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TV Revenue Sharing as a Coordination Device in Sports

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TV Revenue Sharing as a Coordination Device in Sports Leagues

Thomas Peeters*

Abstract

As sports clubs jointly produce contests, they cannot determine contest quality through their private talent investments. Sports leagues therefore try to coordinate talent investments towards the profit-maximizing contest quality. In this paper I analyze how revenue sharing mechanisms may serve this goal when demand comes from hard-core club and neutral sports fans. Performance-based sharing turns out to be an inefficient sharing rule for the cartel, although it is not harmful for social welfare. This inefficient cartel behavior can be rationalized as the result of bargaining with asymmetric outside options. Data from US and European sports leagues illustrate the theoretical findings.

JEL: L41, L83

keywords: cartel behavior, revenue sharing, sports leagues, TV rights

1 Introduction

Consumers of sports competitions come in two kinds. A first type of consumer (the hard-core club fan) is committed to one team only. He prefers to see his favorite win and therefore has a preference for one team dominating the competition. A second type of consumer (the neutral sports fan) is not committed to one team, but instead enjoys a tense competition with a high level of play. As he appreciates uncertainty of outcome in the competition, his willingness-to-pay for a sports competition decreases when one team continuously dominates the contest. Typically, hard-core fans are held responsible for match-day income (i.e. ticket sales, catering in the stadium,...), while neutral fans predominantly determine revenues from the sale of TV rights. A sports club maximizing profits would ideally want to provide a contest that has exactly the right mix of features to get maximal total revenues from both groups of consumers. Unfortunately for them however, clubs are unable to produce sports contests on their own. Rather, they produce contests jointly and decisions by both clubs impact on the features of the contest. More specifically, their talent investments impact strongly on the revenues that may be attained from a sports contest. In a competitive environment clubs therefore compete in talent, which leads them to invest more in talent than the amount maximizing joint profits. The sports industry is however not a typical competitive environment, as clubs join together to form leagues. Although most leagues do not (openly) impose price restrictions, nor agree on the exact amount of talent investments clubs have to make, they are often viewed as textbook examples of business cartels. Sports leagues have created several coordination devices they may use

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to steer talent investment decisions in order to maximize joint profits. Famous examples of such devices are salary caps (where the league limits the amount teams may spend on player wages) and the sharing of broadcast or gate revenues (where the league may set the sharing rule to influence club incentives). Since these devices are in most cases not outlawed by antitrust authorities, sports leagues communicate their existence and application openly. This creates an opportunity to evaluate how leagues have made use of their devices to coordinate club decisions. As such, we may obtain an insight in the functioning of these cartels. In this paper I focus on how a sharing rule for collectively sold broadcast rights may be used to coordinate talent investments by clubs. I evaluate how three different sharing mechanisms (equal sharing, performance-based sharing and sharing based on home market sizes) impact on talent investments, joint profits and social welfare. Equal sharing of broadcast revenues is the system used in the American Major leagues. European soccer leagues on the other hand have used a variety of systems. As can be seen in Table 1, they often combine equal and performance-based sharing with a part of revenues being shared based on TV appearances. This last type of sharing may be thought of as being equivalent to sharing based on home market size, as obviously more popular teams are televised more often.

Table 1: Sharing systems for broadcast revenue in European soccer

Country	Broadcast Revenue/year[1]	Sharing Mechanism[2]
England	£ 672m	50% equal sharing, 25% based on league position, 25% based on TV appearances
France	€ 668m	50% equal sharing, 30% based on league position, 20% based on TV appearances
Germany	€ 413m	50% equal sharing, 50% based on past three and current season performance
Norway	€ 14.54m	40% equal sharing, 30% based on league position, 30% based on TV appearances
Scotland	€ 63.5m	52% equal sharing, 48% based on league position

This paper holds two crucial results about the functioning of revenue sharing as a coordination device. The analysis first shows that performance-based sharing of broadcast revenues is an inefficient way to increase the cartel's joint profit, compared to equal and market size based sharing. This does not necessarily imply that the practice is harmful to social welfare. The paper secondly explains how the existence of performance-based sharing can nevertheless be rationalized by assuming the league makes decisions through negotiations between teams. A simple Nash-bargaining model is developed in which clubs have asymmetric home markets and antitrust authorities prevent sharing purely based on market size. Under these circumstances (which are clearly present in some European soccer leagues) an agreement can only be reached when the cartel implements a sub-optimal solution. Data on European soccer and American major leagues are used to support these results. These findings clearly suggest that sports leagues behave as cartels of independent firms (i.e. clubs) rather than being single economic entities. In this sense, the analysis questions the validity of the single entity

[1] Data taken from www.sportsbusiness.com and Gratton & Solberg (2007). All figures for 2008-2009 season, except Scotland (2004) and Norway (2006).

[2] Based on 2005 figures from Solberg and Gratton (2007).

approach (Neale, 1964), which is often cited to exempt sports leagues from antitrust prosecution[3].

Antitrust concerns about practices in the sports industry have given rise to a significant literature, which goes back to the seminal papers by Rottenberg (1956) and Neale (1964). A large part of these contributions (see e.g. El Hodiri & Quirk (1971) and Szymanski & Késenne (2004)) focus primarily on the issue of competitive balance (i.e. uncertainty of outcome) in a league. Szymanski (2001) introduces the idea of hard-core and neutral fans in this literature. He questions the use of competitive balance as an evaluation criterion for policy measures in sports, because perfectly equal playing strengths are only socially optimal when all teams are equally well supported (which they in practice never are). Forrest et al. (2005) advance these observations by empirically identifying the demand for televised matches as coming from more "neutral" fans than demand for stadium seating. The model in this paper builds on these insights, as it distinguishes between both types of consumers and splits up club revenues by the type of consumers that is responsible for them. By also including both types in a social objective function this paper extends some of the analysis in Falconieri et al. (2004). They examine the effect of collective and individual sales of broadcast rights on social welfare, but only look at the effects on TV viewers. Several earlier contributions have looked at cartel behavior in sports leagues. Ferguson et al. (2000) identify cartel behavior in Major League Baseball, where clubs were kept from hiring free agent players in certain periods. Forrest et al. (2004) explain the English Premier League soccer's supply of televised matches as the outcome of inefficient cartel behavior. A final contribution by Kahn (2007) shows how cartel behavior has allowed earning rents from players in college sports by forcing them into amateurism. The possible use of revenue sharing as an incentive device has been explicitly highlighted by Atkinson et al.(1988). In their contribution sharing all relevant club revenues is evaluated as a tool in a principal-agent problem (league vs. clubs). They present an empirical application using data from the National Football League (NFL), which underlines the significance of revenue sharing in this respect. Palomino and Sakovics (2004) evaluate two distribution mechanisms for broadcast revenues in their effect on the bidding for talented players. They provide a rationale for (partially) performance based sharing by leagues that face international competition for players (e.g. European soccer leagues), while (pure) equal sharing is found to be optimal for isolated leagues (e.g. American major leagues). They do not focus on cartel behavior as such and presume the league to be an efficient profit maximizer. The present paper contributes to this literature in three ways. First, it encompasses the insights of Szymanski (2001) and Forrest et al. (2005) concerning diversified consumers in the analysis of broadcast revenue sharing, as initiated by Palomino and Sakovics (2004). It also furthers their analysis by looking at teams with asymmetric home markets, which is one of the major issues in the economics literature on competitive balance. In doing so, it is capable of considering sharing based on home market size, which cannot be analyzed in the model of Palomino and Sakovics (2004). Second, the model shows in addition to Szymanski (2001) that the distribution of fans across clubs and types is not only important to determine the optimal competitive balance, but also heavily influences the actual talent investments in leagues. Some data on European soccer are highlighted to illustrate this point. Finally, it introduces the idea of inefficient cartel conduct (as put forward in Forrest et al. (2004)) to the analysis of revenue sharing and develops a simple bargaining model to explain how inefficiency may arise.

[3] Neale (1964) proposes to threat leagues as a kind of multi-plant firms. They should then be allowed to engage in horizontal coordination and be exempt from antitrust prosecution. This argument is for example being put forward in the US Supreme Court case NFL vs. American Needle.

In the next section I introduce a model of the sports industry with horizontally diversified consumers. I also look into the pricing decisions of clubs and broadcasters. This is followed in section three by determining the talent investments teams make under different sharing rules. I further provide an overview of some European soccer leagues to show the validity of the model results. A next section holds the evaluation of the sharing mechanisms from the cartel's point of view and looks at the results in terms of the social objective function. Then I present a model to explain cartel inefficiency as the outcome of bargaining with asymmetric fall-back options. I also include a comparison between some US and European leagues to illustrate this point. In the final section I formulate some conclusions and remarks.

2 Model setup

This section presents a simple model of a professional sports league which will be used to study revenue sharing as a coordination device for talent investments. The timing of the game consists in three stages. First, the league sets a distribution rule for broadcast revenues. In a second stage the fully informed clubs decide on talent investments. Finally, their joint product is sold to hard-core fans by means of stadium tickets and to neutral fans through broadcasters. In this stage the clubs and broadcaster take their pricing decisions. Working backward, I first solve the pricing problem and then turn to the other stages of the game in the next sections.

2.1 Clubs and league

In the model two profit-maximizing clubs (k and j) play a competition consisting of two matches, one at each team's venue. As in Palomino and Sakovics (2004) clubs determine talent investments (t_j and t_k) by making a discrete choice. They either invest a high amount, h, or a low amount, l. These investments result in the following win probabilities:

$$w_j(t_j, t_k) : \left\{ \begin{array}{c} w(l,l) = w(h,h) = 1/2 \\ w(h,l) = \beta \\ w(l,h) = 1 - \beta \end{array} \right\} \tag{1}$$

where: $1 > \beta > 1/2$

For reasons of simplicity l is normalized to zero. As in Szymanski and Késenne (2004) players to accommodate both choices are readily available, so there is no bidding for talent between clubs and talent is available at fixed marginal cost. This assumption implies that the supply of talent in the league is not fixed and potential monopsony power of clubs in the player labor market is neglected. For the international labor market of European sports, this seems a very sensible assumption to make. In the American leagues this assumption may seem quite strong. Some authors (e.g. El Hodiri and Quirk, 1971) therefore prefer to assume a fixed talent supply. However, the high amount of clubs in each of the American Major leagues and alternative playing possibilities for players (such as minor leagues and leagues abroad) may still justify regarding clubs as price-takers.

