) = \frac{X(1-\eta K)K}{r-\mu} - \delta K \tag{2.6}$$

$$V_1(X,K) = \frac{X(1-\eta K)K}{r-\mu} - (1-\theta)\delta K \tag{2.7}$$

Using the value functions (2.6) and (2.7) the optimal investment size for a given value of X can be straightforwardly derived. The result is presented in Corollary 1.

Corollary 1. *Let $K_1(X)$ ($K_0(X)$) denote the optimal investment size while the policy is (not) in effect. When the firm decides to invest at X, the optimal investment size is equal to:*

$$K_0(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right) \tag{2.8}$$

$$K_1(X) = \frac{1}{2\eta}\left(1 - \frac{(1-\theta)\delta(r-\mu)}{X}\right) \tag{2.9}$$

The proofs of all corollaries and propositions can be found in Appendix A.

Using similar steps as in Dixit and Pindyck [1994] and Huisman and Kort [2015], the value of the investment option with and without the subsidy can be derived. These are stated in Proposition 1.

Proposition 1. *Let $F_1(X,K)$ ($F_0(X,K)$) denote the value of the option to invest at X while the policy is (not) in effect. When the firm decides to invest at X, it invests in capacity K. The value of the option to invest at X after the subsidy has been retracted is equal to:*

$$F_0(X,K) = \begin{cases} \dfrac{X(1-\eta K)K}{r-\mu} - \delta K & \text{if } X \in [X_0, \infty) \\ A_0 X^{\beta_{01}} & \text{otherwise} \end{cases} \tag{2.10}$$

where A_0 is a (positive) constant and β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0, \beta_{01} > 1$.

The value of the option to invest at X while the subsidy is available is equal to:

$$F_1(X,K) = \begin{cases} \dfrac{X(1-\eta K)K}{r-\mu} - (1-\theta)\delta K & \text{if } X \in [X_1, \infty) \\ A_1 X^{\beta_{11}} + A_0 X^{\beta_{01}} & \text{otherwise} \end{cases} \tag{2.11}$$

where A_1 is a (positive) constant and β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0, \beta_{11} > \beta_{01} > 1$.

When the subsidy is (not) available, it is optimal to invest when $X \geq X_1$ ($X \geq X_0$), yielding Eq. (2.7) (Eq. (2.6)) as the value of the investment option. The firm does not invest, thus waits, when the current output price is too low, i.e. when $X < X_1$ ($X < X_0$) if the subsidy is (not) available. If the subsidy is still present, the value of the investment option consists of two parts: the value of holding the option to invest while the subsidy is available and the option to invest after the subsidy has been retracted. When the subsidy is retracted, the former value is lost as the subsidy will not be re-enacted again in the future.

After the subsidy has been abolished, policy uncertainty will not influence the investment decision anymore. The problem to be solved in such a situation is already analyzed in Huisman and Kort [2015]. Proposition 2 presents the optimal investment decision in this case.

Proposition 2. *When the subsidy is abolished, the optimal investment threshold satisfies:*

$$X_0 = \frac{\beta_{01}+1}{\beta_{01}-1} \cdot \delta(r-\mu) \tag{2.12}$$

whereas the corresponding investment size[13] is given by:

$$K_0^* = [\eta(\beta_{01}+1)]^{-1} \tag{2.13}$$

Proposition 3 presents the firm's optimal investment decision when the subsidy is still available.

Proposition 3. *If the investment subsidy has not been retracted yet, the optimal investment threshold X_1 is implicitly given by:*

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} + (1-\theta)\delta K_1^* = 0 \tag{2.14}$$

in which K_1^* is the optimal capacity under subsidy when investing at $X = X_1$, i.e. eq. (2.9) evaluated at $X = X_1$.

In the special case in which there is no subsidy retraction risk, eq. (2.14) can be solved explicitly. Corollary 2 presents the optimal investment decisions under a lump-sum subsidy without retraction risk.

Corollary 2. *In case of a subsidy with no subsidy retraction risk* (i.e. $\lambda = 0$), *the optimal investment timing and size are given by:*

$$X_1 = \frac{\beta_{01}+1}{\beta_{01}-1} \cdot (1-\theta)\delta(r-\mu) \tag{2.15}$$

and

$$K_1^* = [\eta(\beta_{01}+1)]^{-1} \tag{2.16}$$

Comparing the investment decision under a subsidy and the one without subsidy, we observe that the optimal investment sizes are the same ($K_1^* = K_0^*$), but the timing threshold with subsidy is actually smaller than the one without subsidy ($X_1 = (1-\theta)X_0 < X_0$). The reason behind this is that lower investment costs allow for investment at lower output prices, i.e. earlier. The decrease in investment costs has two effects on the optimal size. First, there is a direct effect. The lower the investment costs, the more the firm likes to invest for a given level of X. Second, there is an indirect effect via the timing. As investment is done sooner, i.e. at a lower output price, the firm can only justify a smaller investment size. The two effects cancel out when the firm invests at the optimal time.

Now, we consider the problem from the perspective of a social planner with the objective to maximize social welfare. The social planner maximizes the total surplus (TS), which consists of the sum of the consumer (CS) and producer surplus (PS)[14] minus the subsidy costs of $\theta\delta K$. We assume the social planner uses the same discount rate r as the firm, following, for example, Huisman and Kort [2015] and Bigerna et al. [2019]. A discussion on alternative assumptions regarding the social planner's discount rate is included in Section 6.

[11] Explicit derivation of the expected time to investment is shown in Appendix C.2.

[12] We write X instead of $X(t)$ for convenience.

[13] For convenience of notation, we use $K_0^* = K_0(X_0)$ and $K_1^* = K_1(X_1)$.

[14] The producer surplus is defined as the value of the firm's project.

The total surplus when investing at X with capacity K is equal to[15]

$$\mathrm{TS}(X,K)=\frac{X(2-\eta K)K}{2(r-\mu)}-\delta K \tag{2.17}$$

Note that the total surplus does not directly depend on the subsidy. This is the result of the fact that the subsidy is solely a welfare-transfer with a zero-sum contribution to total surplus. In other words, each unit of currency used for the subsidy represents on the one hand a cost for the social planner and on the other hand a gain for the producer. Therefore, the net direct impact of the subsidy on total surplus is zero. A subsidy can however impact total surplus indirectly, via influencing the firm's investment decision.

We can determine the socially optimal timing and capacity using similar steps as before. Proposition 4 states the first-best social optimum.

Proposition 4. *The socially optimal capacity for a given level of X is equal to*:

$$K_S(X)=\frac{1}{\eta}\left(1-\frac{\delta(r-\mu)}{X}\right) \tag{2.18}$$

The total surplus (TS) is then given by:

$$\mathrm{TS}(X,K)=\begin{cases}\frac{X(2-\eta K)K}{2(r-\mu)}-\delta K & \text{if } X\in[X_S,\infty)\\ A_S X^{\beta_{01}} & \text{otherwise}\end{cases} \tag{2.19}$$

in which A_S is a (positive) constant, and X_S is the social planner's optimal timing threshold. At this threshold, the social planner is indifferent between investing and not investing. The optimal timing maximizing the total surplus is given by:

$$X_S=\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{2.20}$$

The socially optimal capacity, K_S^*, is given by:

$$K_S^*=2[\eta(\beta_{01}+1)]^{-1} \tag{2.21}$$

We find that the investment timing of the social planner and the firm are identical when there is no subsidy (i.e. $X_S=X_0$). Regarding the size of investment, we conclude that it is socially optimal to invest twice as much as the profit-maximizing firm (i.e. $K_S^*=2K_0^*$). The reason is that the social planner is more eager to invest than the private firm, as the social planner also accounts for consumer surplus. This means that the social planner either invests sooner and adapts size accordingly, or invests more and adapts timing accordingly. We conclude that within our framework the social planner wants to invest more than the profit-maximizing firm. Thus, to obtain the first-best solution, the social planner should stimulate firm investment in such a way that the firm will invest more without changing the investment time. The next section investigates whether introducing a subsidy can achieve this.

3. Investment and subsidy

This section analyzes the effect of an investment subsidy and the probability that the subsidy will be retracted, on the firm's optimal investment decision. The following proposition states how the optimal investment decision is affected by subsidy retraction risk.

Proposition 5. *The optimal investment timing and size are affected by the subsidy retraction risk λ in the following way*:

$$\frac{dX_1}{d\lambda}<0,\ \frac{dK_1}{d\lambda}<0 \tag{3.1}$$

if and only if

$$\frac{(1-\theta)\delta(r-\mu)}{X_1}\geq\frac{(\beta_{01}-1)(\beta_{11}-1)}{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}} \tag{3.2}$$

where β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-r=0$, and β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-(r+\lambda)=0$.

Proposition 5 states that a higher subsidy retraction risk decreases both the optimal investment threshold[16] and the optimal investment size. A firm speeds up investment under a higher subsidy retraction risk in order to make use of the subsidy now, as it is less likely it will be available in the future. Investing at a lower threshold implies that the firm invests when the output price is lower, which leads to a smaller optimal investment size. There is no direct effect of subsidy retraction risk on optimal investment size, but only an indirect effect via the timing, as can be straightforwardly concluded from expression (2.9). The intuition behind this is that the investment subsidy only affects the investment payoff at the moment of the investment, so that the optimal investment size does not depend on whether the subsidy will be withdrawn very soon after investing or remains for a long period of time.

Inequality (3.2) states that when the ratio of costs and the price shock at the moment of investment are above a threshold, then the results in (3.1) hold. Extensive numerical results suggest that this condition is in fact satisfied for any lump-sum subsidy.

The result that a higher probability of retraction of a subsidy speeds up investment is in accordance with findings of Hassett and Metcalf [1999] and Dixit and Pindyck [1994]. Chronopoulos et al. [2016] however find that subsidy retraction risk delays investment for high levels of subsidy retraction risk. This is because Chronopoulos et al. [2016] study a subsidy in the form of a price premium. This keeps on having an effect after the investment has been undertaken, because in case of a price premium a higher retraction probability reduces the expected net present value of the investment. The latter does not happen in our case, because the lump-sum subsidy just affects the investment payoff at the moment of the investment, implying that a retraction of the subsidy occurring at a later date has no effect.

We find that the investment size decreases with subsidy retraction risk. In Chronopoulos et al. [2016] this also holds for low levels of subsidy retraction risk. However, when subsidy retraction risk is high, the fact that the effect of increasing the subsidy retraction risk will delay investment, has the implication that a larger withdrawal risk increases the firm's investment size in Chronopoulos et al. [2016].

Proposition 6 presents the influence of the size of the subsidy on the optimal investment decision.

Proposition 6. *The effects of the subsidy size θ on the optimal investment threshold and the investment size are given by*:

$$\frac{dX_1}{d\theta}<0,\ \frac{dK_1}{d\theta}<0 \tag{3.3}$$

if and only if condition (3.2) holds.

Proposition 6 shows that a larger size of the subsidy speeds up investment and decreases the investment size. Increasing the subsidy size has two different effects on the optimal investment decision. First, providing a larger subsidy gives some incentive to invest more for a given output price. Second, as the lower costs make the investment profitable at lower output prices, it gives also some incentive to invest

[15] See Huisman and Kort [2015] for the details of the derivation of the total surplus.

[16] It can be shown that $X_0>X_1$ holds for any level of subsidy withdrawal risk λ as long as condition (3.2) is met. From Proposition 2 and Corollary 2, it follows that $X_0>X_1$ when λ = 0. By Proposition 3, we have that X_1 decreases if λ increases when condition (3.2) is met.

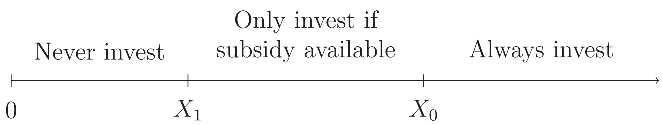

Fig. 1. Optimal investment strategy at different output prices.

earlier, and as result of the dependency between timing and size, invest in a smaller capacity. We find that the second effect always dominates the first, leading to the result in Proposition 6.

From a policy maker's point of view it might be interesting to analyze under which of the following two scenarios the firm's investment is larger: (1) a small subsidy subject to a low probability of retraction, or (2) a larger subsidy subject to a larger probability of retraction. From Propositions 5 and 6, it follows that both a larger retraction risk and a larger subsidy in fact decrease both the investment threshold and the investment size. Therefore, the investment size under the second scenario will be smaller than in the first. However, the firm will have invested sooner under the second scenario compared to under the first.

4. Quantitative analysis

This section contains a numerical analysis of an investment opportunity in a hydro power plant. The parameter values, displayed in Table 1, are taken from Fleten et al. [2016] and Finjord et al. [2018]. The data set in Fleten et al. [2016] consists of 214 licenses to build small hydro power plants granted by the Norwegian Water Resources and Energy Directorate (NVE).

