A new framework for describing the decision behaviour of large groups

Johan Springael*, Yves De Smet†, Pierre Kunsch‡

* Faculty of Applied Economics,
Universiteit Antwerpen,
Prinsstraat 13, B-2000 Antwerp, Belgium

† Service de Mathématiques de la Gestion,
Université Libre de Bruxelles,
Boulevard du Triomphe CP 210-01, B-1050 Brussels, Belgium

‡ Centrum voor Statistiek en Operationeel Onderzoek,
Vrije Universiteit Brussel,
Pleinlaan 2, B-1050 Brussels, Belgium

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Abstract

Many occurrences in real-life result from decisions taken by a very large number of decision makers. Furthermore, these decisions often depend on the optimisation of several conflicting criteria. The decision or behaviour of every microscopic entity results finally in a global macroscopic behaviour of the whole group of decision makers. In such context, the use of classical decision support tools, such as multicriteria based group decision support systems, to model these macroscopic phenomena is not appropriate.

In order to tackle this type of problems, we introduce a tool based on Markov chains to model and manage these large group decisions. Finally, the method results in a statistical distribution of decisions, allowing us to study the impact of policy measures on the global group behaviour. This is illustrated by means of a simple example stemming from the telecommunication sector.

Keywords: Multicriteria Decision Aid, Statistical Modelling, Markov Chains, Pairwise Preference Modelling

AMS Classification: 90B50, 91B06

1 Introduction

One of the major developments in the operations research field of the last decades, although less widely spread in practice, is the conceptual view of multicriteria decision analysis (MCDA). This type of analysis allows us to take into account conflicting criteria relevant for a given decision problem. Several approaches have been presented which can be divided in merely two main classes: multiple attribute value or utility function methods and outranking methods (for a broad overview see e.g. Belton and Steward (2002)). In these multicriteria decision methods (MCDM's) supplementary information is asked to the decision maker with respect to her/his preferences (between the criteria and between the alternatives within a single criterion). Often these preferences are forced into a prescribed structure, which is in most cases the source of the discussion between the conceivers of the different MCDM's. In this paper we will not plead in favour of a specific method, though we will suppose that a MCDA has been performed, leading to a matrix of preference degrees. These points will be clarified in the next section.

A further breakthrough in the field of decision aid arose with the notion of group decision support systems (GDSS's) (Teng and Ramamurthy, 1993), (DeSanctis and Gallupe, 1987) based on a MCDM. Hereby, the scientists working in the MCDM field recognised the fact that some decisions were not made by an individual but by a group of different decision makers with their proper preferences and preference structure.

In this paper, we want to model and manage decisions taken by a huge number of decision makers, typically 1000, 10.000 or more, which do occur in real-life (e.g. the decision with respect to the departure time for different car commuters, the distribution of the market shares of many companies in the same markets, TV channel selection, choice of brands of a product, etc). In such a context, it is practically impossible to perform an individual MCA for every member of the group. Even approaches such as multicriteria group decision aid are not appropriate, due to the high number of entities involved, and due to the fact that the purpose is different. Here the main purpose is to study the global decision behaviour and not to select a single or a set of good alternatives.

In this framework, what we call global decision behaviour is nothing but the observable frequency distribution in the choices of the decision makers involved in the problem. Therefore, our purpose will be to model these frequency distributions by means of an adequate "decision" probability distribution.

In our point of view, decision makers continuously compare and re-compare dynamically pairs of alternatives during their decision process. This kind of behaviour can easily be interpreted as a hidden Markov chain. Starting with a given preference structure, our aim is to build an appropriate transition matrix to characterize this Markov chain. If the necessary conditions for convergence are satisfied, the limiting equilibrium distribution (steady state) can be calculated which we will interpret as the global decision behaviour.

Once able to calculate such a distribution one will increase the ability to understand how this global distribution can be influenced via appropriate policy measures. Applying new measures will change the outcome of the MCA and therefore also the preference matrix, which could lead to a change in the final distribution.

