Physics Letters B 315 (1993) 157–163
North-Holland

PHYSICS LETTERS B

Quark anomalous dimensions at small x ☆

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Received 21 June 1993
Editor: R. Gatto

We discuss the small-x behaviour of the parton anomalous dimensions in perturbative QCD beyond the leading logarithmic approximation. The flavour non-singlet anomalous dimensions are found to be regular at small x. The quark singlet anomalous dimensions are computed by resumming the perturbative series to all orders with next-to-leading logarithmic accuracy.

1. Introduction

Precise quantitative tests of QCD and searches for new physics at present and future hadron colliders are carried out in the small-x kinematic regime. By small x we mean that the ratio $x=p_t^2/S$, between the typical transferred momentum p_t in the process and the centre-of-mass energy $\sqrt{S}$ of the colliding hadrons, is much smaller than unity. For these small-x processes reliable and accurate theoretical predictions are clearly necessary.

As long as p_t is much larger than the QCD scale Λ, the strong coupling $\alpha_S(p_t^2)$ is small and perturbation theory can be applied to compute cross sections and hadron distributions. However in the small-x regime we have to face the theoretical problem of a slowly (or badly) convergent perturbative expansion. The reason is that the perturbative series is plagued by large logarithmic corrections of the type $\alpha_S^n \ln^m x$ $(m\leqslant 2n)$. These corrections have to be estimated to higher orders and, eventually, resummed to all orders in α_S.

At present, the QCD multiparton matrix elements have been computed to double logarithmic accuracy in the small-x region [1–3]. They can be used to study the structure of hadronic final states in small-x processes, thus predicting some new distinctive features [1–3] such as the increase of particle multiplicity and the suppression of large rapidity gaps. Only some phenomenological investigations have been carried out [4], and more detailed analyses are needed. However, it turns out that final results strongly depend on the small-x behaviour of the parton densities of the colliding hadrons.

Parton densities at small x are dominated by the gluon channel. It is known [5] that the leading high-energy corrections to the anomalous dimensions are single-logarithmic terms $(\alpha_S \ln x)^n$ (higher powers of $\ln x$ cancel in this case), which have been resummed to all orders in α_S. The ensuing gluon anomalous dimension [5–7] produces a branch-point singularity (known as perturbative QCD pomeron) in the moment (angular momentum) space, thus leading to a very steep behaviour of the parton densities at small x and large p_t.

The resummation of the leading-order contributions at high energy is a crude (although mandatory) approximation. This is true from both the phenome-

☆ Research supported in part by the Italian Ministero della Università e Ricerca Scientifica.

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nological (terms of relative order α_S are systematically neglected) and the theoretical (the perturbative QCD pomeron violates unitarity to leading order) sides. The evaluation of next-to-leading corrections to parton densities at small x is therefore relevant (i) to set the limits of applicability of the leading-order formalism and to reliably fix the *normalization* and *intercept* of the perturbative QCD pomeron, and (ii) to include corrections necessary to restore unitarity at high energy (although it is likely that the full restoration of unitarity can be achieved only after the inclusion of higher-twist contributions [8]).

The parton densities are more easily extracted from inclusive processes, such as total cross sections or deep inelastic scattering structure functions. The evaluation of the next-to-leading corrections at small x for these processes requires the computation of both the coefficient functions and anomalous dimensions.

In the case of processes that are directly coupled to gluons in the naive parton model, the resummed coefficient functions can be calculated by using the high-energy or $k_\perp$-factorization theorem [9–11]. In particular, a complete resummed calculation for the normalization factor of the perturbative QCD pomeron has been performed in [12] by properly taking care of the factorization issue of mass singularities.

Not much progress has been made so far in the evaluation of subleading contributions to the anomalous dimensions. Most of the efforts have been concentrated on pure-gauge QCD and a complete calculation of the gluon anomalous dimensions now seems feasible [13]. On the contrary, the quark sector (which, beyond leading order, has to be considered on an equal footing with the gluon sector) has received very little attention. This is surprising in view of the fact that the most accurate information on small-x parton densities is expected to come out from HERA data on deep inelastic structure functions, which couple directly to quarks (and not gluons).