Fig. 2 presents the investment timing thresholds X_0 and X_1, and the investment sizes K_0^* and K_1^* as functions of the subsidy retraction risk λ, using the parameter values in Table 1. Fig. 2 is in accordance with the results presented in Proposition 5 in the sense that investment timing X_1 and size K_1^* decrease with subsidy retraction risk λ. Furthermore, as X_0 and K_0^* are the investment threshold and capacity size *after* retraction of the lump-sum subsidy, these do not depend on λ.

More importantly, Fig. 2 shows that the optimal investment size when there is no subsidy available (K_0^*) is in fact larger than the optimal investment size when the subsidy is available (K_1^*) but exposed to retraction risk (i.e. $\lambda > 0$). This means that when there is a risk of subsidy retraction, the firm's optimal investment size at the corresponding investment threshold is *larger* without subsidy than it is with subsidy, but it is equal if there is no subsidy retraction risk. There are three underlying opposing effects of receiving subsidy that influence the firm's optimal investment decision and lead to the aforementioned observation. The first two effects, the direct effect of subsidy on investment size (increasing the optimal size) and the indirect effect of subsidy on investment size via timing (decreasing the optimal size), cancel each other out, as discussed when presenting Corollary 2. The third effect is that retraction risk speeds up investment, as the firm prefers to obtain the subsidy over not obtaining subsidy. Speeding up in fact means investing at a lower threshold where the output price is smaller. This causes the optimal investment size under subsidy to be smaller than without subsidy.

Table 1
Parameter values used in the numerical example.

Notation	Parameter	Value
μ	Electricity price trend	2%
σ	Electricity price volatility	5%
r	Risk-free interest rate	6%
δ	Investment cost per unit of capacity	350 /MWh
η	Slope of the linear demand curve	0.01

Based on Fig. 2, we generate some important policy advice regarding green investment projects. Investors in green investment projects usually have long-term goals and high investment costs. Given that a subsidy has been implemented and the policy maker wants the firm to invest as much as possible, the optimal situation for the policy maker would be that the firm perceives no subsidy retraction risk (i.e. $\lambda = 0$).

To study a situation where the policy risk is large, we set $\lambda = 1$. This means that the firm expects the subsidy to be retracted in about one year. The investment timing thresholds X_0 and X_1, and the investment sizes K_0^* and K_1^* are shown as functions of subsidy size θ in Fig. 3. In accordance with Proposition 6, both timing and size decrease when increasing subsidy size.

To study the effect of subsidy size θ and interpret Fig. 3, it is important to distinguish between two different cases. Firstly, the simple case, in which the firm is in the stopping region at the start of the planning horizon, i.e. the starting value of the GBM X, x, is larger than the investment threshold X_1. Then the firm invests immediately, at the price $P(x)$ and the optimal capacity is equal to $K_1(x)$, i.e. expression (2.9) evaluated at $X = x$. When the government pays for almost all investment costs, that is, the subsidy size θ is close to one, the investment quantity is close to $\frac{1}{2\eta}$, which represents the optimal capacity if investment costs would be equal to zero. That is, the firm maximizes total revenues. Secondly, the firm is in the waiting region at the start of the planning horizon, i.e. $x < X_1$. In this case, the firm waits with investment until the threshold X_1 is hit (or X_0 if the subsidy is withdrawn before investment) and invests in K_1 (K_0) as shown in the right-hand graph in Fig. 3.

Fig. 3 helps to analyze the situation in which a government aims to speed up investment of the waiting firm by threatening to remove the subsidy soon. Whether the firm will invest immediately under large subsidy withdrawal risk, depends on the size of the subsidy and the current output price level. When the government has implemented a large subsidy (i.e. θ close to one), threatening to take away the subsidy soon results in firms investing immediately to still receive the large investment cost subsidy. However, it could happen that then, if the current output price is low, firms will invest in a small capacity.

However, when the subsidy size is relatively small, the approach to make the firm invest immediately by threatening to remove the subsidy soon is not always effective. For example, consider a subsidy size of $\theta = 0.15$. Fig. 3 shows that the optimal timing threshold while the subsidy is available, X_1, is equal to 19.15. Increasing the subsidy withdrawal risk even further than $\lambda = 1$ makes the threshold eventually converge to a value of approximately 18.29 (see Fig. 2). Therefore, when the current value of the demand intercept is smaller than 18.29, trying to let the firm invest immediately by threatening to remove the subsidy, is ineffective as it is never optimal to invest immediately, independent of the subsidy withdrawal risk.

Finally, we study the effect of demand volatility on the investment size and investment threshold. Fig. 4 presents the investment timing threshold X_1 and the investment size K_1^* as functions of the subsidy retraction risk λ for different levels of demand volatility σ, using the parameter values in Table 1. Fig. 5 shows the investment timing threshold X_1 and the investment size K_1^* as functions of the subsidy retraction risk θ for different levels of demand volatility. We observe the standard real options result that a larger demand volatility delays investment and increases investment size (see, e.g., Dangl, 1999 and Huisman and Kort, 2015). However, this effect does not eliminate the effects of subsidy withdrawal risk and subsidy size as shown in Proposition 5 and 6. Even when demand volatility σ is large, both the investment threshold and the investment size decrease with subsidy withdrawal risk and subsidy size.

5. Capacity target and total surplus

We now study how a policy maker can influence and steer the decisions of the firm towards a socially optimal (first-best) decision. In the

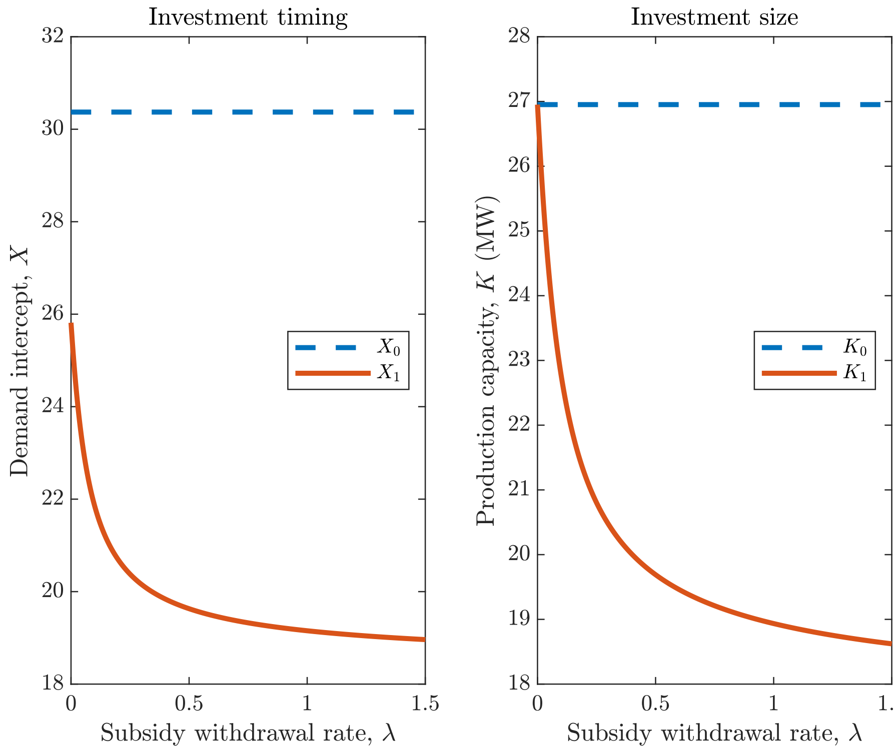

Fig. 2. Investment timing (left) and size (right) as functions of the subsidy withdrawal rate λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\theta = 0.15$.]

following we consider two different types of objectives for the social planner. In Section 5.1, we assume that the policy maker strives to achieve a predetermined capacity target as soon as possible. This is especially relevant considering renewable energy capacity targets. In Section 5.2, we consider a social planner that has the aim to increase total surplus.

5.1. Capacity target

We first focus on the case where the social planner has the aim to reach a certain capacity target $\overline{K}$ as soon as possible. Fig. 6 illustrates the optimal subsidy size required to reach a certain capacity target (left panel) and the resulting investment timing (right panel) as a function of subsidy retraction risk λ.[17]

In case the target is lower than the firm's optimal investment without subsidy (i.e. $\overline{K} < K_0^*$), the social planner can use the policy instrument to speed up the firm's investment. In this scenario a subsidy can be used to reach the capacity target earlier, as illustrated in Fig. 6. The smaller the capacity target, the sooner investment will take place, which is accelerated by offering a larger subsidy. When the subsidy withdrawal risk increases, the subsidy required to reach a certain capacity target decreases. The optimal investment threshold, however, increases as a result of the smaller subsidy size.

Until now we have seen subsidies that are used to speed up investment and, as a side effect, it decreases the firm's optimal investment size. A different matter arises when the capacity target is larger than the firm's optimal investment size if no subsidy is provided. The only way to reach such a target is to implement a conditional subsidy in the sense that such a subsidy is only provided at the moment that the firm invests in a capacity size corresponding to the target.

[17] Note that when $\lambda = 0$, the firm's optimal investment size does not depend on the subsidy size (see equation (2.16)), and thus the social planner cannot influence the firm's optimal size decision. Therefore, the lines in Fig. 6 start for positive λ and not for $\lambda = 0$.

5.2. Total surplus

In this section, we study the question whether a policy maker can increase total economic surplus[18] by use of a subsidy, with a focus on the role of subsidy retraction risk. To analyze the effect of subsidy retraction risk and subsidy size on the total surplus (TS), we study the relative difference between economic surplus generated by the first-best solution and welfare under the investment decision made by the firm. This relative difference is called the relative welfare loss (RWL), and depends on the likelihood of subsidy withdrawal λ and the subsidy size θ. In case there is no subsidy in effect, we can show that the RWL is always equal to:

$$\mathrm{RWL}(X_0, K_0^*) = \frac{\mathrm{TS}(X_S, K_S^*) - \mathrm{TS}(X_0, K_0^*)}{\mathrm{TS}(X_S, K_S^*)} = \frac{1}{4} \tag{5.1}$$

See Appendix C.1 for the derivation details.

This implies that a subsidy only has value in terms of increasing total surplus if it can decrease RWL below 25%. We find that the first-best outcome can in fact not be obtained with a lump-sum subsidy. To achieve the first-best outcome, we learn from Proposition 4 that the subsidy should be such that it should let the firm double the size of the investment without affecting the investment timing. However, providing a subsidy would result in an investment size being less than or equal to the size without subsidy. We conclude that steering the firm towards the first best outcome by providing a subsidy is not possible.

We present further results illustrated by the numerical example with the same parameter values as in Table 1. Fig. 7 plots the total surplus as a function of subsidy retraction risk λ. For any given subsidy level, we find

[18] In this paper, we focus on the question whether a subsidy can increase total economic surplus, assuming no government inefficiencies or market distortions caused by the financing of the subsidy. We use welfare to describe the total economic surplus. Note that in practice, policy makers may need to account for inefficiencies in government spending as well as the costs of obtaining the budget to implement a subsidy. For example, if the subsidy is financed from a distortionary tax, these effects are the consequence of implementing the subsidy.

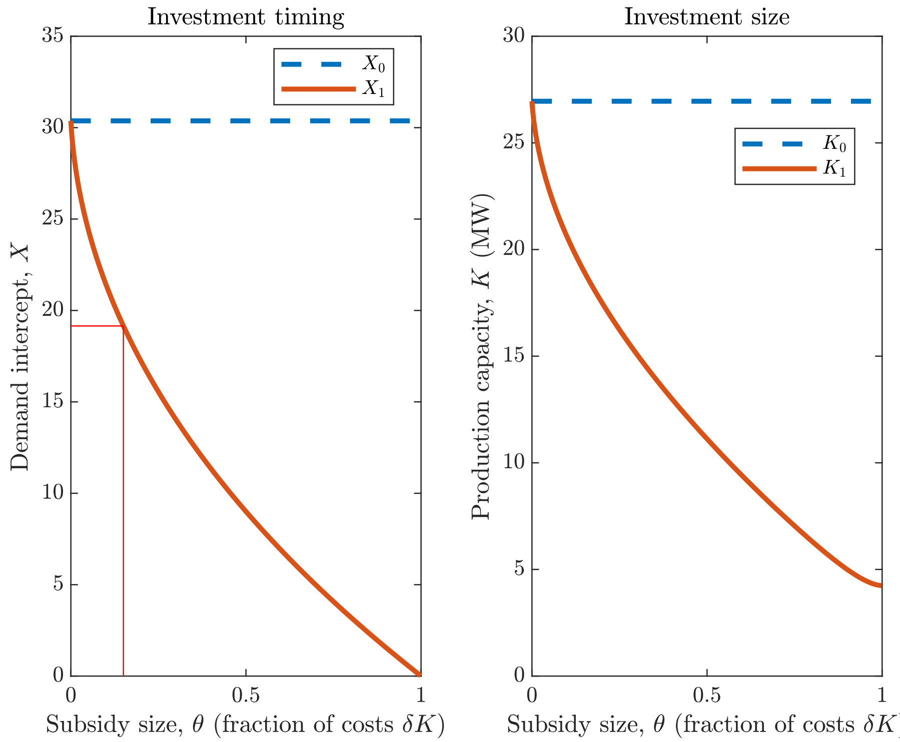

Fig. 3. Investment timing (left) and size (right) as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\lambda = 1$.]

that the higher is the perceived risk of subsidy retraction, the lower the total surplus becomes. The reason is the following. First note that, taking it from a welfare perspective, already under zero retraction risk the firm invests too early in a too low capacity. Fig. 2 learns that the larger the perceived risk of subsidy retraction, the sooner the firm invests in less. So in this way under a subsidy retraction risk the firm's investment decision departs even further away from socially optimal investment. Hence, we conclude that no subsidy retraction risk is optimal in terms of total surplus and a policy maker maximizing total surplus should try to eliminate this risk. Fig. 7 in fact shows that already very small increases in subsidy retraction risk drastically decrease total surplus.

