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THE TIMING OF TECHNOLOGY ADOPTION BY A COST-MINIMIZING FIRM

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The timing of technology adoption by a cost-minimizing firm

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Abstract

This paper deals with the timing of technology adoption by a cost-minimizing firm. Technological progress is assumed to reduce costs and is modeled by a geometric Brownian motion. The firm may have any number of options to switch to a more efficient technology. For each switch, a distinct option value is identified, causing a delay in the adoption of new technologies. As the value of each option increases when the number of remaining technology switches is reduced, under-estimating efficiency gains by future technology adoptions may result in an additional (suboptimal) delay in the adoption of new technologies.

1 Introduction

In a recent publication, Farzin, Huisman & Kort (1998) addressed the problem of the optimal timing of technology adoption by a competitive firm when technology choice is irreversible and the firm faces a stochastic innovation process. Based on the well-documented observation that '*the adoption of new technologies is a slow and incremental process*', the question is posed '*what explains the apparently cautious approach of firms to technology adoption*' (Farzin et al., 1998, p.780). Apparently, firms are often very hesitant

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to adopt new technologies and wait to switch beyond the point where investing in the new technology results in a positive net-present-value (npv). Using a standard production function $h(v, \theta) = \theta v^a$ in which v is a variable input and θ is the (stochastic) factor reflecting technological efficiency, they assume that technology evolves according to a Poisson jump process. Both the timing of the jumps in θ and the size of each jump are considered to be stochastic. The timing of the technology adoption is then derived from the objective of profit-maximization over an infinite time-horizon.

In order to determine the value θ^* triggering a technology switch, a backward recursive solution method is used, starting with a firm that has only one option to switch to a new technology. For the latter case, they find the interesting result that a new technology is adopted considerably later than the traditional npv-rule suggests. The extra delay is caused by the additional option cost at the time of adoption. Since the firm has only one option to switch technology, no future gains can follow from further technological progress after switching. Therefore firms will be hesitant to exercise the option. Unfortunately, when the model is generalized for multiple switches, no option value could be detected except for the last technology switch in a sequence. As a result, it is concluded that new technology will be adopted whenever the npv of the replacement is positive (except of course, for the last switch). This is a somewhat disturbing result, since in general there is no real limit on the number of technology switches. Therefore it could be argued that the model of Farzin et al. does not really solve the problem of delayed technology adoption.

In this paper, an alternative model will be presented to determine the optimal timing of technology adoptions. The model uses a different specification for technological progress and is based on cost-minimization. The purpose of this model is to offer an explanation for the cautious approach to technology adoption as described earlier, for the case in which the operator of the technology has an arbitrary finite or infinite number of switching options. First, we will show that the model identifies an option value for each switch in technology. Such option value constitutes an additional cost to be taken into account at each technology switch and therefore causes a delay in the adoption of new technologies. Second, it will be demonstrated that the value of this option to switch to a superior technology depends largely on the number of switches taken into account by the operator of the technology. As the number of options is reduced, the value of each option to switch to a new technology increases considerably. Therefore, if the operator does not fully take into account future options to switch to superior technology, he will overestimate the value of the first option to replace and therefore cause an additional (suboptimal) delay in the adoption of a new technology.

This paper is further structured as follows. In the next section, some recent publications on replacement and the timing of technology adoption will be reviewed. The main characteristics and the differences with the present analysis will be commented. Section 3 then proceeds with an argumentation for the use of a geometric Brownian motion to model technological progress. The actual problem of technology adoption will be analyzed in section 4, where two distinct cases will be identified. The first involves a single option to switch technology (single-shot technology adoption), discussed in section 5, the second involves a series of options (revolving technology adoption), discussed in section 6. The characteristics of the solution are further examined in section 7, followed by a summary of the main conclusions.

2 Some recent literature

The process of technology adoption is a well-known micro-economic problem, closely related to the theory of capital replacement. The role of technological progress was especially emphasized by authors as Terborgh (1949), Alchian (1952) and Smith (1966). More recently, the problem was addressed by Howe & McCabe (1983), Mauer & Ott (1995) and the author (1998). The growing attention for the role of uncertainty and the use of option-theory to study real investments (See e.g. Dixit & Pindyck, 1994 and Trigeorgis, 1996) highlighted the problem again in recent years.

Contrary to Farzin et al., the work of Mauer and Ott deals with several kinds of uncertainty. Aside from technological progress, the impact of uncertainty about operating costs, tax regimes etc. is examined. Their specification of technological progress by means of a Poisson-process is similar to the single-switch model of Farzin et al., but they only consider a single jump in the state of technology, after which the new technology will remain in operation indefinitely. Grenadier & Weiss (1997) also examine the timing of technology adoption, using a different specification for technological progress. They also treat technological progress as a jump process, but they assume a discrete jump in the available technology appears when some variable following a Brownian motion hits a predefined barrier. Their analysis is restricted to two replacements, but they allow lagging, i.e. it is possible to adopt a technology different from the most recent one.

The approach in this paper is somewhat different from the work mentioned before. Technological progress will be treated as a continuous process (more exactly: a geometric Brownian motion). As in the work of Farzin et al., any number of technology switches will be examined. However, it will be demonstrated that there is a distinct option value at each stage of the adoption process. Consequently the traditional npv-rule does not apply. We will also show that the optimal timing of technology adoption is severely influenced by the number of switches in the chain.

The instruments used in this paper are closely related to the ones used to analyze financial options. The case in which a single technology switch is allowed is comparable to an American financial option with an infinite expiration date. Not exercising the option causes a cost in the form of inefficient production. An important difference is created when allowing for multiple switches. In that case exercising the option to adopt a new technology automatically creates a new option. In the case where multiple subsequent switches are considered, these form a stream of nested options. In order to determine the optimal time to exercise the first option, the value of all subsequent options has to be known. The solution method for this problem is reminiscent of Dixit & Pindyck (1994, p.319) and is similar to the one used by Farzin et al. (1998) for a Poisson-process and Bethuyne (1998) for a deterministic model. Starting from a simple model with only one technology switch, an optimal switching rule will be determined using a recursive solution method.

The mathematical techniques used in this paper are described in various works. We can refer to the standard work of Pindyck (1991), Dixit (1993), Dixit & Pindyck (1994), Kamien & Schwartz (1995) and Ingersoll (1987). The latter work, together with the work of Cox & Miller (1972) and Karlin & Taylor (1971, 1975) can also be helpful for the understanding of some of the more advanced features of stochastic processes.