In this paper we concentrate on the quark sector and present resummed expressions for the quark anomalous dimensions to next-to-leading accuracy. All the results we discuss have been obtained by extending the high-energy factorization theorem [9,10] beyond leading order [14]. Our notation and some general results on anomalous dimensions at small x are discussed in section 2. The problem of evaluating the quark anomalous dimensions beyond two-loop order is considered in section 3, where resummed expressions to all orders in the effective expansion parameter $\alpha_S \ln x$ are given. Our main results are summarized in section 4.

2. Parton densities and anomalous dimensions

The leading-twist parton densities $f_a(x,\mu^2)$ ($a=q_i$, $\bar{q}_i$, g, $i=1, \ldots, N_f$, N_f being the number of flavours) entering the factorization theorem of mass singularities for hadron collisions [15] fulfil the renormalization group evolution equations

$$\frac{\mathrm{d}f_a(x,\mu^2)}{\mathrm{d}\ln\mu^2} = \sum_b \int_x^1 \frac{\mathrm{d}z}{z} P_{ab}(\alpha_S(\mu^2), z) f_b(x/z, \mu^2) . \tag{1}$$

Here $P_{ab}(\alpha_S, z)$ are generalized Altarelli–Parisi splitting functions, which are computable in QCD perturbation theory as power series expansions in α_S

$$P_{ab}(\alpha_S, x) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n P_{ab}^{(n-1)}(x) . \tag{2}$$

The x-dependence of the evolution equations (1) can be diagonalized by introducing the N-moments [#1] of the parton densities

$$f_{a,N}(\mu^2) \equiv \int_0^1 \mathrm{d}x\, x^N f_a(x,\mu^2) \tag{3}$$

and the anomalous dimension matrix

$$\gamma_{ab,N}(\alpha_S) \equiv \int_0^1 \mathrm{d}x\, x^N P_{ab}(\alpha_S, x) . \tag{4}$$

The ensuing evolution equations read as follows:

$$\frac{\mathrm{d}f_{a,N}(\mu^2)}{\mathrm{d}\ln\mu^2} = \sum_b \gamma_{ab,N}(\alpha_S(\mu^2)) f_{b,N}(\mu^2) . \tag{5}$$

Although the parton densities cannot be evaluated in QCD perturbation theory, their μ-dependence is completely predictable in perturbative QCD via the evolution equations (1) (or (5)) once the splitting

[#1] Note that our definition of N-moments differs from the standard one by the replacement $N-1 \to N$.

functions P_{ab} (or γ_{ab}) have been computed to some order in α_S.

The leading-order splitting functions $P^{(0)}_{ab}(x)$ (which have been known for a long time [16]) are factorization theorem invariants, i.e. they do not depend on the explicit factorization procedure of mass singularities. The physical reason for this is that they are directly related to observable scaling violations in deep inelastic scattering (DIS) processes. On the contrary, splitting functions and anomalous dimensions beyond one-loop order to depend on the regularization and factorization schemes of mass singularities. Nonetheless, due to charge conjugation invariance and $SU(N_f)$ flavour symmetry of QCD, they satisfy the following scheme-independent properties:

$$\gamma_{q_ig}=\gamma_{\bar{q}_ig}\equiv\gamma_{qg}\,,$$
$$\gamma_{gq_i}=\gamma_{g\bar{q}_i}\equiv\gamma_{gq}\,,$$
$$\gamma_{q_iq_j}=\gamma_{\bar{q}_i\bar{q}_j}\equiv\gamma^{NS}_{qq}\delta_{ij}+\gamma^{S}_{qq}\,,$$
$$\gamma_{q_i\bar{q}_j}=\gamma_{\bar{q}_iq_j}\equiv\gamma^{NS}_{q\bar{q}}\delta_{ij}+\gamma^{S}_{q\bar{q}}\,. \tag{6}$$

The symmetry properties (6) imply that the anomalous dimensions matrix γ_{ab} has only seven independent components. Correspondingly three flavour non-singlet $(f^{(V)}, f^{(-)}_{q_i}, f^{(+)}_{q_i})$ and two flavour singlet (f_S, f_g) parton densities can be introduced so that the evolution equations (5) are completely diagonalized (in the partonic space) for the non-singlet sector. One explicitly finds (we drop the overall dependence on N, μ, α_S)

$$\frac{d\ln f^{(V)}}{d\ln\mu^2}=\gamma^{(V)}\,,$$
$$\frac{d\ln f^{(-)}_{q_i}}{d\ln\mu^2}=\gamma^{(-)}\,,\quad \frac{d\ln f^{(+)}_{q_i}}{d\ln\mu^2}=\gamma^{(+)}\,, \tag{7}$$