Next, we turn our analysis to the socially optimal subsidy size θ. Fig. 8 plots the total surplus as a function of subsidy size θ. We obtain that providing subsidy can increase welfare as illustrated in both the left and

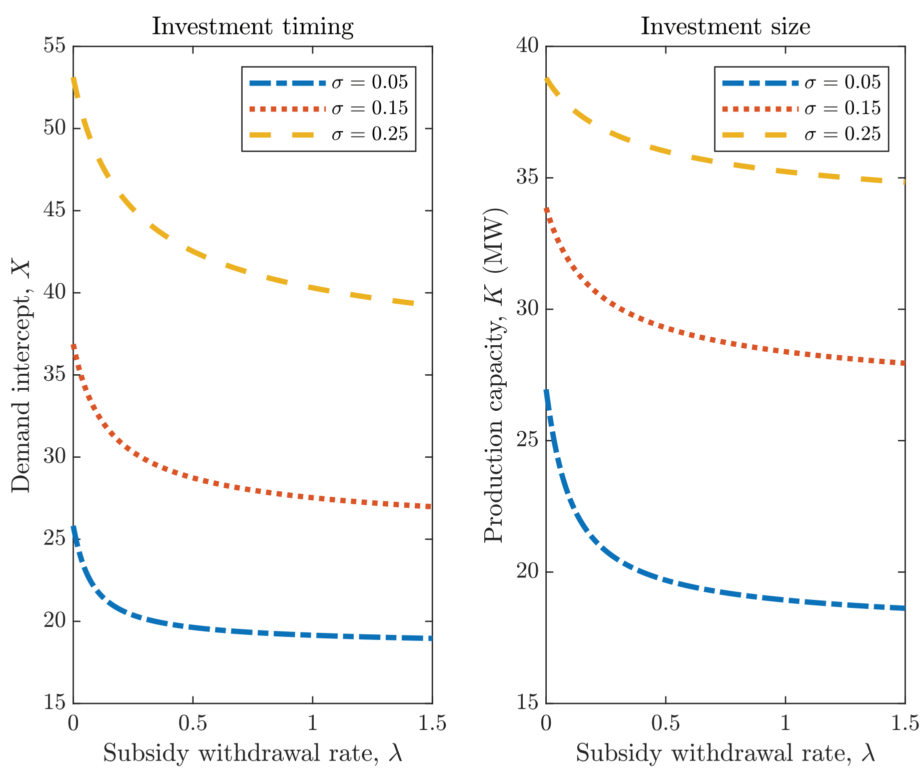

Fig. 4. Investment timing (left) and size (right) as functions of the subsidy withdrawal rate λ. [Parameter values: $\mu = 0.02$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\theta = 0.15$.]

middle panel of Fig. 8. The left panel of Fig. 8 shows that in case of no subsidy retraction risk the total surplus is highest when $\theta = 0.156$, i.e. the lump-sum subsidy is equal to 15.6% of the firm's total investment costs. At $\theta = 0.156$, the total surplus is equal to 429.79, while the first-best outcome leads to a total surplus of 543.25. This results in a RWL of 20.9% opposed to the 25% when the subsidy is not provided. By implementing the subsidy, the relative welfare loss decreases by approximately 16.4%. The increase in welfare is the result of the fact that, under no withdrawal risk, the firm invests earlier and in the same size. This increases both the discounted consumer surplus and the discounted producer surplus, and these increases outweigh the costs of providing the subsidy. This result holds when there is no policy risk. We now study how policy risk affects this result.

The middle panel of Fig. 8 shows the total surplus if there is a low subsidy retraction risk. If we introduce only a small probability of subsidy withdrawal by setting $\lambda = 0.0001$, the optimal subsidy size is slightly smaller and equal to $\theta^* = 0.135$ compared to when there is no risk of subsidy retraction ($\theta^* = 0.156$). Introducing a probability of a subsidy retraction, results in that the investment is done sooner and, therefore, with a smaller capacity. Decreasing the subsidy size makes the firm postpone investment. When it invests, it, therefore, invests in a larger size. Thus, decreasing the subsidy size counters the effect of the increased probability of subsidy retraction. Comparing the middle panel with the left panel in Fig. 8, we observe that for any given subsidy size the total surplus decreases when there is subsidy retraction risk.

Assuming a slightly larger subsidy withdrawal risk by setting $\lambda = 0.001$, it in fact becomes optimal not to introduce a subsidy at all. This is because the firm has a strong incentive to invest early, but therefore, in a small capacity. The investment is done too early and at a too small scale from a welfare-maximizing point of view. Therefore, when policy risk is large, it is best for social welfare not to offer a subsidy at all.

6. Discussions

Next, we discuss the effect of alternative assumptions on our results. We discuss the effect of different types of subsidies, the effect of the firm having the option to expand, and the effect of the social planner's discount rate in this section. A detailed analysis of the effect of a different demand function is included in Appendix B, in which we assume an isoelastic demand function.

Firstly, we compare our results under a lump-sum subsidy with the (expected) results under two different types of subsidies: the feed-in tariff (FIT) and feed-in premium (FIP). Feed-in policies (i.e. tariffs and premiums) are still widely used. By the end of 2019, they were in place in 113 jurisdictions at the national, state or provincial levels [REN21, 2020]. The main difference between a lump-sum subsidy on the one hand and the FIT and FIP on the other hand is that the lump-sum subsidy is a one-time transfer at the time of investment, while both the FIT and FIP payments happen during the project life-time. This difference is also the key explanatory factor in the difference in conclusions.

We find that, under a lump-sum subsidy, an increase in the subsidy withdrawal risk, lowers the firm's investment threshold and decreases its investment size. Chronopoulos et al. [2016] studies investment under subsidy withdrawal risk under a FIP and draws the same conclusion when the risk of subsidy withdrawal is low. This is the result of a firm wanting to obtain subsidy and it is being threatened the subsidy may disappear in the near future. When the risk of subsidy withdrawal is high, this effect disappears for the FIP, but not for the lump-sum subsidy. In case of a FIP, a firm increases its investment threshold and increases its investment size when the subsidy withdrawal risk of withdrawal increases. The firm's gain from a feed-in premium is obtained from production, hence a firm only invests when either the output price is high or when the expected lifetime of the feed-in premium is substantial. This is different from the lump-sum subsidy, for which the gain is fully obtained at the moment of investment.

Boomsma et al. [2012] studies the effect of FITs on investment. Assuming there is no risk of subsidy withdrawal, Boomsma et al. [2012] conclude that FITs encourage earlier investment. The firm invests earlier under a FIT as it is protected from risk on the market. When accounting for the risk of subsidy withdrawal, the firm faces a trade-off similar to the scenario in which the subsidy available is a FIP. We would expect both the investment threshold and investment size to go down (up)

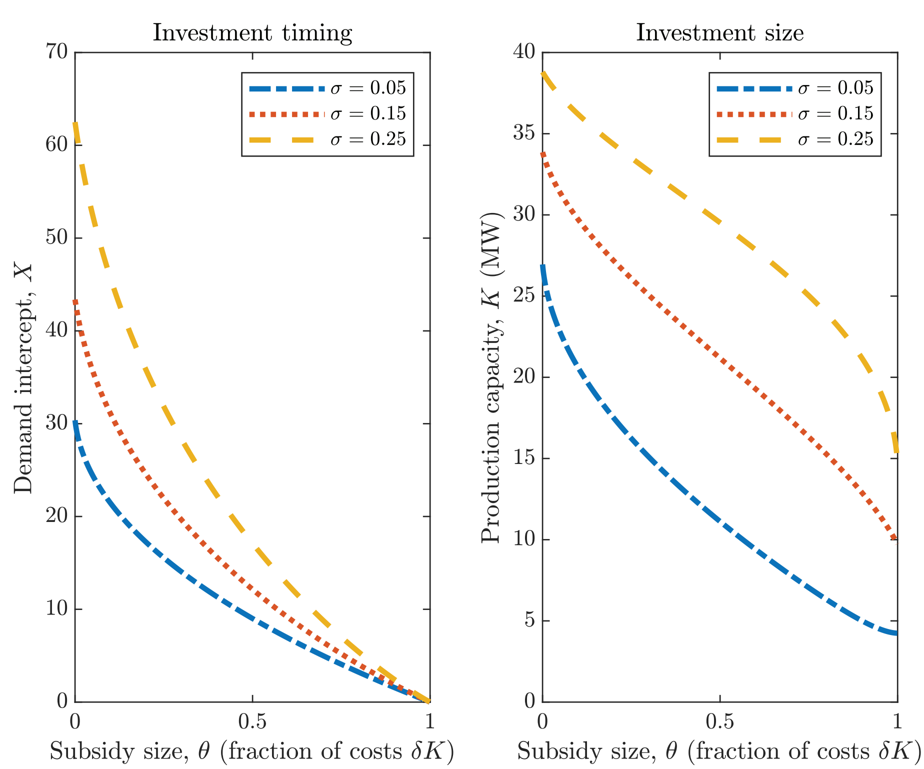

Fig. 5. Investment timing (left) and size (right) as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\lambda = 1$.]

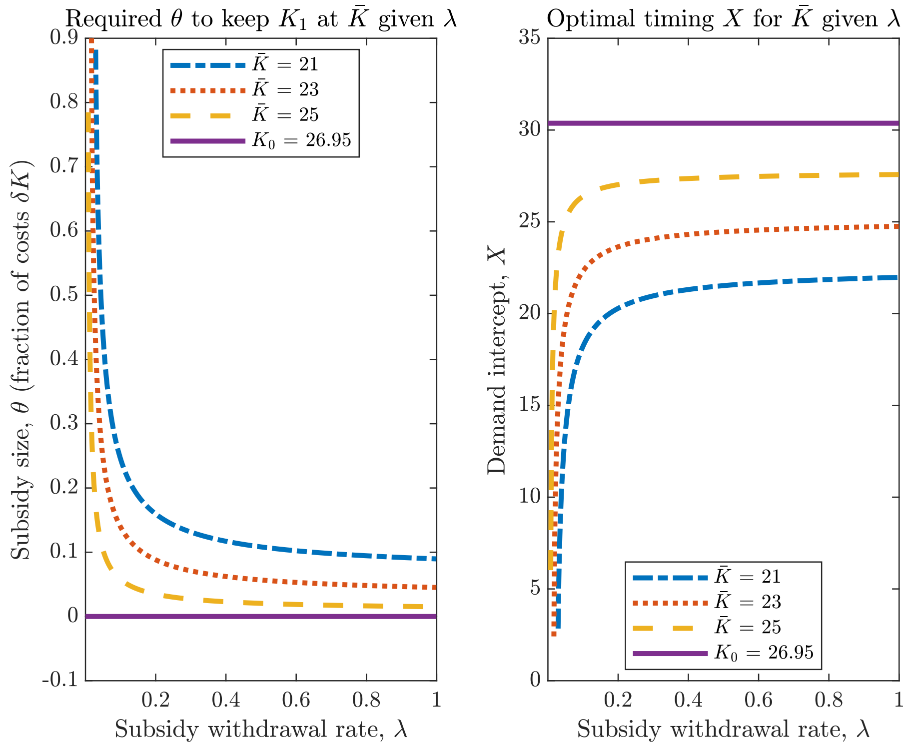

Fig. 6. Subsidy size (left) and optimal investment timing (right) as functions of subsidy withdrawal rate λ for different capacity targets. [Parameter values: $\mu = 0.02, \sigma = 0.05, r = 0.06, \eta = 0.01, \delta = 350$.]

with retraction risk when the risk of retraction is low (high). The trade-off consists of two opposing effects. Firstly, the firm has an incentive to invest sooner in order to still obtain the subsidy. The firm would then also invest in a smaller size. Secondly, it wants to keep its revenue high also in the case when the FIT is retracted. Hence, it has the incentive to increase its investment threshold to make sure output prices are sufficiently high. In this case, the firm would increase its investment size.