3 Technological progress

Technological progress has an impact on production in various ways, often also in ways that are very difficult to analyze in a purely mathematical form. Technology can improve the output of equipment while costs remain unchanged, or it can reduce costs while leaving production unaffected. Also, it could modify the characteristics (quality, durability, ...) of the output, ... The kind of technological progress considered here only affects costs. We will assume that the output of the equipment is unaffected, in quantity as well as in quality. Furthermore we will assume that the effect of technology on costs can be described by an index θ which follows a geometric Brownian motion: $d\theta = -g\theta\, dt + \sigma\theta\, dz$, where dz describes a Wiener process. In this sense, technological progress can be considered as a form of input cost uncertainty as described by Pindyck (1993), who also uses a Brownian motion to model uncertainty. The negative sign of the drift-rate refers to the fact that technological progress reduces costs. Of course the Brownian motion may not always be the best type of stochastic process to model technological progress. For instance, in the case where technology advances in a less continuous way and is more susceptible to sudden jumps caused by inventions, Poisson processes may indeed be the better choice.

However, in other instances the use of a Brownian motion to model technological progress can easily be motivated. A variable following a Brownian motion will change continuously following a certain trendline. This is clearly the case with certain types of technology, where knowledge is gathered gradually. However, the Brownian motion also allows brief movements against the trendline. This would imply a regression of technology, making more recent technology less efficient than older technology. At first sight this may seem counter-intuitive, but on further inspection it is not that unrealistic. Indeed, although it could be argued that knowledge is cumulative and a regression in knowledge hardly ever occurs, it should not be forgotten that θ represents the cost of the latest technology. A certain technology can incorporate more knowledge and be technically more efficient than a previous one and still cost more. Therefore over sufficiently short intervals of time, small regressions in the effect of technology on costs may seem very well in line with reality, while over larger periods of time it can be expected that the most recent technology will be also the most cost-efficient. A geometric Brownian motion has the additional advantage that the rate of technological progress remains constant. This is not the case with for instance a Poisson process, where the absolute size of the jumps is constant. The Poisson-specification of Farzin et al. implies that over a long period of time, the relative advantages of technological progress diminish and tend to zero.

4 Technology adoption

Consider an operator that currently uses equipment of technology θ_0. Assume that the cost of production with this equipment can be expressed as:

$$m(\theta_0) = m\theta_0 \tag{1}$$

where m is a constant. Let $C(\theta_t; \theta_0)$ be the expected present value of the production cost at t over an infinite time-horizon, for an operator using technology θ_0, when the state of the most recent technology is θ_t (θ_0 is a parameter and will be mentioned only when useful). If the firm cannot switch to the more advanced technology, $C(\theta_t)$ will simply be $\frac{m\theta_0}{i}$ (in which i is the interest rate), i.e. the perpetual operating cost of the existing equipment. However, we will assume that the operator has one or more options to switch to a more efficient technology $m(\theta_t) = m\theta_t$, at a cost $P(\theta_t) = P\theta_t$. Therefore $C(\theta_t)$ will be smaller than $\frac{m\theta_0}{i}$ because of potential gains from the technology switch.

Assume that the state of technology θ_t follows a stochastic process. At any time before the replacement the expected present value of all future costs $C(\theta_t)$ can be divided in immediate operating cost $m\theta_0\, dt$ over the next infinitesimal period of time and (expected) future costs $\varepsilon\left[C\left(\theta_{t+dt}\right)\right]$ in the following manner:

$$C(\theta_t) = m\theta_0\, dt + \frac{\varepsilon\left[C\left(\theta_{t+dt}\right)\right]}{1 + i\, dt} \tag{2}$$

Because the right-hand side expresses the cost as it appears before replacement, we will call it the *continuation cost.* The first term (the immediate operating cost $m\theta_0\, dt$) is certain because it depends on the known state of technology θ_0. The second term is the expected present value of all future cost at $t+dt$. Expectations are taken with respect to the state of technology dt units of time from the present.

Multiplying by $1 + i\, dt$ and omitting terms of order 2 in dt, the objective function takes the form:

$$C(\theta_t)\left(1 + i\, dt\right) = m\theta_0\, dt + \varepsilon\, C\left(\theta_{t+dt}\right) \tag{3}$$

Or after rearranging and omitting subscript t:

$$iC(\theta) = m\theta_0 + \frac{\varepsilon\left[dC\left(\theta\right)\right]}{dt} \tag{4}$$

Eq.(4) is a differential equation, expressing the evolution of costs in relation to the underlying stochastic variable θ. The total cost of operating the present equipment and its future challengers over an infinite time horizon has an expected present value of $C(\theta)$, with a corresponding opportunity cost of $iC(\theta)$. The cost of keeping the defending equipment in operation can be split in immediate (operating) costs and the expected change in future costs due to technological progress and deterioration. Notice the analogy with the return on stocks and bonds (which of course is maximized instead of minimized). The expected return of stocks or bonds can also be split in an immediate return (dividends or coupons) and the expected change in value (capital gains). The source of $\varepsilon\left[dC\left(\theta\right)\right]$ is to be found in changes in the state of available technology. If there were no opportunity to replace the defending technology with a technologically more advanced challenger, $\varepsilon\left[dC\left(\theta\right)\right]$ would simply be zero.

Let $\Omega(\theta)$ be the (expected) present value of all future costs at the time a new technology θ is adopted, including the switching cost $P\theta$ itself and all costs incurred afterwards. The specific nature of $\Omega(\theta)$ will be of major importance for the optimal timing of the switch and will depend on the number of technology switches available to the operator. The cost of the operator can now be expressed as:

$$C\left(\theta\right)=\begin{cases}\frac{m\theta_0}{i}+\frac{1}{i}\frac{\varepsilon[dC(\theta)]}{dt} & \text{for } \underline{\theta}>\theta^* \\ \\ \Omega(\theta^*) & \text{for } \underline{\theta}\leq\theta^*\end{cases} \tag{5}$$

in which at any time, $\underline{\theta}$ is the lowest value of θ reached before that moment and θ^* is the critical value of θ triggering a technology switch.