where

$$f^{(V)}\equiv\sum_{j=1}^{N_f}(f_{q_j}-f_{\bar{q}_j})\,,$$
$$f^{(\pm)}_{q_i}\equiv f_{q_i}\pm f_{\bar{q}_i}-\frac{1}{N_f}\sum_{j=1}^{N_f}(f_{q_j}\pm f_{\bar{q}_j})\,, \tag{8}$$

and the non-singlet anomalous dimensions are given by

$$\gamma^{(V)}=\gamma^{NS}_{qq}-\gamma^{NS}_{q\bar{q}}+N_f(\gamma^{S}_{qq}-\gamma^{S}_{q\bar{q}})\,,$$
$$\gamma^{(\pm)}=\gamma^{NS}_{qq}\pm\gamma^{NS}_{q\bar{q}}\,. \tag{9}$$

The evolution equations are instead still coupled in the singlet sector

$$\frac{df_S}{d\ln\mu^2}=[\gamma^{NS}_{qq}+\gamma^{NS}_{q\bar{q}}+N_f(\gamma^{S}_{qq}+\gamma^{S}_{q\bar{q}})]f_S+2N_f\gamma_{qg}f_g\,,$$
$$\frac{df_g}{d\ln\mu^2}=\gamma_{gq}f_S+\gamma_{gg}f_g\,, \tag{10}$$

where the quark singlet density is defined by $f_S=\sum_{i=1}^{N_f}(f_{q_i}+f_{\bar{q}_i})$.

In the small-x limit the generalized Altarelli–Parisi splitting functions $P_{ab}(\alpha_S, x)$ in eq. (2) behave in general as $1/x$ modulo $\ln x$ corrections. Using the high-energy factorization theorem it is straightforward to show that the highest power of these $\ln x$ corrections appearing to $(n-1)$-loop order is $n-1$:

$$P^{(n-1)}_{ab}(x)\sim\frac{1}{x}[\ln^{n-1}x+O(\ln^{n-2}x)]\,,$$
$$x\to 0\,. \tag{11}$$

This small-x behaviour corresponds, in N-moment space, to singularities for $N\to 0$ in the form

$$\gamma_{ab,N}(\alpha_S)=\sum_{k=1}^{\infty}\left[A^{(k)}_{ab}\left(\frac{\alpha_S}{N}\right)^k+B^{(k)}_{ab}\alpha_S\left(\frac{\alpha_S}{N}\right)^k+\dots\right],$$
$$N\to 0\,, \tag{12}$$

where $A^{(k)}_{ab}$, $B^{(k)}_{ab}$, ... are the *leading*, *next-to-leading*, ... coefficients in the high-energy regime.

As discussed before, the higher-loop contributions to the anomalous dimensions are factorization-scheme-dependent. Therefore, in order to compute $A^{(k)}_{ab}$, $B^{(k)}_{ab}$ and so on, we have to specify the factorization scheme we use. In the rest of this section we consider one of the most commonly used schemes, namely the $\overline{MS}$ scheme, because in this case the factorization procedure is uniquely defined to all orders in α_S. The partonic matrix elements are first computed in $n=4+2\epsilon$ space–time dimensions, considering $(n-2)$ helicity states for gluons and 2 helicity states for quarks. Collinear and ultraviolet singularities are thus automatically regularized and show up as single poles in $1/\epsilon$. Then the renormalized parton densities f_a are defined by subtracting the pole factors $(\alpha_S S_\epsilon/\epsilon)^n$ $(S_\epsilon=\exp\{-\epsilon[\psi(1)+\ln 4\pi]\})$ from the partonic matrix elements.

We start the presentation of our small-x results by considering the (quark) non-singlet sector. Using the high-energy factorization theorem we can show [14] that in the $\overline{\rm MS}$ scheme the non-singlet anomalous dimensions (9) are *regular* for $N\to 0$ order by order in α_S. In other words, the corresponding n-loop splitting functions $P^{(n-1)}$ in eq. (2) are less singular than $1/x$ for $x\to 0$. This is not surprising, because high-energy singularities are due to multiple gluon exchanges in the t-channel and the gluon channel is not coupled to the flavour non-singlet sector. All the high-energy contributions α_S^n/N^k ($n\geqslant k\geqslant 1$) are thus associated with the singlet sector, and more precisely with the gluon anomalous dimensions γ_{gg}, γ_{gq} and with the quark anomalous dimensions $\gamma_{qq}^{\rm S}\propto\gamma_{q\bar{q}}^{\rm S}$, γ_{qg}.

