DEPARTMENT OF ENGINEERING MANAGEMENT

Multi-exchange neighborhoods for the capacitated ring tree problem

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Abstract. A *ring tree* is a tree graph with an optional additional edge that closes a unique cycle. Such a cycle is called a *ring* and the nodes on it are called *ring nodes*. The *capacitated ring tree problem* (CRTP) asks for a network of minimal overall edge cost that connects given customers to a depot by ring trees. Ring trees are required to intersect in the depot which has to be either a ring node in a ring tree or a node of degree one if the ring tree does not contain a ring. Customers are predefined as of *type 1* or *type 2*. The type 2 customers have to be ring nodes, whereas type 1 customers can be either ring nodes or nodes in tree sub-structures. Additionally, optional Steiner nodes are given which can be used as intermediate network nodes if advantageous. Capacity constraints bound both the number of the ring trees as well as the number of customers allowed in each ring tree. In this paper we present first advanced neighborhood structures for the CRTP. Some of them generalize existing concepts for the TSP and the Steiner tree problem, others are CRTP-specific. We also describe models to explore these multi-node and multi-edge exchange neighborhoods in one or more ring trees efficiently. Moreover, we embed these techniques in a heuristic multi-start framework and show that it produces high quality results for small and medium size literature instances.

Keywords: capacitated ring tree problem, network design, local search

1 Introduction

The design of cost efficient networks under capacity constraints is of undoubted importance for applications in various industries. Especially in the field of transportation and telecommunication significant cost savings were achieved through the application of appropriate optimization models in the last decades. Topologically, many networks are based on fundamental structures such as trees or rings. The extensively studied *minimum weight spanning trees* (MSTs) assure connectivity such that a unique path between any two nodes in the network exists, whereas the *capacitated minimum spanning tree problem* (CMSTP) [2] asks for such a tree of minimal total edge costs while limiting the number of nodes of sub-trees connected to a depot by a single edge. In practice, the integration of optional intermediate *Steiner nodes* is highly relevant and is facilitated by the well-known *Steiner tree problem* (STP) [7]. On the contrary, a prominent ring based optimization problem is the *travelling salesman problem* (TSP), asking for

a travel cost minimizing sequence in which each customer of a given set should be visited before returning to a depot. Such a *tour* is required for each vehicle starting from the depot in the *vehicle routing problem* (VRP) [3]. The need for multiple vehicles arises from the commonly limited transport capacity to deliver or pick up goods from or to the customers. Beyond these concepts, the recent *capacitated ring tree problem* (CRTP) [5] integrates the ring structure and the tree structure into an optimization model under consideration of capacities and the useful Steiner nodes. The implemented *ring tree* structure is defined to be either a tree, a ring or a ring with additional disjoint trees attached to some of its nodes. Moreover, certain customers are prespecified to be of type 2 and thus required to be contained in sub-rings in ring trees. The remaining type 1 customers can be such *ring nodes* or nodes in sub-trees. Additional capacity constraints bound the total number of customers on each ring tree as well as the number of ring trees originating from the depot. Figure 1 shows a feasible network that satisfies these requirements and minimizes the overall edge costs, i.e. the objective function. The CRTP is NP-hard as are its special cases, the STP and the TSP, but computationally even more challenging [5]. For most real world applications heuristic solution approaches are indispensable due to the size limits for efficient exact algorithms. Therefore, in this paper we generalize known neighborhood structures for the purely tree [1] and purely ring based [6] special cases by treating the ring tree case. Furthermore, the CRTP gives rise to interesting structured neighborhoods on its own that we introduce and show how to efficiently explore. We embed these techniques in a multi-start heuristic framework and show its efficiency on a set of literature instances.
After a formal definition of the CRTP in section 2 we introduce the novel neighborhoods and corresponding exploration techniques in section 3. The embedding of these ideas in a multi-start heuristic is described in section 4 before we close with our conclusion in section 5.