It should be remarked that the problem considered in this paper has already been studied by the authors. Actually, in a previous paper De Smet et al. (2002) the authors have introduced some of the main ideas developed here below and have illustrated them on a first example, based on car commuters' behaviour. The model presented in that paper was empirically developed and tested. Our goal here is, above all, to develop a theoretical framework for this new approach, which is based on a fundamental mathematical structure. Furthermore, let us stress that we were not the first to apply Markov chains in a multicriteria setup. Works of Glineur (2002) testify that this idea was already considered, however, in the context of a choice problem, which is totally different from the one studied in this paper.

In the next section we will present the general scheme of the method together with the necessary assumptions and notations. The ideas based on the theory of Markov chains are introduced in order to determine the statistical distribution of decisions. Though this will end in a mathematical problem which is solved in a constructive manner, starting from the case in which the decision has to be made between two alternatives, which is done in section 2.2. Section 2.3 consists in generalising the result for two alternatives to the general case of any number of alternatives. An application of the methodology is presented in section 3. In the last section we present some conclusions and further remarks for future research.

2 Theoretical framework

2.1 Introductory notations

Let us consider a large group of decision makers confronted with the choice between N alternatives A_i, $i = 1, \ldots, N$ (each decision maker is supposed to choose a single alternative among all the potential alternatives). In this framework, we assume that the global decision behaviour can be deduced from a single preference matrix representing the general preference structure of the group $P^{(N)}$. Hence, we implicitly assume that the preferences of the members of the studied group are sufficiently homogeneous. It will be possible to depart from this restriction by considering several different homogeneous groups.

The starting point of our approach is thus: perform a "single" multicriteria analysis in order to obtain this preference structure $P^{(N)}$. The elements of this matrix: $P_{ij}^{(N)}$ stand for the preference of alternative A_i over the alternative A_j and satisfy the following conditions:

$$P_{ij}^{(N)} \geq 0 \tag{1}$$

$$P_{ij}^{(N)} + P_{ji}^{(N)} \leq 1 \tag{2}$$

$$P_{ii}^{(N)} = 0 \tag{3}$$

As already mentioned we assume that while making his choice the decision maker is dynamically comparing and re-comparing the different alternatives with eachother. In this case one may consider the probability that the decision maker will change his mind with respect to alternative A_i in favour of alternative A_j. Let us denote this transition probability $T_{ij}^{(N)}$. All the pairwise comparison transition probabilities can be gathered in the so-called transition matrix $T^{(N)}$.

The idea we propose is to see the transition probability matrix $T^{(N)}$ as a transition matrix related to a hidden Markovian proces, which under some conditions (ergodicity), generates a stationary distribution starting from an initial distribution. It can be shown for ergodic Markov chains (EMC) that the stationary distribution $Q^{(N)}$ solves the matrix equation:

$$Q^{(N)} \cdot \left[\mathbb{1}_N - T^{(N)} \right] = 0 \tag{4}$$

with $\mathbb{1}_N$ the $N \times N$ identity matrix and $Q^{(N)}$ a N-dimensional vector, of which the components have to satisfy the following normalisation relationship:

$$\sum_{i=1}^{N} Q_i^{(N)} = 1 \tag{5}$$

Equations (4) together with eq.(5) lead in the case of an EMC to a unique solution and therefore also to a unique distribution of choices.

There is however a step in the reasoning which has not yet been discussed, namely the question on how to determine the transition probabilities $T_{ij}^{(N)}$, having only presumed that the preference matrix $P^{(N)}$ is known. Hence, a transformation is needed to transform pairwise preferences into transition probabilities, which will be denoted as follows:

$$H_N : \mathcal{P}_N \to \mathcal{T}_N : P^{(N)} \mapsto T^{(N)} = H_N \left(P^{(N)} \right) \tag{6}$$

with $\mathcal{P}_N$ the set of $N \times N$ matrices with elements satisfying relations (1-3) and $\mathcal{T}_N$ the set of $N \times N$ stochastic matrices.