The gluon anomalous dimensions contain leading contributions of the type $(\alpha_S/N)^k$, which can be traced back to the work of Balitskii, Fadin, Kuraev and Lipatov [5,6]. More precisely, we have ($\bar{\alpha}_S\equiv C_A\alpha_S/\pi$)

$$\gamma_{gg,N}(\alpha_S)=\gamma(\bar{\alpha}_S/N)+{\rm O}\left(\alpha_S\left(\frac{\alpha_S}{N}\right)^k\right),$$

$$\gamma_{gq,N}(\alpha_S)=\frac{C_F}{C_A}\,\gamma(\bar{\alpha}_S/N)+{\rm O}\left(\alpha_S\left(\frac{\alpha_S}{N}\right)^k\right), \qquad (13)$$

and the BFKL anomalous dimension $\gamma(\bar{\alpha}_S N)$ is obtained by the following implicit equation:

$$1=\frac{\bar{\alpha}_S}{N}\chi\left(\gamma\left(\frac{\bar{\alpha}_S}{N}\right)\right), \qquad (14)$$

where $\chi(\gamma)$ is expressed in terms of the Euler ψ-function

$$\chi(\gamma)=2\psi(1)-\psi(\gamma)-\psi(1-\gamma)\,. \qquad (15)$$

By solving eq. (14) in power series of the coupling constant one resums the leading $(\alpha_S/N)^k$ contributions for the gluon anomalous dimension to all orders in α_S. The first perturbative terms are [$\zeta(n)$ is the Riemann zeta-function: $\zeta(3)\simeq 1.202$, $\zeta(5)\simeq 1.037$]

$$\gamma\left(\frac{\bar{\alpha}_S}{N}\right)=\frac{\bar{\alpha}_S}{N}+2\zeta(3)\left(\frac{\bar{\alpha}_S}{N}\right)^4+2\zeta(5)\left(\frac{\bar{\alpha}_S}{N}\right)^6 +{\rm O}\left(\left(\frac{\bar{\alpha}_S}{N}\right)^7\right). \qquad (16)$$

Note that $\gamma(\bar{\alpha}_S/N)$ slowly departs from its one-loop contribution. Actually, the coefficients of the orders α_S^2, α_S^3 and α_S^5 vanish. This is due to strong cancellations between real emission diagrams and virtual corrections. Starting from ${\rm O}(\alpha_S^6)$ similar cancellations are no longer effective and the BFKL anomalous dimension increases, reaching the saturation value $\bar{\gamma}=\frac{1}{2}$. Therefore the resummation of the singular terms $(\alpha_S/N)^k$ build up a stronger singularity at $\bar{N}=4\alpha_S\ln 2$ (perturbative QCD pomeron), which is responsible for the $x^{-(\bar{N}+1)}$ behaviour of the gluon density $f_g(x,\mu^2)$ at high μ^2.

The next-to-leading contributions ${\rm O}(\alpha_S(\alpha_S/N)^k)$ in eq. (13) are not yet known, apart from the first non-trivial term ($k=1$) [17]. A first step towards their evaluation has been performed recently [13].

The quark anomalous dimensions $\gamma_{qq}^{\rm S}$ and γ_{qg} are known up to two-loop order [17,18]. In the $\overline{\rm MS}$ scheme, considering the $N\to 0$ limit, one gets

$$\gamma_{qg,N}^{(\overline{\rm MS})}(\alpha_S)=\frac{\alpha_S}{2\pi}\,T_R\cdot\frac{2}{3}\left(1+\frac{5}{3}\frac{\bar{\alpha}_S}{N}+\dots\right),$$

$$\gamma_{qq,N}^{{\rm S}(\overline{\rm MS})}(\alpha_S)=\tfrac{5}{9}C_FT_R\left(\frac{\alpha_S}{\pi}\right)^2\frac{1}{N}+\dots\,. \qquad (17)$$

The expressions (17) have no $(\alpha_S/N)^k$ leading contributions but do have next-to-leading terms α_S^2/N. Next-to-leading corrections are also present to higher orders in α_S, and their resummation is considered in the following section.

