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Location Theory and Multi-Criteria Decision Making: An Application of the MOORA Method

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ABSTRACT The first systematic research on Location Theory dates back to 1826. Quantitative approaches came much later. On the supply side extensive Input-Output Tables can be mentioned and on the demand side the optimization by Multi-Criteria Decision Making. The advantages of Input-Output Tables for location opportunities on a regional and urban basis have to be emphasized, whereas the link is made between Input-Output and Multi-Criteria Optimization. MOORA, Multi-Objective Optimization by Ratio Analysis, is composed of two methods: Ratio Analysis and Reference Point Theory and responds to the different conditions of robustness needed for optimization. This approach attempts to localize in an optimal way a certain project facing different indicators, criteria or objectives sometimes originating from different groups or individuals. Here however type and importance of objectives and alternatives were only simulated. The real stakeholders to be considered are rather the national and local authorities, the contributing firms and their personnel. In the production sphere consumer sovereignty was only indirectly involved. If consumers, via consumer organizations and trade unions, were directly involved, other claims could emerge. The simulation used was limited in its applications. Clearly if this simulation has no practical consequences, it still provides a learning experience with the use of the MOORA Method in its double composition.

KEY WORDS: Location Theory, Input-Output Analysis, MOORA, MULTIMOORA, Objectives, Alternatives, Simulation, Ratio Analysis, Reference Point Method

JEL Classification: C12, C13, C44, C54, C61, R41

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1. Quantification in Location Theory

1.1. The Beginning

In 1826 Von Thünen wrote the first systematic work on Location Theory in his publication on the "Isolierte Staat", (translated by Wartenberg, 1966). For the first time he remarked that transportation costs may correct the Comparative Costs of Ricardo. However, real quantification came much later by Input-Output Analysis (Leontief, 1936; 1941) and Multi-Criteria Decision Making (MacCrimmon, 1968; Roy, Benayoun, & Sussman, 1966).

Nevertheless one should be aware that there is considerable confusion and overlapping with Regional and Urban Economics. For instance Voogd (1983) treats urban and regional planning, but he also includes Location Theory. For instance he presents an evaluation for potential sites for new housing (p. 239).

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Location Theory has a supply side and a demand side. The supply side provides an inventory of location possibilities for production and distribution. It is assumed that the supply side is supported by extensive Input-Output Tables with many sectors and sub-sectors. The demand side, in turn, is characterized by many criteria or objectives specifying the demand more effectively.

1.2. Location Theory on Basis of Input-Output Tables

An Input-Output Table forms the basis of the Input-Output Theory with, vertically, all the materials, services and value added necessary for the production of an industrial or service sector and, horizontally, all the clients. The table is the most interesting for the choice of a location of an enterprise when the table shows a large number of sectors and a limited area of application such as regions and urban centers.

Isard was the first to introduce empirical results of Regional Input-Output Analysis as a model of a Space Economy (Isard). Brauers composed Input-Output Tables for the three Belgian Regions: Flanders, Wallonia and Brussels (1973; 1980). The more an I/O model is extensive, the more corresponding location theory will be valuable. At this point, China is the most advanced at this moment. For example, a model on water supply uses a simulation period ranging from 2012 to 2025 including more than 80 mathematical functions with 8828 variables and 7878 constraints (Ke et al., 2016).

Urban location is another example of Input-Output application, such as a study for Stockholm (Artle, 1959). Still another Input-Output study concerns the harbor city of Antwerp with a large concentration of chemical industry (Van Straelen, Puuylaert, & Brauers, 1964). Finally has to be mentioned the construction of a new port for the export of natural gas in Algeria, known as the port of Arzew. The government decided that the port would come between Oran and Algers. An Input- Output pre-study was made to measure the social and economic impact on the whole region (Brauers & Hurt, 1975).

The support of an authority for a well defined project is another example at the supply side of location. This support would be observed if, for instance the Thai government were to ask for the construction of a new seaport in the Gulf of Thailand. Any construction firm can subsequently introduce its project.