Hypothesis 1 *Each element $T_{ij}^{(N)}$ of the transition matrix only depends on the pairwise preferences $(P_{ij}^{(N)}, P_{ji}^{(N)})$ and on the total number of alternatives N.*

Without being too restrictive, this assumption has been made in order to simplify the model. Weaker assumptions may be considered but this would lead us beyond the scope of this paper and might lead to a non- Markovian process (see later), due to the additional dependencies with respect to other alternatives.

In order to determine this transformation we chose to follow a constructive approach starting with the case of two alternatives, which is elaborated here below.

2.2 A preference-transition transformation for two alternatives

In order to build the transformation H_2, we consider the three extreme occurences which are represented in figure 1.

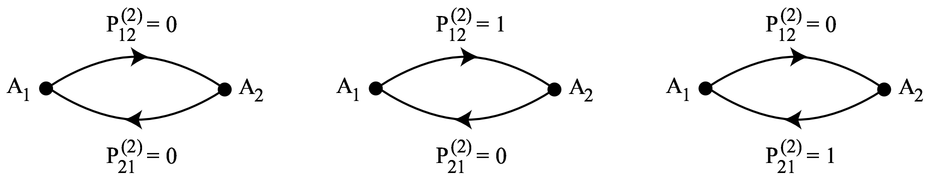

Figure 1: Representation of the preference structure for three extreme situations: complete indifference (left); A_1 dominates A_2 completely (middle); A_2 dominates A_1 completely (right)

The first case is the situation in which a decision maker is completely indifferent between the two alternatives. The preference matrix is given by:

$$P^{(2)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

It is clear in this case that the probability to remain in alternative A_1, or to leave this alternative in favour of alternative A_2 must be equal to 1/2. The same reasoning holds when starting from alternative A_2. Hence, the transition matrix is written as:

$$T^{(2)} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \tag{8}$$

A second extreme situation occurs when one of the alternatives is strictly prefered to the other, which is represented in the middle and right part of figure 1. Considering that A_1 is strictly prefered to A_2, we obtain the following preference matrix:

$$P^{(2)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{9}$$

Since the decision maker has the maximal preference degree to choose A_1 over A_2, the probability of changing his mind when considering A_1 as starting alternative will be zero. In the same way one may argue that the probability of changing his mind in favour of alternative A_1, when considering A_2 as initial alternative, will be equal to 1. Hence, we obtain the transition probability matrix:

$$T^{(2)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \tag{10}$$

A similar reasoning will yield the preference matrix:

$$P^{(2)} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{11}$$

in case alternative A_2 is prefered over A_1 with preference degree equal to 1. As a result the following transition probability matrix is obtained:

$$T^{(2)} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{12}$$

Hypothesis 2 *H_2 is affine.*

Again for simplicity reasons, we will assume that the transformation H_2 is affine such that one may represent $T^{(2)}_{12}$ and $T^{(2)}_{21}$ in terms of $P^{(2)}_{12}$ and $P^{(2)}_{21}$ graphically as follows:

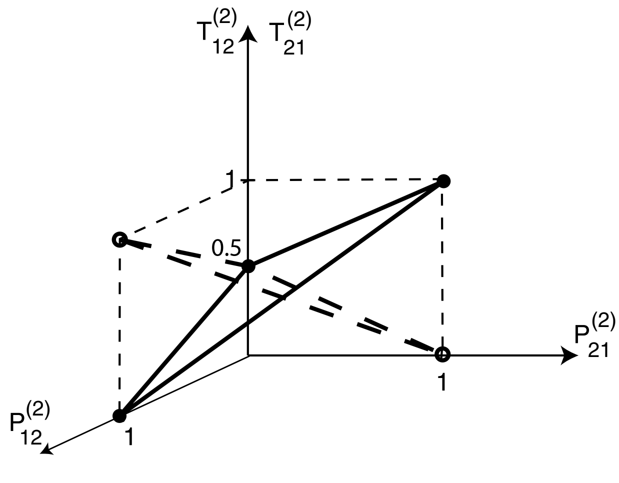

Figure 2: Graphical representation of the transformation H_2. Full lines and dots represent the result of the transformation H_2 for the transition probability $T^{(2)}_{12}$, whereas the dashed line and hollow dots show the result for $T^{(2)}_{21}$

