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Dynamic Capital Structure Choice and Investment Timing*†

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Abstract

The paper considers the problem of an investor that has the option to acquire a firm. Initially this firm is run as to maximize shareholder value, where the shareholders are risk averse. To do so it has to decide each time on investment and dividend levels. The firm's capital stock can be financed by equity and debt, where less solvable firms pay a higher interest rate on debt. Revenue is stochastic.

We find that the firm is run such that capital stock and dividends develop in a fixed proportion to the equity. In particular, it turns

*This paper was started several years ago at the moment that Engelbert Dockner got the idea to combine Hartl et al. (2002) with the real options approach. Unfortunately, before we were able to finish the paper, he passed away. His coauthors feel honored to finish the paper in his memory.

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out that more dividends are paid if the economic environment is more uncertain. We also derive that the relationship between the levels of risk aversion of the current shareholders and the potential investor is a significant determinant in establishing whether the firm is a profitable takeover target for the investor.

Key words: real options, optimal control, capital accumulation

1 Introduction

A huge literature deals with the topic of mergers and acquisitions (see, e.g., Loukianova et al. (2017)). In the real options area there is a considerable amount of papers focusing on takeover timing and sharing of the surplus under uncertainty (Lambrecht (2004), Lambrecht and Meyers (2008), Alvarez and Stenbacka (2006), Thijssen (2008), Hackbart and Morellec (2008), Lukas and Welling (2012)). Our paper extends this literature by considering the value of the takeover target to be endogenous. In particular, this value is obtained as the result of determining the optimal dividend and capital accumulation policies over time. In this way the present paper extends the theoretical framework of analyzing when to acquire a firm.

An outside investor, who is risk averse, has an option to acquire the firm. Before being acquired by the investor, the firm is run as to maximize shareholder value. The shareholders are risk averse as well, and the objective of the firm is to maximize a discounted utility stream of dividends. At each moment in time the firm has to decide how much to invest and pay out dividends, which at the same time determines how much the firm will borrow. Borrowing is more expensive the higher the debt-equity ratio is.

We start out formulating the dynamic model of the firm, which is based on Steigum (1983) (see also Hartl et al. (2002)). Where the Steigum model is deterministic we extend it to a stochastic framework by considering a stochastic revenue process. We show that after an initial impulse investment, the firm invests such that capital stock and equity develop over time in a proportional way. Part of the capital stock is financed by debt. Also the debt-equity ratio is fixed over time, and this ratio is high in case of large marginal revenue, a low depreciation rate, and a low interest rate. Moreover, also the dividend is paid out in fixed proportion to the equity. This proportion is higher in case the economic environment is more uncertain and the shareholder time preference rate is large. The dependence of dividend payout on the degree of risk aversion is less clear.

The investor is able to acquire the firm once it offers a price to the current shareholders that is at least equal to the (current) shareholder value of the firm. We obtain an explicit expression based on which it can be determined whether the underlying firm is a suitable takeover target to the investor. It turns out that especially the risk aversion parameters of both current shareholders and the investor are significant determinants in this respect. Time is not a factor in the sense that, if the takeover takes place, it will be done immediately. Hence, we could not confirm the result in Hugonnier and Morellec (2007), who find that risk aversion provides an incentive for the investor to delay investment.

Technically, the analysis of the model is new in the sense that it combines continuous and lumpy investments in one framework. Continuous investments are for the first time considered in the capital accumulation model in Jorgensen (1963). Lucas (1967) and Gould (1968) are among the first to introduce a convex adjustment cost function of investment in such a framework that spreads out the growth of the capital stock over time. Later on, authors introduce non-concavities in the revenue function, which results in history-dependent long run equilibria separated by Skiba points (see, e.g., Davidson and Harris (1981) and Dechert (1983)). A dynamic model of the firm with a completely convex revenue function, caused by for instance increasing returns to scale, is studied in Barucci (1998) (see also Hartl and Kort (2000)).Next to an adjustment cost function being convex, also adjustment costs with a (partly) concave shape are considered (see, e.g., Jorgensen and Kort (1993)). The just mentioned contributions all assume a perfect capital market, indicating that firms can lend and borrow as much as they want against a fixed interest rate. Jorgensen and Kort (1997) depart from this assumption by letting the interest rate increase with debt. Alternatively, Steigum (1983) and also the present paper considers a framework in which the interest rate increases with the debt-equity ratio, reflecting that it is more risky to lend to firms that are less solvable, so a higher interest rate is demanded then.