The differential $dC\left(\theta\right)$ in this expression deserves some further attention, since it involves the stochastic variable θ. Following Itô's lemma (see the appendix for more details):

$$dC\left(\theta\right)=\left(-g\theta C'\left(\theta\right)+\frac{1}{2}\sigma^2\theta^2C''\left(\theta\right)\right)dt+\sigma\theta C'\left(\theta\right)dz \tag{6}$$

As long as $\underline{\theta}>\theta^*$, substituting eq.(6) in objective function (5) expresses $iC\left(\theta\right)$ as:

$$iC\left(\theta\right)=m\theta_0-g\theta C'(\theta)+\frac{1}{2}\sigma^2\theta^2C''(\theta) \tag{7}$$

The result is a Cauchy differential equation. In appendix a general solution to this equation is determined, of the form:

$$C(\theta)=\frac{m\theta_0}{i}+F_1\theta^{\gamma_1}+F_2\theta^{\gamma_2} \tag{8}$$

In which F_j represent the (still unknown) constants of integration and γ_j are the (real) roots of the characteristic equation corresponding to this differential equation:

$$\gamma_j=\frac{1}{2}+\frac{g}{\sigma^2}\pm\sqrt{\left(\frac{1}{2}+\frac{g}{\sigma^2}\right)^2+\frac{2i}{\sigma^2}} \tag{9}$$

Notice that the smallest of both roots $\gamma_1<0$ and the largest $\gamma_2>1$.

The cost $C(\theta)$ of operating the present equipment and all succeeding challengers over an infinite time horizon breaks down in two components: $\frac{m\theta_0}{i}$ and $F_1\theta^{\gamma_1}+F_2\theta^{\gamma_2}$. If the state of technology did not change, replacement would never occur and the cost would be the simple constant perpetuity $\frac{m\theta_0}{i}$ reflected by the first component. The second component however is caused by the fact that at some point, when a Brownian motion hits a barrier, action may be undertaken. In this case, a replacement with cost $\Omega(\theta^*)$ will occur when θ hits a lower barrier θ^*. When the state of technology degenerates ($\theta \to \infty$) the operator will simply keep the defender perpetually.

This information can be used to determine the constants of integration F_j and the barrier θ^*. When $\theta = \theta^*$ the following boundary condition must hold (*value-matching condition*):

$$C(\theta^*) = F_1\theta^{\gamma_1} + F_2\theta^{\gamma_2} + \frac{m\theta_0}{i} = \Omega\left(\theta^*\right) \tag{10}$$

If the state of technology degenerates $(\theta \to \infty)$, no action will occur and the cost will be the perpetual operating cost of the defender:

$$\lim_{\theta\to\infty} C(\theta) = \lim_{\theta\to\infty} \left(F_1\theta^{\gamma_1} + F_2\theta^{\gamma_2} + \frac{m\theta_0}{i}\right) = \frac{m\theta_0}{i} \tag{11}$$

which serves as the second boundary condition. Since $\gamma_2 > 1$, eq.(11) implies that $F_2 = 0$. For simplicity, we will denote the remaining constant as F instead of F_1 and the remaining root as γ instead of γ_1.

This still leaves eq.(10) with two unknown variables, namely the remaining constant of integration F and the unknown barrier θ^*. However, for the barrier θ^* to be an optimal stopping point, the continuation cost is required to be tangent to the terminal cost function: $C'(\theta^*) = \Omega'\left(\theta^*\right)$ (also called the *smooth-pasting condition* - see e.g. Dixit, 1993, p.34). Therefore F and θ^* can then be found solving the system:

$$\begin{aligned} C(\theta^*) &= F\theta^{*^{\gamma}} + \frac{m\theta_0}{i} = \Omega\left(\theta^*\right) &(12)\\ C'(\theta^*) &= \gamma F\theta^{*^{(\gamma-1)}} = \Omega'\left(\theta^*\right) &(13)\end{aligned}$$

Notice that $F\theta^{*^{\gamma}}$ is the cost reduction due to the option to replace the defender. Obviously when the state of technology degenerates $(\theta \to \infty)$, the option becomes worthless[1]. A further comment on the endpoint function $\Omega\left(\theta\right)$ is in order now. At this stage of the analysis, we still assume that the operator considers only one technology adoption.

[1] Notice also that the value of the option to replace is only defined for $\theta \geq \theta^*$, since F is determined using the information that the diffusion process followed by θ is terminated at θ^*.

5 Single-shot technology adoption

If the operator has only one option to adopt a new technology, the terminal cost $\Omega(\theta)$ consists of the net-installation cost of the new technology and the cost of operating the new technology perpetually:

$$\Omega(\theta) = \left(\frac{m}{i} + P\right)\theta \tag{14}$$

Using this information, from eq.(13) we can determine the value of the constant $F = \left(\frac{m}{i} + P\right)\frac{1}{\gamma}\theta^{*(1-\gamma)}$. Notice that for values of $\theta^* > 0$, F is negative. This is indeed what could be expected, since the option to replace should be a cost-reduction vis-à-vis the situation where the old technology is maintained perpetually. Substituting the result of F in eq.(12) shows the following switching value of θ:

$$\theta^* = \frac{\gamma}{\gamma - 1}\left(1 + \frac{iP}{m}\right)^{-1}\theta_0 = \xi^{-1}\theta_0 \tag{15}$$

Substituting the constant F and the previous result for θ^* in eq.(12) gives the minimal expected cost at the moment the switch is made ($\theta = \theta^*$):

$$C^* = \frac{m\theta_0}{i}\left[\frac{1}{\gamma}\left(\frac{\gamma}{\gamma - 1}\right)^{1-\gamma}\left(1 + \frac{iP}{m}\right)^{\gamma} + 1\right] \tag{16}$$

The optimal switching value θ^* in eq.(15) is homogenous of degree 0 in P and m. Let $\phi = \frac{\gamma - 1}{\gamma}$, then eq.(15) can also be written as:

$$\frac{m}{i}\theta_0 = \phi\left(\frac{m}{i} + P\right)\theta^* \tag{17}$$

This format allows an interesting interpretation. The left-hand side of eq.(17) represents the cost of operating the original technology perpetually. The right-hand side contains (apart from the factor ϕ) the cost of adopting the new technology P and the cost of operating it perpetually.

The result implies that the new technology will be adopted considerably later than the normal npv-rule suggests. Indeed, applying the npv-rule would lead to a technology switch at $\theta = \theta_{NPV}$ satisfying $\frac{m\theta_0}{i} = \left(\frac{m}{i} + P\right)\theta_{NPV}$. Due to uncertainty and future growth-potential, replacement will be postponed until the cost of the old technology is a factor $\phi > 1$ higher than the cost of the new technology. For reasonable values of the different parameters in the problem, this implies that the new technology has to be substantially more efficient before it is adopted. The result even remains when the evolution of technology is certain instead of stochastic, since:

$$\lim_{\sigma \to 0} \phi = \lim_{\sigma \to 0} \frac{\gamma - 1}{\gamma} = \frac{i+g}{i} \tag{18}$$

which transforms the result of eq.(17) into a result determined elsewhere for deterministic replacement models (Bethuyne, 1998).