In figure 2 it is clear that the transition probabilities can be expressed as:

$$T_{21}^{(2)} = \frac{1}{2}\left(P_{12}^{(2)} - P_{21}^{(2)} + 1\right) = 1 - T_{12}^{(2)} = T_{11}^{(2)} \tag{13}$$

$$T_{12}^{(2)} = \frac{1}{2}\left(P_{21}^{(2)} - P_{12}^{(2)} + 1\right) = 1 - T_{21}^{(2)} = T_{22}^{(2)} \tag{14}$$

From the relations (13) and (14) we have the remarkable property that

$$T_{12}^{(2)} + T_{21}^{(2)} = 1. \tag{15}$$

This enables us to determine the stationary distribution of choices between two alternatives using eqs.(4), (5), (13) and (14):

$$Q^{(2)} = \begin{bmatrix} T_{21}^{(2)} \\ T_{12}^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(P_{12}^{(2)} - P_{21}^{(2)} + 1) \\ \frac{1}{2}(P_{21}^{(2)} - P_{12}^{(2)} + 1) \end{bmatrix} \tag{16}$$

It is worth noticing that different preference matrices can be related to the same transition matrix. This follows immediately from relations (13) and (14). Indeed,

$$T_{ij}^{(2)} = \frac{1}{2}\left(P_{ji}^{(2)} - P_{ij}^{(2)} + 1\right)$$

which only depends on the difference $P_{ji}^{(2)} - P_{ij}^{(2)}$.

Example:

The matrices $P^{(2)}$ and $\tilde{P}^{(2)}$ are such that $H_2(P^{(2)}) = H_2(\tilde{P}^{(2)}) = T^{(2)}$:

$$P^{(2)} = \begin{bmatrix} 0 & 0.5 \\ 0.4 & 0 \end{bmatrix} \quad \tilde{P}^{(2)} = \begin{bmatrix} 0 & 0.3 \\ 0.2 & 0 \end{bmatrix} \quad T^{(2)} = \begin{bmatrix} 0.55 & 0.45 \\ 0.55 & 0.45 \end{bmatrix} \tag{17}$$

2.3 Generalisation towards any number of alternatives

In order to generalise the transformation between pairwise preference degrees and transition probabilities to the case of N alternatives, we have to introduce two additional assumptions regarding the way decision makers are comparing more than two alternatives.

Hypothesis 3 *(Sequential Decision Assumption) When facing the decision between N alternatives, we assume that the decision maker first randomly chooses an alternative and then, compares it with the current choice.*

This assumption is an extension of the idea of pairwise comparisons to the general case. Hence, we suppose that a decision maker, when having an alternative A_i in mind, will first choose an alternative A_j of the remaining ones. Secondly, he/she will compare it with the original one. In this way, the problem of choosing between alternatives is reduced to the situation explained in the former section.

Hypothesis 4 *(Equidistribution) The decision maker chooses randomly between the $N-1$ remaining alternatives using a uniform distribution.*

Let us note that this assumption can be weakened in the sense that other statistical distributions may be used. Though, this will be the topic of future works.

Using both assumptions and imposing independence between both choice probabilities, we may rewrite $T_{ij}^{(N)}$ as follows:

$$T_{ij}^{(N)} = \frac{1}{N-1} T_{ij}^{(2)} \tag{18}$$

where the factor $\frac{1}{N-1}$ stands for the probability to choose alternative A_j within the set of $N-1$ remaining alternatives. Since decision makers only perform pairwise comparisons in our setup, the transition probability from alternative A_i to alternative A_j will be nothing but the one in which the system of both alternatives would be isolated.