The literature stated in the previous paragraph has in common that all work in a deterministic framework, and, as stated, investments are continuous or incremental. We extend these contributions, first, by also considering the lumpy investment of the investor needed to acquire the firm. Second, we introduce uncertainty by letting revenue be stochastic. As such we combine capital accumulation models with real options analysis. The seminal work in this area is Dixit and Pindyck (1994). In a real options model firms determine the optimal time to undertake a lumpy investment, where it is assumed that investment is irreversible and subject to ongoing uncertainty. The key result is that firms delay investing in a more uncertain world since uncertainty generates a value of waiting with investment. Later on this theory was extended to include competition, in which several firms have the option to invest in the same market (see Grenadier (2000) for a survey). The standard real options literature considers an investment problem as a timing problem, where the firm should find the optimal time to undertake an investment of given size. More recently, contributions arise that, besides the timing, also optimize the size. This literature is surveyed in Huberts et al. (2015).

The paper is organized as follows. The model is formulated in Section 2. Section 3 contains the analysis of the firm model while the investor's problem is solved in Section 4. Section 5 concludes.

2 The Model

We consider the problem of an investor that has the option to acquire a firm. To obtain the firm, it has to incur an investment cost equal to the value of the firm for the current shareholders plus some profit margin. The value of the firm fluctuates over time, and is given by $V(X)$, where X is the amount of equity of the targeted firm. The firm's purchase price is equal to P.

The firm under consideration operates such that it maximizes its shareholder value. The shareholder value equals a discounted utility stream of dividend payouts. Shareholders are risk averse, so that the utility function is concavely increasing with dividends, D. We impose that its utility function is given by $U(D) = \frac{1}{\gamma}D^{\gamma}$, where $\gamma \in (0,1)$ is the risk-aversion parameter, in the sense that the decision maker is more risk averse for a lower value of γ. The shareholder time preference rate is constant and equal to i. Hence, the value of the firm is given by

$$V(X_0) = \max E\left[\frac{1}{\gamma}\int_0^{\infty} e^{-it} D^{\gamma} dt\right]. \tag{1}$$

The investor has to incur an amount being equal to P to obtain the firm. To keep the problem tractable we make a simplifying assumption. In particular, we assume that all negotiation power is in the hand of the investor in the sense that the investor makes a take-it-or-leave-it offer. Therefore, the shareholders accept the offer if P is that large that it equals $(\gamma V(X))^{1/\gamma}$ plus one cent. In utility terms we get

$$U(P) = V(X) + \xi,$$

where ξ is the tiny excess utility coming from the premium of one cent. The value of ξ is that small that in determining the optimal investment and dividend rates the shareholders do not need to anticipate a possible offer by the outside investor. This implies that the objective is just maximizing the discounted utility stream of dividends over an infinite planning period, as expressed by (1).[1]

Equity, X, increases over time due to revenue obtained from selling products on the market, and decreases due to depreciation, δK, where K is the capital stock, interest payment $r(B/X)B$, where B is the amount of debt and $r(B/X)$ is the interest rate, and paying dividends to the shareholders. Due to demand uncertainty, revenue, $R(K)$, is stochastic:

$$R(K)\,dt = aKdt + \tilde{\sigma}Kdz.$$

This means that the revenue over an infinitesimal time interval of length dt, consists of a deterministic part being linearly dependent on the capital stock, K, and a stochastic part governed by the increment of a Wiener process dz, where the standard deviation is equal to $\tilde{\sigma}K$. The linearity of the deterministic part of revenue, aK, is valid e.g. if the firm is a price taker on the output market and there is constant returns to scale in the production process.[2] Revenue is stochastic because of demand shocks.

Following e.g. Steigum (1983), we impose that the interest rate $r(B/X)$ is increasing in the debt to equity ratio, B/X. To avoid that arbitrageurs will wipe out a premium that comes without risk, we assume some market friction in the form that the entire economy is completely cash constrained, so that cash is scarce and a higher demand for debt financing will drive prices up. Later on, we use the specification

$$r(B/X) = r_0 + \beta\frac{B}{X}. \tag{2}$$

The expression (2) creates a soft constraint for the amount of debt. Alternatively, a hard constraint could have been imposed where debt is strictly

[1]We like to thank an anonymous referee for providing this model suggestion.

[2]Alternatively, the parameter a could be defined as the "Total Factor Productivity"(TFP) like in an AK model.

bounded from above. At the same time we have to admit that, for reasons of analytical tractability, we disregard the bankruptcy issue, which makes the firm's debt essentially riskless. However, we still want to cover the fact that it is more difficult/expensive to borrow when the firm has more debt.