3. Quark singlet anomalous dimensions

Higher-order QCD calculations for hadron collisions are usually performed in two different factorization schemes of mass singularities, the $\overline{\rm MS}$ scheme and the DIS scheme [19]. In order to evaluate higher-loop contributions to the quark anomalous dimensions it is convenient to consider the DIS scheme. As explained in [14], this scheme offers some computational advantages. Moreover, the next-to-leading contributions to the gluon anomalous dimensions being still unknown, the knowledge of the quark anomalous dimensions in the DIS scheme may help in preliminary phenomenological investigations of the small-x behaviour of the DIS structure function $F_2(x,Q^2)$. This is because in the DIS scheme gluons are not directly coupled to F_2. As a matter of fact, after having regularized the parton matrix elements using dimensional regularization, the DIS-scheme parton densities $f_a^{(\mathrm{DIS})}$ are defined by enforcing the constraint that the customary structure function $F_2(x, Q^2)$ has the same expression as in the naive parton model. For instance, in the one-photon approximation of deep inelastic lepton–hadron scattering, we have (e_i are the quark charges):

$$F_2(x,Q^2) = \sum_{i=1}^{N_f} e_i^2 [f_{q_i}^{(\mathrm{DIS})}(x,Q^2) + f_{\bar{q}_i}^{(\mathrm{DIS})}(x,Q^2)] . \tag{18}$$

The price which has to be paid for this simplification is that the DIS-scheme factorization procedure defines unequivocally (to all orders in α_S) only the quark densities. The DIS-scheme gluon density is still ambiguous and is given by an arbitrary combination of gluon and quark singlet densities in the $\overline{\mathrm{MS}}$ scheme

$$f_{g,N}^{(\mathrm{DIS})}(\mu^2) = U_{g,N}(\alpha_S(\mu^2)) f_{g,N}(\mu^2) + U_{q,N}(\alpha_S(\mu^2)) f_{S,N}(\mu^2) , \tag{19}$$

with the only constraints $U_{g,N}(\alpha_S(\mu^2)) = 1 + \mathrm{O}(\alpha_S)$, $U_{q,N} = \mathrm{O}(\alpha_S)$ (eq. (19) must reduce to the identity to leading order) and $U_{g,N} + U_{q,N} = 1$ for $N=1$ (momentum conservation).

For the purposes of our all-order calculation, it is not necessary to specify the actual form of the two coefficient functions $U_{g,N}$, $U_{q,N}$ in eq. (19). We just assume that they are chosen not to be pathologically singular at high energies, i.e. they should not contain leading-order contributions of the type $(\alpha_S/N)^k$ for $N \to 0$ [#2]. This is sufficient to ensure that the main $\overline{\mathrm{MS}}$-scheme results discussed in the previous section remain valid in the DIS scheme: the non-singlet anomalous dimensions (9) are not singular for $N \to 0$ and the leading-order gluon anomalous dimensions are still given by eq. (13).

We are now in a position to present our results on the quark anomalous dimensions γ_{qg} and γ_{qq}^S. We start considering γ_{qg}. By using the high-energy factorization theorem we obtain [14] the following resummed expression for $N \to 0$ in the DIS scheme:

[#2] This property is fulfilled by the standard definition to $\mathrm{O}(\alpha_S)$ [19,20] and can be implemented in a simple way to all orders [14].

$$\gamma_{qg,N}^{(\mathrm{DIS})}(\alpha_S) = \frac{\alpha_S}{2\pi} T_R \frac{2+3\gamma_N - 3\gamma_N^2}{3-2\gamma_N} \frac{\Gamma^3(1-\gamma_N)\Gamma^3(1+\gamma_N)}{\Gamma(2+2\gamma_N)\Gamma(2-2\gamma_N)} R(\gamma_N) + \mathrm{O}\left(\alpha_S^2 \left(\frac{\alpha_S}{N}\right)^k\right) , \tag{20}$$

where $R(\gamma_N)$ is the normalization factor of the perturbative QCD pomeron computed in [12]:

$$R(\gamma_N) = \left(\frac{\Gamma(1-\gamma_N)\chi(\gamma_N)}{\Gamma(1+\gamma_N)[-\gamma_N \chi'(\gamma_N)]} \right)^{1/2} \times \exp\left(\gamma_N \psi(1) + \int_0^{\gamma_N} \mathrm{d}\gamma \frac{\psi'(1) - \psi'(1-\gamma)}{\chi(\gamma)} \right) , \tag{21}$$

and χ, χ' are the characteristic function in eq. (15) and its first derivative, respectively.