Relation (18) together with (13) yields an expression for the transition probilities $T_{ij}^{(N)}$ in terms of the pairwise preference degrees $P_{ij}^{(2)}$, which are thanks to the aforementioned hypothesis nothing but $P_{ij}^{(N)}$:

$$T_{ij}^{(N)} = \frac{1}{2(N-1)} \left(P_{ji}^{(2)} - P_{ij}^{(2)} + 1 \right) \qquad j \neq i \tag{19}$$

$$T_{ii}^{(N)} = 1 - \sum_{j \neq i} T_{ij}^{(N)} \tag{20}$$

A direct consequence of the above relations is the following property:

Theorem 1

$$T_{ii}^{(N)} = \sum_{j \neq i} T_{ji}^{(N)} \tag{21}$$

Theorem 2 *If the pairwise preference degrees $P_{ij}^{(N)}$ are computed according to the PROMETHEE framework we have:*

$$\forall i : \sum_{j \neq i} T_{ij}^{(N)} = \frac{1 - \phi_i}{2} \tag{22}$$

where ϕ_i is nothing but the PROMETHEE II net flow related to alternative A_i.

The previous theorem clearly establishes a link between the presented model and existing multicriteria methodologies. Its proof follows immediately from relation (19) and the definition of ϕ_i (Brans and Vincke, 1985) expressed in our notation:

$$\phi_i = \frac{1}{N-1} \sum_{j \neq i} \left[P_{ij}^{(2)} - P_{ji}^{(2)} \right]$$

A major assumption of our framework is the ergodicity of the Markov chain, or more precisely that the Markov chain has a unique aperiodic final class. This ensures that the resulting probability distribution $Q^{(N)}$ will not be affected by the initial distribution and therefore that the Markov chain will converge towards a stationary distribution.

Though it should be remarked that this condition is not always fulfilled. It therefore remains a challenge to examine those preference structures which give rise to a transition probability matrix of which the powers do not converge. However, for most practical problems it can be admitted that the convergence property is fulfilled. One may formulate the conjecture that the probability is zero for the occurence in real-life of a transition matrix which does not lead to an equilibrium distribution.

With expressions (19) and (20) one can easily calculate the stationary distribution $Q^{(N)}$ (if it exists) through the relations (4) and (5) for numeric examples.

3 Application to a real-life problem

3.1 The context of the example

The following example is inspired by the Belgian mobile phone sector. Our goal is to study the segmentation of prepaid cards among the three operators present on this market. Let us call these operators for convenience operator A, B and C.

Table 1 summarises the data used to model the preferences with respect to the choice between the operators. The first four columns indicate the price per minute communication (€/min) for the different operators depending on the call destination (i.e. the same operator or not) and the time schedule (during working hours or not). The quality of the operator is an index aggregating information such as network quality , covering of the network, . . . Finally a rough evaluation of the market shares of the operators in the market is given in the last column.

As a first observation let us note that the prices proposed by operator A are only differentiating with respect to the call destination. In addition operator B

	Same Operator		Different Operator			
	During Working hours	Outside Working hours	During Working hours	Outside Working hours	Quality	Market share
Operator A	0.124	0.124	0.62	0.62	9	35%
Operator B	0.37	0.245	0.74	0.245	10	50%
Operator C	0.13	0.13	0.5	0.13	6	15%

Table 1: Data: cost per minute structure, in terms of call destination and time period, and quality

distinguishes the time schedule of the call; the price remains the same for any destination when calling outside the working hours. Finally operator C proposes a fixed price except for calls to other operators during working hours.

Before illustrating our method with these data, it should be noticed that the context presented here, involves huge simplifications with respect to the real case:

- Every operator on the market proposes several types of prepaid cards. To keep things clear, which is not always the case in reality, we restricted ourselves to consider a single type of prepaid card per operator; the one that appeared to us to be the most representative for the company.
- No time component was taken into account, though it should be remarked that operator C arrived much later on the market than operator A and B.
- No subjective criteria such as the impact of marketing, the inertia to stay with the same operator, ... have been incorporated in this analysis.