We conclude that the evolution of equity satisfies the following stochastic differential equation:

$$dX = [aK - \delta K - r(B/X)B - D]\, dt + \tilde{\sigma} K dz. \tag{3}$$

Capital stock follows the standard evolution such that it increases with investment, I, and decreases with depreciation, δK, i.e.

$$dK = (I - \delta K)\, dt.$$

The balance equation of the firm is such that capital stock can be financed by equity and debt:

$$K = X + B.$$

From the last three equations it can be derived that

$$dB = [I - aK + r(B/X)B + D]\, dt - \tilde{\sigma} K dz.$$

Hence, given that the firm holds no cash, this equation represents the cash balance: at each moment in time money is spent on investments, paying interest and dividends, while the cash inflow consists of revenue and extra borrowing (if $dB > 0$). On the other hand, dB can also be negative, in which case the firm pays off debt.

Of course we need some upper bound on B which we introduce indirectly by imposing that equity needs to be non-negative:

$$X \geq 0. \tag{4}$$

We conclude that the dynamic model of the firm can be presented as follows:

$$\begin{aligned} V(X_0) &= \max_{I,D} E\left[\frac{1}{\gamma}\int_0^\infty e^{-it} D^\gamma dt\right], && (5)\\ \dot{K} &= I - \delta K, && (6)\\ dX &= \left[aK - \delta K - r_0 B - \beta \frac{B^2}{X} - D\right] dt + \tilde{\sigma} K dz, && (7)\\ B &= K - X, && (8)\\ D &\geq 0, X \geq 0. && (9) \end{aligned}$$

The above model is based on Steigum (1983), where our interest rate function is a simplified version. On the other hand the depreciation rate δ is not explicitly included in the Steigum formulation, and our revenue function is stochastic, while in Steigum (1983) it is deterministic[3].

The problem of the investor is when, if ever, to acquire the firm, where the value of the firm for the investor equals $V_I(X)$. In Section 4, where the investor's problem is analyzed, we explain how this value is determined. To optimize his acquisition decision, the investor has to solve

$$\max_T E\left[e^{-i_I T}\left(V_I\left(X\left(T\right)\right) - U_I\left(P\right)\right)\right], \tag{10}$$

in which T is the acquisition time, $U_I(P)$ is the investor's utility function, and i_I is the time preference rate of the investor.

The next section analyzes the optimization problem of the firm, $(5-9)$, while the investor's problem (10) will be solved in the subsequent section.

3 Analysis of the Firm Model

Like Steigum (1983), we apply a two-step approach. A two-step approach has the aim to simplify the optimization problem by reducing the number of state or control variables (see Hartl (1988)). To reduce the number of state variables from two to one in a two-state optimal control problem, two conditions need to hold. First, one of the two state variables appears either in the objective or in the state equation of the other state variable. Second, the control variable governing the first mentioned state variable is unconstrained and only appears in the state equation of that state variable.

Considering the problem $(5-9)$, we observe that these conditions are fulfilled, because K appears in the state equation for X and not in the objective, whereas the control variable I is unconstrained and appears only in the state equation of K. For this model the two-step approach is set up as follows. In Step 1, a static optimization problem is solved to determine, for every value of equity, X, the optimal level of the capital stock, K. In this way, the function $K(X)$ is obtained. In Step 2 the remaining stochastic optimal control problem is solved using the function $K(X)$ as input.

Mathematically, the two steps can be described as follows.

[3] A deterministic variant of the Steigum [24] model is studied in Hartl et al. [12].

3.1 Step 1: Capital Accumulation

For a given equity level, X, we maximize the deterministic instantaneous profit rate with respect to K. This leads to the following problem:

$$\begin{aligned} \pi(X) &= \max_K \left[aK - \delta K - r_0 B - \beta \frac{B^2}{X} \right], \\ \text{s.t. } B &= K - X. \end{aligned} \tag{11}$$

The optimal level of K is therefore given by

$$K(X) = \left(1 + \frac{a - \delta - r_0}{2\beta}\right) X. \tag{12}$$

which implies that debt $B = K - X$ is always positive:

$$B(X) = \frac{a - \delta - r_0}{2\beta} X > 0. \tag{13}$$

If expected marginal revenue, a, is large relative to depreciation, δ, and/or if the cost of debt, β, is small, the firm has a high debt-equity ratio. In such a situation it is good for the firm to borrow a substantial amount to enhance further growth.

Results (12) and (13) mean that the firm should always keep a constant capital to equity ratio, and a constant debt to equity ratio. Since the model (5) to (9) has no adjustment costs for capital stock, K, impulse investments or disinvestments are performed to reach these levels.