It is an important observation that the optimal switching value θ^* is uniquely determined by the ratio of switching cost and instantaneous operating costs P/m, which allows to characterize a technology by a single ratio and creates a direct and easy to interpret link between the timing of technology adoption and the intensity[2] the equipment is used with. Indeed, as equipment of given technology is used less intensively, the ratio of capital over operating costs will rise. The cost ratio of old and new technology $\frac{\theta^*}{\theta_0} = \xi^{-1}$ is a decreasing function of P/m, implicating that new technology will be adopted first by the most intensive users. Notice that for very low intensities the adoption of new technology can be postponed quite long. Indeed, in the limit $\lim_{P/m \to \infty} (\theta^*/\theta_0) = 0$. When switching costs P becomes marginal compared to operating costs m, there will be still be a considerable delay in the adoption of new technology, since $\lim_{P/m \to 0} (\theta^*/\theta_0) = \phi^{-1} < 1$. The value of the option to adopt the new technology is expressed as:

$$F\theta^\gamma = \left(\frac{m}{i} + P\right) \frac{1}{\gamma} \theta^{*^{1-\gamma}} \theta^\gamma \tag{19}$$

Since a decrease of the intensity of utilization lowers both operating cost m and the value of the switching value θ^*, the option looses value for low-intensity users.

For a numerical illustration, we assume a drift-rate of technological progress of $g = 3\%$ and a standard deviation of the Wiener-process $\sigma = 5\%$. The interest-rate is set at 5%. The operator has the option the switch to a new technology, but this technology adoption is of the single-shot type (i.e. no further technology switches are allowed afterwards) and the switch itself costs $P\theta$. The value of θ^* is determined for P/m taking values 0 , 1, 4 , 10 and 25, as illustrated in the next table. We also report values of $\frac{iF}{m}$. Since $\theta_0 = 1$, the value of F can be directly interpreted as the value of the option to replace at $t = 0$ and $\frac{iF}{m}$ expresses the option value relative to the

[2] One way to define intensity of utilization is the rate at which output is produced using equipment of a certain technology, relative to its maximum output rate.

	$P/m = 0$	$P/m = 1$	$P/m = 4$	$P/m = 10$	$P/m = 25$
θ^*	60.142%	57.278%	50.119%	40.095%	26.730%
iF/m	-18.506%	-17.192%	-14.055%	-10.037%	-5.444%

Figure 1: Table 1. Single-shot technology adoption

cost of using the existing technology perpetually (or the relative expected cost-savings due to the option to adopt a better technology later). For completeness, $\gamma = -1.509$ and $\phi^{-1} = 0.601$ (which are unaffected by P/m).

For known values of P and m, the problem can also be represented graphically as in figure 1. It illustrates the value of the $C(\theta)$ (i.e. the present value of all future costs) and the stochastic factor θ. In order not to complicate the figure unduly, we only represent the case for $P = 1000$ and $m = 100$. In this case, the horizontal represents the present value of

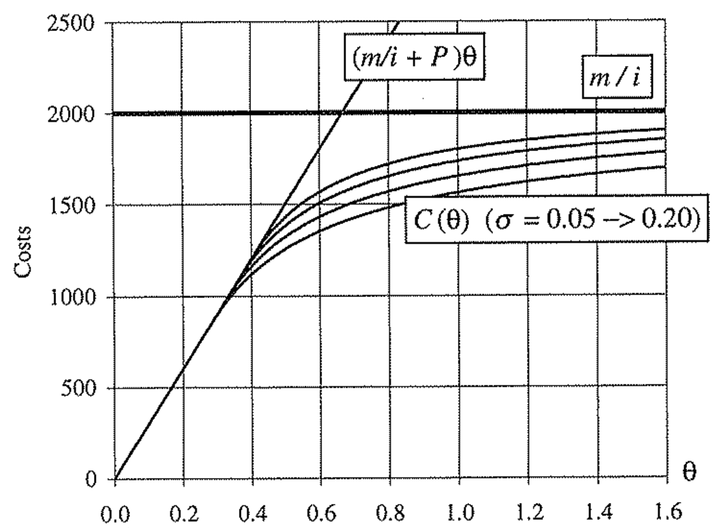

Figure 2: Continuation and terminal cost in the single-shot technology adoption model

the operating costs if the old were to be maintained indefinitely, namely $\frac{m}{i} = 2000$. The upward sloping straight line represents the present value of the costs incurred when the more advanced technology is installed instead: $\left(P + \frac{m}{i}\right)\theta = 3000.\theta$. To the right of $\theta^* = 0.40095$ it is optimal to maintain the old technology. The value of the $C(\theta)$ is then $\frac{1}{\gamma}\left(\frac{m}{i} + P\right)\theta^{*(1-\gamma)}\theta^{\gamma} + \frac{m\theta_0}{i}$ (the curved lines in figure 1). The first term represents the value of the option to replace, represented by the vertical difference between the horizontal line and the curved line. Notice that there are four different levels of $C(\theta)$ in figure 1, corresponding to different values of the standard deviation in the stochastic process ($\sigma = .05, .10, .15$ and $.20$). The lower the standard deviation, the higher the level of the objective function. In $\theta = \theta^*$, the curved line is tangent to the upward-sloping line, as is required in the value-matching and the smooth-pasting conditions. An increase in the standard deviation shifts the point of tangency to the left, thus postponing the technology switch.

Finally, it is now also possible to analyze the sensitivity of θ^* to changes in the degree of uncertainty, the rate of technological growth and the ratio of P/m. The results of θ^* for g and σ ranging from 0 to 10% and for P/m ranging from 0 to 25 are given in figure 2. If $g = \sigma = 0$, there will be no option value and $\theta^* = \theta_{NPV}$. As g and σ increase, the corresponding effect on θ^* can easily be found in figure 2. As expected, the switching technology θ^* decreases for higher values of P/m. It turns out that the level of the switching technology is more sensitive to changes in the growth-rate of technological progress than to changes in its standard deviation. Also, the relative effect of uncertainty seems to decrease with higher values of g.

6 Revolving technology adoption

The next step will be to generalize the model by increasing the number of possible technology switches, thus transforming the one-shot model in a revolving technology adoption model. We will consider a finite chain of n subsequent replacements and derive optimality conditions for each technology switch in the process in a backward recursive way. Obviously, the last switch in the chain can be considered as a simple single-shot replacement problem.