All the results presented below can be criticised with respect to these remarks. It is however not our purpose in this paper to make an analysis of the market share of the three Belgian mobile phone operators, but to give a pedagogical example to illustrate the framework we have presented in the former section.

Let us remark that the market share is based on the total number of clients for each operator, which is not the same as the number of prepaid cards sold by each operator. But, this market share information, of which the data have been roughly estimated, is still needed due to the dependence of the cost structure with respect to the call destination.

Due to the complexity of the cost structure we have decided to build two new criteria in order to evaluate each operator. To perform this we will implement two client profiles: a professional and a private one. Professional clients are supposed to use their mobile phones mainly during the peak hours (80% of the calls during working hours). On the other hand private users are supposed to use their phone mainly outside the working hours (70% of the calls outside the working hours). With these assumptions we are now able to define the expected cost for both profiles (see table 2). As an example the expected cost for operator B – private profile is given by:

$$50\% \times (0.37 \times 30\% + 0.245 \times 70\%) + 50\% \times (0.74 \times 30\% + 0.245 \times 70\%) = 0.338$$

	Operator A	Operator B	Operator C
Professional	0.4464	0.493	0.3816
Private	0.4464	0.338	0.22435

Table 2: Evaluations of the expected costs in €/min

In order to give a better notion of the price, we express the aforementioned costs on an hour basis (table 3)

	Operator A	Operator B	Operator C
Professional	26.784	29.58	22.896
Private	26.784	20.28	13.461

Table 3: Evaluations of the expected costs in €/h

3.2 Preference modelling

In this example we have decided to use the PROMETHEE method (Brans and Vincke, 1985) for the preference modelling. In particular we will use the so-called general linear preference function (see fig. 3). The p and q parameters respectively denote the strict preference and indifference tresholds, the values of which are listed in table 4.

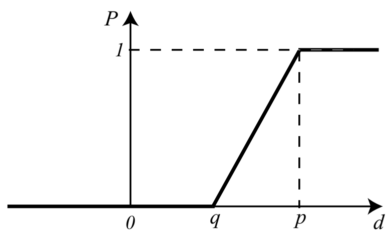

Figure 3: The general linear preference function

The parameters q and p have been defined as follows: a difference of 1€/h does not permit to discriminate one alternative above another. On the other hand a difference of 5€/h on an hour basis is jugded to be sufficient to choose between

	w	q	p
exp.cost prof.	0.1	1	5
exp.cost priv.	0.5	1	5
quality	0.4	0	3

Table 4: Preference modelling parameters: q, p being expressed in €/h except for the criterion quality. The weights of the criteria are denoted by w.

two alternatives. The weights of 0.5 has been assigned to the expected cost for private use, while it is only 0.1 for professional purposes. By this we take into account the fact that few prepaid card users have a high mobile phone activity during working hours. Finally the weight assigned to quality is 0.4 since this aspect appears to be an important issue. The quality stands essentially for the network connectivity and has been been modelled by means of scores.

As a result we obtain the following preference matrix:

$$P^{(3)} = \begin{bmatrix} 0 & 0.0449 & 0.4 \\ 0.6333 & 0 & 0.4 \\ 0.5722 & 0.6 & 0 \end{bmatrix}$$

Let us stress that the parameters characterizing the preference modelling have been roughly estimated, based on the authors' experience, rational constraints and prejudices,... We do not claim that these are representing the most faithfull picture of the reality, but they allow us to illustrate our model in a pedagogical manner.