Secondly, we discuss the case in which the firm has the option to expand the renewable energy capacity by investment in new locations after. This means it faces a sequential investment decision. In the case of sequential investment, a firm can invest early to take advantage of the available subsidy, while still being able to scale up investment later if output prices are high. This provides it with more flexibility. We expect that this leads to the firm investing sooner to obtain subsidy and also investing more in the long-run if output prices are high.

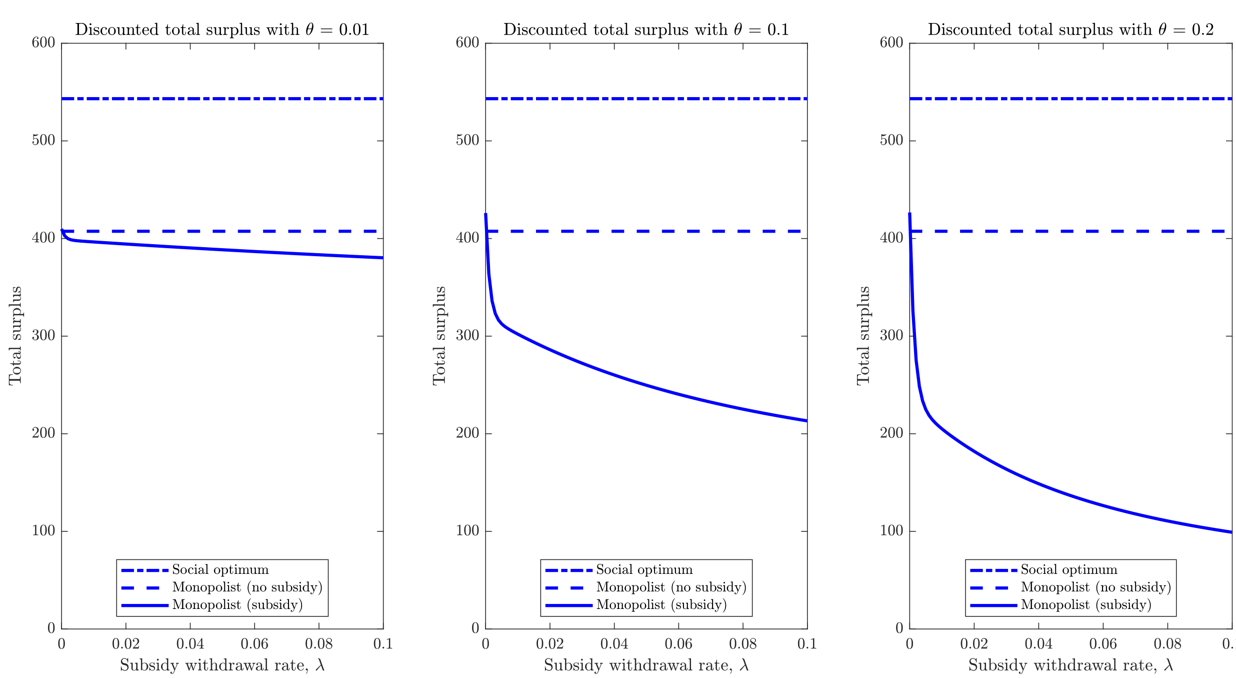

Fig. 7. Total surplus as a function of the subsidy retraction risk λ for different subsidy sizes θ. [Parameter values: $\mu = 0.02, \sigma = 0.05, r = 0.06, \eta = 0.01$ and $\delta = 350$.]

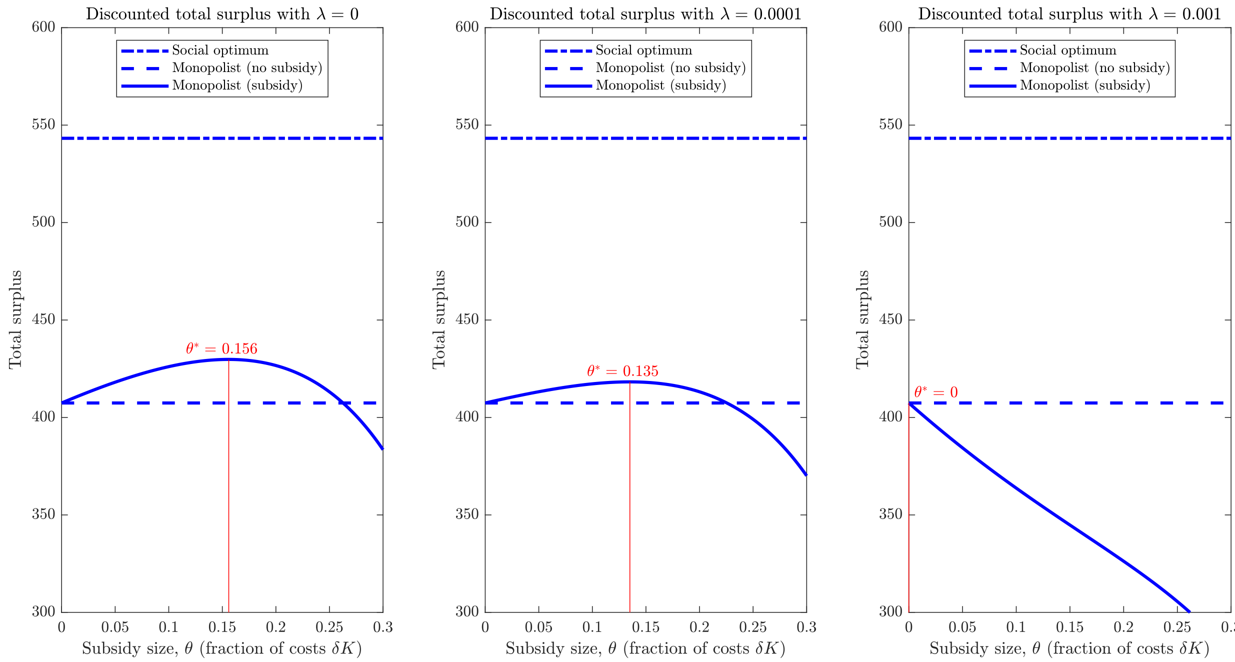

Fig. 8. Total surplus as a function of the subsidy size θ for different levels of subsidy withdrawal risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$ and $\delta = 350$.]

Lastly, we discuss the effect of a difference between the social planner's and the firm's discount rate. The firm's investment size and quantity are affected in the same way by a lump-sum subsidy under withdrawal risk as discussed in Sections 3 and 4: both a higher withdrawal risk and a higher subsidy size speed up investment and decrease the investment size. In case the social planner maximizes total surplus and has a higher discount rate than the firm, it prefers that the firm invests sooner than the firm would without subsidy. Therefore, the larger the social planner's discount rate, the larger its optimal subsidy.

7. Conclusions

This paper studies the effect of a lump-sum subsidy subject to risk of retraction on optimal investment decisions in terms of timing and capacity size installed. We find that increasing the likelihood of subsidy withdrawal gives the firm an incentive to invest sooner to still obtain the subsidy. As the firm invests sooner, it also invests in a smaller size. The same effect, i.e. investing sooner in a smaller size, is obtained by increasing the subsidy size under positive subsidy withdrawal risk.

Since the firm does not take into account the consumer surplus when investing, it has less incentives to invest than a social planner maximizing total surplus. When demand is linear, a profit-maximizing firm invests at the right time but in a too small capacity. When demand is isoelastic, the firm does invest in the same capacity as the social planner, but the profit-maximizing firm invests later. We find that in both cases a lump-sum subsidy can increase welfare when there is no subsidy retraction risk, but it harms welfare when there is substantial subsidy retraction risk. Therefore, a social planner maximizing welfare should try to minimize the subsidy retraction risk. If subsidy retraction risk increases, the socially optimal subsidy size decreases, and welfare decreases rapidly as the firm invests in a much too small size from a socially optimal point of view.

In case the policy maker aims to reach a capacity target that is smaller than the firm's optimal investment size without subsidy, implementing a lump-sum subsidy can speed up the firm's investment. If the policy maker sets a capacity target that is larger than the firm's optimal investment size, the only way to achieve the target is to implement a subsidy that is provided conditional on the firm investing in the right capacity size.

Our model can be extended for the case in which the firm is able to receive signals on future government decisions, so that it can update its beliefs about the possibility of a subsidy retraction. Pawlina and Kort [2005] propose a model with consistent authority behavior, which takes into account that the government will only intervene at a certain price level, but they only consider the investment timing decision and not the investment size decision. Dalby et al. [2018] provide a model in which firms receive signals and can learn about the timing of subsidy revision. However, their model does not account for a firm's investment timing and capacity size decisions.

Appendix A. Proofs of theorems and propositions

A.1. Proof of corollary 1

Proof of Corollary 1. This proof shows that the expression for $K_1(X)$ (expression (2.9)) holds for $X > X_1$. The proof that Eq. (2.8) is correct for $X > X_0$ follows the same steps.

The optimal investment size $K = K_1^*$ maximizes $V_1(K,X)$ for $X > X_1$. Since $\frac{d^2V_1}{dK^2} = -\frac{2\eta X}{r-\mu} < 0$ for $X > 0$, it holds that $V_1(K,X)$ is concave in K as $X > X_1 > 0$. Therefore the first order condition, $\frac{dV_1}{dK} = 0$, can be applied here.

$$\frac{dV_1}{dK} = 0 \Leftrightarrow \frac{X(1-2\eta K)}{r-\mu} - (1-\theta)\delta = 0 \tag{A.1}$$

$$\Leftrightarrow K_1(X) = \frac{1}{2\eta}\left(1 - \frac{(1-\theta)\delta(r-\mu)}{X}\right) \tag{A.2}$$

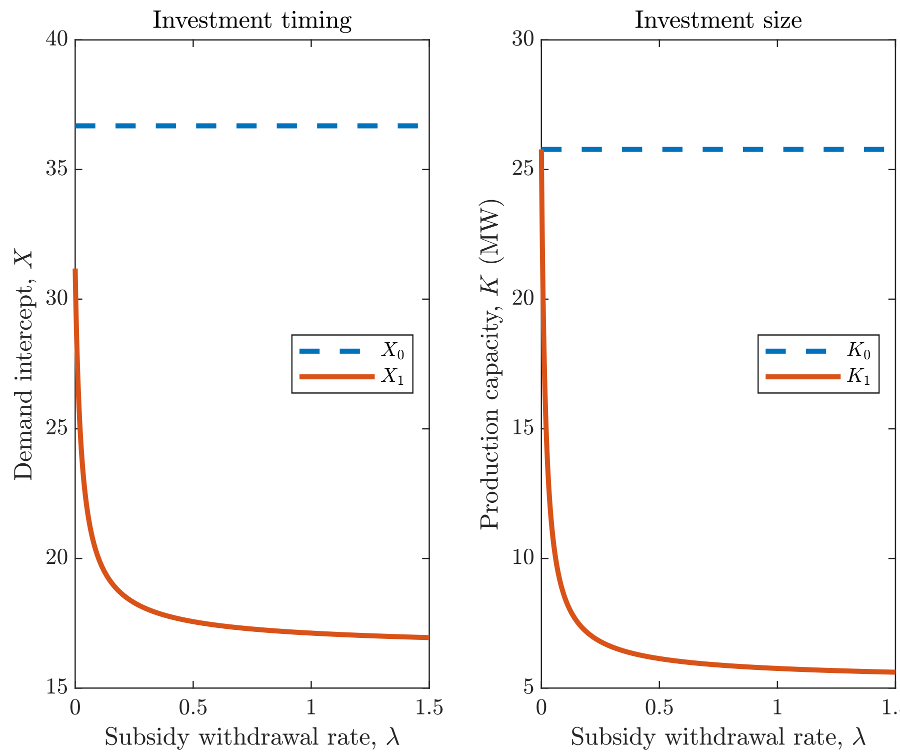

Fig. 9. Investment timing (left) and size (right) under iso-elastic demand as functions of the subsidy retraction risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$, $\gamma = 0.4$ and $\theta = 0.15$.]