3.2 Step 2: Optimal Dividend Policy

Next, we proceed to solve the following stochastic optimal control problem in order to obtain the dividend rate:

$$\max E\left[\frac{1}{\gamma}\int_0^\infty e^{-it} D^\gamma dt\right],$$

subject to

$$\begin{aligned} dX &= \left[(a-\delta) K(X) - r_0 B(X) - \beta \frac{B(X)^2}{X} - D\right] dt + \tilde{\sigma} K(X)\, dz, \\ D &\geq 0, \end{aligned} \tag{14}$$

$$B = K(X) - X.$$

Substitution of (12) and (13) into expression (14) gives

$$dX = \left[\left(a - \delta + \frac{(a - \delta - r_0)^2}{4\beta} \right) X - D \right] dt + \sigma X dz, \tag{15}$$

with

$$\sigma = \tilde{\sigma} \left(1 + \frac{a - \delta - r_0}{2\beta} \right). \tag{16}$$

To derive the optimal dividend rate, we employ the Bellman equation

$$\begin{aligned} iV(X) &= \max_D \left[\frac{1}{\gamma} D^\gamma + V'(X) \left[\left(a - \delta + \frac{(a - \delta - r_0)^2}{4\beta} \right) X - D \right] \right] \\ &\quad + \frac{1}{2} \sigma^2 X^2 V''(X), \end{aligned} \tag{17}$$

where we recall that $V(X)$ is the value of the firm. The dividend rate is, therefore, given by

$$D = V'(X)^{\frac{1}{\gamma-1}}.$$

To solve the Bellman equation, we postulate that

$$V(X) = \frac{c}{\gamma} X^\gamma, \tag{18}$$

where c is some constant to be determined. This gives a linear dividend payment rule

$$D = c^{\frac{1}{\gamma-1}} X. \tag{19}$$

After substitution of (19) in (17), we obtain that

$$c = \left(\frac{\left(\frac{i}{\gamma} + \frac{\sigma^2}{2}(1-\gamma) \right) - \left(a - \delta + \frac{(a-\delta-r_0)^2}{4\beta} \right)}{\frac{1}{\gamma} - 1} \right)^{\gamma-1}. \tag{20}$$

This implies that the dividend payment rule is given by

$$D = \frac{\left(\frac{i}{\gamma} + \frac{\sigma^2}{2}(1-\gamma) \right) - \left(a - \delta + \frac{(a-\delta-r_0)^2}{4\beta} \right)}{\frac{1}{\gamma} - 1} X. \tag{21}$$

From (20) and (21) we see that we have to assume that

$$i + \frac{\sigma^2}{2}\gamma(1-\gamma) > \gamma\left(a - \delta + \frac{(a-\delta-r_0)^2}{4\beta}\right), \tag{22}$$

must hold in order to ensure that dividend is positive. Note that this condition is always satisfied for a sufficiently large time preference rate. This implies that the shareholders are sufficiently impatient that dividend payments are required from the beginning. Note also that the inequality is more difficult to satisfy for larger values of γ, provided that $\gamma \in (0.5, 1)$. Hence, the less risk averse the shareholders are, the more they have the incentive to inject money into the firm instead of receiving it from the firm in the form of dividends. This makes sense given the uncertainty of the firm's revenue stream.

Summing up the results so far, we have obtained a complete characterization of the firm's investment and dividend policy as follows:

Proposition 1. *If the time preference rate is sufficiently large* (22), *the firm always pays dividends proportional to equity, given by the explicit rule* (21). *Consequently, also debt is proportional to equity, as given by* (13). *This results in a concave value function, explicitly given by* (18).

In order to perform sensitivity analysis, first we see that the dividend-equity ratio increases in the shareholder time preference rate, i. This makes sense since a large value of i means that the shareholder has high opportunity costs. A risk-averse shareholder does not like to be subject to uncertainty, σ. Therefore, the shareholder prefers a higher dividend payout when the firm operates in a more uncertain economic environment. This effect (D/X increases with σ) is more pronounced if the shareholder is more risk averse, which is reflected by a smaller γ.

From the previous section we recall that the firm wants further growth especially when expected marginal revenue, a, is large relative to depreciation, δ, and/or if the cost of debt, r_0 as well as β, is small. It makes sense that in such a situation the firm pays fewer dividends per unit of equity.

4 The Investor's Problem

Here we consider the problem of the investor that has the opportunity to acquire this firm at a value that is assigned to it by the current shareholders.