Let t_{n-1} be the time when the one but last technology was put in operation, and let the level of technological progress at that moment be θ_{n-1}. At this moment the operator faces a single-shot technology adoption model

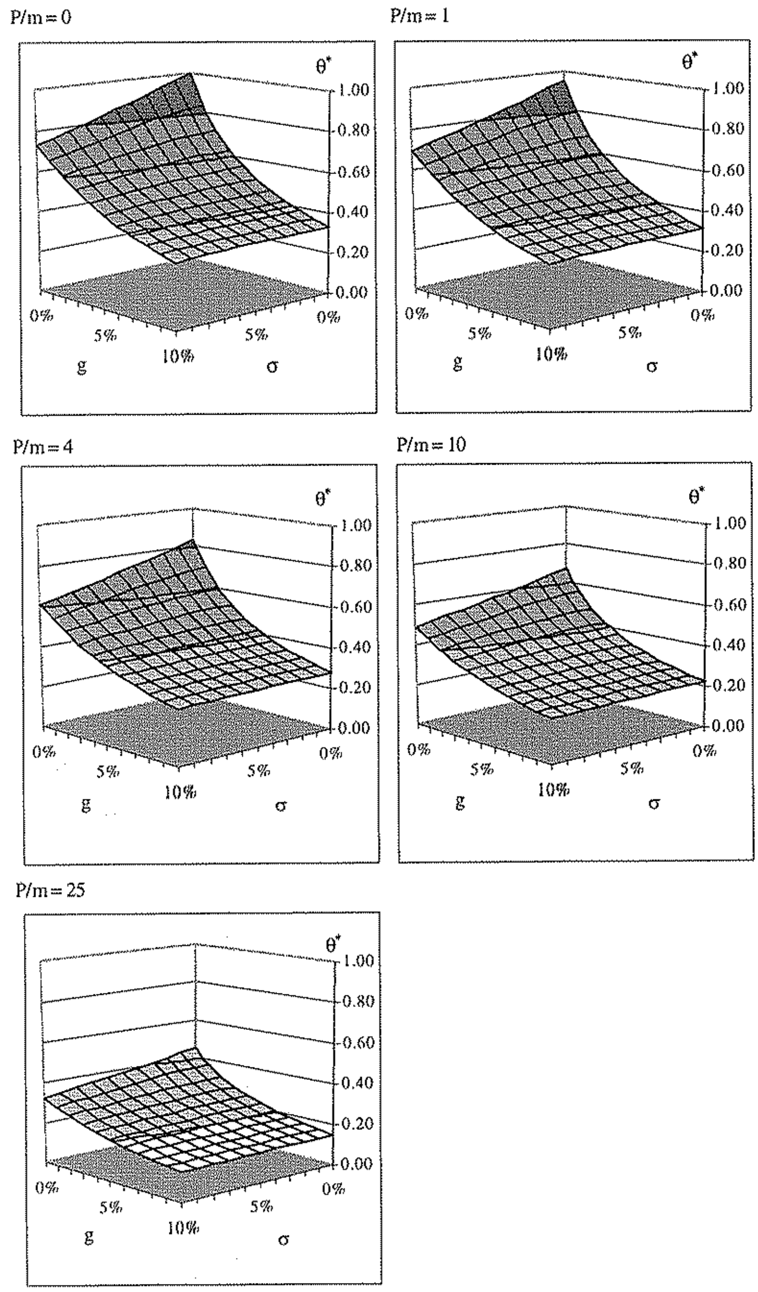

Figure 3: Switching technology in function of g and σ for different values of P/m

with optimal values of θ and C:

$$\theta_n^* = \frac{\gamma}{\gamma-1}\left(\Phi_0 + \frac{iP}{m}\right)^{-1}\theta_{n-1} \tag{20}$$

$$C_{n-1}^* = \frac{m\theta_{n-1}}{i}\Phi_1 \tag{21}$$

as in the previous section, in which:

$$\Phi_0 = 1 \tag{22}$$

$$\Phi_1 = \frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_0 + \frac{iP}{m}\right)^{\gamma} + 1 \tag{23}$$

The role of the coefficient Φ_0 will become clear when evaluating the preceding replacement decisions.

Next assume that at t_{n-2}, the operator has just installed new equipment and has left two options to adopt new technology. The objective will now be to choose the timing of both switches in order to minimize costs. In the continuation region (i.e. as long as no replacement is carried out) the minimal cost will again be of the form:

$$C_{n-2}(\theta_{n-1}) = G\theta_{n-1}^{\gamma} + \frac{m\theta_{n-2}}{i} \tag{24}$$

(G is a constant). As in the single-shot problem, we assume that the operating cost at time t_{n-2} is $m\theta_{n-2}$. For the first technology switch, the terminal cost is now determined by the optimal solution of the single-shot problem C_{n-1}^* and the switching cost $P\theta_{n-1}$:

$$\Omega_{n-1}(\theta_{n-1}) = C_{n-1}^* + P\theta_{n-1} = \left(\frac{m}{i}\Phi_1 + P\right)\theta_{n-1} \tag{25}$$

Hence, the value-matching and smooth-pasting condition look as follows:

$$G\theta_{n-1}^{*\gamma} + \frac{m\theta_{n-2}}{i} = \left(\frac{m}{i}\Phi_1 + P\right)\theta_{n-1}^* \tag{26}$$

$$\gamma G\theta_{n-1}^{*(\gamma-1)} = \frac{m}{i}\Phi_1 + P \tag{27}$$

The value-matching condition states that at the time of replacement, the cost involved with continuing to operate technology θ_{n-2} has to be equal to the cost of switching and operating the new technology θ_{n-1}. Solving eq.(26) and (27) for G and θ^*_{n-1} and substituting in the cost function (24) gives the following results:

$$\theta^*_{n-1} = \frac{\gamma}{\gamma-1}\left(\Phi_1 + \frac{iP}{m}\right)^{-1}\theta_{n-2} \tag{28}$$

$$C^*_{n-2} = \frac{m\theta_{n-2}}{i}\Phi_2 \tag{29}$$

With:

$$\Phi_2 = \frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_1 + \frac{iP}{m}\right)^{\gamma} + 1 \tag{30}$$

We can push the problem even further to 3, 4, ... n replacements, using the same solution method. Soon a pattern in the solutions will become apparent. The results are illustrated in the following table.