1.3. Demand side of Location Theory

Up till now, locations were considered attractive for an investment. Next will be to find the best location but for a given project. In both cases, advice has to be given for selection among several options and consequently for an optimal choice. Each project will be characterized by several criteria and these criteria have to be fulfilled in an optimal way.

A single criterion-objective is not advisable for any planning. It could be General Well-being, but what does it mean? If general well-being is the top objective, economic welfare, individual well-being and sustainable development could be the objectives. At that level measurability is still absent but not at a lower level such as income and employment for economic welfare; public goods, life expectancy and security for individual well-being and abatement of water, air and noise pollution and rationing of natural resources at the sustainable development level. Therefore a Method of Multi-Criteria has to be chosen.

1. 4. Six Conditions to complete a Study on Multi-Objective Optimization

A Study on Multi-Objective Optimization needs the presence of six conditions:

1) the choice of objectives (criteria)
2) number of objectives at least two
3) the choice of alternative solutions at least two
4) normalization of the units of the objectives
5) importance of the objectives
6) all stakeholders are involved. The whole operation could be made by one person, such as an expert, but an expert is not considered to be neutral. One decision maker like a captain of industry will focus on his own objectives. In certain industrial countries the large companies are obliged to have directors in the board of directors from outside the company. Even this group of decision makers will adhere to their own limited objectives. Rather all stakeholders, which mean all persons interested in a certain issue, have to be found, which may present a difficult issue. The choice of stakeholders will take place in a Nominal Group Technique exercise (Brauers & Lepkova, 2003). Contrary to Delphi, convergence is

Table 1. Decision Matrix Composition

	Obj. 1	Obj. 2	....	Obj. i	....	Obj. n
Alternative 1	X_{11}	X_{21}	....	X_{i1}	....	X_{n1}
Alternative 2	X_{12}	X_{22}	....	X_{i2}	....	X_{n2}
....	....	....	....	....	....	....
Alternative j	X_{1j}	X_{2j}	....	X_{ij}	....	X_{nj}
....	......	....	....	....	....	....
Alternative m	X_{1m}	X_{2m}	....	X_{im}	....	X_{nm}

not aimed at, but final voting is used in the Ameliorated Nominal Group Technique. In this way, the Nominal Group Technique could be considered as exploring any idea about objectives, advisable for a preliminary version of Delphi, where convergence could be reached regarding the list of objectives. Delphi and the Ameliorated Nominal Group Technique are explained in Brauers, (2004), with Dalkey and Helmer (1963) as basis for the first one and Van de Ven and Delbecq (1971) for the second.

In a simulation exercise the author takes the place of the stakeholders.

The method MOORA is selected given its robustness in realization of the six conditions.

2. MOORA Method (Multi-Objective Optimization by Ratio Analysis) applied for the Location of a Project

2.1. Decision Matrix

Multiple Objective Optimization will count at least two objectives and two alternative solutions.

A **Decision Matrix** assembles raw data with vertically numerous objectives, criteria (a weaker form of objectives) or indicators and horizontally alternative solutions, such as projects.

2. 2. Horizontal reading of the Decision Matrix

The Additive Weighting Procedure (MacCrimmon, 1968), which was called SAW, Simple Additive Weighting Method by Hwang and Yoon (1981, p. 99) starts from the following formula:

$$Max\ x_j = w_1 x_{1j} + w_2 x_{2j} + \cdots + w_i x_{ij} + \cdots + w_n x_{nj} \qquad (1)$$

$$\sum_{i=1}^{i=n} w_i = 1 \qquad (2)$$

creates a Super-Objective on basis of the sum of weights = 1

Weights: mixture of normalization and importance. What is what?

Numerous Numbers of objectives would ask for many, many weights, how to choose? It is impossible to fix weights for a huge number, like 20, of objectives or criteria.

2.3. Vertical Reading of the Decision Matrix

SAW reads the Decision Matrix horizontally. Reading vertically means creating **Dimensionless Measurements,** i.e. there is no longer a need for Normalization and no more problems with the number of objectives.

Consequently of the 5 problems, only 3 remain:

i. choice of objectives
ii. importance of objectives
iii. choice of alternative solutions.