After having observed talent investments, clubs set prices to hard-core fans. As is common in the literature, clubs do not compete for each other's fans and are consequently monopolists in their home markets. Both teams differ in the size of their home market and therefore in the maximum amount of hard-core fans they may serve. The large market club, from now on club 1, may serve m_1 fans, while the small market club, further on called club 2, has only m_2 potential hard-core fans, with $0 < m_2 < m_1$. After clubs have taken their pricing decisions, they sell tickets for the match in their home venue. I assume that thereafter no sharing of gate revenues takes place.

In order to serve neutral fans clubs pool their broadcast rights and sell them to the highest bidding broadcaster in an auction setting. Revenues from these sales may be distributed in three different ways, given by the sharing mechanism d_i. The league either sets an equal sharing rule ($d_i = 1/2$), a performance-based sharing rule ($d_i = w_j$) or a sharing rule based on market sizes ($d_i = \frac{m_j}{m_j+m_k}$). Clubs know the sharing rule before making talent investments.

2.2 Sports consumers

A crucial innovation in the present model is that sports fans are not a homogeneous group. Instead, they come in two distinct groups that have a different appreciation for several aspects of the sports product. This shows in the model through the fact that both groups attach different quality levels to the same product. Quality for both types depends on the talent investments chosen by clubs. As hard-core fans primarily enjoy a high winning percentage, their quality, $f(t_j, t_k)$,is given by:

$$f(h,l) > f(h,h) = f(l,l) > f(l,h) \tag{2}$$

where:

$$f(h,l) - f(l,l) = f(l,l) - f(l,h) \tag{3}$$

This last condition implies that hard-core fans consider a similar fall in winning percentage as a similar quality decline whether or not it is in the "winning" region or the "losing" region. In other words, avoiding a loss has the same importance as obtaining a win.

Neutral sports fans enjoy a high level of fielded playing talent and tension in a sports match. This is indicated by their quality variable $b(t_j, t_k)$:

$$b(h,h) > b(h,l) = b(l,l) = b(l,h) \tag{4}$$

When both tension and level of play increase, the quality of the competition goes up. In case one aspect improves at the expense of the other, quality remains at the same level. As neutral fans have no preference for one team over the other they attach the same value to domination of the contest by either team. Both groups of fans have a demand for sports contest which depends on quality and price. As in Falconieri et al. (2004), fans in both groups have individual specific preferences (x_v^b and x_v^f) that are uniformly distributed along the interval $[0, 1]$. As such, each fan is, depending on his type, confronted with the maximization problem:

$$\begin{aligned}
&Max\left\{x_v^b b(t_j, t_k) - p_b, 0\right\}\\
&or\\
&Max\left\{x_v^f f(t_j, t_k) - p_f^j, 0\right\}
\end{aligned}$$

This leads to a market demand for one fixture from both groups given by:

$$D_b = n\frac{b(t_j,t_k)-p_b}{b(t_j,t_k)} \tag{5}$$

$$D_j^j = m_j\frac{f(t_j,t_k)-p_f^j}{f(t_j,t_k)} \tag{6}$$

Demand from hard-core fans is given separately for both clubs, as index j shows, while neutral fan demand is equal to market demand.

2.3 Broadcasters

The demand for televised matches cannot be met directly by clubs. The league therefore sells broadcasting rights in an auction to the highest bidding broadcaster, who in turn faces demand from neutral fans. As such, it is the broadcaster which takes the pricing decision. From the specification of neutral consumers and normalizing the cost of broadcasting to zero, the profit for a monopolist broadcaster is given as:

$$\pi_b = 2p_b n\frac{b(t_j,t_k)-p_b}{b(t_j,t_k)} \tag{7}$$

In the model the broadcasting sector is assumed to be competitive in nature. The auction is then a common value auction with a sufficient number of bidders. Consequently, broadcasters will be willing to pay every amount up to the monopoly profit when bidding for the broadcast rights. The league thus succeeds in capturing the entire monopoly rents.

2.4 Pricing decisions and profits

Determining the Nash-equilibria in the game first involves equating the pricing decisions. As mentioned before, this is the final stage in the game and occurs after both clubs have taken talent investment decisions that are common knowledge to all players. Both the broadcaster and the clubs maximize revenues from fans for a given quality level. Their maximization problem is:

$$\underset{p_b}{Max}\left\{2p_b n\frac{b(t_j,t_k)-p_b}{b(t_j,t_k)}\right\}$$
$$\underset{p_f^j}{Max}\left\{p_f^j m_j\frac{f(t_j,t_k)-p_j}{f(t_j,t_k)}\right\}$$

Solving these problems for both prices leads to:

$$p_b = \frac{b(t_j,t_k)}{2}$$
$$p_f^j = \frac{f(t_j,t_k)}{2}$$

This translates into club revenues from both groups given by:

$$R_b = n\frac{b(t_j,t_k)}{2} \quad (8)$$
$$R_f^j = m_j\frac{f(t_j,t_k)}{4} \quad (9)$$

Revenues from neutral fans are then divided following the sharing rule, which was agreed on up-front by both teams and the league. Expected club profits for the large club (1) and small club (2) can now be determined as a function of talent investments and the distribution scheme:

$$\pi_1^i(t_1,t_2) = m_1\frac{f(t_1,t_2)}{4} + d_i n\frac{b(t_1,t_2)}{2} - t_1 \quad (10)$$
$$\pi_2^i(t_2,t_1) = m_2\frac{f(t_2,t_1)}{4} + d_i n\frac{b(t_2,t_1)}{2} - t_2 \quad (11)$$

2.5 League profits and social welfare

As a cartel of clubs, the league is supposed to set a sharing rule which maximizes joint profits. Therefore the league's objective function boils down to:

$$\pi_L = \frac{1}{4}\left[m_1 f(t_1,t_2) + m_2 f(t_2,t_1) + 2nb(t_1,t_2)\right] - (t_1+t_2) \quad (12)$$

which she should maximize by implementing an optimal distribution scheme d_i, and where $t_j(d_i)$ is written simply as t_j.

Since the preferences and demand curves of both types of consumers are specified in the model I may establish a social welfare function based on these specifications. From the demand and pricing decisions it is clear that consumer surplus in both markets equals:

$$CS_b = \frac{1}{4}nb(t_j,t_k)$$
$$CS_f^j = \frac{1}{8}m_j f(t_j,t_k)$$

Social welfare in the industry as a whole is the sum of consumer surplus, club profits and broadcaster profits (which equal zero), which results in:

$$SW = \frac{3}{8}\left[m_1 f(t_1,t_2) + m_2 f(t_2,t_2) + 2nb(t_1,t_2)\right] - (t_1+t_2) \quad (13)$$

All elements are now present to evaluate revenue sharing rules in their impact on talent investments, club profits, league profits and social welfare.

3 Club investments

This section presents the results of the talent investment stage. It is clear that four different outcomes may arise:

1. mutually high investments:

$$t_1 = h, t_2 = h \quad (14)$$

2. large market domination:

$$t_1 = h, t_2 = l \tag{15}$$

3. small market domination:

$$t_1 = l, t_2 = h \tag{16}$$

4. mutually low investments:

$$t_1 = l, t_2 = l \tag{17}$$

The approach adopted here consists in determining under which conditions on m_1, m_2 and n each of these four outcomes is a Nash-equilibrium of the investment stage of the game. First proposition 1 looks into the case of small market domination. In order to ensure unique Nash-equilibria at all parameter values (except for the thresholds at which clubs are indifferent) it is necessary to impose $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$. In an extension, which is available on request, I examine which model results hold when this assumption is relaxed. This case is however only interesting for completeness and adds little to the insights of the analysis. Therefore it is not given in full detail here.

Proposition 1 *A situation in which the small market club invests more in talent than the large market club is under none of the three distribution schemes a Nash-equilibrium of the talent investment stage.*

Proof. see appendix. ■

Proposition 1 contains a very intuitive result, as in practice leagues in which talent investments from smaller clubs dominate the investments made by larger clubs are indeed hard to think of. It is important to stress perhaps that proposition 1 by no means implies that small clubs may never win the sports contest. Even under large market domination the small club obtains a part $(1-\beta)$ of total wins, while under equal investments this amounts to half of the wins. The driving force behind the result is that under none of the distribution schemes the small club's incentives to make high talent investments are as great as the large club's. Redistributing revenues from neutral fans must always give the same incentives to both clubs. When they aim to inspire high investments in the small club, they inevitably do the same in the large club. It is then clear that these incentives will never be able to compensate for the greater incentives the large club receives from hard-core fans. Consequently, it always pays for the large club to match high investments, when it pays for the small club to make them and small market domination is never an equilibrium.