j	$\xi_j^{-1} = \frac{\theta^*_{n-j+1}}{\overline{\theta_{n-j}}}$	C^*_{n-j}	Φ_j
0		$\frac{m\theta_n}{i}$	1
1	$\frac{\gamma}{\gamma-1}\left(\Phi_0 + \frac{iP}{m}\right)^{-1}$	$\frac{m\theta_{n-1}}{i}\Phi_1$	$\frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_0 + \frac{iP}{m}\right)^{\gamma} + 1$
2	$\frac{\gamma}{\gamma-1}\left(\Phi_1 + \frac{iP}{m}\right)^{-1}$	$\frac{m\theta_{n-2}}{i}\Phi_2$	$\frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_1 + \frac{iP}{m}\right)^{\gamma} + 1$
...			
$n-1$	$\frac{\gamma}{\gamma-1}\left(\Phi_{n-2} + \frac{iP}{m}\right)^{-1}$	$\frac{m\theta_1}{i}\Phi_{n-1}$	$\frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_{n-2} + \frac{iP}{m}\right)^{\gamma} + 1$
n	$\frac{\gamma}{\gamma-1}\left(\Phi_{n-1} + \frac{iP}{m}\right)^{-1}$	$\frac{m\theta_0}{i}\Phi_n$	$\frac{1}{\gamma}\left(\frac{\gamma}{\gamma-1}\right)^{1-\gamma}\left(\Phi_{n-1} + \frac{iP}{m}\right)^{\gamma} + 1$

The pattern in the solutions is now clear and is in a format that can easily be incorporated and analyzed on its numerical characteristics in a spreadsheet program. The counter j represents the number of remaining technology switches at each stage, so $n-j$ represents the rank number of the installed technology (starting with 0). The most important result is the series of values of ξ_j^{-1} in the second column. This coefficient compares the levels of the replacing and installed technology at the time of replacement. Notice the bar over the denominator $\overline{\theta_{n-j}}$, indicating that the value of the technological cost-index of the installed technology is known a priori, whereas the state of technological progress triggering replacement θ^*_{n-j+1} has to be determined in the optimization process.

7 Main characteristics of the solution

In a situation where the operator has n options to switch technology, the optimal timing of the first switch is determined solely by the state of the installed technology relative to the replacing technology. As can be found in the second column of the last row in the table, when the existing equipment is of technology $\overline{\theta_0}$, it will be replaced at the time technology has become a factor $\xi_n^{-1} = \frac{\gamma}{\gamma-1}\left(\Phi_{n-1} + \frac{iP}{m}\right)^{-1}$ more efficient (i.e. less costly). Applying the npv-rule would lead to a clearly different result. Indeed, the npv of the technology switch would become positive, if $C_1\left(\theta^*_{1_{NPV}}\right) + P\theta^*_{1_{NPV}} = \frac{m\overline{\theta_0}}{i}$. Using the results from the table, applying npv to determine the first triggering value $\theta^*_{1_{NPV}}$, this would lead to a ratio:

$$\xi^{-1}_{n_{NPV}} = \frac{\theta^*_{1_{NPV}}}{\overline{\theta_0}} = \left(\Phi_{n-1} + \frac{iP}{m}\right)^{-1} \tag{31}$$

The difference in both ratios (the factor $\phi^{-1} < 1$) reflects the option-value of postponing the technology switch at each stage.

To obtain a better understanding of the mechanism involved in this revolving technology adoption model, we can resume the numerical example. In the limit, when an infinite number of replacements is allowed and when replacement costs are marginal in relation to operating costs ($n \to \infty$, $P/m \to 0$), it is optimal to use the most efficient technology at all times. Indeed, as the number of replacements increases:

$$\lim_{n\to\infty, P/m\to 0} \frac{\overline{\theta_0}}{\theta_1} = \lim_{n\to\infty, P/m\to 0} \xi_n^{-1} = 1$$

This is an expected result, because when there are no costs involved in switching technology and an infinite number of switches is allowed, the best solution is to work with the most advanced technology at all times. As the ratio of $\overline{\theta_0}$ and θ_1 tends to unity, this means that replacement will take place each time there is an improvement in the technology at hand. However, this does not imply that the time-interval between replacements will tend to zero nor that the most recent equipment will be used at all times. Indeed, it can never be optimal to replace at times when the state variable θ is increasing. Therefore replacement will take place whenever θ is decreasing and remain stationary otherwise. The limiting case without replacement costs is of course mainly of academic interest. For further analysis and graphical representation, we will resume the example from section 5.

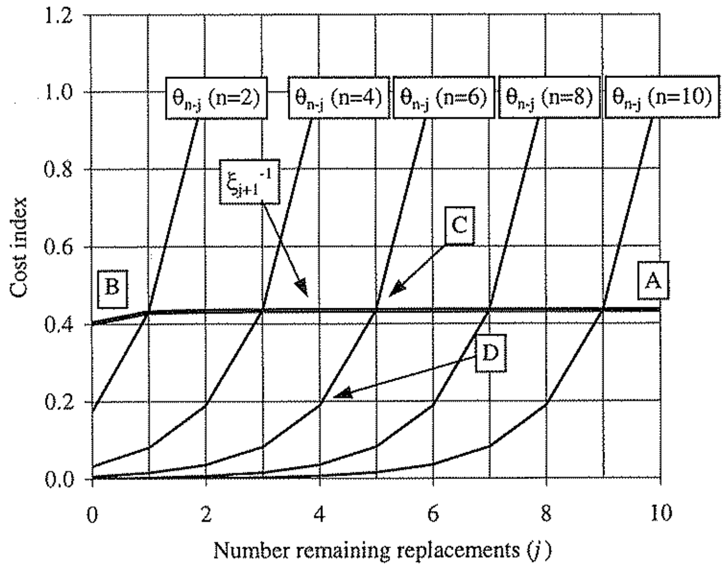

Figure 4: Absolute and relative cost-index of the switching technology for $n = 2 \rightarrow 10$.

Values for ξ_{j+1}^{-1} are determined and illustrated in figure 3 for j ranging from 0 to 10. Also the evolution of θ_{n-j}^{*} is depicted for starting values $\overline{\theta}_0 = 1$ when the operator has 10, 8, 6, 4, and 2 options to switch technology. Figure 3 can be interpreted as follows. Curve AB represents ξ_{j+1}^{-1}, which is the relative advantage of the switching technology as opposed to the existing technology. If we take e.g. the case with 6 switches, the first switch will take place when technology has reduced costs to 43.360% of their original starting level (point C in figure 3). Notice that the absolute cost-index θ_{n-j} and the relative cost-index ξ_{j+1}^{-1} coincide in point C since the first technology in the cycle has unit-cost: $\theta_0 = 1$. The second replacement in the cycle occurs when technology reduced costs again to 43.360% of their original value, which reduces the absolute cost-index to 0.188, as indicated by point D. The value of ξ_{j+1}^{-1} depends on the number of switches, although it is clear in the figure that for this case its value converges very quickly as the number of switches increases. In fact, in this example there is only a small noticeable endpoint-effect for the last technology switch.