The vertical reading of the Response Matrix is applied in the Ratio Analysis of MOORA and in the Reference Point Methods.

2.4. Ratio Analysis of MOORA (Multiple Objectives Optimization by Ratio Analysis)

Simple averages are inconsistent as they may change the sign and even lead to non-sense results. A study of 2006 showed several other solutions (Brauers & Zavadskas, 2006) with this conclusion being the best one:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=i}^{m} x_{ij}^2}} \quad (3)$$

with no problem for the number of objectives and with all objectives of the same importance leading to:

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^* \quad (4)$$

i = 1,2,…,g, objectives to maximized

i = g+1, g+2,…, n objectives to minimized

y_j^*= alternative j concerning all objectives and showing the final preference.

2.5. Second Part of MOORA: the Method of Reference Point

Which Reference Point has to be chosen?

1) ***Maximal Objective Reference Point***
   Suppose 2 points: A (100,20) and B (50,100)
   Dominating coordinates r_m***(100;100)***
   or in general $\{r_m\} = \{r_1, r_2,..., r_n\}$
2) ***Utopian Objective Reference Point***
   is farther away than the Maximal Objective Reference Point
3) ***Aspiration Objective Reference Point*** is closer than the Maximal Objective Reference Point.

The most general synthesis of the Reference Point is the Minkowski Metric (Minkowski, 1896, 1911):

$$Min.M_j = \left\{ \left\{ \sum_{i=1}^{i=n} \left(r - x_{ij}^*\right)^{\alpha} \right\}^{1/\alpha} \right\} \quad (5)$$

with M_j = Minkowski metric for solution j

r = reference point each time with its ith coordinate

x^*_{ij} = objective i of solution j

α = 1 → **Rectangular**

For two attributes or objectives leads to ∞ solutions:

$$Min.M_j = (r - x_{1j}^*) + (r - x_{2j}^*) \quad (6)$$

In order to come to a single solution VIKOR introduces **Significance Coefficients**: *s*, which the authors (Opricovic & Tzeng, 2004, p. 452) call wrongly weights (see above):

$$(s_i r_i - s_i x_{ij}^*) \quad (7)$$

α = 2 → **Euclidean** → ∞ solutions

$$Min.M_j = \left\{ \left\{ \sum_{i=1}^{i=n} \left(r - x_{ij}^*\right)^2 \right\}^{1/2} \right\} \quad (8)$$

In order to come to a single solution TOPSIS, originally using Euclidean distances (Hwang & Yoon, 1981, p. 132), introduces **Significance Coefficients**: *s* , which the authors call wrongly weights (Hwang & Yoon, 1981, p. 133).

The Euclidean Distance Metric, characterized by three attributes, is represented by radii of concentric spheres, with the reference point being the center.

This convex outcome does not produce evidence of optimality for non-convex manifolds possible for *more than three attributes*.

α = 3

negative results are possible if some co-ordinates of the alternatives exceed the coordinates of the Reference Point, possible with an ***Aspiration Objective Reference Point***.

It is also not clear if non-convex manifolds will eventually have a chance for optimality.

The same for the case with *α > 3* and following.

Continuing in that direction, difficulties arise in imagining further outcomes. Therefore Tshebycheff sees the best fit in the Max.-Min. Norm with *α*→ ∞ (Chebyshev, 1947; Karlin & Studden, 1966, p. 279).

Only one distance per point, viz. the largest one away from r_i, is kept in the running. Finally, the smallest outcome is chosen. This outcome is similar to a chain that is only as strong as its weakest link.

The Minkowski metric becomes then the Tchebycheff *Max - Min Metric:*

$$\underset{(j)}{Min}\left\{ \underset{(i)}{max} \sqrt[2]{(r_i - x_{ij}^*)^2} \right\} \quad (9)$$

r_i = the i^{th} co-ordinate of the reference point
x_{ij}*= the dimensionless measurement of objective i for alternative j
$i = 1,2......n$; n the number of objectives or attributes
$j = 1,2.....m$; m the number of alternatives

2.6. Only the importance of an objective in comparison to the other objectives still has to be solved.

With MOORA the introduction of importance coefficients does not change the result (see Appendix A).