Proposition 2 *A situation of mutually high investments is the unique Nash-equilibrium of the investment stage when m_2 is higher than the threshold value $\theta^h_{d_i}(h,n)$, which is increasing in h and decreasing in n. The threshold is lowest under performance-based sharing ($\theta^h_{win}(h,n)$) and highest under sharing based on market size ($\theta^h_{market}(h,n)$), while equal sharing ($\theta^h_{equal}(h,n)$) is in the middle.*

Proof. see appendix. ■

To gain some intuitive insights into the results of proposition 2, it is crucial to see that, from proposition 1, it is never the large team which deviates in the mutually high investment situation. Therefore mutually high investments constitute a Nash-equilibrium when it is profitable for the small team to invest heavily in talent rather than to undergo the large club's domination. The small club's assessment in this case is simply whether increased willingness-to-pay from neutral and hard-core fans when moving to mutually high investments is enough to pay for high investments. Consequently, lower investments costs and more fans of both kinds increase the probability of a Nash-equilibrium in mutually high investments. Hence, the intuition behind the influence of h and n on $\theta^h_{d_i}(h,n)$.

To explain differences across distribution schemes, again the small club's incentives are crucial. Each scheme impacts differently on the small club's revenue from mutually high and unbalanced investments. Essentially, the increase in the small club's revenues when moving to mutually high investments consists in three parts. First, revenues from hard-core fans increase. As these revenues are not shared in the model, they cannot explain differences between sharing mechanisms. Second, total league broadcast revenues from neutral fans rise. The choice in distribution scheme determines how large a part of this increase goes to the small club. While under mutually high investments, equal and performance-based sharing grant the small club half of the revenues, sharing based on market size provides a smaller share. Hence, this scheme motivates less to invest heavily in talent. Third, the share of broadcast revenues a club is entitled to, may also rise when moving to high investments. The only scheme which has this feature is performance-based sharing. Therefore small clubs receive a triple dividend from high investments under this scheme. Revenues from hard-core fans, total league broadcast revenues and the share of broadcast revenues all increase. Consequently, performance-based sharing provides the strongest incentive to match high investments.

Proposition 3 *A situation of mutually low talent investments is the unique Nash-equilibrium of the investment stage, when m_1 is below the threshold value $\theta^l_{d_i}(h,n)$, which is increasing in h and decreasing in n in the case of performance-based sharing. Under both other systems $\theta^l_{d_i}(h,n)$ is increasing in h, but independent of n. The threshold is the same for equal sharing and sharing based on market size ($\theta^l_{equal}(h) = \theta^l_{market}(h)$), yet lower for performance-based sharing ($\theta^l_{win}(h,n)$).*

Proof. see appendix. ■

As in the case of mutually high investments, proposition 1 again allows ruling out unilateral deviation by one of the clubs (i.e. the small club). Therefore I focus attention on the large club in order to understand the intuition behind proposition 3. The large club's choice problem is between mutually low and unbalanced investments and the crucial issue is whether higher revenues from dominating the league may offset high talent costs. Obviously, higher talent costs reduce the temptation to invest heavily and as such h positively impacts on the threshold.

On the revenue side the situation is different from before. Willingness-to-pay from hard-core fans still goes up when moving to higher talent levels. The same cannot be said of neutral fans, as this group has no preference for an unbalanced contest over a low level contest. Consequently, total broadcast revenues fail to increase upon the large club's decision to raise investments. This implies that larger investments can only be offset by enhanced revenues from hard-core fans or a larger share in broadcast revenues. When the league introduces sharing based on market size or equal sharing, only soaring revenues from hard-core fans may motivate unilaterally high talent investments by the large club. Subsequently, under these systems the amount of neutral fans has no impact on the threshold value. Since performance-based sharing allocates a larger share to the dominating club, it provides extra incentives to deviate from mutually low investments. Consequently, mutually low talent investments are less likely under this system.

Proposition 4 *Large market domination is the unique Nash-equilibrium of the investment stage when m_1 is above the threshold $\theta^l_{d_i}(h,n)$, while m_2 is lower than the threshold $\theta^h_{d_i}(h,n)$. The necessary difference in fan base, given by $\theta^u_{d_i}(n)$, is increasing in n and larger under equal and performance-based sharing than under sharing based on market size.*

Proof. see appendix. ■

From proposition 4, unbalanced investments appear as an equilibrium when both mutually low and high investments are unstable. The amount of hard-core fans for the large club has to be too large to allow for mutually low investments, while at the same time the small team has too little fans for mutually high investments. As a result it is necessary (though not sufficient) that the amount of hard-core fans differs substantially between clubs. Under all three schemes the threshold $\theta^l_{d_i}(h,n)$ is higher than $\theta^h_{d_i}(h,n)$. This implies that under none of the distribution schemes unbalanced investments can be ruled out as a possible equilibrium. However, the necessary difference in market size between both clubs differs across schemes and therefore the probability of an equilibrium in unbalanced investments is affected by the choice of distribution scheme. Since $\theta^l_{equal}(h,n) = \theta^l_{market}(h,n)$ while $\theta^h_{equal}(h,n) < \theta^h_{market}(h,n)$, it is clear that sharing based on market size allows unbalanced investments more easily and protects competitive balance less than equal sharing. Observe also that $\theta^l_{equal}(h,n) > \theta^l_{win}(h,n)$ and $\theta^h_{equal}(h,n) > \theta^h_{win}(h,n)$, meaning that unbalanced high investments arise at lower levels of m_1, yet matching these investments happens at lower levels of m_2 under performance-based sharing. Comparing both systems reveals that the necessary difference between market sizes is exactly the same under both. In other words, performance-based sharing not necessarily means an improvement of competitive balance compared to equal sharing.

Table 2: Comparison of European Soccer Leagues 2005-2008

League	average competitive balance[4]	average UEFA points	average attendance/ fixture	average bottom 3 attendance[5]	average top 3 attendance	average total attendance/TV households[6]
England	0.39	67.75	34,548	19,961	58,621	3.27
France	0.33	51.64	21,688	8,800	42,467	1.73
Germany	0.41	47.61	39,479	20,178	66,589	2.03
Belgium	0.47	29.44	10,472	4,799	24,415	4.04
Netherlands	0.53	39.69	17,463	5,571	40,763	4.49
Scotland	0.54	31.50	15,896	4,874	40,913	9.00
Austria	0.37	20.36	7,945	4,222	12,396	2.34
Norway	0.35	20.83	9,719	5,275	16,895	6.68
Sweden	0.39	13.22	9,027	4,160	13,207	3.14

[4] Competitive balance is measured by NAMSI, see Peeters (2009) for more information on this. This measure increases when competitive balance decreases. Data were taken from RSSSF. The RSSSF is the Rec. Sport. Soccer Statistics Foundation. More information on this organisation and the full archives can be found at http://www.rsssf.org

[5] This statistic is calculated as the average attendance of the three best attended clubs in each season 2005-2008. Data are taken from European Football Statistics, online retrievable at http://www.european-football-statistics.co.uk/.

[6] Sum of all club average attendances per fixture, divided by the number of TV households, based on European Football Statictics data and World Development Indicators.

Propositions 1 to 4 suggest it should be possible to find sports leagues that exhibit mutually high, mutually low and unbalanced talent investments. Assuming that talent investment costs are equal across leagues in the same sport, the driving factors behind such a classification should be the distribution of hard-core fans between clubs and the relative importance of both fan groups for the league. Table 2 compares several European soccer leagues which apply collective sales of broadcasting rights for the period 2005-2008. The table depicts the average competitive balance and UEFA points to categorize the leagues in terms of talent investments. Leagues exhibiting a high level of competitive balance (i.e. showing a low figure in column 2) and a high amount of UEFA points may be thought of as exhibiting mutually high investments. England, France and Germany are clearly in this situation. Following the results of propositions 1 to 4, smaller teams in these leagues should be well supported and/or they should serve a relatively large amount of neutral fans. This result is easily confirmed for England and Germany as these leagues show a very high average attendance for their bottom 3 clubs. France may have somewhat less supported small clubs, but the French TV market turns out to be more important relative to stadium attendances. Belgium, the Netherlands and Scotland best fit the situation of large club domination, as they appear to have an average level of play, combined with a low competitive balance. According to the model this should imply that smaller clubs have low amounts of hard-core fans and do not cross the relevant thresholds therefore investing low amounts. At the same time the well supported large clubs do cross their thresholds and invest heavily. On top of this, the amount of neutral fans should be too low to compensate for this difference. Since these countries appear to have a high average top 3 attendance, but a low bottom 3 average attendance, combined with a relatively low amount of TV viewers, the predictions of the model again appear to be reasonable. Finally, a third group of leagues, Sweden, Austria and Norway may be characterized as showing mutually low talent investments. They show a low amount of UEFA points and a high competitive balance. The model explains this as the result of top teams in these countries failing to reach enough hard-core fans to cross their threshold. The table shows that average attendance of the three best supported clubs in this countries is indeed significantly below that in the other countries. In this case the model predicts that the relative importance of the TV market has less influence on the decisions of large clubs, which also shows in the data.

4 League profits and social welfare

I now look into the question which distribution scheme an efficient cartel would choose and which scheme is preferable from a social welfare point-of-view. I define the threshold values on n, $m_1$and m_2 for the cartel's optimal talent investment outcome as $\lambda^{l,h}, \lambda^{u,l}$ and $\lambda^{u,h}$. The superscripts indicate which talent investment outcomes are being compared, for example $\lambda^{u,l}$ gives the threshold at which unbalanced investments are preferred over mutually low investments. $\lambda^{u,h}$ and $\lambda^{u,l}$ are thresholds on the difference between the clubs' home market sizes $(m_1 - m_2)$. A large difference causes unbalanced investments to be preferable over equal investments. $\lambda^{l,h}$ is a threshold on the amount of neutral fans the league may reach (n). A high amount of neutral fans leads to high investments being preferred over low investments. Mutually low investments are then optimal for the league when both $n \leq \lambda^{l,h}$ and $m_1 - m_2 \leq \lambda^{u,l}$, mutually high investments when $n \geq \lambda^{l,h}$ and $m_1 - m_2 \leq \lambda^{u,h}$ and unbalanced investments when $m_1 - m_2 \geq \lambda^{u,h}$ and $m_1 - m_2 \geq \lambda^{u,l}$. In exactly the same way, the thresholds on n and $m_1 - m_2$ for socially optimal investments are given by $\sigma^{l,h}$, $\sigma^{u,l}$ and $\sigma^{u,h}$. Proposition 5 compares the cartel and social thresholds.