The sensitivity of the results for a cycle of 6 replacements to changes in P/m are represented in figure 4. The relative improvement in costs at the

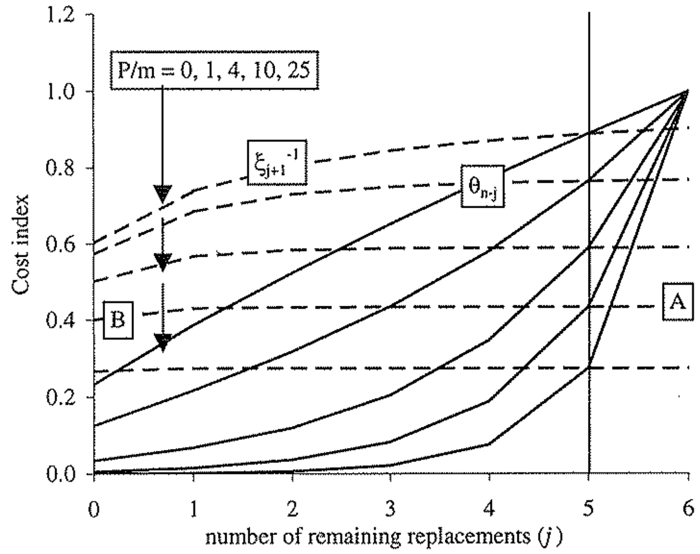

Figure 5: Sensitivity of θ^* to changes is in the ratio P/m

time of switching ξ_{j+1}^{-1} is represented by the dashed lines in the figure. The highest line represents the values for $P/m = 0$, the lowest for $P/m = 25$. The line AB is equivalent to the line AB in the previous figure. It turns out that convergence of ξ_{j+1}^{-1} appears very soon for relatively high values of P/m. However, as P/m approaches zero, convergence (in this case to unity) appears only for a large number of replacements. For these cases, with small switching costs or large operating costs, it is extremely important that the operator of the technology uses a correct estimate of the number of switching options. If the operator does not take fully into account future options to switch to superior technology (n is underestimated), then θ^* will be underestimated and the technology switch will be delayed unduly. This could be an additional explanation for the apparent delay in the timing of technology adoptions.

In general, as the number of switches in the cycle increases, the successive steps in technology become smaller. Higher values of the capital cost P and lower values of the operating cost m increase the steps and will thus prolong the time between successive technology switches. The value of the technological cost-index θ_{n-j} is represented in figure 4 by the full lines. Notice that $\xi_n^{-1} = \theta_1$, as indicated by the vertical line. As an illustration, a

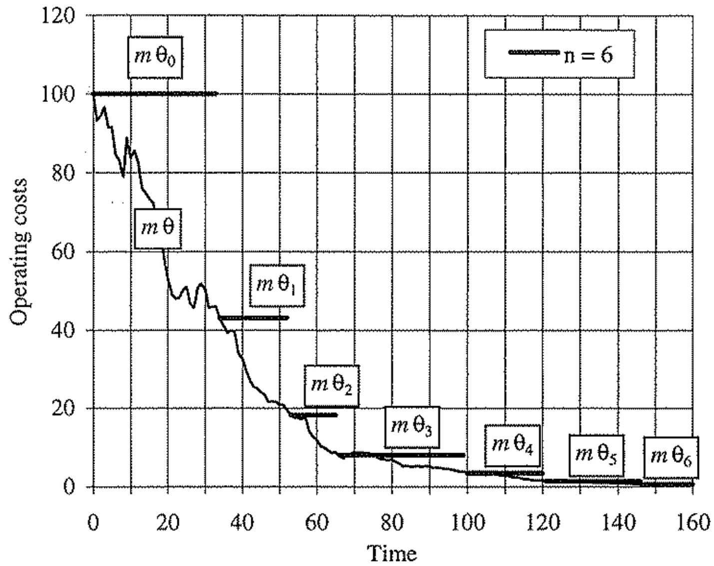

Figure 6: Example of a stochastic technology replacement cycle ($n = 6$)

(possible) outcome of such cycle of 6 technology adoptions is depicted in figure 5 for $P = 1000$ and $m = 100$. The chart depicts the evolution of the operating cost of the most recent technology in function of time. The horizontals show operating cost of the installed technology, when technology switches are chosen in an optimal way. For instance, the operator will use technology θ_0 until θ has decreased to 43.360% of its original level. Operating cost until that moment are 100 per unit of time, afterwards 43.360 per unit of time (until the next switch).

The exact time of a technology adoption in a stochastic case like this, cannot be determined in advance. However, the expected economic life of an existing technology can be determined as the solution of a first-passage time problem. For a variable following a geometric Brownian motion, the distribution of the first time this variable passes a certain barrier is well known in the literature. We can refer to Cox & Miller (1972), Ingersoll (1987), and Mauer & Ott (1995) for the solution of the first-passage time problem. Therefore we also calculated the point at which ξ_j^{-1} converges for values of g ranging from 1.5% to 15% for some different values of P/m (for practical calculations, we assumed convergence when the numerical value of

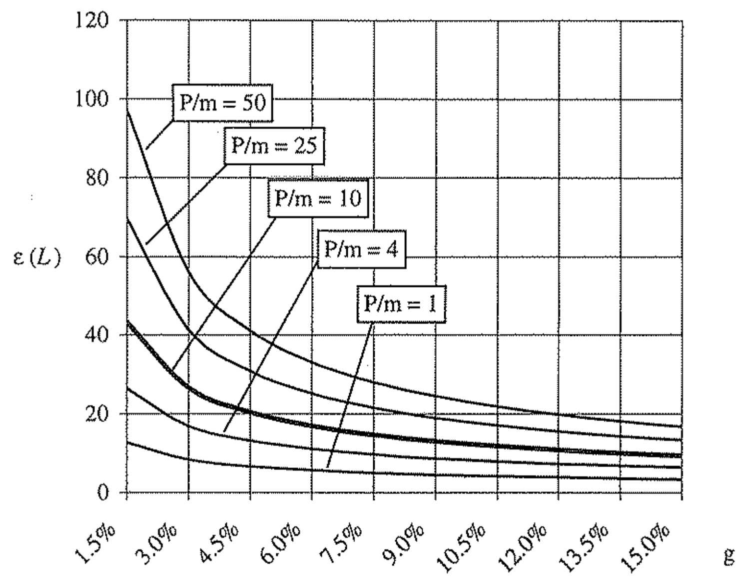

Figure 7: Expected economic life for different values of g and P/m (numerical example)