The result in eq. (20) is conveniently written in terms of the BFKL anomalous dimension $\gamma_N \equiv \gamma(\bar{\alpha}_S/N)$. The resummation of the contributions $\alpha_S(\alpha_S/N)^k$ to all orders in α_S is incorporated in eq. (20) through the α_S/N-dependence of γ_N [known from the BFKL equation (14)] and the γ_N-dependence of $R(\gamma_N)$ as given by eq. (21).

Using the expansion (16) for the BFKL anomalous dimension, the first perturbative terms of the quark anomalous dimension (20) can easily be computed:

$$\begin{aligned} \gamma_{qg,N}^{(\mathrm{DIS})} &= \frac{\alpha_S}{2\pi} T_R \left[\frac{2}{3} + \frac{13}{9}\frac{\bar{\alpha}_S}{N} + \tfrac{2}{3}[\tfrac{71}{18} - \zeta(2)] \left(\frac{\bar{\alpha}_S}{N}\right)^2 + \tfrac{2}{3}[\tfrac{233}{27} - \tfrac{13}{6}\zeta(2) + \tfrac{8}{3}\zeta(3)] \left(\frac{\bar{\alpha}_S}{N}\right)^3 + \mathrm{O}\left(\frac{\bar{\alpha}_S}{N}\right)^4 \right] \\ &\simeq \frac{\alpha_S}{2\pi} T_R \frac{2}{3} \left[1 + 2.17 \frac{\bar{\alpha}_S}{N} + 2.30 \left(\frac{\bar{\alpha}_S}{N}\right)^2 + 8.27 \left(\frac{\bar{\alpha}_S}{N}\right)^3 + \mathrm{O}\left(\frac{\bar{\alpha}_S}{N}\right)^4 \right] . \end{aligned} \tag{22}$$

The coefficients of the first two terms in the large square brackets agree with the known one- and two-loop anomalous dimensions in the DIS scheme [17,18,20]. Higher-order contributions can be used to estimate the effect of these small-x corrections for intermediate values of x. In particular, the $\mathrm{O}(\alpha_S^3)$ term in (22) can be combined with the existing $\mathrm{O}(\alpha_S^2)$ calculations of the coefficient functions for the

Drell–Yan [21] and DIS [20] processes in order to check the stability of the fixed-order perturbative expansion in the x-range accessible at present ($x \sim 10^{-2}$). Note that, unlike the case of the gluon anomalous dimensions (13), (16), all the perturbative coefficients in eq. (22) are non-vanishing. Therefore in the quark sector one may expect a quicker departure from the fixed-order perturbative behaviour.

At smaller values of x ($x \lesssim 10^{-3}$), the complete resummed result (20) has to be considered. Note that the expression (20) is analytic for $0 \leqslant \gamma_N < \frac{1}{2}$. Thus, independently of the value of α_S, the leading trajectory in N-moment space is still given by the BFKL pomeron. As the BFKL anomalous dimension γ_N increases towards its saturation value at $\gamma_N = \frac{1}{2}$, the quark anomalous dimension (20) quickly increases, approaching a singularity due to the pomeron normalization factor $R(\gamma_N)$:

$$R(\gamma_N) \simeq \text{const.} \cdot \left(\frac{1}{1-2\gamma_N(\alpha_S)} \right)^{1/2}, \quad \gamma_N \to \tfrac{1}{2}. \qquad (23)$$

The increase of γ_{qg} leads to strong scaling violations, although the singularity at $\gamma_N = \frac{1}{2}$ is likely to be cancelled [12,14], in physical observables, by analogous contributions to resummed coefficient functions.

The quark anomalous dimension γ^{S}_{qq} starts in order α_S^2. Its resummed expression at high energy is however related in a simple way to that for γ_{qg}. For $N \to 0$ we find [14]

$$\gamma^{\mathrm{S}}_{qq,N}(\alpha_S) = \frac{C_F}{C_A} \left(\gamma_{qg,N}(\alpha_S) - \frac{\alpha_S}{2\pi} T_R \cdot \tfrac{2}{3} \right) + \mathrm{O}\left(\alpha_S^2 \left(\frac{\alpha_S}{N} \right)^k \right). \qquad (24)$$

We see that beyond $\mathrm{O}(\alpha_S^2)$ the coefficients of the terms $\alpha_S(\alpha_S/N)^k$ in γ_{qa} ($a = q$, g) are equal, apart from an overall factor given by the ratio of the colour charges of the initial-state partons a.