3.3 Analysis

As presented in the previous section the first step is to transform the preference matrix into a transition matrix by means of formulas (19,20). Applying this on the above preference matrix $P^{(3)}$ leads to:

$$T^{(3)} = \begin{bmatrix} 0.30984 & 0.39711 & 0.29305 \\ 0.10289 & 0.59711 & 0.3 \\ 0.20695 & 0.2 & 0.59305 \end{bmatrix}$$

which on account of eqs. (4, 5) results in the following distribution of the equilibrium market shares:

$$Q^{(3)} = (0.1852, 0.3923, 0.4225).$$

These results are conform to the data listed in table 1: operator B and C are more or less equally represented while operator A clearly has the smallest market share of the three operators. This is due to the fact that operator B has a high quality and that its cost structure is low, except for the calls to a different operator during working hours. However, this weakness is smoothed out by its high market share and by the fact that the professional profile, which is sensible to this issue, is underestimated in this example. On the other hand operator C is also well represented because it compensates its medium quality by its low cost structure. Finally, operator A suffers clearly from its proper cost policy with respect to calls towards other operators.

3.4 Testing a few policy measures by means of a parametric analysis

In this subsection our aim is to analyse the impact of certain policy measures. Let us again stress that the scenarios presented below are only of pedagogical nature.

As already stated the weak point of operator C is related to its medium quality. In a first test we will measure the possible impact of an increase of its quality index (8 instead of 6). Performing analogue calculations as those presented in the previous subsections, leads to:

$$Q^{(3)} = (0.1426, 0.3309, 0.5265).$$

As expected operator C gains an important piece of the market (+10%), to the detriment of the others. This is however under the assumption that all cost structures remain unchanged.

Let us now assume that operator B changes its cost for a call to another operator during working hours. Indeed, he is the most expensive for this specific time period and call destination. His strategy is to replicate the cost of operator A, i.e. 0.62€/min instead of 0.74€/min. By doing so, the expected cost for the professional and private profiles are respectively 0.445 and 0.32€/min instead of 0.493 and 0.338€/min (all other parameters are the same as those of the original problem). This leads to the following stationary distribution:

$$Q^{(3)} = (0.1758, 0.4103, 0.4139).$$

As expected this change has a positive impact for operator B. However the impact is rather limited due to the fact that it affects merely the professional profile which, as already mentioned, is underestimated.

Another point of interest is related to the choice of the weights on the criteria. Let us for example lower the weight related to the quality and therefore increase the weight of the costs (this could for example reflect the opinion of younger people who are more sensible to the price of a prepaid card). The weights considered here are respectively 0.125 for the professional profile, 0.625 for the private profile and 0.25 for the quality (the weights for the professional and private profile have been chosen in such a way that their relative importance remains the same). The following distribution is obtained, confirming intuition, as B benefits even more from its higher quality:

$$Q^{(3)} = (0.1322, 0.2778, 0.59).$$

Let us now assume as final scenario that operator A and C are in a merging phase. At this moment both operators want to keep their proper cost structure. However, the calls between them will be considered as calls to the same operator. This assumption modifies the expected costs for both professional and private profiles (see table 5).

	Operator A	Operator B	Operator C
Professional	22.32	29.58	16.68
Private	22.32	20.28	11.13

Table 5: Evaluations of the expected costs in €/h for the merging scenario

Applying our method to this scenario leads to the following stationary distribution, taking the same original parameters:

$$Q^{(3)} = (0.2565, 0.3149, 0.4286).$$

From this it is clear that operator A is the winning partner of this decision. Indeed, as already stated, this operator was suffering from his cost policy with respect to a call to a different operator. This solely affects operator B since operator C maintains his level.

4 Conclusion

In this paper we have presented a new method to describe and analyse the decision behaviour of a large group of decision makers. The approach starts from a preference matrix, which enables the comparison between each pair of alternatives and which is transformed into a stochastic transition matrix; the group decision behaviour being considered as a result of a hidden Markov chain.

A theoretical framework has been presented and linked to existing MCDA tools such as for example the PROMETHEE method. In addition a pedagogical example has been treated and shows encouraging results.

A number of questions still remain open. Among them several are related to the transformation of the preference matrix, for which we made to our opinion the most intuitive assumptions.

Other applications with real data should be considered to further investigate the applicability of the proposed scheme.

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