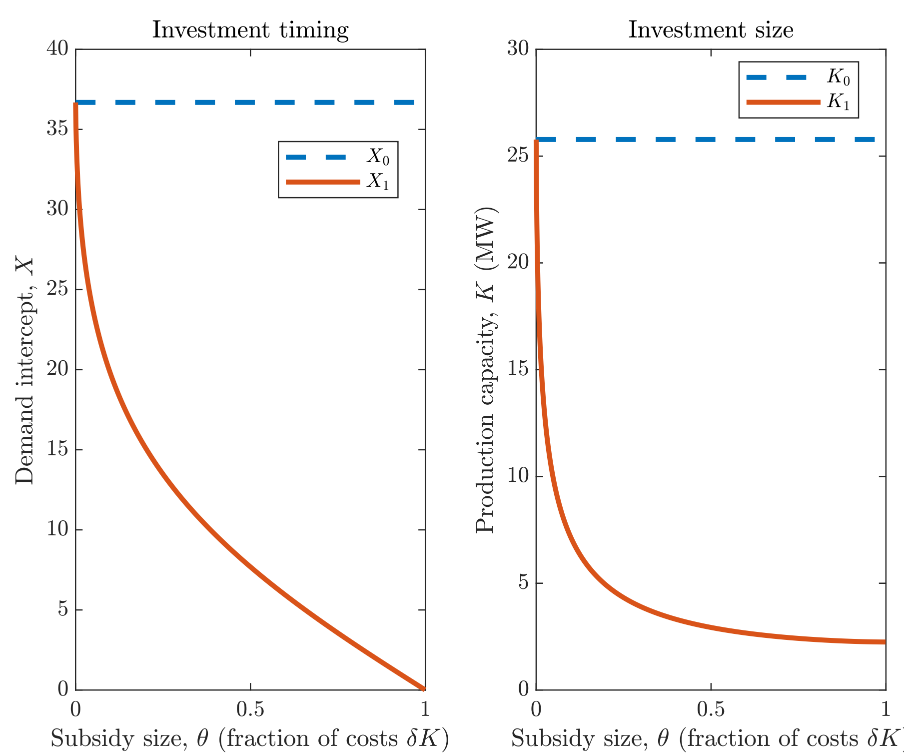

Fig. 10. Investment timing (left) and size (right) under iso-elastic demand as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$, $\gamma = 0.4$ and $\lambda = 1$.]

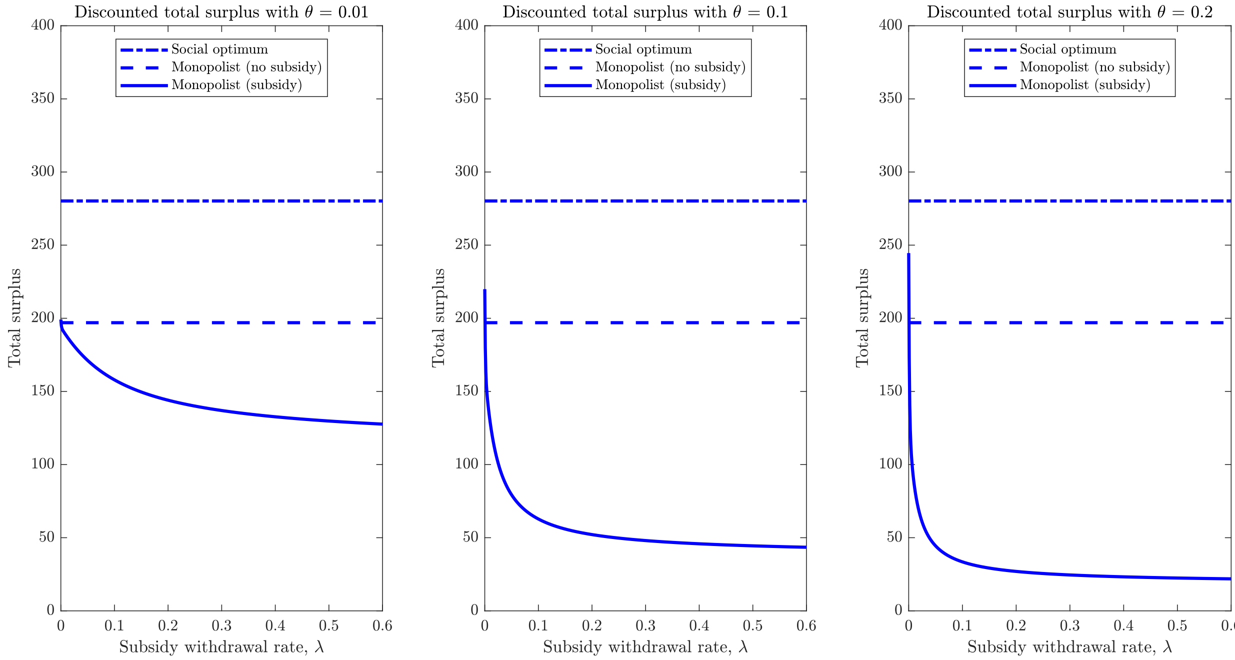

Fig. 11. Total surplus under iso-elastic demand as a function of the subsidy retraction risk λ for different subsidy sizes θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

A.2. Proof of proposition 1

Proof of Proposition 1. *Firstly, looking at the value of the investment option without the subsidy, we can follow Huisman and Kort [2015] as there is no subsidy uncertainty in this case. When $X > X_0$, it is optimal to invest, and we have*:

$$V_0(X,K) = \frac{X(1-\eta K)K}{r-\mu} - \delta K \tag{A.3}$$

When $X < X_0$, it is optimal to wait with investing. It can be shown that the following holds for $V_0(X)$, the value of the investment at level X when the policy has been withdrawn (see, e.g., Dixit and Pindyck, 1994):

$$\frac{1}{2}\sigma^2 X^2 V_0''(X) + \mu X V_0'(X) - r V_0(X) = 0 \tag{A.4}$$

Solving this ordinary differential equation yields $V_0(X) = A_0 X^{\beta_{01}} + B_0 X^{\beta_{02}}$. In this expression, A_0 and B_0 are constants that remain to be determined. β_{01} (β_{02}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. Since $V_0(0) = 0$ and $\beta_{02} < 0$, it follows that $B_0 = 0$, hence:

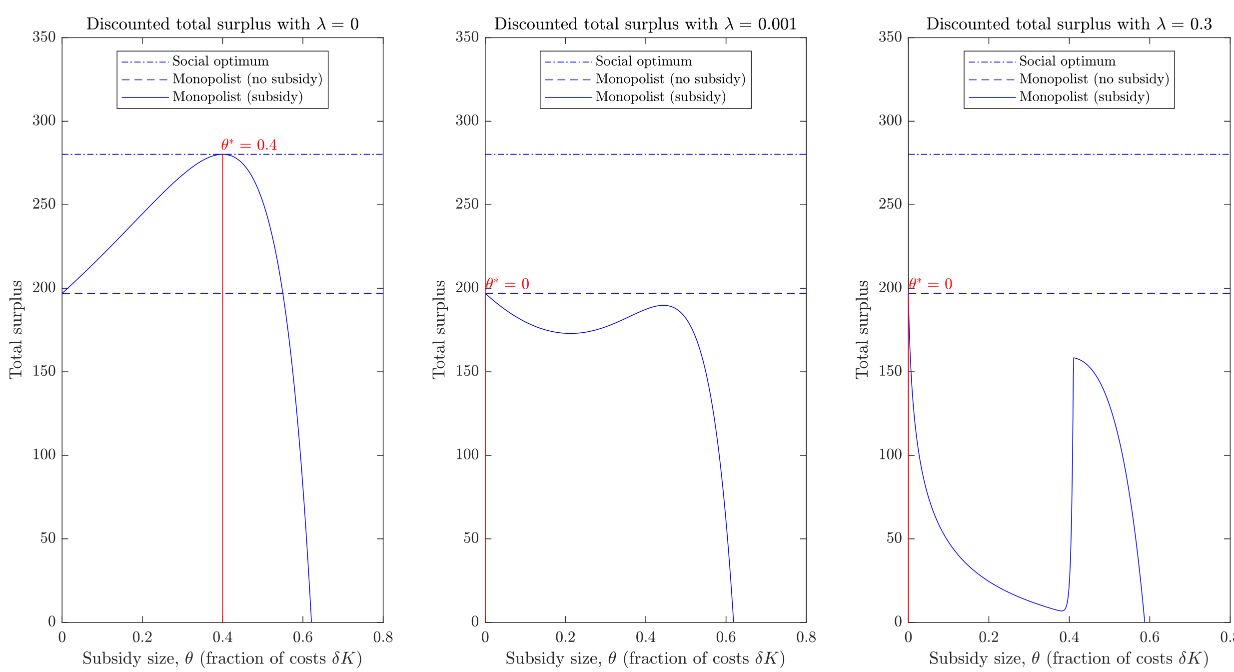

Fig. 12. Total surplus under iso-elastic demand as a function of the subsidy size θ for different levels of subsidy withdrawal risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

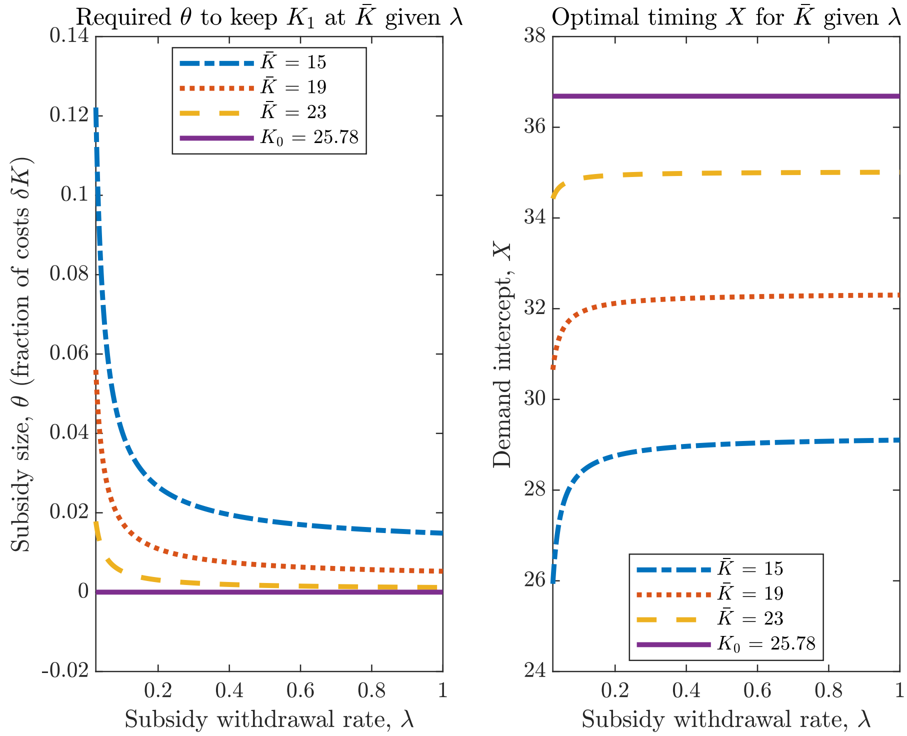

Fig. 13. Subsidy size (left) and optimal investment timing (right) under iso-elastic demand as functions of the subsidy withdrawal rate λ for different capacity targets. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

$$V_0(X) = A_0 X^{\beta_{01}} \tag{A.5}$$

Combining expressions (A.3) and (A.5) yields the expression (2.10) for V_0.

Secondly, we derive expression (2.11) for V_1. When $X > X_1$, it is optimal to invest and the value of the option to invest when the subsidy is in effect is equal to:

$$V_1(X, K) = \frac{X(1-\eta K)K}{r-\mu} - (1-\theta)\delta K \tag{A.6}$$

For $X < X_1$, it holds that it is best to wait. The investment option while the policy is active satisfies the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 V_1''(X) + \mu X V_1'(X) - rV_1(X) + \lambda(V_0(X) - V_1(X)) = 0 \tag{A.7}$$

The main difference with Eq. (A.4) is the addition of the term $\lambda(V_0(X) - V_1(X))$, which has been added as the value of the option to invest can drop from V_1 to V_0 if the subsidy is retracted while we wait. Since $X < X_1$ means $X < X_0$, we have $V_0(X) = A_0 X^{\beta_{01}}$ for $X < X_1$. Solving the homogeneous part of the above ordinary differential equation yields solution $V_1^H(X) = A_1 X^{\beta_{11}} + B_1 X^{\beta_{12}}$. β_{11} (β_{12}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r + \lambda) = 0$.

To find a particular solution to the ordinary differential equation in (A.7), one can try $V_1^P(X) = C_1 X^{\beta_{01}}$, as the in-homogeneous part is $A_0 X^{\beta_{01}}$. From this it follows that $C_1 = A_0$. Combining the homogeneous and particular solution gives $V_1(X) = A_1 X^{\beta_{11}} + B_1 X^{\beta_{12}} + A_0 X^{\beta_{01}}$. However, as $V_1(0) = 0$ and $\beta_{12} < 0$, it follows that $B_1 = 0$.