Let us assume that the investor's utility function, $U_I(D)$, is of the same structure as the one of the current shareholders but then with different risk aversion parameter $\gamma_I \in (0,1)$:

$$U_I(D) = \frac{1}{\gamma_I} D^{\gamma_I}.$$

The question is whether the investor should buy the firm, and if yes, when. To get the answer, he has to solve the optimization problem (10).

As we have stated in the beginning of the Model section, the current shareholders accept the investor's offer if the purchase price is such that

$$P = (\gamma V(X))^{1/\gamma} + \text{one cent}.$$

From (18) we obtain that

$$P = \left(\gamma \frac{c}{\gamma} X^{\gamma}\right)^{1/\gamma} + \text{one cent},$$

so that

$$P = c^{\frac{1}{\gamma}} X + \text{one cent}, \tag{23}$$

with c from (20).

Since the investor's utility function is of the same structure as the one of the current shareholders, it holds that, as soon as he takes over the firm, the investor will copy the dividend and investment policy of the existing shareholders, but then with parameters γ and i replaced by γ_I and i_I, respectively. The resulting value of the firm for the investor will therefore be equal to

$$V_I(X) = \frac{c_I}{\gamma_I} X^{\gamma_I}, \tag{24}$$

with

$$c_I = \left(\frac{\left(\frac{i_I}{\gamma_I} + \frac{\sigma^2}{2}(1-\gamma_I)\right) - \left(a - \delta + \frac{(a-\delta-r_0)^2}{4\beta}\right)}{\frac{1}{\gamma_I} - 1} \right)^{\gamma_I - 1}. \tag{25}$$

Typically, the investor represents a larger firm than the one under consideration, and therefore the investor is likely to be less risk averse than the current shareholders. Therefore, we impose that

$$\gamma_I > \gamma.$$

This inequality ensures that the investor is interested to buy the firm when equity, X, is large, because it holds that $V_I(X)$ is larger (smaller) than $V(X)$ for X large (small) enough. This implies that the investment timing problem can be modeled as an optimal stopping problem (see, e.g., Dixit and Pindyck (1994)), with a stopping region for large values of X and a continuation region for small values of X.

After taking into account that dividend satisfies (21), where after the acquisition again parameters γ and i are replaced by γ_I and i_I, we obtain from (15) that equity, X, develops according to

$$dX = AXdt + \sigma Xdz,$$

with

$$A = \frac{1}{1-\gamma}\left[a - \delta + \frac{(a-\delta-r_0)^2}{4\beta} - \left(i + \frac{\sigma^2}{2}\gamma(1-\gamma)\right)\right].$$

before the acquisition, whereas

$$dX = A_I Xdt + \sigma Xdz,$$

with

$$A_I = \frac{1}{1-\gamma_I}\left(a - \delta + \frac{(a-\delta-r_0)^2}{4\beta} - \left(i_I + \frac{\sigma^2}{2}\gamma_I(1-\gamma_I)\right)\right)$$

after the investor has bought the firm. From the theory of real options (see, e.g., Dixit and Pindyck, 1994) it is known that investment is delayed until infinity, thus will never be undertaken, when

$$A \geq i_I. \tag{26}$$

In case $A < i_I$, a threshold value for X, say X^*, can be identified, above which it is optimal to invest. So, for $X > X^*$ the investor invests immediately and the payoff of this transaction equals

$$V_I(X) - U_I(P) = \frac{c_I}{\gamma_I}X^{\gamma_I} - \frac{1}{\gamma_I}\left(c^{\frac{1}{\gamma}}X\right)^{\gamma_I} - \xi_I = \frac{1}{\gamma_I}\left(c_I - c^{\frac{\gamma_I}{\gamma}}\right)X^{\gamma_I} - \xi_I, \tag{27}$$

in which $-\xi_I$ is the (tiny) excess disutility for the investor coming from this premium of one cent. We can already conclude that *the investor never buys if* $c_I - c^{\frac{\gamma_I}{\gamma}} \leq 0$.

For $X < X^*$ the investor refrains from investment, so that he keeps the investment option alive. Let us denote the value of the investment option by $F(X)$. Following e.g. Dixit and Pindyck (1994), application of Ito´s lemma and the Bellman equation implies that

$$i_I F(X) = AXF'(X) + \frac{1}{2}\sigma^2 X^2 F''(X).$$

The solution to this differential equation is given by

$$F(X) = GX^{\varepsilon}, \tag{28}$$

where G is an unknown constant and ε is the positive root of the fundamental quadratic

$$\frac{1}{2}\sigma^2\varepsilon[\varepsilon - 1] + A\varepsilon - i_I = 0,$$

i.e.

$$\varepsilon = \frac{-2A + \sigma^2 + \sqrt{4A^2 + \sigma^4 - 4A\sigma^2 + 8i_I\sigma^2}}{2\sigma^2}, \tag{29}$$

Note that the negative root cancels due to the boundary condition

$$F(0) = 0,$$

and that the positive root always exceeds one.