Two alternative solutions are possible

- The introduction of exponents (see Appendix B), which is not very advisable because the increase is exponential:
  - with the importance coefficients the increase is as follows for two: 2; 4; 6; 8; 10 etc.
  - instead for exponents for two: 2; 4; 8; 16; 32 etc.
- The introduction of sub-objectives:
  - for instance instead of given an importance coefficient of 3 to pollution three kinds of pollution, each with their own criteria are introduced.
  - The importance coefficient 2 of employment is compensated by the introduction of objectives direct and indirect employment.

3. Two Applications of Location Theory with the use of MOORA

Location Theory has many possible applications. Two simulations are presented: one concerning the location of a Department Store (Brauers & Zavadskas, 2008) and one on the location of a container terminal in a seaport (Brauers, 2013).

3.1. Simulation Exercise

The studies are limited to simulation exercises. Contrary to many other definitions, simulation is defined in this study in a rather broad sense. Gordon, Enzer and Rochberg (1970, p. 241) give the most complete description of simulation as mechanical, metaphorical, game or mathematical analogs. These authors conclude that simulations: "are used where experimentation with an actual system is too costly, is morally impossible, or involves the study of problems which are so complex that analytical solution appears impractical".

The simulations explained here and based on MOORA are acceptable as no other multiple objective method based on dimensionless measures studied the problem, neither in reality nor as a fictive example.

3.2. Location of a Department Store

The problem posed is a problem of location theory: where to install department stores? There are several objectives for a department store: the turnover, either with deepening of the home market or with penetration abroad; the profitability and productivity aspects; government support; increasing employment level and Value Added and eventually a positive influence on the Balance of Payments. In addition, support is needed of the personnel, the trade unions, the shareholders and the clients. All these stakeholders like to enjoy an optimum position, as well.

In order to better define an objective, we have to focus on the notion of *Attribute*. For instance, if the objective to be maximized concerns the generating of new employment, the attribute could be: guaranteeing at least 1,000 new jobs over a certain period. An attribute should always be measurable.

Assume a number of mutually competitive alternatives, called here *Projects,* facing the attributes and objectives. A simulation for a large department store takes into account four projects, called A, B, C and D. First the alternatives have to pass a filtering stage. Afterwards an optimization method is used, namely MOORA.

All the stakeholders together decided on a hard constraint on the Internal Rate of Return of 12% and Project D did not reach that rate. On the contrary, projects A, B and C passed all the constraints. Project A expands less in the domestic market, but rather in one country of Southern Europe and in one country of Eastern Europe. This expansion means a higher risk ratio, mainly because import prices for these countries will rise more than their domestic and export prices, which means a deterioration of the terms of trade. Project C is more interested in the domestic market. Project B expands in the domestic market and in another large industrialized country, but is in fact mainly situated between the two other projects. In other words, projects A and C take more extreme positions than B. After examining the available information none of the remaining projects seems to dominate the others for all objectives simultaneously.

The following objectives were proposed:

- Entrepreneurial economics

1. NPV MAX
2. IRR MAX.
3. Payback Period MIN.
4. Taxes min. Subsidies (incl. para-fiscal) MIN.
5. Penetration Domestic MAX.
6. Penetration International MAX.
   6.1. Other Industrialized Country MAX.
   6.2. Country Southern Europe MAX.
   6.3. Country Eastern Europe MAX.
7. Deterioration Terms of Trade possible MIN.

- Macro-economics

1. Direct Employment (national) MAX.
2. Indirect + Secondary Employment (national) MAX.
3. Gross VA MAX
4. Positive influence Balance of Payments MAX.