This results in the following expression for $V_1(X)$:

$$V_1(X) = A_1 X^{\beta_{11}} + A_0 X^{\beta_{01}} \tag{A.8}$$

where A_1 and A_0 are constants that needs be determined. As before, β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. Combining expressions (A.6) and (A.8) yields expression (2.11) for V_1.

A.3. Proof of proposition 2

Proof of Proposition 2. *The constant A_0 and thresholds X_0 satisfy the value matching and smooth pasting condition for V_0. The value matching equation for V_0 is (A.9), which guarantees that the value for $V_0(X_0, K_0^*)$ is uniquely defined.*

$$A_0 X_0^{\beta_{01}} = \frac{X_0(1-\eta K_0^*)K_0^*}{r-\mu} - \delta K_0^* \tag{A.9}$$

Apart from value matching condition, there is also a smooth pasting condition for V_0. Eq. (A.10) guarantees that $\frac{dV_0}{dX}$ has a unique value at $X = X_0$.

$$A_0 \beta_{01} X_0^{\beta_{01}-1} = \frac{(1-\eta K_0^*)K_0^*}{r-\mu} \tag{A.10}$$

Multiplying (A.9) by β_{01} and subtracting X_0 times (A.10) from it yields:

$$0 = (\beta_{01} - 1) \cdot \frac{X_0(1-\eta K_0^*)K_0^*}{r-\mu} - \beta_{01}\delta K_0^* \tag{A.11}$$

$$\Leftrightarrow X_0(1-\eta K_0^*) = \frac{\beta_{01}}{\beta_{01}-1} \cdot \delta(r-\mu) \tag{A.12}$$

Plugging the expression for the optimal capacity K_0^* (see expression (2.8)) into (A.12) and rewriting this equation results in:

$$X_0 = \frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{A.13}$$

Substituting the expression (A.13) for X_0 into (2.8) yields an expression for the optimal capacity when the subsidy is not available.

$$K_0(X_0) = [\eta(\beta_{01}+1)]^{-1} \tag{A.14}$$

A.4. Proof of proposition 3

Proof of Proposition 3. *The constant A_1 and threshold X_1 satisfy the value matching and smooth pasting conditions for V_1. The value matching equation is (A.15), which guarantees that the value for $V_1(X_1,K_1^*)$ is uniquely defined.*

$$A_1X_1^{\beta_{11}} + A_0X_1^{\beta_{01}} = \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - (1-\theta)\delta K_1^* \tag{A.15}$$

Apart from value matching condition, there is also smooth pasting condition (A.16), which guarantees that $\frac{dV_1}{dX}$ has a unique value at $X = X_1$.

$$A_1\beta_{11}X_1^{\beta_{11}-1} + A_0\beta_{01}X_1^{\beta_{01}-1} = \frac{(1-\eta K_1^*)K_1^*}{r-\mu} \tag{A.16}$$

Subtracting $\frac{X_1}{\beta_{11}}$ times Eq. (A.16) from (A.15) yields:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}} = \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - (1-\theta)\delta K_1^* \tag{A.17}$$

Rearranging terms in (A.17) leads to:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} + (1-\theta)\delta K_1^* = 0 \tag{A.18}$$

In the above, an expression for A_0 can be derived by rewriting Eq. (A.10) and subsequently substituting the derived expressions for X_0 and K_0^*:

$$A_0 = \frac{\delta}{\eta\left(\beta_{01}^2-1\right)}\cdot X_0^{-\beta_{01}} \tag{A.19}$$

A.5. Proof of proposition 4

Proof of Proposition 4. To derive the optimal capacity from a social welfare point of view, we take the first order condition of TS with respect to K, similar to deriving the optimal capacity for the profit-maximizing firm, see the proof in Appendix A.1.

We take the same steps as the proof in Appendix A.2 when determining the expression for V_0 to derive the value of the option to invest for the social planner.

The threshold for the social planner X_S satisfies the value matching and smooth pasting conditions. The value matching equation is:

$$A_SX_S^{\beta_{01}} = \frac{X_S(2-\eta K_S(X_S))K_S(X_S)}{2(r-\mu)} - \delta K_S(X_S) \tag{A.20}$$

and the smooth pasting condition is:

$$A_S\beta_{01}X_S^{\beta_{01}-1} = \frac{(2-\eta K_S(X_S))K_S(X_S)}{2(r-\mu)} \tag{A.21}$$

The interpretation of the value matching and smooth pasting conditions are the same as the value matching and smooth pasting conditions for the profit-maximizer, which are discussed in Section 2.

The threshold X_S can be derived using the same steps as in Appendix A.3 and even yields:

$$X_S = \frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{A.22}$$

A.6. Proof of proposition 5

Proof of Proposition 5. *We start by proving the first statement of this proposition*:

$$\frac{dX_1}{d\lambda}<0 \Leftrightarrow X_1 \le \frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \tag{A.23}$$

To derive the effect of subsidy retraction risk λ on timing threshold X_1, we only have to look at the direct effect of λ on X_1, as there is no indirect effect via investment size, since $\frac{\partial K_1^*}{\partial\lambda} = 0$. Therefore:

$$\frac{dX_1}{d\lambda} = \frac{\partial X_1}{\partial\lambda} \tag{A.24}$$

Let implicit Eq. (2.14) be denoted by f. To derive $\frac{\partial X_1}{\partial\lambda}$, we apply total differentiation to f:

$$0 = \frac{df}{d\lambda} = \frac{\partial f}{\partial\lambda} + \frac{\partial f}{\partial X}\cdot\frac{\partial X_1}{\partial\lambda} \Leftrightarrow \frac{\partial X_1}{\partial\lambda} = -\frac{\left(\frac{\partial f}{\partial\lambda}\right)}{\left(\frac{\partial f}{\partial X}\right)} \tag{A.25}$$

We are going to show that $\frac{\partial f}{\partial\lambda}<0$ always holds, and $\frac{\partial f}{\partial X}<0$ if and only if condition (3.2) holds.

To derive $\frac{\partial f}{\partial\lambda}$, we can use that $\frac{\partial K_1}{\partial\lambda} = 0$. This gives:

$$\begin{aligned}\frac{\partial f}{\partial\lambda} &= \frac{\beta_{11}\cdot\frac{d\beta_{11}}{d\lambda} - (\beta_{11}-\beta_{01})\frac{d\beta_{11}}{d\lambda}}{\beta_{11}^2}\cdot A_0X_1^{\beta_{01}} \\ &- \frac{\beta_{11}\cdot\frac{d\beta_{11}}{d\lambda} - (\beta_{11}-1)\frac{d\beta_{11}}{d\lambda}}{\beta_{11}^2}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} \\ &= \frac{1}{\beta_{11}^2}\cdot\frac{d\beta_{11}}{d\lambda}\left(\beta_{01}A_0X_1^{\beta_{01}} - \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu}\right)\end{aligned} \tag{A.26}$$

where

$$\frac{d\beta_{11}}{d\lambda} = \frac{1}{\sigma^2\left(\beta_{11}-\frac{1}{2}\right)+\mu}>0$$

We rewrite the smooth pasting condition (see Eq. (A.16)) as:

$$\beta_{01}A_0X_1^{\beta_{01}} = \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - \beta_{11}A_1X_1^{\beta_{11}} \tag{A.27}$$

and plug (A.27) into (A.26). This gives:

$$\frac{\partial f}{\partial\lambda} = -\frac{1}{\beta_{11}}\cdot\frac{d\beta_{11}}{d\lambda}\cdot A_1X_1^{\beta_{11}}<0 \tag{A.28}$$

To prove $\frac{\partial f}{\partial X}<0$ if and only if condition (3.2) holds, we start with taking the partial derivative of f with respect to X. Note that we also need to account for the derivative of the optimal investment size under subsidy with respect to the timing evaluated at the optimal timing threshold, which we denote by $\frac{dK_1^*}{dX}$. Taking the partial derivative of f with respect to X gives the following, after using that $\frac{X_1(1-2\eta K_1^*)}{r-\mu} = (1-\theta)\delta$ can be derived from the expression for K_1^* (substituting $X = X_1$ into (2.9)), and rearranging terms:

$$\frac{\partial f}{\partial X} = \frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\beta_{01}A_0X_1^{\beta_{01}-1} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\left(\frac{(1-\eta K_1^*)K_1^*}{r-\mu} + \frac{X_1(1-2\eta K_1^*)}{r-\mu}\cdot\frac{dK_1}{dX}\right) + (1-\theta)\delta\cdot\frac{dK_1^*}{dX} \quad (A.29)$$

$$= \frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\beta_{01}A_0X_1^{\beta_{01}-1} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{(1-\eta K_1^*)K_1^*}{r-\mu} + \frac{1}{\beta_{11}}\cdot(1-\theta)\delta\cdot\frac{dK_1^*}{dX} \quad (A.30)$$

$$= \frac{\beta_{01}}{X_1}\left(\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu}\right) + (\beta_{01}-1)\cdot\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{(1-\eta K_1^*)K_1^*}{r-\mu} + \frac{1}{\beta_{11}}\cdot(1-\theta)\delta\cdot\frac{dK_1^*}{dX} \quad (A.31)$$

The term $\frac{dK_1^*}{dX}$ is the derivative of $K_1(X)$ with respect to X evaluated at X_1 and can be rewritten into terms of X_1 and K_1^* as follows:

$$\frac{dK_1^*}{dX} = \left.\frac{dK_1}{dX}\right|_{X=X_1} = \frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2} = \frac{1}{X_1}\left(\frac{1}{2\eta} - K_1^*\right) \quad (A.32)$$

Note that the term between the brackets in the first line of Eq. (A.31) can be substituted out by using the implicit Eq. (2.14), i.e. $\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} = -(1-\theta)\delta K_1^*$. Using this fact, combined with Eq. (A.32) for $\frac{dK_1^*}{dX}$ and Eq. (2.9) evaluated at $X = X_1$ for K_1^* reduces (A.31) after some algebra to:

$$\frac{\partial f}{\partial X} = \frac{1}{4\eta X_1^2(r-\mu)}\cdot g(X_1) \quad (A.33)$$

where

$$g(X_1) = (\beta_{01}-1)(\beta_{11}-1)X_1^2 - 2\beta_{01}\beta_{11}(1-\theta)\delta(r-\mu)X_1 + (\beta_{01}+1)(\beta_{11}+1)(1-\theta)^2\delta^2(r-\mu)^2. \quad (A.34)$$

Since $\frac{1}{4\eta X_1^2(r-\mu)}>0$, we conclude that $\frac{\partial f}{\partial X}<0$ if and only if $g(X_1) < 0$. It is straightforward that g is a parabola that opens upward for $\beta_{11} \geq \beta_{01} > 1$. The two zeros are at:

$$X_{g,L} = \frac{\beta_{01}\beta_{11} - \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \quad (A.35)$$

$$X_{g,R} = \frac{\beta_{01}\beta_{11} + \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \quad (A.36)$$

Since $\frac{dX_1}{d\lambda}\leq 0$ if and only if $g(X_1) < 0$, we can conclude that $\frac{dX_1}{d\lambda}\leq 0$ if and only if $X_1 \in (X_{g,L}, X_{g,R})$ always holds. Since $X_1 \leq X_{g,R}$ is the condition (3.2), only a lower bound on X_1, $X_{\min}$, meeting the requirement $X_{g,L} \leq X_{\min}$ needs to be shown.

A lower bound on X_1 is found by assuming all value is lost after subsidy withdrawal, i.e. $A_0 = 0$. Then solving implicit (2.14), we find $X_{\min} = \frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)$. To show $X_{g,L} \leq X_{\min}$, we rewrite it as follows:

$$X_{g,L}\leq X_{\min} \Leftrightarrow \frac{\beta_{01}\beta_{11} - \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)} \leq \frac{\beta_{11}+1}{\beta_{11}-1} \quad (A.37)$$

$$\Leftrightarrow \beta_{01}\beta_{11} - \sqrt{\beta_{01}^2+\beta_{11}^2-1} \leq (\beta_{11}+1)(\beta_{01}-1) \quad (A.38)$$

$$\Leftrightarrow \beta_{11} - \beta_{01} + 1 \leq \sqrt{\beta_{01}^2+\beta_{11}^2-1} \quad (A.39)$$

Since $\beta_{11} - \beta_{01} + 1 > 0$, we can square both sides, and the inequality still holds. Therefore:

$$X_{g,L}\leq X_{\min} \Leftrightarrow (\beta_{11}-\beta_{01}+1)^2 \leq \beta_{01}^2+\beta_{11}^2-1 \quad (A.40)$$

$$\Leftrightarrow -2\beta_{01}\beta_{11} + 2\beta_{11} - 2\beta_{01} + 2 \leq 0 \quad (A.41)$$

$$\Leftrightarrow 2(\beta_{11}+1)(1-\beta_{01}) \leq 0 \quad (A.42)$$

which holds since $\beta_{11} \geq \beta_{01} \geq 1$.