The threshold value X^* can be obtained by the value matching and the smooth pasting conditions. From the expressions (27) and (28) it follows that the value matching condition is given by

$$GX^{*\varepsilon} = \frac{1}{\gamma_I}\left(c_I - c^{\frac{\gamma_I}{\gamma}}\right)X^{*\gamma_I} - \xi_I,$$

The smooth pasting condition equals

$$\varepsilon GX^{*\varepsilon-1} = \left(c_I - c^{\frac{\gamma_I}{\gamma}}\right)X^{*\gamma_I-1}.$$

From these two equalities it is obtained that

$$X^* = \left(\frac{\xi_I}{\left(c_I - c^{\frac{\gamma_I}{\gamma}}\right)\left(\frac{1}{\gamma_I} - \frac{1}{\varepsilon}\right)}\right)^{\frac{1}{\gamma_I}}.$$

Note that, since $\gamma_I < 1$ and $\varepsilon > 1$, the term in the denominator, $\frac{1}{\gamma_I} - \frac{1}{\varepsilon}$, is positive. Hence, we can conclude that, since ξ_I is a small number, either the investor buys immediately (if $c_I - c^{\frac{\gamma_I}{\gamma}} > 0$) or never (if $c_I - c^{\frac{\gamma_I}{\gamma}} \leq 0$).

We state our finding in the following proposition.

Proposition 2. *It is optimal for the investor to acquire the firm immediately, if*

$$c_I - c^{\frac{\gamma_I}{\gamma}} > 0,$$

whereas the investor will not acquire the firm if

$$c_I - c^{\frac{\gamma_I}{\gamma}} \leq 0.$$

in which c and c_I are given by (20) and (25), respectively.

We illustrate this result in a short example. We consider the following parameters:

$$\begin{aligned} \gamma &= 0.7; \quad \gamma_I = 0.9; \quad \tilde{\sigma} = 0.2; \quad i = 0.1; \quad i_I = 0.07; \quad \delta = 0.2; \\ a &= 0.25; \quad r_0 = 0.01; \quad \beta = 0.064. \end{aligned}$$

The risk aversion parameters γ and γ_I should be between zero and one and the considered values seem reasonable (see , e.g., Henderson and Hobson (2002), Chronopoulos et al. (2014)). A 20% standard deviation is quite realistic as argued in Dixit and Pindyck (1994, p.153), and the latter reference also considers discount rates up to 10% (Dixit and Pindyck (1994, p.139). A depreciation rate of 0.2 indicates a five-year lifetime of the capital good, and for a capital good to be profitable the marginal revenue, a, should at least be larger than the sum of the depreciation rate, δ and the smallest interest rate on debt, r_0. The value for β is chosen such that, together with r_0, the interest rate on debt stays in between 1 and 8%, as long as debt does not exceed equity.

From this we derive that

$$\begin{aligned} c &= 1.562; \quad \sigma = 0.2625; \quad A = -0.170; \quad \varepsilon = 6.258; \\ c_I &= 1.161; \quad A_I = -0.169, \end{aligned}$$

from which we obtain the outcome

$$y = c_I - c^{\frac{\gamma_I}{\gamma}} = -0.613\,26.$$

Based on Proposition 2 we conclude that in this scenario the investor is not interested in acquiring the firm.

We now carry out some comparative statics analysis, where we keep on using the notation $y = c_I - c^{\frac{\gamma_I}{\gamma}}$.

Figure 1 shows that, since y is increasing in i, the acquisition becomes more attractive for larger values of the shareholder time preference rate, because the value of the firm is decreasing in i for the current shareholders. However, even for very large values of the time preference rate of the current shareholders, the investor still will not acquire the firm.

Figure 2 shows the opposite behavior of Figure 1, where now the investor is less eager to buy the firm when his time preference rate increases. This makes sense because when his time preference rate is larger, it gives a lower expected value of the discounted stream of dividend utility. Still the firm will not be acquired by the investor for all relevant values of i_I.

Figure 3 reflects that also for varying levels of the demand uncertainty the investor does not want to acquire the firm. The increasing slope tells us that the firm becomes relatively more attractive to the investor when demand is more uncertain, the reason of which is that the investor is less risk averse than the current shareholders.