ξ_j^{-1} changed less than 0.01%). Say ξ_c^{-1} is the value to which ${\xi_j}^{-1}$ converges as the number of remaining replacements increases to infinity (formally: $\lim_{n\to\infty} \xi_n^{-1} = \xi_c^{-1}$). In that case it is possible to determine the expected economic life of the first technology when infinite switches are allowed, as the expected first-passage time for θ to evolve from 1 to ξ_c^{-1}:

$$\varepsilon(L) = \left(-g - \frac{\sigma^2}{2}\right)^{-1} \ln\left(\xi_c^{-1}\right) \tag{32}$$

In figure 6 the expected first-passage time for this example is represented in function of g for five different values[3] P/m. The figure shows some interesting results. As could be expected, an increase in the rate of technological progress accelerates technology switches. However, a reduction in the ratio of capital and operating costs decelerates technology adoption. It should be clear from this that the optimal timing of technology adoption does not only depend on the characteristics of the technologies involved, but also on

[3]The curve for $P/m = 0$ is not depicted because economic life for this case was too close to zero for all values of g.

the characteristics of operation: technology that should be replaced for a high-intensity user may still have value for low-intensity users.

8 Conclusions

In this paper, the optimal timing of technology adoption was examined when technology evolves stochastically, using stochastic dynamic programming and option theory. A technology adoption model was developed using a geometric Brownian motion to model the effect of advances in technology on the cost of equipment.

The conditions for optimal technology adoption were derived, using a dynamic program of nested optimal stopping problems. In the case where there is only one technology switch (the single-shot problem), the result shows some similarities with an American financial option with infinite expiration date. The single-shot case then served as the terminal cost for the preceding problem, with two options to replace. Nesting more of these problems resulted in the revolving technology adoption model, for which an optimum was determined. The main result of this procedure is the conclusion that at the time a new technology is adopted, the capital and operating cost of the challenging technology have to be considerably lower than suggested by the npv-rule before replacement is optimal, due to the value of the option to replace. Contrary to the work of Farzin et al. (1998), a distinct option-value could be determined at each stage of the switching cycle.

Furthermore, for short planning horizons the exact timing of the replacement proved to be rather sensitive to the number of remaining replacement options in the cycle. This conclusion is important for several reasons. First, an underestimation of the number of available switching options by the operator of the technology will result in lower values of the switching technology and hence in result in an additional (suboptimal) delay in the timing of technology adoption. This seems most likely to occur when the switching cost is low relative to the operating cost. The stated result is also important, because in the literature on technology adoption, use is often made of an extremely limited number of technology switches (We referred e.g. to Mauer & Ott (1995), who use only one technology switch, and Grenadier & Weiss (1997), using two switches). Obviously, omitting further options to switch results in biased switching values.

A numerical example was developed, demonstrating that the timing of the technology adoption was especially sensitive to changes in the growth-rate of technological progress and to a lesser extend to the uncertainty surrounding this process. The expected economic life of a technology using an infinite number of switches could be determined as the solution of an expected first-passage time problem. Since the switching technology is largely determined by the ratio of capital and operating costs, and since this ratio depends (among others) on the intensity with which equipment is used, the timing of technology adoption depends on both the characteristics of the technology and its operation.

9 Appendix

If θ follows a stochastic process of the form $d\theta = -g\theta dt + \sigma\theta dz$ in which dz represents a Wiener process, then according to Itô's lemma, the differential of a function of θ - say $C(\theta)$ - is, contrary to 'normal' deterministic calculus:

$$dC(\theta) = C'(\theta)\, d\theta + \frac{1}{2}C''(\theta)\, d\theta^2 \tag{33}$$

As dz represents a Wiener process, $dz = n\sqrt{dt}$ in which n is a standard-normally distributed variable, the expression of $dC(\theta)$ includes an additional term in $d\theta^2$. Substituting dz in $d\theta$ and $d\theta$ in $dC(\theta)$, omitting all terms in dt of a degree higher then 1 and using the fact that $dz^2 = dt$, we find:

$$dC(\theta) = \left(-g\theta C'(\theta) + \frac{1}{2}\sigma^2\theta^2 C''(\theta)\right) dt + \sigma\theta C'(\theta)\, dz \tag{34}$$

For more details on Itô's lemma, see e.g. Dixit [4], Dixit & Pindyck [5] or Kamien & Schwartz [10].

Substituting this result in eq.(4), we find the differential equation:

$$\frac{1}{2}\sigma^2\theta^2 C''(\theta) - g\theta C'(\theta) - iC(\theta) = -m\theta_0 \tag{35}$$

which is a special form of a second-order differential equation with variable coefficients, known as a Cauchy equation. This equation can be transformed in a form with constant coefficients by substituting $\theta = e^\tau$. In that case:

$$\frac{dC}{d\theta} = \frac{dC}{d\tau}\frac{d\tau}{d\theta} = \frac{1}{\theta}\frac{dC}{d\tau} \tag{36}$$

$$\frac{d^2C}{d\theta^2} = \frac{d}{d\theta}\left(\frac{1}{\theta}\frac{dC}{d\tau}\right) = \frac{1}{\theta^2}\left(\frac{d^2C}{d\tau^2} - \frac{dC}{d\tau}\right) \tag{37}$$

Substituting (36) and (37) in the differential equation transforms it into:

$$\frac{\sigma^2}{2}\left[C''(\tau) - C'(\tau)\right] - gC'(\tau) - iC(\tau) = -m\theta_0 \tag{38}$$

Which can be rearranged as:

$$\frac{\sigma^2}{2}C''(\tau) - \left(g + \frac{\sigma^2}{2}\right)C'(\tau) - iC(\tau) = -m\theta_0 \tag{39}$$

Let $A = \frac{\sigma^2}{2}$ and $B = -\left(g + \frac{\sigma^2}{2}\right)$, then the characteristic equation of the corresponding homogenous differential equation is:

$$A\gamma^2 + B\gamma - i = 0 \tag{40}$$

This equation has two distinct real roots:

$$\gamma_j = \frac{1}{2} + \frac{g}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} + \frac{g}{\sigma^2}\right)^2 + \frac{2i}{\sigma^2}} \tag{41}$$

The solution to the homogenous differential equation is then:

$$C_h = F_1\theta^{\gamma_1} + F_2\theta^{\gamma_2} \tag{42}$$

where F_j are constants. The complete solution to the general differential equation consists of C_h plus a particular solution of the general problem C_p. Fortunately the right-hand side of the general differential equation is of a well-known type, so we can state immediately C_p is of the form $C_p = F_3$. Substitution of this particular solution in the general differential equation reveals that $F_3 = \frac{m\theta_0}{i}$.

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