The filtering process excluded Project D already, but kept Projects A, B and C in the running. In the final test on the given data and with MOORA, project C is excluded as expanding only in the domestic market. In this study, the common sense of investment remains: "you must not put all your eggs in one basket", or in other words spread your risks. Moreover, the possibilities of growth in the domestic market concerning department stores are very restrained in Western Europe. Therefore, Project A, expands mainly in foreign countries, whereas Project B expands in the domestic market and abroad. In fact, Project B brings a midway solution between A and C with no extreme positions of its responses to the objectives. Finally, MOORA will chose Project B as solution.

3.3. The Location of a Container Terminal in a Seaport

The "Annals of Operations Research, Vol. 2020 of 2013" give all calculations made by MOORA concerning the Location of a Container Terminal in a seaport.

On this subject no other studies are known that are based on Multi Criteria Decision Making. Adler (1967) explains in detail Cost-Benefit in transportation and Coto-Millán, Pesquera and Castenado are the editors of a book on Essays on Port Economics (2010), consisting of nineteen papers using Cost-Benefit Analysis for an economic evaluation of the feasibility of building new ports or enlarging existing ones. Furthermore, Cost-Benefit presents a materialistic approach, whereby for instance unemployment and health care are degraded to monetary items. People are more likely to be solution minded rather than objective-oriented. Cost-Benefit Analysis is a product of this way of thinking.

A developing country or a transition economy wishes to install a container terminal of at least 500,000 TEU.

Five alternatives are proposed.

The first alternative, *Project A*, consists of the installation at a riverside port, 100 km inland, but the installation is on the river itself, capable of receiving large container ships. The possibility to bring large container ships so far inland is an important advantage of this project, reflected in the willingness of the ship owners to pay high demurrage and local taxes for this solution. The installation remains, however, part of a tidal harbor.

Project B possesses the same advantages as project A, belonging also to a riverside port, but installed behind locks. This project also means fewer problems with low and high tide, but investment costs are higher, given the necessity to foresee locks and docks.

Project C is located at a seaport immediately near the sea, but behind locks, which means fewer tidal problems, but again with considerable investment costs.

Project D consists of a terminal also immediately near the sea but in open docks i.e. without locks. This means fast delivery of the goods but with a severe problem of salinity, caused by the open dock system at the seaside.

Project E consists of a container terminal on an island in the sea, meaning fast delivery of the goods. However investment costs are extremely high translated into high depreciation costs for the island.

Seven objectives or criteria have to be fulfilled: two for *micro-economics*; three for *macro-economics* and two for *consumer sovereignty* (Brauers, 2013, pp. 13-14).

The Filtering Stage in Seaport Planning

- Project D is excluded from the side of the national government, as the degree of saltiness is too high.
- Project E is rejected on basis of prohibitive investment costs. In this way only projects A, B and C remain in the running.

Nevertheless in China instead of the extension of the seaports at the seaside like Shanghai, already used intensively, one could think of an inland seaport with the advantages described above. Suppose that there is an interest for a seaport in industrial Wuhan. The fact that sea ships could come far inside China is an important advantage. Indeed Wuhan is located approximately 2,400 km inland on the Chang Jiang River (Yangtze), which flows directly in the East China Sea without passing Shanghai.

The bottlenecks are rather of a technical nature. If a city such as Wuhan seeks to become an inland seaport, together with other industrial regions along the river, different existing too low bridges have to be changed and the course of the Chang Jiang River (Yangtze) has to be corrected. Nevertheless other expensive alternative investment opportunities in the country have to be taken into account too (Adler, 1967). Perhaps a choice has to be made, for instance to abandon the also cost-intensive project of a New Silk Road.

Ranking Stage in Seaport Planning

A ranking of alternatives (projects) is derived from the objectives per alternative. However, before considering a ranking, the problem has to be solved if one alternative does not dominate all the others for all objectives. The ranking only takes place for a set of non-dominated alternatives. Neither projects A, B or C is dominating completely, which means that a ranking has to bring the solution.

A summary of the ranking of the two MOORA methods was made on view. If there would be no unique classification at that moment, other additional methods could bring support. First of all, we thought of another method also based on dimensionless measurements, namely the Full Multiplicative method. Brauers and Zavadskas made this link under the name of MULTIMOORA. As MULTIMOORA consists of 3 approaches an Ordinal Dominance Method will bring the final ranking (Brauers & Zavadskas, 2010a).