If γ is larger, the current shareholder is less risk averse. Figure 4 tells us that the current shareholders are willing to sell the firm to the investor, if they are less risk averse.

Figure 5 shows that the investor is especially interested to acquire the firm when he is more risk averse. Combining this result with Figure 4 learns that the difference in the level of risk aversion of the current shareholders and the investor plays a significant role in the decision of whether such an acquisition will go through.

5 Conclusion

The paper considers the firm as a takeover target. New to the literature is that the value of the takeover target is endogenously determined. In particular, it is the result of an optimal control model in which the firm maximizes a discounted utility stream of dividends over time by optimally establishing capital accumulation and dividend payout.

The paper is innovative in the sense that it combines continuous investment for capital accumulation with the lumpy investment needed to acquire the firm, while the considered framework is stochastic. As such it combines optimal control with the theory of real options. This paper is the first in a series of analyses that combines both approaches. As the model is now, in optimizing its behavior the manager maximizes shareholder value, whereas the outside investor has all negotiation power. The latter implies that the investor can make a take-it-or-leave-it offer where the current shareholders accept this offer if the purchase price exceeds the shareholder value of the firm by just a tiny amount. Consequently, in managing the firm the current manager does not need to take into account the possibility that the firm will be acquired by an outside investor. It will be interesting to extend this paper by addressing a similar framework where the manager of the firm has to explicitly anticipate the existence of this outside investor that has the aim to acquire the firm once its underlying value is big enough.

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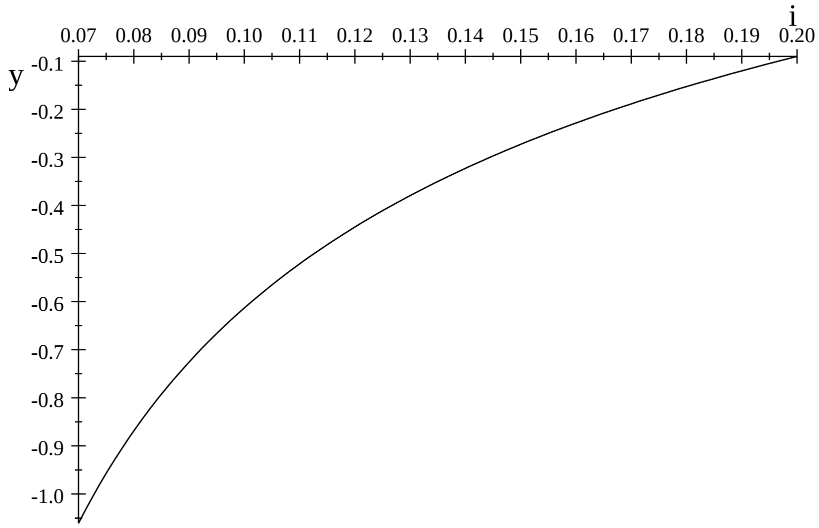