The resummation of the high-energy contributions to the quark anomalous dimensions in the $\overline{\mathrm{MS}}$ scheme is more cumbersome. Using the high-energy factorization theorem we have checked the known results in eq. (17) [17,18] and we have computed [14] the three-loop coefficient

$$\gamma^{(\overline{\mathrm{MS}})}_{qg,N}(\alpha_S) = \frac{\alpha_S}{2\pi} T_R \cdot \frac{2}{3} \left[1 + \frac{5}{3} \frac{\bar{\alpha}_S}{N} + \frac{14}{9} \left(\frac{\bar{\alpha}_S}{N} \right)^2 + \mathrm{O}\left(\left(\frac{\bar{\alpha}_S}{N} \right)^3 \right) \right]$$
$$\simeq \frac{\alpha_S}{2\pi} T_R \cdot \frac{2}{3} \left[1 + 1.67 \frac{\bar{\alpha}_S}{N} + 1.56 \left(\frac{\bar{\alpha}_S}{N} \right)^2 + \mathrm{O}\left(\left(\frac{\bar{\alpha}_S}{N} \right)^3 \right) \right] \qquad (25)$$

The calculation of higher-order contributions is in progress and the results (hopefully, including a complete resummation) will be reported elsewhere. Note that the first two coefficients in the square brackets of eq. (25) are systematically (although slightly) smaller than the corresponding DIS-scheme coefficients in (22). We do not know whether such a behaviour persists in higher orders. Note also that the difference between the DIS-scheme and the $\overline{\mathrm{MS}}$-scheme anomalous dimensions in eqs. (22) and (25) is related to the coefficient functions of the deep inelastic scattering structure functions. One can easily check that our results (22) and (25) reproduce the small-x behaviour of these coefficient functions recently computed up to $\mathrm{O}(\alpha_S^2)$ [20].

The all-order relation (24) between γ^{S}_{qq} and γ_{qg} is still valid in the $\overline{\mathrm{MS}}$ scheme [for this reason we have dropped the superscripts $\overline{\mathrm{MS}}$ and DIS in (24)]. One can also show [14] that this relation and its leading-order analogue (13) do not depend on the details of the dimensional regularization prescription. In particular they remain true using dimensional reduction [22], a dimensional regularization scheme which explicitly respects supersymmetric Ward identities. In this sense, eqs. (13) and (24) can be considered as high-energy limits of the supersymmetry identity $\gamma_{qq} + \gamma_{gq} = \gamma_{qg} + \gamma_{gg}$ valid in $N = 1$ supersymmetric Yang–Mills theory, i.e. for $C_F = T_R = C_A$. It follows that in the supersymmetric case the gluon anomalous dimensions γ_{gg} and γ_{gq} coincide also to the next-to-leading order $\alpha_S(\alpha_S/N)^k$. This property may be useful as a technical tool to check and simplify the calculation of the next-to-leading-order corrections in the gluon sector.

4. Summary

In this paper we have presented the theoretical results of a first attempt to go beyond the leading logarithmic approximation in the evaluation of the parton anomalous dimensions at small x. We have considered in detail the quark anomalous dimensions in both the flavour non-singlet and flavour singlet sectors. We have shown that in the factorization schemes commonly used ($\overline{\mathrm{MS}}$ and DIS), the non-singlet splitting functions $P_{ab}(\alpha_S, x)$ are less singular than $1/x$ order by order in perturbation theory.

The quark singlet splitting functions are instead more singular than $1/x$ because of the presence of enhancing logarithmic factors. These contributions have been computed with next-to-leading logarithmic accuracy up to three loops; in the case of the DIS scheme we have obtained a complete resummed expression to all orders in α_S.

Although a consistent next-to-leading analysis of hadronic cross sections requires also the computation of the gluon anomalous dimensions, we think that the results presented here can be used for more accurate phenomenological investigations of parton distributions and, in particular, of deep inelastic scattering structure functions.

Acknowledgement

We thank M. Ciafaloni, G. Marchesini and B.R. Webber for valuable discussions. One of us (F.H.) is grateful for the hospitality of the DESY-Institut für Hochenergiephysik at Zeuthen, where part of this work was carried out.

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