Nevertheless the simulation side of the example has to be stressed.

The final ranking for Projects A, B and C which passed the Filtering Stage is derived from the simulated numbers as given in: Table 3 of Brauers, *Annals of Operations Research,* Vol. 2020 of 2013 p. 15.

The ranking is then as follows for the two methods:

1. Project A, namely a container terminal at a riverside port on the river itself;
2. Project B, viz., a container terminal at riverside port behind locks;
3. Project C, viz., a container terminal near the sea behind locks. Previously, projects D and E were already excluded:

4. Is There No Link Possible between MOORA and Input-Output for Application in Location Theory?

Is it possible to make a link between MOORA and Input- Output for Location Theory but only to show the location possibilities? The answer is positive. The exercise was made for Tanzania on basis of an updated 2002 Input-Output table (Brauers & Zavadkas, 2010b). It was shown that for 2002 and the following years, light industry under the form of a sugar factory and a cotton mill has to be promoted on the first place. This finding is understandable, as Tanzania possesses a comparative advantage in cotton and sugar.

Secondly, tourism and the construction of new roads is classified. Intensified promotion of tourism is handicapped without a network of new roads. This new roads program needs more imports, counterbalancing the balance of payments surplus of tourism revenues. Two tendencies exist. On one side, Tanzania could follow the example of Kenya in the promotion of tourism. On the other side, the status quo could be maintained with respect for the wildlife in the national parks. Moreover, the quietness of the tribes in the interior is also guaranteed. Reform of agriculture comes on the third place and finally heavy industry could be promoted under the form of a steel plant and a new hydro-electrical power station.

5. Conclusion

If Location Theory was somewhat theoretical in the beginning, it became more quantitative at the supply side with Input-Output Tables and by Multi-Criteria Optimization at the demand side.

For a researcher in multi-criteria decision making the choice between many methods of Multi-Criteria Optimization is not easy. We intended to assist the researcher with several guidelines to make an effective choice. In order to distinguish the various multi-criteria methods from each other we used a qualitative definition of robustness whereby the most robust multi-objective method has to satisfy different conditions.

MOORA, Multi-Objective Optimization by Ratio Analysis, composed of two methods: Ratio Analysis and Reference Point Theory, which starts from the previous found ratios, responds to the different conditions of robustness.

Two simulations of Location Theory illustrated the MOORA research. The suggested planning followed the MOORA method with its two parts, the Ratio System itself and the Reference Point part. As we are only concerned with a simulation, we determined the type and importance of the criteria and the alternatives ourselves, instead of the stakeholders concerned. Being in the production sphere consumer sovereignty was only indirectly involved. Nevertheless the authorities were also taken as the legitimate representatives of the consumers. If consumers, via consumer organizations and trade unions, would be directly involved perhaps other claims could still emerge.

The simulations were limited in their application. Even if the simulation has no practical consequences, in any case it provides a learning experience with MOORA in its double composition.

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Appendix A

Table. A MOORA Simulation Lithuanian Sustainable Development (2006-2012)

a - Matrix of Responses of Alternatives on Objectives: (x_{ij})

Yearly figures	1. Inflation (in %) MIN.	2. Increase Public Debt (% GDP) MIN.	3. Def.Public Budget (% GDP) MIN.	4. Unemploym. (in % labor force) MIN.	5. Increase real wages in% MAX.	6. Shop time (in weekly hours) MIN.	7. Increase GDP (in%) MAX.	8. Minus % Energy consumpt. MAX (a)	9. CO2 ton/cap. MIN.	10. Other Pollution MIN.
EMU	**2**	**3**	**3**	**17**	**3**	**38**	**6.88**	**7.3**	**8.53**	**2**
EEU	**4**	**1.9**	**1.9**	**8.3**	**1**	**40**	**7**	**7.3**	**8.53**	**2**
Secession	**3**	**1**	**1**	**12**	**2**	**45**	**5.5**	**0.5**	**8.70**	**4**
Totals	**9**	**5.9**	**5.9**	**37.3**	**6**	**123**	**19.38**	**15.1**	**25.756**	**8**