Figure 1. as a function of the shareholder time preference rate ~

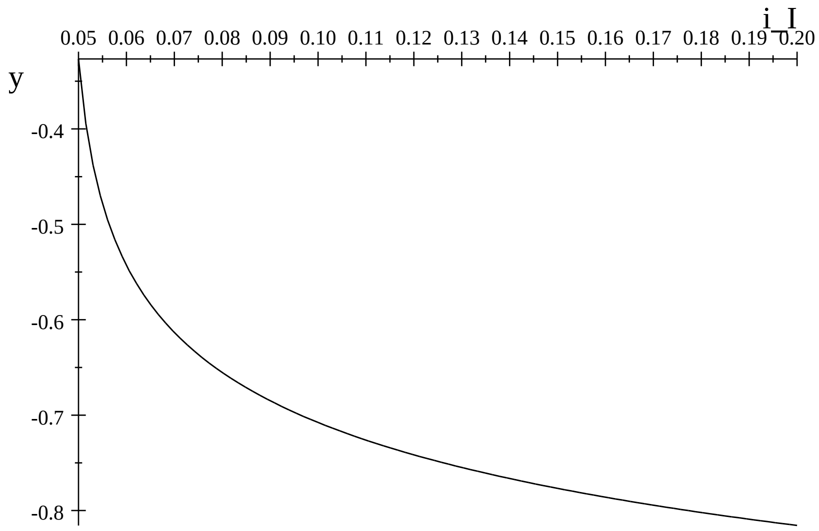

Figure 2. as a function of the investor's time preference rate ~

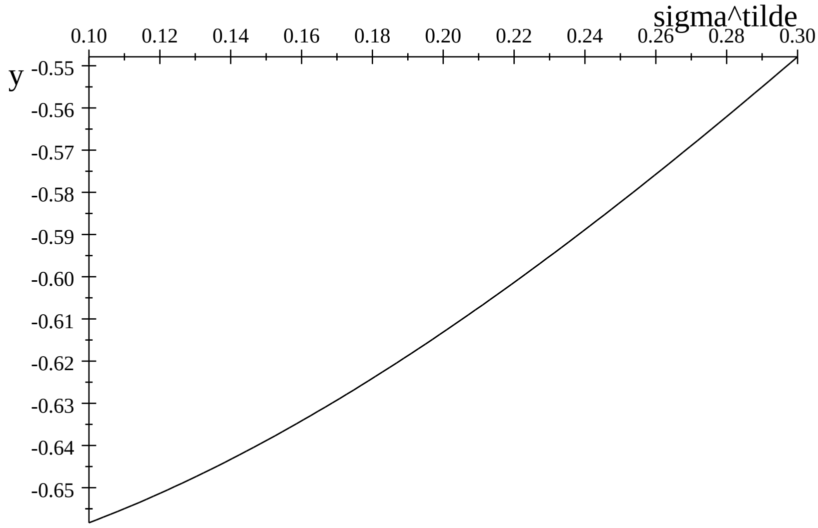

Figure 3. as a function of the demand uncertainty parameter sigma^tilde.

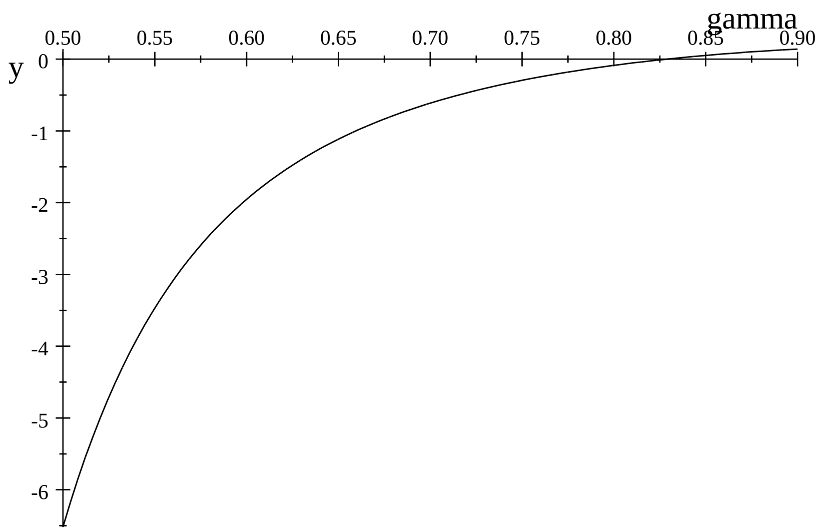

Figure 4. as a function of the shareholder risk aversion parameter gamma.

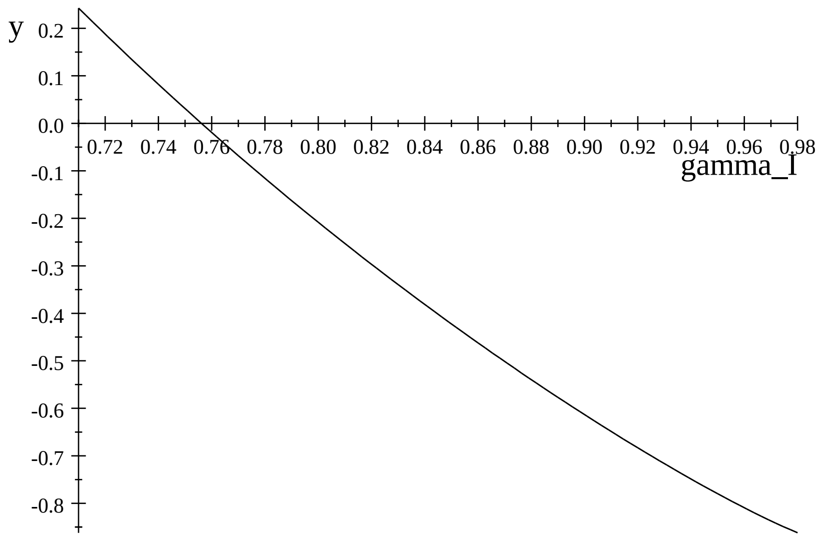

Figure 5. as a function of the investor's risk aversion parameter gamma_I∼