Next, we prove the second part of Proposition 5:

$$\frac{dK_1}{d\lambda} < 0 \Leftrightarrow X_1 \leq \frac{\beta_{01}\beta_{11} + \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \quad (A.43)$$

We apply total differentiation to K_1^* to get:

$$\frac{dK_1}{d\lambda} = \frac{\partial K_1}{\partial\lambda} + \frac{\partial K_1}{\partial X}\cdot\frac{\partial X_1}{\partial\lambda} \quad (A.44)$$

Since $K_1(X) = \frac{1}{2\eta}\left(1-\frac{(1-\theta)\delta(r-\mu)}{X}\right)$, we have $\frac{\partial K_1}{\partial\lambda} = 0$ and $\frac{\partial K_1}{\partial X} = \frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X^2}>0$.

As shown previously, if and only if $X_1 \leq \frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu)$, we conclude:

$$\frac{\partial X_1}{\partial\lambda} \leq 0 \quad (A.45)$$

Therefore, if and only if $X_1 \leq \frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu)$, we have that

$$\frac{dK_1}{d\lambda} = 0 + \frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2}\cdot\frac{\partial X_1}{\partial\lambda} \leq 0 \quad (A.46)$$

A.7. Proof of proposition 6

Proof of Proposition 6. *We start the proof by showing that*

$$\frac{dX_1}{d\theta} < 0 \Leftrightarrow X_1 \leq \frac{\beta_{01}\beta_{11} + \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \quad (A.47)$$

Taking the total differential of X_1 with respect to θ yields:

$$\frac{dX_1}{d\theta} = \frac{\partial X_1}{\partial\theta} + \frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial\theta} \quad (A.48)$$

We can directly derive $\frac{\partial K_1}{\partial\theta}$ from the closed-form expression of K_1^*, Eq. (2.9), yielding:

$$\frac{\partial K_1}{\partial\theta} = \frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} > 0 \quad (A.49)$$

Furthermore, after rewriting (Eq. (2.9)) to

$$X_1(K) = \frac{(1-\theta)\delta(r-\mu)}{1-2\eta K} \quad (A.50)$$

and using (Eq. (2.9)) evaluated at $X = X_1$ for K_1^*, it follows that:

$$\frac{\partial X_1}{\partial K} = \frac{2\eta X_1^2}{(1-\theta)\delta(r-\mu)} > 0 \tag{A.51}$$

Thus, the indirect effect of subsidy size on timing is captured by:

$$\frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial \theta} = \frac{2\eta X_1^2}{(1-\theta)\delta(r-\mu)}\cdot\frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} = \frac{X_1}{1-\theta} > 0 \tag{A.52}$$

Therefore, $\frac{dX_1}{d\theta} < 0$ if and only if

$$\frac{\partial X_1}{\partial \theta} < -\frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial \theta} = -\frac{X_1}{1-\theta} \tag{A.53}$$

Let f be the implicit Eq. (2.14). To derive the $\frac{\partial X_1}{\partial \theta}$, we apply total differentiation to f:

$$0 = \frac{df}{d\theta} = \frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial X}\cdot\frac{\partial X_1}{\partial \theta} \Leftrightarrow \frac{\partial X_1}{\partial \theta} = -\frac{\left(\frac{\partial f}{\partial \theta}\right)}{\left(\frac{\partial f}{\partial X}\right)} \tag{A.54}$$

A larger subsidy size decreases the investment threshold if and only if:

$$\frac{dX_1}{d\theta} = -\frac{\left(\frac{\partial f}{\partial \theta}\right)}{\left(\frac{\partial f}{\partial X}\right)} + \frac{X_1}{1-\theta} < 0 \tag{A.55}$$

We will show that $\frac{\partial f}{\partial \theta} < 0$ always holds, and, therewith, condition (A.55) can only hold if $\frac{\partial f}{\partial X} < 0$. $\frac{\partial f}{\partial \theta}$ is derived via partial differentiation on implicit equation f:

$$\frac{\partial f}{\partial \theta} = -\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-2\eta K_1^*)}{r-\mu}\cdot\frac{\partial K_1}{\partial \theta} + (1-\theta)\delta\cdot\frac{\partial K_1}{\partial \theta} - \delta K_1^* \tag{A.56}$$

From the first order condition with respect to capacity, it can be shown that $\frac{X_1(1-2\eta K_1^*)}{r-\mu} = (1-\theta)\delta$. Therefore, we can derive the following:

$$\frac{\partial f}{\partial \theta} = \left(-\frac{\beta_{11}-1}{\beta_{11}} + 1\right)(1-\theta)\delta\cdot\frac{\partial K_1}{\partial \theta} - \delta K_1^* \tag{A.57}$$

$$= \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta} \tag{A.58}$$

We first note $\frac{\partial f}{\partial \theta}$ is monotonically decreasing in X_1 for $X_1 > 0$. As shown in the proof in Appendix A.6, $X_1 \geq \frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)$ holds. Therefore, we can show that $\frac{\partial f}{\partial \theta} < 0$:

$$\frac{\partial f}{\partial \theta} = \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta} \tag{A.59}$$

$$\leq \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{\left(\frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)\right)} - 1\right)\frac{\delta}{2\eta} \tag{A.60}$$

$$= -\frac{1}{\beta_{11}}\cdot\frac{\delta}{2\eta} \tag{A.61}$$

$$< 0 \tag{A.62}$$

Assuming (3.2) holds, we have that $\frac{\partial f}{\partial X} < 0$, as shown in Proposition 5. Then, (A.55) can be rewritten as:

$$\frac{\partial f}{\partial \theta} - \frac{X_1}{1-\theta}\cdot\frac{\partial f}{\partial X} < 0 \tag{A.63}$$

Plugging in Eqs. (A.58) for $\frac{\partial f}{\partial \theta}$ and (A.33) for $\frac{\partial f}{\partial X}$ into (A.63), condition (A.63) can be rewritten to:

$$\frac{\partial f}{\partial \theta} - \frac{X_1}{1-\theta}\cdot\frac{\partial f}{\partial X} < 0 \Leftrightarrow \frac{1}{4\eta(1-\theta)(\beta_{01}-1)\beta_{11}X_1(r-\mu)}\cdot h(X_1) < 0 \tag{A.64}$$

where

$$h(X_1) = -(\beta_{11}-1)X_1^2 + 2\beta_{11}(1-\theta)\delta(r-\mu)X_1 - (\beta_{11}+1)(1-\theta)^2\delta^2(r-\mu)^2 \tag{A.65}$$

Since $\frac{1}{4\eta(1-\theta)(\beta_{01}-1)\beta_{11}X_1(r-\mu)} > 0$, we have that $\frac{dX}{d\theta} < 0$ if and only if $h(X_1) < 0$. h is a parabola that opens downward with the following two zeros:

$$X_{h,L} = (1-\theta)\delta(r-\mu) \tag{A.66}$$

$$X_{h,R} = \frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu) \tag{A.67}$$

We have shown that $X_1 > X_{h,R}$ in the proof of Proposition 5 as $X_{h,R}$ is the lower bound on X_1 by assuming all value is lost after subsidy withdrawal. Therefore, $h(X_1) < 0$ and we conclude that $\frac{dX}{d\theta} < 0$.

Deriving the conditions for $\frac{dK_1}{d\theta} < 0$ if condition (3.2) holds, can be shown by starting with total differentiation:

$$\frac{dK_1}{d\theta} = \frac{\partial K_1}{\partial \theta} + \frac{\partial K_1}{\partial X}\cdot\frac{\partial X_1}{\partial \theta} \tag{A.68}$$

As previously derived:

$$\frac{\partial K_1}{\partial \theta} = \frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} > 0 \tag{A.69}$$

$$\frac{\partial K_1}{\partial X} = \frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2} > 0 \tag{A.70}$$

$$\frac{\partial X_1}{\partial \theta} = -\frac{\left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta}}{\frac{\partial f}{\partial X}} \tag{A.71}$$

We can rewrite expression (A.68) to:

$$\frac{dK_1}{d\theta} = \left(1 + \frac{1-\theta}{X_1}\cdot\frac{\partial X_1}{\partial \theta}\right)\cdot\frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} \tag{A.72}$$

When $\frac{dX_1}{d\theta} < 0$ holds, it follows that $\frac{\partial X_1}{\partial \theta} < -\frac{1-\theta}{X_1}$ from (A.53), hence $\frac{dK_1}{d\theta} < 0$.

Appendix B. Robustness under iso-elastic demand

This appendix performs a robustness analysis on the results of Sections 3 and 5 by replacing the linear demand curve (2.1) with an iso-elastic curve. In case of iso-elastic demand, the output price at time t, $P(t)$, is given by:

$$P(t) = X(t)K^{-\gamma}, \tag{B.1}$$

where K is the firm's installed capacity, and $\gamma \in (0,1)$ is the elasticity parameter. X follows the GBM, defined in (Eq. (2.2)).

For this analysis we make two additional assumptions (see also Huisman and Kort, 2015). Firstly, the costs of investing in a capacity of size K are $\delta_1 K + \delta_2$, where $\delta_2 > 0$ is the fixed cost component. Secondly, we assume $\beta_{01}\gamma > 1$, where β_{01} is defined as before. Under these assumptions the firm's optimal investment decision again consists of a threshold that determines the timing without subsidy, X_0, and an investment size without subsidy, K_0^*.

The firm's optimization problem is given by:

$$F(x,\theta) = \sup_{\{\tau,K\}} \mathbb{E}\left[\int_\tau^\infty P(t)Ke^{-rt}dt - \left(1-\theta\cdot 1_{\xi(\tau)}\right)\cdot(\delta_1 K + \delta_2)e^{-r\tau} \,|\, X(0) = x, \xi(0) = 1\right] \quad \text{(B.2)}$$

with $P(t)$ as in (B.1) and

$$\xi(t) = \begin{cases} 0 & \text{if subsidy retraction has occurred at time } t \text{ or earlier} \\ 1 & \text{otherwise} \end{cases} \quad \text{(B.3)}$$

We take the same steps as in Section 2 to solve the optimization problem in (B.2). To do so, we can derive the firm's optimal investment decision when the subsidy has been abolished. The result is stated in Proposition 7.

Proposition 7. *When the subsidy is abolished, the optimal investment threshold is given by*:

$$X_0 = \frac{(K_0^*)^\gamma}{1-\gamma}\cdot\delta_1(r-\mu) \quad \text{(B.4)}$$

whereas the corresponding investment size is given by:

$$K_0^* = \frac{\beta_{01}(1-\gamma)}{\beta_{01}\cdot\gamma-1}\cdot\frac{\delta_2}{\delta_1} \quad \text{(B.5)}$$

where β_{01} is the positive solution to the fundamental quadratic, as defined in Proposition 1.

Proof. The proof takes the same steps as the proof of Proposition 2 in Appendix A.3 and is therefore omitted.

Proposition 8 presents the firm's optimal investment decision under isoelastic demand when the subsidy is still available, but subject to subsidy retraction risk.

Proposition 8. *If the investment subsidy has not been retracted yet, the optimal investment threshold X_1 is implicitly given by*:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1\cdot(K_1^*)^{1-\gamma}}{r-\mu} + (1-\theta)\cdot(\delta_1 K_1^* + \delta_2) = 0 \quad \text{(B.6)}$$

in which K_1^* is the optimal capacity under subsidy when investing at $X = X_1$, i.e.:

$$K_1^* = \left(\frac{(1-\gamma)X_1}{(1-\theta)\delta_1(r-\mu)}\right)^{\frac{1}{\gamma}} \quad \text{(B.7)}$$

Proof. The proof takes the same steps as the proof of Proposition 3 in Appendix A.4 and is therefore omitted.

Proposition 9 contains the socially optimal investment decision.

Proposition 9. *The social planner maximizes the total surplus, which under iso-elastic demand, is given by*:

$$\text{TS}(X,K) = \frac{XK^{1-\gamma}}{r-\mu} - (\delta_1 K + \delta_2) \quad \text{(B.8)}$$

The optimal timing maximizing the total surplus, X_S, is equal to:

$$X_S = (K_S^*)^\gamma\cdot\delta_1(r-\mu) \quad \text{(B.9)}$$

The socially optimal capacity, K_S^*, is given by:

$$K_S^* = \frac{\beta_{01}(1-\gamma)}{\beta_{01}\cdot\gamma-1}\cdot\frac{\delta_2}{\delta_1} \quad \text{(B.10)}$$

Proof. The proof is omitted as it takes the same steps as the proof of Proposition 4 in Appendix A.5.