b - Sum of squares and their square roots

Projects										
EMU	4	9	9	289	9	1444	47.3344	53.29	72.692676	4
EEU	16	3.61	3.61	68.89	1	1600	49	53.29	72.7609	4
Secession	9	1	1	144	4	2025	30.25	0.25	75.69	16
sum of squares	29	13.61	13.61	501.89	14	5069	126.5844	106.83	221.14358	24
square roots	5.38516481	3.68917335	3.6891733	22.4029016	3.741657387	71.19691	11.250973	10.3358599	14.870897	4.898979486

c - Objectives divided by their square roots and MOORA											**sum**	**rank**
EMU	0.37139068	0.813190	0.8131903	0.75883028	0.801783726	0.533731	0.61150	0.70627892	0.5733346	0.40824829	-2.1523502	**2**
EEU	0.74278135	0.515021	0.5150205	0.370488	0.267261242	0.5618221	0.6221684	0.706279	0.5736036	0.40824829	-2.091276	**1**
Secession	0.55708601	0.271063	0.2710634	0.5356449	0.534522484	0.6320499	0.4888466	0.04837527	0.5850353	0.816496581	-2.596695	**3**

d - Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal objective values

ri	0.37139068	0.271063	0.2710634	0.37049	0.801783726	0.533731	0.62217	0.70627892	0.5733346	0.40824829

e - Reference Point Theory: Deviations from the reference point											**max.**	**rank min.**
EMU	0	0.54213	0.5421269	0.38834	0	0	0.01067	0	0	0	0.5421269	**2**
EEU	0.37139068	0.243957	0.2439571	0.000000	0.534522484	0.0280911	0.00000	0.000000	0.000269	0	0.5345225	**1**
Secession	0.18569534	-	0	0.16516	0.267261242	0.0983189	0.13332	0.65790365	0.0117007	0.40824829	0.6579037	**3**

Table. A MOORA Simulation Lithuanian Sustainable Development (2006-2012) with increase in real wages x 2 (n°5)

a - Matrix of Responses of Alternatives on Objectives: ***(x_{ij})***

Yearly figures	1. Inflation (in %) MIN.	2. Increase Public Debt (% GDP) MIN.	3. Def.Public Budget (% GDP) MIN.	4. Unemploy. (in % labor force) MIN.	5. Increase real wages in% MAX.	6. Shop time (in weekly hours) MIN.	7. Increase GDP (in%) MAX.	8. Minus % Energy consumpt. MAX (a)	9. CO2 ton/cap. MIN.	10. Other Pollution MIN.
EMU	**2**	**3**	**3**	**17**	**6**	**38**	**6.88**	**7.3**	**8.53**	**2**
EEU	**4**	**1.9**	**1.9**	**8.3**	**2**	**40**	**7**	**7.3**	**8.53**	**2**
Secession	**3**	**1**	**1**	**12**	**4**	**45**	**5.5**	**0.5**	**8.70**	**4**
Totals	**9**	**5.9**	**5.9**	**37.3**	**12**	**123**	**19.38**	**15.1**	**25.756**	**8**

b - Sum of squares and their square roots

Projects										
EMU	4	9	9	289	36	1444	47.3344	53.29	72.692676	4
EEU	16	3.61	3.61	68.89	4	1600	49	53.29	72.7609	4
Secession	9	1	1	144	16	2025	30.25	0.25	75.69	16
sum of squares	29	13.61	13.61	501.89	56	5069	126.5844	106.83	221.14358	24
square roots	5.3851648	3.6891733	3.6891733	22.402902	7.483314774	71.19691	11.250973	10.33586	14.870897	4.8989795

c - Objectives divided by their square roots and MOORA

											sum	**rank**
EMU	0.3713907	0.813190	0.8131903	0.7588303	0.801783726	0.533731	0.61150	0.7062789	0.5733346	0.4082483	-2.1523502	**2**
EEU	0.7427814	0.515021	0.5150205	0.370488	0.267261242	0.5618221	0.6221684	0.706279	0.5736036	0.4082483	-2.091276	**1**
Secession	0.557086	0.271063	0.2710634	0.5356449	0.534522484	0.6320499	0.4888466	0.0483753	0.5850353	0.8164966	--2.596695	**3**

d - Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal objective values

ri	0.3713907	0.271063	0.2710634	0.37049	0.801783726	0.533731	0.62217	0.7062789	0.5733346	0.4082483

e - Reference Point Theory: Deviations from the reference point

											max.	**rank min.**
EMU	0	0.54213	0.5421269	0.38834	0	0	0.01067	0	0	0	0.5421269	**2**
EEU	0.3713907	0.243957	0.2439571	0.000000	0.534522484	0.0280911	0.00000	0.000000	0.000269	0	0.5345225	**1**
Secession	0.1856953	-	0	0.16516	0.267261242	0.0983189	0.13332	0.6579037	0.0117007	0.4082483	0.6579037	**3**

Appendix B

Table. A MOORA Simulation Lithuanian Sustainable Development (2006-2012) with increase in real wages with exponent 2 (n°5)

a - Matrix of Responses of Alternatives on Objectives: (x_{ij})

Yearly figures	1. Inflation (in %) MIN.	2. Increase Public Debt (% GDP) MIN.	3. Def.Public Budget (% GDP) MIN.	4. Unemploy. (in % labor force) MIN.	5. Increase real wages in% MAX.	6. Shop time (in weekly hours) MIN.	7. Increase GDP (in%) MAX.	8. Minus % Energy consumpt. MAX (a)	9. CO2 ton/cap. MIN.	10. Other Pollution MIN.
EMU	**2**	**3**	**3**	**17**	**9**	**38**	**6.88**	**7.3**	**8.53**	**2**
EEU	**4**	**1.9**	**1.9**	**8.3**	**1**	**40**	**7**	**7.3**	**8.53**	**2**
Secession	**3**	**1**	**1**	**12**	**4**	**45**	**5.5**	**0.5**	**8.70**	**4**
Totals	**9**	**5.9**	**5.9**	**37.3**	**14**	**123**	**19.38**	**15.1**	**25.756**	**8**

b - Sum of squares and their square roots

Projects										
EMU	4	9	9	289	81	1444	47.3344	53.29	72.692676	4
EEU	16	3.61	3.61	68.89	1	1600	49	53.29	72.7609	4
Secession	9	1	1	144	16	2025	30.25	0.25	75.69	16
sum of squares	29	13.61	13.61	501.89	98	5069	126.5844	106.83	221.14358	24
square roots	5.38516481	3.68917335	3.6891733	22.4029016	9.899494937	71.19691	11.250973	10.3358599	14.870897	4.898979486

c - Objectives divided by their square roots and MOORA

											sum	**rank**
EMU	0.37139068	0.813190	0.8131903	0.75883028	0.90913729	0.533731	0.61150	0.70627892	0.5733346	0.40824829	-2.0449966	**1**
EEU	0.74278135	0.515021	0.5150205	0.370488	0.101015254	0.5618221	0.6221684	0.706279	0.5736036	0.40824829	-2.257522	**2**
Secession	0.55708601	0.271063	0.2710634	0.5356449	0.404061018	0.6320499	0.4888466	0.04837527	0.5850353	0.816496581	-2.727157	**3**

d - Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal objective values

ri	0.37139068	0.271063	0.2710634	0.37049	0.90913729	0.533731	0.62217	0.70627892	0.5733346	0.40824829

e - Reference Point Theory: Deviations from the reference point

											max.	**rank min.**
EMU	0	0.54213	0.5421269	0.38834	0	0	0.01067	0	0	0	0.5421269	**1**
EEU	0.37139068	0.243957	0.2439571	0.000000	0.808122036	0.0280911	0.00000	0.000000	0.000269	0	0.8081220	**3**
Secession	0.18569534	-	0	0.16516	0.505076272	0.0983189	0.13332	0.65790365	0.0117007	0.40824829	0.6579037	**2**

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