Proposition 5 *Small club domination is never optimal, both for the league and from a social perspective. The threshold values $\lambda^{l,h}, \lambda^{u,l}$, $\sigma^{l,h}$ and $\sigma^{u,l}$ are increasing in h, where $\lambda^{l,h} > \sigma^{l,h}$ and $\lambda^{u,l} > \sigma^{u,l}$. The thresholds $\lambda^{u,h}$ and $\sigma^{u,h}$ are increasing in n and decreasing in h, where $\lambda^{u,h} < \sigma^{u,h}$ When $n = \lambda^{l,h} \Rightarrow \lambda^{u,l} = \lambda^{u,h}$ and when $n = \sigma^{l,h} \Rightarrow \sigma^{u,l} = \sigma^{u,h}$.*

Proof. see Appendix. ■

Not surprisingly, it is neither optimal for the league nor from a social perspective that the competition be dominated by the small market team. This result may easily be understood by pointing out that the earnings potential and consumer surplus of the small club's hard-core fans is always lower than that of the large club's fans. As such, it never pays to raise the small team's fans' utility at the expense of the utility of the large club's fans. Fortunately, small market domination is never a Nash-equilibrium in the investment stage.

The choice between mutually low and mutually high investments boils down to evaluating whether the costs of high talent investments can be offset by increased revenues/consumer surplus from neutral fans. The utility of both groups of hard-core fans remains unchanged when moving from mutually low to mutually high investments. It is then clear why the thresholds $\lambda^{l,h}$ and $\sigma^{l,h}$ only imply a restriction on the amount of neutral fans and not on the amount of hard-core fans. Evidently, rising investment costs push up the value of these thresholds.

Large club domination involves a trade-off between reduced utility for the small club's hard-core fans and higher utility for the large club's hard-core fans. Therefore it turns out to be optimal when the difference in the amount of hard-core fans $(m_1 - m_2)$ both clubs may reach, attains a certain threshold. Less obvious is the intuition behind the factors impacting on $\lambda^{u,l}, \lambda^{u,h}, \sigma^{u,l}$ and $\lambda^{u,h}$. Crucial to see is that at high values of n (i.e. $n > \lambda^{l,h}$ or $\sigma^{l,h}$), the relevant alternative for large market domination is mutually high investments, while at low values of n (i.e. $n < \lambda^{l,h}$ or $\sigma^{l,h}$) the relevant alternative is mutually low investments. Unbalanced investments involve less talent investments than mutually high investments. Hence, the negative impact of h on $\lambda^{u,h}$ and $\sigma^{u,h}$. Yet, large club domination involves higher talent investments than mutually low investments. Therefore the influence of h turns from negative to positive when n falls. Moving from unbalanced investments to mutually high investments leads to an increase in utility for neutral fans. This explains why $\lambda^{u,h}$ and $\sigma^{u,h}$ are both increasing in n.

Since from proposition 5, $\lambda^{l,h} > \sigma^{l,h}$ and $\lambda^{u,l} > \sigma^{u,l}$ it is clear that mutually low investments are less likely to be preferable from a social point-of-view than from the league's perspective. Mutually high investments on the other hand are more often the optimal outcome under the social objective than from the league's private point-of-view. The situation for large club domination is more ambiguous. While this is from a social perspective more easily preferred over mutually low investments, it is less attractive compared to mutually high investments. The driving force behind these results is clearly that talent investments are evaluated against the total surplus they deliver, instead of solely against private club profits. Using the thresholds from proposition 5, proposition 6 evaluates the effects of the different sharing rules on league profits.

Proposition 6 *Under performance-based and equal sharing, mutually high investments always arise when they are optimizing league profits. Sharing based on market size cannot guarantee this outcome.*

When they occur in equilibrium, mutually low investments are optimal for league profits under all distribution schemes. Under none of the distribution schemes the minimal necessary difference $\theta^u_{d_i}(n)$ stops the unbalanced outcome from arising, when this is optimal.

Proof. see appendix. ■

From proposition 6 it is clear that the league, when she introduces equal sharing, guarantees mutually high investments to arise when this is optimal for league profits. In fact, under equal sharing mutually high investments may even arise, when they are not the optimal outcome. It follows that the threshold value under equal sharing $\theta^h_{equal}(h,n)$ is certainly low enough. Lowering the threshold to $\theta^h_{win}(h,n)$ by introducing performance-based sharing is then unnecessary. Furthermore, it is even harmful for league-wide profits, as it widens the range of values of n and m_2, at which mutually high investments occur without being optimal. It follows that performance-based sharing is clearly less effective than equal sharing in matching the leagues' optimum with club behavior here. Since the threshold value $\theta^h_{market}(h,n)$ is higher than $\theta^h_{equal}(h,n)$, the same cannot be said of sharing based on market size. Under sharing based on market size, the league cannot guarantee that mutually high investments always arise when this is the optimal solution. On the other hand, there is less chance of them occurring when they should not. Consequently, it is indeterminate whether equal sharing or sharing based on market size, is preferable from the leagues' point of view.

The second part of proposition 6 implies that the league should never bother to prevent mutually low investment from arising. Under all three distribution schemes clubs only make mutually low investments when this is optimal for league profits. On the other hand, the league cannot guarantee that mutually low investments arise whenever this is desirable. She should therefore simply strive to minimize the range of values at which low investments fail to occur in equilibrium. It then follows that the league's optimal strategy is to choose the scheme that most encourages them. As proposition 4 shows that $\theta^j_{equal}(h) = \theta^l_{market}(h) > \theta^l_{win}(h,n)$, this is clearly not performance-based revenue sharing, while both other schemes perform exactly the same in this respect.

Finally, proposition 6 shows that under each scheme the difference in hard-core fans which is minimally necessary for the unbalanced outcome to appear, is smaller than the necessary difference for it to be the optimal outcome. In other words this necessary difference never stands in the way of the occurrence of an optimal outcome. Further, it is clear from proposition 4 that this difference can never constitute a sufficient, yet only a necessary condition. Therefore no scheme is outperformed by any other in this regard.

The previous analysis clearly shows why performance-based sharing only has disadvantages compared to equal sharing and is therefore dominated in matching the cartel's objectives with club behavior. The league would do best to ignore this sharing mechanism and not even use it partially. In order to approach the optimal thresholds as close as possible the league may possibly combine equal sharing and sharing based on market size.

It is not possible to show that the results of proposition 6 also apply to the social optima. This implies that none of the distribution schemes can be shown to dominate from a social welfare point-of-view. So, harmful as performance-based sharing may be for clubs, it should not necessarily bother us. It is even quite possible that clubs who are overinvesting in players and losing money, actually deliver more social welfare in the form of consumer surplus than in the case where they would keep a tight budget. Another important implication for social welfare is that increasing competitive balance is not always preferable. Indeed, certainly when h and n are relatively small, an unbalanced competition would quite often be preferred to a balanced one. This point once again shows why increasing competitive balance in itself should never be a criterion when evaluating different policy options. Revisiting the statistics given in table 2, it is now clear that though they are unbalanced, the investment decisions of Scottish, Belgian and Dutch teams may actually benefit society. As such, no action should be taken to prevent clubs from making these choices.

5 Bargaining on a sharing rule

An evident question is why numerous sports leagues around the world have been implementing performance-based sharing, when it is not an efficient way to maximize joint profits. Palomino and Sakovics (2004) suggest that performance-based sharing is a way of inducing clubs to attract star players when leagues compete in a bidding game to attract talent. In the context of the present model I propose that performance-based sharing is the result of inefficient cartel behavior. A sports league, acting as a cartel of clubs, must reach its decisions through negotiation. Therefore it makes sense to model the leagues' decisions as a bargaining game played between clubs, rather than as the result of efficient profit-maximizing. In the context of the present model clubs negotiate on the share of broadcast revenues they should be entitled to have. Their fall-back position is the revenue they could obtain when negotiations break down and they have to sell individually. The revenues they may divide are given by the model as:

$$R_{b,\text{col}} = n\frac{b\left(t_1(d_i), t_2(d_i)\right)}{2} \tag{18}$$

The equilibrium of the bargaining game is then a set of positive revenues $(R^1_{b,\text{col}}, R^2_{b,\text{col}})$, given as:

$$\begin{aligned} R^1_{b,\text{col}} &= d^1_i n\frac{b\left(t_1(d_i), t_2(d_i)\right)}{2} \\ R^2_{b,\text{col}} &= (1-d^1_i)n\frac{b\left(t_1(d_i), t_2(d_i)\right)}{2} \end{aligned}$$

where d^1_i is the share of the large team under sharing rule d_i.