We find that the investment threshold of the social planner is lower than the one of the firm when there is no subsidy (i.e. $X_S = (1-\gamma)X_0$). The firm without subsidy and the social planner do optimally invest in the same size (i.e. $K_S^* = K_0^*$). Like with linear demand, also here the social planner is more eager to invest than the firm. However, where in case of linear demand this results in more investment than the firm at the same optimal time, under iso-elastic demand the social planner invests sooner than the firm in the same investment size.

In order to study the robustness of our results in Sections 3 and 5, we provide a numerical example. We consider the following parameter values as in Table 2.

Table 2
Parameter values used in the iso-elastic demand scenario.

Notation	Parameter	Value
μ	Electricity price trend	2%
σ	Electricity price volatility	5%
r	Risk-free interest rate	6%
δ_1	Variable investment cost	150 €/MWh
δ_2	Fixed investment cost	200 €
γ	Demand elasticity	0.4

First, we discuss the results that are robust under iso-elastic demand. In short, the effect of subsidy withdrawal risk and subsidy size on the optimal investment timing and size as well as the effect of subsidy withdrawal risk effect on total surplus remains the same. Furthermore, it also holds that under iso-elastic demand a lump-sum subsidy can only speed up investment and decrease the investment size as a result. The only difference in results between linear and iso-elastic demand, is the optimal subsidy size to maximize welfare under subsidy withdrawal risk.

The effect of subsidy withdrawal risk and subsidy size on the firm's optimal investment decisions are shown in Figs. 9 and 10. Fig. 9 shows the optimal investment timing threshold with subsidy (X_1) and without subsidy (X_0) as functions of subsidy retraction risk λ, as well as the optimal investment size with subsidy (K_1^*) and without subsidy (K_0^*) as functions of subsidy retraction risk λ. Firstly, the results in Fig. 9 are conform Corollary 2. In case of no subsidy retraction risk (i.e. $\lambda = 0$) we have $X_0 < X_1$ and $K_0^* = K_1^*$, which was also the case for linear demand. Secondly, Fig. 9 is conform Proposition 5: the larger the subsidy retraction risk, the lower the optimal investment threshold and the lower the optimal capacity. In Fig. 10, the optimal investment decisions subject to large subsidy retraction risk are shown, where $\lambda = 1$. It shows that increasing subsidy size decreases both the investment threshold and the optimal investment size. These results are similar to Fig. 3.

Next, we present two figures representing total surplus as a function of the welfare retraction probability and the subsidy size. Fig. 11 shows that increasing subsidy retraction risk harms welfare, independent of the size of the subsidy, which coincides with the conclusion drawn from Fig. 7 in the linear demand case. The only difference between the linear demand and iso-elastic demand case is that it could still be optimal in the linear demand case to implement a subsidy in case of a positive but not too significant subsidy withdrawal risk, which is not the case under iso-elastic demand (see Fig. 12).

As can be seen in Fig. 11, a larger subsidy withdrawal risk decreases total surplus and no subsidy withdrawal risk is optimal; both results are identical to the results in Section 5. The sensitivity of the total surplus with respect to the subsidy size parameter is slightly different from before, as the investment cost structure between the linear demand case in Section 5 and the iso-elastic demand case differ. As before, the maximum total surplus is largest when the subsidy withdrawal risk is zero. In this case, the maximum total surplus is attained by setting subsidy size $\theta = 0.4$. It is optimal not the implement a subsidy when the subsidy withdrawal risk is positive, as can be seen from both the middle and right panel of Fig. 11. From the middle and right panel of Fig. 11, it shows that the subsidy size θ has a non-monotonic effect on the total surplus. This is the result of two different effects that work in opposite direction. Firstly, increasing subsidy size lowers the firm's optimal investment threshold (left panel of Fig. 10) causing the expected time to investment to decrease and, therefore, increases the total surplus. Secondly, a larger subsidy size lowers the firm's optimal investment size (right panel of Fig. 10) decreasing consumer surplus, which has a negative effect on total surplus. The upward jump in the total surplus just after $\theta = 0.4$ is caused by the fact that the firm's expected time to invest drops to zero. As the subsidy becomes larger, there is a point at which the firm's optimal investment threshold is equal to or smaller than the starting value of the GBM, which is assumed to be equal to 10. This means that investment is done immediately, which is beneficial for total surplus.

In Fig. 13, we study the role of subsidy withdrawal risk on the social planner's ability to reach certain capacity targets as soon as possible. Similarly to the results under linear demand shown in Fig. 6, a lump-sum subsidy can only speed up investment at the cost of a lower investment size.

Appendix C. Additional derivations

C.1. Derivation of constant relative welfare loss under no subsidy

Let X_S and K_S^* denote the socially optimal timing and capacity, and let X_0 and K_0^* be the firm's optimal timing and capacity without any subsidy. Using that $X_S = X_0$ and $K_S^* = 2K_0^*$, and expressions (2.12) and (2.13) for X_0 and K_0^*, the relative welfare loss is equal to:

$$\begin{aligned}
\text{RWL} = \frac{\text{TS}(X_S, K_S^*) - \text{TS}(X_0, K_0^*)}{\text{TS}(X_S, K_S^*)} &= \frac{\frac{X_S(2-\eta K_S^*)K_S^*}{2(r-\mu)} - \delta K_S^* - \left(\frac{X_0(2-\eta K_0^*)K_0^*}{2(r-\mu)} - \delta K_0^*\right)}{\frac{X_S(2-\eta K_S^*)K_S^*}{2(r-\mu)} - \delta K_S^*} \\
&= \frac{\frac{4X_0(1-\eta K_0^*)K_0^*}{2(r-\mu)} - 2\delta K_0^* - \frac{X_0(1-\eta K_0^*)K_0^* + X_0K_0^*}{2(r-\mu)} + \delta K_0^*}{\frac{4X_0(1-\eta K_0^*)K_0^*}{2(r-\mu)} - 2\delta K_0^*} \\
&= \frac{3X_0(1-\eta K_0^*) - X_0 - 2\delta(r-\mu)}{4X_0(1-\eta K_0^*) - 4\delta(r-\mu)} \\
&= \frac{\frac{3}{2}(X_0 + \delta(r-\mu)) - (X_0 + 2\delta(r-\mu))}{2(X_0 + \delta(r-\mu)) - 4\delta(r-\mu)} \\
&= \frac{1}{4}
\end{aligned}$$

C.2. Stochastic discount factor and expected time to investment

When analyzing the effect of a subsidy on welfare, we need to take into account that a subsidy speeds up investment, and thus investment is done at a different time under subsidy than without the subsidy. As we compare welfare outcomes under different times, we need to discount both the welfare with and without subsidy properly. This subsection shows that the discount factor for investment without subsidy is equal to:

$$S_0 = \left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.1}$$

We also derive that when investment is influenced by a subsidy subject to subsidy retraction risk, the discount factor is equal to:

$$S_1 = \text{P}[s > \tau_1] \cdot \left(\frac{x}{X_1}\right)^{\beta_{01}} + (1 - \text{P}[s > \tau_1]) \cdot \left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.2}$$

where

$$\text{P}[s > \tau_1] = \exp\left\{\frac{(X_1 - x)}{\sigma}\left(\frac{\mu}{\sigma} - \sqrt{\frac{\mu^2}{\sigma^2} + 2\lambda}\right)\right\} \tag{C.3}$$

The discount factor for discounting investment without subsidy risk has been derived in Dixit and Pindyck [1994] and has been addressed in Huisman and Kort [2015].

To derive the stochastic discount factor for investment under subsidy subject to subsidy retraction risk, we need to derive the expected time to investment. We define the first hitting times of the thresholds as follows:

$$\tau_0 = \min\{t : X(t) \geq X_0\} \tag{C.4}$$

$$\tau_1 = \min\{t : X(t) \geq X_1\} \tag{C.5}$$

Furthermore, let s be the time at which the exponential jump process with parameter λ has its first jump. Then the expected time to investment τ^* can be written as follows:

$$\tau^* = \text{P}[s > \tau_1] \cdot \mathbb{E}[\exp(-r \cdot \tau_1)] + (1 - \text{P}[s > \tau_1]) \cdot \mathbb{E}[\exp(-r \cdot \tau_0)] \tag{C.6}$$

The first part of the sum takes the scenario in which the first exponential jump occurs after the first hitting time of investment threshold X_1. In that case, the first hitting time of X_1 is relevant for our solution. The second part of the sum takes the scenario in which the first exponential jump occurs before the first hitting time of investment threshold X_1. Then, the policy is withdrawn before we invest and we are no longer interested in the first time the GBM process reaches X_1, but the first hitting time of threshold X_0 is the relevant stochastic variable.

In Eq. (C.6), the analytic expressions for $\mathbb{E}[\tau_0]$ and $\mathbb{E}[\tau_1]$ are known from, e.g., Dixit and Pindyck [1994, p. 315–316]:

$$\mathbb{E}[\exp(-r \cdot \tau_0)] = \left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.7}$$

$$\mathbb{E}[\exp(-r \cdot \tau_1)] = \left(\frac{x}{X_1}\right)^{\beta_{01}} \tag{C.8}$$

$\mathbb{P}[s > \tau_1]$ is the probability that the exponential jump occurs after the first time the GBM process X hits the threshold X_1. Thus, we compare two first passage times of two independent random processes. In general, this problem is solved by solving the following integral:

$$\int_0^\infty e^{-\lambda t} f_{\tau_1}(t) dt \tag{C.9}$$

where $f_{\tau 1}(t)$ is the density function of the hitting time of the GBM. Valenti et al. [2007] state that the distribution of time τ_1 for a GBM process X starting at x (see Eq. (2.2)) to reach threshold X_1 is given by the inverse Gaussian:

$$f(X_1,x)=\frac{X_1-x}{\sqrt{2\pi\sigma^2\tau_1^3}}\cdot e^{\frac{-(X_1-x-\mu\tau_1)^2}{2\sigma^2\tau_1}} \tag{C.10}$$

To simplify the derivation, we rewrite (C.10) into the standard form of an inverse Gaussian pdf:

$$f(\tau_1;X_1,x)=\frac{X_1-x}{\sqrt{2\pi\sigma^2\tau_1^3}}\cdot\exp\left\{\frac{-(X_1-x-\mu\tau_1)^2}{2\sigma^2\tau_1}\right\} \tag{C.11}$$

$$=\sqrt{\frac{\left(\frac{X_1-x}{\sigma}\right)^2}{2\pi\tau_1^3}}\cdot\exp\left\{-\frac{(X_1-x)^2}{\sigma^2}\cdot\frac{\left(\tau_1-\frac{X_1-x}{\mu}\right)^2}{2\tau_1\left(\frac{X_1-x}{\mu}\right)^2}\right\} \tag{C.12}$$

Expression (C.12) is an inverse Gaussian pdf with parameters $\widehat{\lambda}$ and $\widehat{\mu}$, where $\widehat{\lambda}=\left(\frac{X_1-x}{\sigma}\right)^2$ and $\widehat{\mu}=\frac{X_1-x}{\mu}$. So, from now on, we use

$$f(\tau_1)=\sqrt{\frac{\widehat{\lambda}}{2\pi\tau_1^3}}\cdot\exp\left\{-\widehat{\lambda}\cdot\frac{(\tau_1-\widehat{\mu})^2}{2\tau_1\cdot\widehat{\mu}^2}\right\} \tag{C.13}$$

for the pdf of the first hitting time.

Now the integral can be solved as follows:

$$\int_0^\infty \exp(-\lambda t)f_{\tau_1}(t)dt$$
$$=\int_0^\infty \exp(-\lambda t)\sqrt{\frac{\widehat{\lambda}}{2\pi t^3}}\cdot\exp\left\{-\widehat{\lambda}\cdot\frac{(t-\widehat{\mu})^2}{2t\widehat{\mu}^2}\right\}dt \tag{C.14}$$

$$=\exp\left\{\frac{\widehat{\lambda}}{\widehat{\mu}}\left(1-\sqrt{1+\frac{2\lambda\widehat{\mu}^2}{\widehat{\lambda}}}\right)\right\} \tag{C.15}$$

Plugging in the expressions for $\widehat{\lambda}$ and $\widehat{\mu}$ into (C.15), we get:

$$\exp\left\{\frac{\widehat{\lambda}}{\widehat{\mu}}\left(1-\sqrt{1+\frac{2\lambda\widehat{\mu}^2}{\widehat{\lambda}}}\right)\right\}$$
$$=\exp\left\{\frac{\mu(X_1-x)}{\sigma^2}\left(1-\sqrt{1+2\lambda\cdot\frac{\sigma^2}{\mu^2}}\right)\right\} \tag{C.16}$$

$$=\exp\left\{\frac{X_1-x}{\sigma}\left(\frac{\mu}{\sigma}-\sqrt{\frac{\mu^2}{\sigma^2}+2\lambda}\right)\right\} \tag{C.17}$$

The probability the exponential jump occurs after threshold X_1 is hit, is equal to the expression (C.17), in which x is the starting value of the GBM.

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