Experience in Spain and Italy suggests that the earnings potential of clubs under individual sales is strongly related to the size of their local market[7]. I presume therefore that the distribution of revenues under individual sales is similar to that under sharing based on market sizes. The fall-back position of both clubs is then given as:

$$R^1_{b,ind} = \alpha n\frac{m_1}{m_1+m_2}\frac{b\left(t_1(market), t_2(market)\right)}{2} \tag{19}$$

$$R^2_{b,ind} = \alpha n\frac{m_2}{m_1+m_2}\frac{b\left(t_1(market), t_2(market)\right)}{2} \tag{20}$$

[7] In Spain earnings by the top clubs, FC Barcelona and Real Madrid amounted to 65 million euro, compared to 8 million euro for the lowest earning club, Racing Santander in the 2005/2006 season. In Italy the difference between AC Milan (66 million euro) and Siena (10 million euro) was almost identical in the 2006/2007 season. The correlation between dp, a local market size measure (see Peeters, 2009) and broadcast revenues ammounted to 0.88 in Spain and 0.96 in Italy. Sources: Gratton and Solberg (2007) and European Football Statistics.

where $\alpha \leq 1$ indicates a fall in club revenues when teams compete among each other and lose (a part of) their monopoly rents. The Nash-bargaining equilibrium of this game is the solution of the well-known equations:

$$\begin{aligned} R^1_{b,\text{col}} + R^2_{b,\text{col}} &= R_{b,\text{col}} \\ R^1_{b,\text{col}} - R^1_{b,ind} &= R^2_{b,\text{col}} - R^2_{b,ind} \end{aligned}$$

which solves as:

$$\begin{aligned} R^{1*}_{b,\text{col}} &= \frac{1}{2}(R_{b,\text{col}} + R^1_{b,ind} - R^2_{b,ind}) \\ R^{2*}_{b,\text{col}} &= \frac{1}{2}(R_{b,\text{col}} + R^2_{b,ind} - R^1_{b,ind}) \end{aligned}$$

After filling in (18), (19) and (20), this may be rewritten to find:

$$R^{1*}_{b,\text{col}} = \frac{n}{4(m_1+m_2)} \left\{ \begin{array}{c} m_1 \left[b\left(t_1(d_i), t_2(d_i)\right) + \alpha b\left(t_1(market), t_2(market)\right) \right] \\ +m_2 \left[b\left(t_1(d_i), t_2(d_i)\right) - \alpha b\left(t_1(market), t_2(market)\right) \right] \end{array} \right\} \quad (21)$$

$$R^{2*}_{b,\text{col}} = \frac{n}{4(m_1+m_2)} \left\{ \begin{array}{c} m_2 \left[b\left(t_1(d_i), t_2(d_i)\right) + \alpha b\left(t_1(market), t_2(market)\right) \right] \\ +m_1 \left[b\left(t_1(d_i), t_2(d_i)\right) - \alpha b\left(t_1(market), t_2(market)\right) \right] \end{array} \right\} \quad (22)$$

Observe from (21) and (22) that sharing based on market size is a stable equilibrium when α approaches 1. Equal sharing on the other hand becomes a stable outcome in case α is zero. More importantly perhaps, the equations show that for any positive value of α, the bargained solution should deviate more from equal sharing when the difference in local markets sizes is larger. In other words, when market sizes differ substantially and α is not too low we should not expect to see equal sharing. In those cases, the large teams' outside options are so strong that they permit them to negotiate a sharing rule which allocates more revenue to them than equal sharing does. In the present model this may be done in two different ways. First, clubs may introduce sharing based on market sizes. This system has one important setback however. Antitrust authorities permit collective sales to enable leagues to redistribute revenues in a more egalitarian way. They therefore often not approve of a sharing rule, which has a large component of sharing based on market size. A second way to grant more expected revenues to the large teams is performance-based sharing. As proposition 1 shows, small teams may never dominate the competition in terms of sporting results. Large teams should then expect to earn at least half of the revenues, which are shared based on performance. In many instances however, they earn more than half of these revenues. Following this line of reasoning, performance-based sharing may arise when local markets are sufficiently different, the loss of monopoly rents is not too high and antitrust authorities prevent having a large portion of sharing based on market size in the arrangement. In extreme cases of asymmetry between teams negotiations may also brake down when sharing based on market size is not allowed. In those cases large teams refuse to enter a collective arrangement, as they cannot be rewarded enough through the sharing rule.

Table 3 presents an overview of some European soccer leagues and American major leagues to illustrate the reasoning I developed in this section. The table first shows the distribution of local market sizes among clubs. The statistic is calculated as the standard deviation of average club attendances over 4 seasons divided by the league average attendance over this period. This procedure aims to avoid overestimating the local market when one time sporting results have driven up attendance. The use of attendance data allows to calculate it for a large amount of leagues, whereas other measures are often poorly comparable between leagues[8]. Then, the table gives the percentage of revenues that were shared equally, based on market size/TV appearances and performance. Finally the table provides a figure for total broadcast revenues in the league. When comparing leagues across the Atlantic, it is striking that all American leagues have adopted equal sharing, while European leagues only use it partially (France, Germany, England) or not at all (Italy, Spain).[9] In terms of the model, this is explained by the fact that home market sizes are more homogeneously distributed in the USA. The table indicates that this is indeed the case, with figures around 0.1 for all leagues, compared to a minimum of 0.38 in Europe. Within Europe, Spain and Italy show the most heterogeneous distribution of home markets. In line with the predictions of the model, this renders the bargaining on a sharing rule more difficult, which has led these countries to abandon the collective system. In the other European leagues teams have come to negotiated solutions. Their sharing rules partially involve performance-based sharing and sharing based on market size. A final observation from table 3 may be that individual sales apparently have not harmed overall broadcast revenues in Spain and Italy too much in comparison to the collectively selling leagues. This suggests that the value of α in the model may be closer to one than zero for European soccer.

Table 3: Revenue sharing and national broadcast revenue in European soccer and US major leagues

League	Local market size distribution	Percentage equal	Percentage market size	Percentage performance	Total Broadcast Revenue (in \$1m) [10]
NFL	0.098	100	0	0	3,700
NBA	0.116	100	0	0	930
NHL	0.108	100	0	0	675
Premier League	0.380	50	25	25	1,092
Bundesliga	0.420	50	0	50	578
Ligue 1	0.547	50	20	30	934
Serie A	0.647	0	0	0	645
La Liga	0.625	0	0	0	470

6 Conclusion and final remarks

In this contribution I have built a model of a team sports league which includes two horizontally diversified types of consumers, hard-core and neutral fans. The league operating as a profit-maximizing cartel may use its sharing rule for broadcast revenues to steer talent investments by clubs to maximize the profit made from both types. I analyze three ways in which revenue sharing may take place, equal sharing, performance-based sharing and sharing based on market size. Sharing based on performance turns out to be an inefficient way to maximize joint profits in the cartel, while not necessarily implying a loss of social welfare. On the other hand, the leagues' choice between equal sharing and sharing based on market size is indeterminate, as well from a social perspective as from the leagues'

[8] See Peeters (2009) for more information on the use of this measure.

[9] One important remark to add is that American clubs often have retained the right to individually sell their local broadcast rights. As such, they also partially introduce market-based sharing.

[10] Exchange rates on 28/01/2010. All data from www.sportbusiness.com and Gratton & Solberg (2007). Data from 2008-2009 season, except Serie A (2007) and Liga (2006).

private point-of-view. The intuition for the existence of performance-based sharing comes from the bargaining on a sharing rule within the cartel, which never leads to equal sharing when clubs have asymmetric fall-back positions. Data from European soccer and American major leagues illustrate the main theoretical findings in the paper.

An apparently limiting assumption made in the model is that talent investments are discrete choices. However, the threshold values on the amounts of fans (m_1, m_2 and n) given in the model can easily be rewritten as thresholds on the talent investment costs (h). They may then be used to show how differences in talent investments between both clubs depend on the amounts of hard-core and neutral fans. As such, the model may be made more flexible, but again never leads the competition to be dominated by the small market team. A second remark concerns the presumed competitive nature of the broadcasting market. In many smaller countries this may seem to be a strong assumption. Practice has shown that in cases where only a small number of broadcasters is interested in buying sports rights, most often some kind of bargaining between the league and the broadcaster takes place. The result is that monopoly rents are only partially absorbed by the sports league. This would mean that revenues originating from neutral fans should have been divided by some factor depending on the strength of both bargaining parties (the same is done in section 5 to model individual sales). Essentially, this would only diminish the impact of the amount of neutral fans vis-a-vis the other factors in the model without dramatically changing any of the other results.

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8 Appendix

In this appendix I provide proof of all propositions.

8.1 Proposition 1

Proof. I show that $t_1 = l, t_2 = h$ can never arise in equilibrium by showing that the necessary conditions for this lead to a contradiction, namely that $m_1 \leq m_2$.

$t_1 = l, t_2 = h$ is a Nash equilibrium

$$\Leftrightarrow \left\{ \begin{array}{c} \pi_1(l,h) \geq \pi_1(h,h) \\ and \\ \pi_2(l,l) \leq \pi_2(h,l) \end{array} \right\}$$

This means under:

1. $d_i = 1/2$

   from (10) and (11):

$$\Leftrightarrow \left\{ \begin{array}{c} \frac{1}{4}m_1 f(l,h) + \frac{1}{4}nb(l,h) \geq \frac{1}{4}m_1 f(h,h) + \frac{1}{4}nb(h,h) - h \\ and \\ \frac{1}{4}m_2 f(l,l) + \frac{1}{4}nb(l,l) \leq \frac{1}{4}m_2 f(h,l) + \frac{1}{4}nb(h,l) - h \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} h \geq \frac{1}{4}m_1 \left[f(h,h) - f(l,h)\right] + \frac{1}{4}n\left[b(h,h) - b(l,h)\right] \\ and \\ h \leq \frac{1}{4}m_2\left[f(h,l) - f(l,l)\right] \end{array} \right\}$$

$$\Rightarrow \frac{1}{4}m_1\left[f(h,h) - f(l,h)\right] + \frac{1}{4}n\left[b(h,h) - b(l,h)\right] \leq \frac{1}{4}m_2\left[f(h,l) - f(l,l)\right]$$

   from (3):

$$\Leftrightarrow m_1 + n\frac{b(h,h)-b(l,h)}{f(h,h)-f(l,h)} \leq m_2$$

   since $b(h,h) - b(l,h) \geq 0$ and $f(h,h) - f(l,h) \geq 0$:

$$\Leftrightarrow m_1 \leq m_2$$

2. $d_i = w_i$

   Again from (10) and (11):

$$\Leftrightarrow \left\{ \begin{array}{c} \frac{1}{4}m_1 f(l,h) + \frac{1}{2}nw_1(l,h)b(l,h) \geq \frac{1}{4}m_1 f(h,h) + \frac{1}{2}nw_1(h,h)b(h,h) - h \\ and \\ \frac{1}{4}m_2 f(l,l) + \frac{1}{2}nw_2(l,l)b(l,l) \leq \frac{1}{4}m_2 f(h,l) + \frac{1}{2}nw_2(h,l)b(h,l) - h \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} h \geq \frac{1}{4}m_1 \left[f(h,h) - f(l,h)\right] + \frac{1}{2}n\left[\frac{1}{2}b(h,h) - (1-\beta)b(l,h)\right] \\ and \\ h \leq \frac{1}{4}m_2 \left[f(h,l) - f(l,l)\right] + \frac{1}{2}n\left[\beta b(h,l) - \frac{1}{2}b(l,l)\right] \end{array} \right\}$$

$$\Rightarrow m_1 \left[f(h,h) - f(l,h)\right] + 2n\left[\frac{1}{2}b(h,h) - (1-\beta)b(l,h)\right] \leq m_2 \left[f(h,l) - f(l,l)\right] + 2n\left[\beta b(h,l) - \frac{1}{2}b(l,l)\right]$$

$$\Leftrightarrow m_1 \left[f(h,h) - f(l,h)\right] + 2n\left[\frac{1}{2}b(h,h) - b(l,h) + \beta b(l,h) - \beta b(h,l) + \frac{1}{2}b(l,l)\right] \leq m_2 \left[f(h,l) - f(l,l)\right]$$

   From (4):

$$\Leftrightarrow m_1 \left[f(h,h) - f(l,h)\right] + n\left[b(h,h) - b(l,h)\right] \leq m_2 \left[f(h,l) - f(l,l)\right]$$

   From (3):

$$\Leftrightarrow m_1 + n\frac{b(h,h)-b(l,h)}{f(h,l)-f(l,l)} \leq m_2$$

   since $b(h,h) - b(l,h) \geq 0$ and $f(h,h) - f(l,h) \geq 0:$

$$\Leftrightarrow m_1 \leq m_2$$

3. $d_i = \frac{m_j}{m_k + m_j}$ with $k \neq j$

   Again from (10) and (11):

$$\Leftrightarrow \left\{ \begin{array}{c} \frac{1}{4}m_1 f(l,h) + \frac{1}{2}\frac{m_1}{m_1+m_2}nb(l,h) \geq \frac{1}{4}m_1 f(h,h) + \frac{1}{2}\frac{m_1}{m_1+m_2}nb(h,h) - h \\ and \\ \frac{1}{4}m_2 f(l,l) + \frac{1}{2}\frac{m_2}{m_1+m_2}nb(l,l) \leq \frac{1}{4}m_2 f(h,l) + \frac{1}{2}\frac{m_2}{m_1+m_2}nb(h,l) - h \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} h \geq \frac{1}{4}m_1 \left[f(h,h) - f(l,h)\right] + \frac{1}{2}\frac{m_1}{m_1+m_2}n\left[b(h,h) - b(l,h)\right] \\ and \\ h \leq \frac{1}{4}m_2 \left[f(h,l) - f(l,l)\right] + \frac{1}{2}\frac{m_2}{m_1+m_2}n\left[b(h,l) - b(l,l)\right] \end{array} \right\}$$

   From (4):

$$\Rightarrow m_1 \left[f(h,h) - f(l,h)\right] + 2\frac{m_1}{m_1+m_2}n\left[b(h,h) - b(l,h)\right] \leq m_2 \left[f(h,l) - f(l,l)\right]$$

   From (3):

$$\Leftrightarrow m_1 + 2\frac{m_1}{m_1+m_2}n\frac{b(h,h)-b(l,h)}{f(h,l)-f(l,l)} \leq m_2$$

   since $b(h,h) - b(l,h) \geq 0$ and $f(h,h) - f(l,h) \geq 0:$

$$\Leftrightarrow m_1 \leq m_2$$

$$\Rightarrow t_1 = l, t_2 = h \text{ is never an equilibrium.}$$

■

8.2 Proposition 2 & 3

Proof. Since from proposition 1 $t_1 = l, t_2 = h$ is never an equilibrium, unilateral deviation of club 1 from $t_1 = h, t_2 = h$ need not be considered. I define the threshold value on market size of the small club for $t_1 = h, t_2 = h$ to arise as $\theta^h_{d_i}(h,n)$. Then, $t_1 = h, t_2 = h$ is a Nash equilibrium $\Leftrightarrow \pi_2(h,h) \geq \pi_2(l,h)$. This implies under:

1. $d_i = 1/2$

   $\Leftrightarrow \frac{1}{4}m_2 f(h,h) + \frac{1}{4}nb(h,h) - h \geq \frac{1}{4}m_2 f(l,h) + \frac{1}{4}nb(h,l)$

   $\Leftrightarrow \frac{4h-n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{equal}(h,n) \leq m_2$

2. $d_i = w_i$

   $\Leftrightarrow \frac{1}{4}m_2 f(h,h) + \frac{1}{4}nb(h,h) - h \geq \frac{1}{4}m_2 f(l,h) + \frac{1}{2}n(1-\beta)b(h,l)$

   $\Leftrightarrow \frac{4h-n[b(h,h)-2(1-\beta)b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{win}(h,n) \leq m_2$

3. $d_i = \frac{m_j}{m_k+m_j}$ with $k \neq j$

   $\Leftrightarrow \frac{1}{4}m_2 f(h,h) + \frac{1}{2}\frac{m_2}{m_1+m_2}nb(h,h) - h \geq \frac{1}{4}m_2 f(l,h) + \frac{1}{2}\frac{m_2}{m_1+m_2}nb(h,l)$

   $\Leftrightarrow \frac{4h-2\frac{m_2}{m_1+m_2}n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{market}(h,n) \leq m_2$

   Since $\beta > \frac{1}{2} > \frac{m_2}{m_1+m_2}$, it follows that $\theta^h_{market}(h,n) > \theta^h_{equal}(h,n) > \theta^h_{win}(h,n)$.

Again from proposition 1, $t_1 = l, t_2 = h$ is never an equilibrium, so unilateral deviation of club 2 from $t_1 = l, t_2 = l$ need then not be considered. I define the threshold value, this time on the large club's home market size, $\theta^l_{d_i}(h,n)$. Then, $t_1 = l, t_2 = l$ is a Nash equilibrium $\Leftrightarrow \pi_1(h,l) \leq \pi_1(l,l)$ or under:

1. $d_i = 1/2$

   $\Leftrightarrow \frac{1}{4}m_1 f(h,l) + \frac{1}{4}nb(h,l) - h \leq \frac{1}{4}m_1 f(l,l) + \frac{1}{4}nb(l,l)$

   From (4)

   $\Leftrightarrow m_1 \leq \theta^l_{equal}(h,n) = \frac{4h}{f(h,l)-f(l,l)}$

2. $d_i = w_i$

   $\Leftrightarrow \frac{1}{4}m_1 f(h,l) + \frac{1}{2}\beta nb(h,l) - h \leq \frac{1}{4}m_1 f(l,l) + \frac{1}{4}nb(l,l)$

   From (4)

   $\Leftrightarrow m_1 \leq \theta^l_{win}(h,n) = \frac{4h-n(2\beta-1)b(h,l)}{f(h,l)-f(l,l)}$

3. $d_i = \frac{m_j}{m_k+m_j}$ with $k \neq j$

   $\Leftrightarrow \frac{1}{4}m_1 f(h,l) + \frac{1}{2}\frac{m_1}{m_1+m_2}nb(h,l) - h \leq \frac{1}{4}m_1 f(l,l) + \frac{1}{2}\frac{m_1}{m_1+m_2}nb(l,l)$

   From (4)

   $\Leftrightarrow m_1 \leq \theta^l_{market}(h,n) = \frac{4h}{f(h,l)-f(l,l)}$

   Since $\beta > \frac{1}{2}$, it follows that $\theta^l_{market}(h,n) = \theta^l_{equal}(h,n) > \theta^l_{win}(h,n)$

At both thresholds, clubs are indifferent between high and low investments. Therefore multiple equilibria are possible at these values. In order to have unique Nash-equilibria at all other values for the model parameters, it is necessary that both thresholds are never met simultaneously. Observe this is the case under

1. $d_i = 1/2$

   $\Leftrightarrow \left\{ \begin{array}{c} m_1 < \theta^l_{equal}(h,n) = \frac{4h}{f(h,l)-f(l,l)} \\ and \\ \frac{4h-n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{equal}(h,n) < m_2 \end{array} \right\}$ is never true. This is guaranteed by $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ as follows:

plugging $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ in the second condition:

$\Rightarrow \frac{4h}{f(h,h)-f(l,h)} - (m_1 - m_2) < m_2$

$\Leftrightarrow \frac{4h}{f(h,h)-f(l,h)} - (m_1) < 0$

$\Leftrightarrow \frac{4h}{f(h,h)-f(l,h)} < m_1$

which violates the first line.

2. $d_i = w_i$

$\Leftrightarrow \left\{ \begin{array}{c} m_1 < \theta^l_{win}(h,n) = \frac{4h-n(2\beta-1)b(h,l)}{f(h,l)-f(l,l)} \\ and \\ \frac{4h-n[b(h,h)-2(1-\beta)b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{win}(h,n) < m_2 \end{array} \right\}$ is never true. This is guaranteed by $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ as follows:

plugging $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ in the second condition:

$\Rightarrow \frac{4h-n(2\beta-1)b(h,l)}{f(h,h)-f(l,h)} - (m_1 - m_2) < m_2$

$\Leftrightarrow \frac{4h-n(2\beta-1)b(h,l)}{f(h,h)-f(l,h)} - (m_1) < 0$

$\Leftrightarrow \frac{4h-n(2\beta-1)b(h,l)}{f(h,h)-f(l,h)} < m_1$

which violates the first line.

3. $d_i = \frac{m_j}{m_k+m_j}$ with $k \neq j$

$\Leftrightarrow \left\{ \begin{array}{c} m_1 < \theta^l_{market}(h,n) = \frac{4h}{f(h,l)-f(l,l)} \\ and \\ \frac{4h-2\frac{m_2}{m_1+m_2}n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} < m_2 \end{array} \right\}$ is never true. This is guaranteed by $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ as follows:

plugging $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ in the second condition and using

$2\frac{m_2}{m_1+m_2} < 1 \Rightarrow 2\frac{m_2}{m_1+m_2}n\left[b(h,h) - b(h,l)\right] < \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$:

$\Rightarrow \frac{4h}{f(h,h)-f(l,h)} - (m_1 - m_2) < m_2$

$\Leftrightarrow \frac{4h}{f(h,h)-f(l,h)} - (m_1) < 0$

$\Leftrightarrow \frac{4h}{f(h,h)-f(l,h)} < m_1$

which violates the first line.

■

8.3 Proposition 4

Proof. As proposition 2 and 3 show the threshold value under which either $t_1 = t_2 = h$ or $t_1 = t_2 = l$ prevail as a unique equilibrium instead of $t_1 = h, t_2 = l$. It is clear that $t_1 = h, t_2 = l$ is a Nash-equilibrium when both these conditions are simultaneously not satisfied. The necessary minimum differences between both fan sizes may be calculated under:

1. $d_i = 1/2$

$\Leftrightarrow \left\{ \begin{array}{c} \theta^l_{equal} \leq m_1 \\ and \\ \theta^h_{equal} \geq m_2 \end{array} \right\} \Rightarrow m_1 - m_2 \geq \theta^l_{equal} - \theta^h_{equal}$

$\Leftrightarrow m_1 - m_2 \geq \frac{4h}{f(h,l)-f(l,l)} - \frac{4h-n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)}$
$= \frac{n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^u_{equal}(n)$

2. $d_i = w_i$
$\Leftrightarrow \left\{ \begin{array}{c} \theta^l_{win} \leq m_1 \\ and \\ \theta^h_{win} \geq m_2 \end{array} \right\} \Rightarrow m_1 - m_2 \geq \theta^l_{win} - \theta^h_{win}$
$\Leftrightarrow m_1 - m_2 \geq \frac{4h-n(2\beta-1)b(h,l)}{f(h,l)-f(l,l)} - \frac{4h-n[b(h,h)-2(1-\beta)b(h,l)]}{f(h,h)-f(l,h)}$
$= \frac{n[b(h,h)-2(1-\beta)b(h,l)]-n(2\beta-1)b(h,l)}{f(h,l)-f(l,l)}$
$= \frac{nb(h,h)+nb(h,l)-2nb(h,l)+2n\beta b(h,l)-2n\beta b(h,l)}{f(h,l)-f(l,l)}$
$= \frac{nb(h,h)-nb(h,l)}{f(h,l)-f(l,l)} = \frac{n[b(h,h)-b(h,l)]}{f(h,l)-f(l,l)} = \theta^u_{win}(n)$

3. $d_i = \frac{m_j}{m_k+m_j}$ with $k \neq j$
$\Leftrightarrow \left\{ \begin{array}{c} \theta^l_{market} \leq m_1 \\ and \\ \theta^h_{market} \geq m_2 \end{array} \right\} \Rightarrow m_1 - m_2 \geq \theta^l_{market} - \theta^h_{market}$
$\Leftrightarrow m_1 - m_2 \geq \frac{4h}{f(h,l)-f(l,l)} - \frac{4h-2\frac{m_2}{m_1+m_2}n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = 2\frac{m_2}{m_1+m_2}\frac{n[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^u_{market}(n)$
Observe that $\theta^u_{d_i}(n) > 0$ for every d_i. This is again a confirmation of the fact that small market domination never occurs, as this would mean that the difference $m_2 - m_1 \geq \theta^l_{market} - \theta^h_{market} > 0$ which contradicts $m_2 < m_1$. As $\frac{m_2}{m_1+m_2} < \frac{1}{2}$, it is clear that the necessary difference is smallest under sharing based on market size.

■

8.4 Proposition 5

Proof. League profits in each talent investment situation are given by:

$$\pi^{h,h}_L = \frac{1}{4}\left((m_1+m_2)f(h,h) + 2nb(h,h)\right) - 2h \tag{23}$$
$$\pi^{h,l}_L = \frac{1}{4}\left(m_1 f(h,l) + m_2 f(l,h) + 2nb(h,l)\right) - h \tag{24}$$
$$\pi^{l,h}_L = \frac{1}{4}\left(m_1 f(l,h) + m_2 f(h,l) + 2nb(l,h)\right) - h \tag{25}$$
$$\pi^{l,l}_L = \frac{1}{4}\left((m_1+m_2)f(l,l) + 2nb(l,l)\right) \tag{26}$$

As before, I establish the conditions on n, m_1, m_2 and h under which each talent investment situation is optimal. Note that small market domination is optimal for league profits, in case: $\pi^{l,h}_L \geq \pi^{h,h}_L$, $\pi^{l,h}_L \geq \pi^{h,l}_L$ and $\pi^{l,h}_L \geq \pi^{l,l}_L$. However, from (24) and (25):
$\pi^{l,h}_L \geq \pi^{h,l}_L$
$\Leftrightarrow \frac{1}{4}\left(m_1 f(l,h) + m_2 f(h,l) + 2nb(l,h)\right) - h \geq \frac{1}{4}\left(m_1 f(h,l) + m_2 f(l,h) + 2nb(h,l)\right) - h$
$\Leftrightarrow m_2\left(f(h,l) - f(l,h)\right) \geq m_1\left(f(h,l) - f(l,h)\right)$
$\Leftrightarrow m_2 \geq m_1$

which is a contradiction. Therefore, small market domination is never optimal for league profits and it should not be considered as an alternative. Now I turn to the three other talent investment outcomes.

1. $t_1 = t_2 = h$ is optimal

$$\Leftrightarrow \left\{ \begin{array}{c} \pi_L^{h,h} \geq \pi_L^{h,l} \\ and \\ \pi_L^{h,h} \geq \pi_L^{l,l} \end{array} \right\}$$

From (23), (24) and (26):

$$\Leftrightarrow \left\{ \begin{array}{c} \frac{1}{4}\left((m_1+m_2)f(h,h)+2nb(h,h)\right)-2h \geq \frac{1}{4}\left(m_1 f(h,l)+m_2 f(l,h)+2nb(h,l)\right)-h \\ and \\ \frac{1}{4}\left((m_1+m_2)f(h,h)+2nb(h,h)\right)-2h \geq \frac{1}{4}\left((m_1+m_2)f(l,l)+2nb(l,l)\right) \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} (m_1+m_2)f(h,h)+2nb(h,h)-4h \geq m_1 f(h,l)+m_2 f(l,h)+2nb(h,l) \\ and \\ (m_1+m_2)f(h,h)+2nb(h,h)-8h \geq (m_1+m_2)f(l,l)+2nb(l,l) \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} 2n\left(b(h,h)-b(h,l)\right)-4h \geq m_1\left(f(h,l)-f(h,h)\right)-m_2\left(f(h,h)-f(l,h)\right) \\ and \\ 2nb(h,h)-8h \geq 2nb(l,l) \end{array} \right\}$$

From (2) and (3):

$$\Leftrightarrow \left\{ \begin{array}{c} 2n\left(b(h,h)-b(h,l)\right)-4h \geq (m_1-m_2)\left(f(h,l)-f(h,h)\right) \\ and \\ n\left(b(h,h)-b(l,l)\right) \geq 4h \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} \lambda^{u,h} = \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \geq m_1-m_2 \\ and \\ n \geq \frac{4h}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\}$$

2. $t_1 = t_2 = l$ is optimal

$$\Leftrightarrow \left\{ \begin{array}{c} \pi_L^{l,l} \geq \pi_L^{h,l} \\ and \\ \pi_L^{l,l} \geq \pi_L^{h,h} \end{array} \right\}$$

From part 1 of this proof (24) and (26):

$$\Leftrightarrow \left\{ \begin{array}{c} \frac{1}{4}\left((m_1+m_2)f(l,l)+2nb(l,l)\right) \geq \frac{1}{4}\left(m_1 f(h,l)+m_2 f(l,h)+2nb(h,l)\right)-h \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} (m_1+m_2)f(l,l) \geq m_1 f(h,l)+m_2 f(l,h)-4h \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\}$$

Again from (2):

$$\Leftrightarrow \left\{ \begin{array}{c} 4h \geq (m_1-m_2)\left(f(h,l)+f(h,h)\right) \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{array}{c} \lambda^{u,l} = \frac{4h}{f(h,l)+f(h,h)} \geq m_1-m_2 \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\}$$

3. $t_1 = h, t_2 = l$ is optimal

$$\Leftrightarrow \left\{ \begin{array}{c} \pi_L^{h,l} \geq \pi_L^{h,h} \\ and \\ \pi_L^{h,l} \geq \pi_L^{l,l} \end{array} \right\}$$

From part 1 and 2 of this proof:

$$\Leftrightarrow \left\{ \begin{array}{c} \lambda^{u,h} = \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \leq m_1 - m_2 \\ and \\ \lambda^{u,l} = \frac{4h}{f(h,l)+f(h,h)} \leq m_1 - m_2 \end{array} \right\}$$

From the observation that $n = \frac{4h}{b(h,h)-b(l,l)} \Leftrightarrow \frac{4h}{f(h,l)+f(h,h)} = \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)}$, it follows that both conditions converge at the point where the league is indifferent between mutually high and mutually low investments. Notice as well that the condition $m_1 - m_2 \geq \frac{n(b(h,h)-b(l,l))}{f(h,l)-f(l,l)}$ never prevents a balanced outcome being preferred by the league.

Social welfare in each talent investment outcome is given by:

$$\begin{aligned} SW^{h,h} &= \frac{3}{8}((m_1+m_2)f(h,h) + 2nb(h,h)) - 2h \\ SW^{h,l} &= \frac{3}{8}(m_1 f(h,l) + m_2 f(l,h) + 2nb(h,l)) - h \\ SW^{l,h} &= \frac{3}{8}(m_1 f(l,h) + m_2 f(h,l) + 2nb(l,h)) - h \\ SW^{l,l} &= \frac{3}{8}((m_1+m_2)f(l,l) + 2nb(l,l)) \end{aligned}$$

Following a completely similar logic as before, it is easy to show that:

1. small club domination is never socially optimal.
2. $\sigma^{l,h} = \frac{8}{3}\frac{h}{b(h,h)-b(l,l)}$.
3. $\sigma^{u,l} = \frac{8}{3}\frac{h}{f(h,l)-f(l,l)}$.
4. $\sigma^{u,h}(h,n) = \frac{2n[b(h,h)-b(h,l)]-\frac{8}{3}h}{f(h,l)-f(h,h)}$.

Comparison of these thresholds reveals: $\sigma^{l,h} < \lambda^{l,h}$, $\sigma^{u,l} < \lambda^{u,l}$ and $\sigma^{u,h} > \lambda^{u,h}$. ■

8.5 Proposition 6

Proof. In order to provide evidence for proposition 6 I first investigate the case of equal sharing and then compare both other schemes with this benchmark.

1. $d_i = 1/2$

   - if $t_1 = t_2 = h$ is optimal, it always occurs.

     Suppose $t_1 = t_2 = h$ is optimal, then from proposition 5:

$$\Rightarrow n \geq \frac{4h}{b(h,h)-b(l,l)} = \lambda^l(h) \tag{27}$$

From proposition 2: $t_1 = t_2 = h$ is the unique Nash-equilibrium under $d_i = 1/2$

$$\Leftrightarrow \frac{4h - n\,[b(h,h)-b(h,l)]}{f(h,h)-f(l,h)} = \theta^h_{equal}(h,n) < m_2 \tag{28}$$

Filling in (27) into (28) leads to $t_1 = t_2 = h$ occurring

$$\Leftrightarrow \quad \frac{4h - n\left[b(h,h) - b(h,l)\right]}{f(h,h) - f(l,h)} \leq \frac{4h - \frac{4h}{b(h,h)-b(l,l)}\left[b(h,h) - b(h,l)\right]}{f(h,h) - f(l,h)} < m_2$$

$$\Leftrightarrow \quad \frac{4h - 4h}{f(h,h) - f(l,h)} = 0 < m_2$$

Since $0 < m_2$, $t_1 = t_2 = h$ is always the unique Nash-equilibrium of the investment stage when this is optimal for league profits.

- if $t_1 = t_2 = l$ occurs, it is optimal.

  Suppose $t_1 = t_2 = l$ occurs, then from proposition 3

$$\Rightarrow m_1 \leq \frac{4h}{f(h,l) - f(l,l)} \tag{29}$$

  From proposition 5, $t_1 = t_2 = l$ is optimal for league profits

$$\Leftrightarrow \left\{ \begin{array}{c} \lambda^{l,h}(h) = \frac{4h}{f(h,l)+f(h,h)} \geq m_1 - m_2 \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} = \lambda^h(h) \end{array} \right\} \tag{30}$$

  Filling in (29) in the right hand-side of the first condition of (30), yields: $m_1 - m_2 \leq \frac{4h}{f(h,l)-f(l,l)} - m_2 < \frac{4h}{f(h,l)+f(h,h)}$, because $0 < m_2$. So the first condition of (30) is satisfied. To see why (29) also implies that the second part of (30) is satisfied, observe that $n > \frac{4h}{b(h,h)-b(l,l)}$ and the second part of (30) can never be satisfied simultaneously. On the contrary, when $n > \frac{4h}{b(h,h)-b(l,l)}$ is not satisfied (30) is always satisfied. On the other hand it is clear that the occurrence of $t_1 = t_2 = l$ and $t_1 = t_2 = h$ also mutually exclude each other. Since $n > \frac{4h}{b(h,h)-b(l,l)}$ is a (more than) sufficient condition for $t_1 = t_2 = h$ to occur, it follows that when $t_1 = t_2 = l$ occurs (and consequently $t_1 = t_2 = h$ fails to occur and (29) is satisfied) $n > \frac{4h}{b(h,h)-b(l,l)}$ can never be satisfied. Therefore in such cases the second part of (30) is always satisfied. In this way, (29) also implies that the second part of (30) is met.

- if $t_1 = h, t_2 = l$ is optimal, $\theta^u_{equal} < m_1 - m_2$

  Suppose $t_1 = h, t_2 = l$ is optimal, then from proposition 5 it follows that

  - either $\left\{ \begin{array}{c} \frac{4h}{f(h,l)+f(h,h)} \leq m_1 - m_2 \\ and \\ n \leq \frac{4h}{b(h,h)-b(l,l)} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \frac{4h}{f(h,l)+f(h,h)} \leq m_1 - m_2 \\ and \\ n\left(b(h,h) - b(l,l)\right) \leq 4h \end{array} \right\}$

    (plugging the second line into the first)

    $\Rightarrow \theta^u_{equal} = n\frac{(b(h,h)-b(l,l))}{f(h,l)+f(h,h)} \leq \frac{4h}{f(h,l)+f(h,h)} \leq m_1 - m_2$

  - or $\left\{ \begin{array}{c} \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \leq m_1 - m_2 \\ and \\ n \geq \frac{4h}{b(h,h)-b(l,l)} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \leq m_1 - m_2 \\ and \\ n\left(b(h,h) - b(l,l)\right) \geq 4h \end{array} \right\}$

    (plugging the second line into the first)

    $\Rightarrow \theta^u_{equal} = n\frac{(b(h,h)-b(l,l))}{f(h,l)+f(h,h)} \leq n\frac{(b(h,h)-b(l,l))}{f(h,l)+f(h,h)} + \frac{n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \leq \frac{2n(b(h,h)-b(h,l))-4h}{f(h,l)-f(h,h)} \leq m_1 - m_2$

  So, the minimal necessary difference in market size can never prevent large market domination to arise, when it is optimal.

2. $d_i = w_i$

- if $t_1 = t_2 = h$ is optimal, it always occurs

  From proposition 2:
$$\theta^h_{win} < \theta^h_{equal} \tag{31}$$
  While from part 1 of this proof: $t_1 = t_2 = h$ is optimal $\Rightarrow \theta^h_{equal} < m_2$

  Plugging (31) into this gives:

  $t_1 = t_2 = h$ is optimal $\Rightarrow \theta^h_{win} < \theta^h_{equal} < m_2$ and consequently $t_1 = t_2 = h$ occurs under $d_i = w_i$.

- if $t_1 = t_2 = l$ occurs, it is optimal.

  From proposition 3:
$$\theta^l_{win} < \theta^l_{equal} \tag{32}$$
  While again from part 1 of this proof: $m_1 \leq \theta^l_{equal} \Rightarrow t_1 = t_2 = l$ is optimal.

  Combining (32) with this yields:

  $m_1 \leq \theta^l_{win} \Rightarrow m_1 \leq \theta^l_{equal} \Rightarrow t_1 = t_2 = l$ is optimal.

- if $t_1 = h, t_2 = l$ is optimal, $\theta^U_{win} < m_1 - m_2$

  From proposition 4: $\theta^u_{win} = \theta^u_{equal}$, so the reasoning of part 1 applies.

3. $d_i = \frac{m_j}{m_k + m_j}$ with $k \neq j$

- if $t_1 = t_2 = h$ is optimal, it cannot be guaranteed to occur

  Suppose $t_1 = t_2 = h$ is optimal, then as before from proposition 5
$$\Rightarrow n \geq \frac{4h}{b(h,h) - b(l,l)} = \lambda^l(h) \tag{33}$$
  From proposition 2 $t_1 = t_2 = h$ occurs as an equilibrium under $d_i = \frac{m_j}{m_k + m_j}$ with $k \neq j$
$$\Leftrightarrow \frac{4h - 2\frac{m_2}{m_1+m_2} n\,[b(h,h) - b(h,l)]}{f(h,h) - f(l,h)} = \theta^h_{market}(h,n) < m_2 \tag{34}$$
  Plugging (33) in (34) leads to:

  $\frac{4h - 2\frac{m_2}{m_1+m_2} n[b(h,h) - b(h,l)]}{f(h,h) - f(l,h)} \leq \frac{4h - \frac{m_2}{m_1+m_2} 8h}{f(h,h) - f(l,h)} = \frac{4h\left(1 - 2\frac{m_2}{m_1+m_2}\right)}{f(h,h) - f(l,h)} < m_2$. As $\frac{1}{2} > \frac{m_2}{m_1+m_2}$, this cannot be guaranteed to be satisfied for all positive values of m_2

- if $t_1 = t_2 = l$ occurs, it is optimal.

  From proposition 3:
$$\theta^l_{market} = \theta^l_{equal} \tag{35}$$
  While from part 1 of this proof: $m_1 \leq \theta^l_{equal} \Rightarrow t_1 = t_2 = l$ is optimal. Plugging (35) into this expression leads to:$m_1 \leq \theta^l_{market} \Rightarrow m_1 \leq \theta^l_{equal} \Rightarrow t_1 = t_2 = l$ is optimal

- if $t_1 = h, t_2 = l$ is optimal, $\theta^u_{market} < m_1 - m_2$,

  From proposition 4: $\theta^u_{market} < \theta^u_{equal}$, so the reasoning of part 1 applies.

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