2 The capacitated ring tree problem

In the following we give a formal definition of the CRTP using basic graph theoretic notation. We consider a network $\mathcal{N}$ synonymous with an undirected simple graph with node set $V[\mathcal{N}]$ and edge set $E[\mathcal{N}]$. The graph obtained after the removal of a node $v \in V[\mathcal{N}]$ is denoted by $\mathcal{N} \setminus v$.
Definition. *We are given a set of nodes $V = U_2 \dot\cup U_1 \dot\cup W \dot\cup \{d\}$ where the nodes in U_t correspond to type t customers, nodes in W are Steiner nodes and d represents a central depot. The cost of connecting two nodes $u \neq v$ in V by an edge $e = \{u, v\}$ is $c_e > 0$. A solution for the CRTP is a network $\mathcal{N}$ obtained from the union of a set of rings $\mathcal{R} = \{R_1, ..., R_k\}$ and a set of trees $\mathcal{T} = \{T_1, ..., T_l\}$ on V such that*

- *each type 2 customer is contained in exactly one ring,*
- *each type 1 customer is contained in exactly one ring or tree,*
- *each Steiner node is contained in at most one ring or tree,*
- *each ring contains the depot d,*
- *each tree contains either the depot d or a node of a ring,*

and $\mathcal{N}$ is capacity feasible, i.e.

- *the number of connected components in $\mathcal{N} \setminus d$ is at most m and*
- *the number of type 1 and type 2 customers in each connected component of $\mathcal{N} \setminus d$ does not exceed q.*

The CRTP asks for such a network of minimal total edge cost $c(\mathcal{N}) = \sum_{e \in E[\mathcal{N}]} c_e$.
From each connected component of $\mathcal{N} \setminus d$ we obtain a *ring tree* Q by adding the depot d and the edges connecting d and Q in $\mathcal{N}$. Such a ring tree forms either a tree or a ring with disjoint trees attached to it. Figure 1 illustrates a solution network based on 2 rings and 4 trees according to our definition of the CRTP.

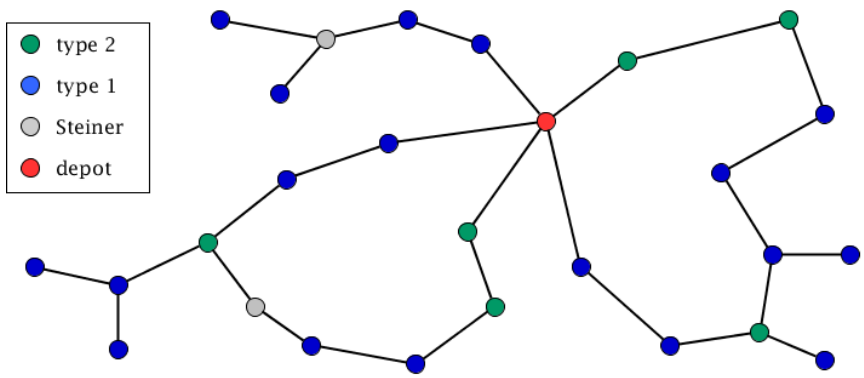

Fig. 1. A CRTP solution with 24 customers in 3 ring trees.

3 Neighborhood structures

In the following we elaborate several structured neighborhoods for the CRTP and explain how to efficiently explore them. They partially generalize existing concepts for the TSP, VRP, STP and CMSTP but we also introduce CRTP-specific neighborhoods that do not have non-trivial counterparts in these specializations. For the sake of simplified descriptions we introduce some notation which refers to a CRTP solution network $\mathcal{N}$ unless explicitly stated differently. For a ring tree $\mathcal{Q} \subseteq \mathcal{N}$ we denote the set of neighbors of a node $v \in V[\mathcal{Q}]$ in Q as $N_{\mathcal{Q}}[v]$. Let $P_{\mathcal{Q}}[u, v]$ be the set of paths that connect two distinct nodes $u, v \in V[\mathcal{Q}]$. We recall that if $\mathcal{Q}$ contains a ring then $|P_{\mathcal{Q}}[u, v]| \leq 2$, otherwise $\mathcal{Q}$ is a tree and thus $|P_{\mathcal{Q}}[u, v]| = 1$. Then we define $T_{\mathcal{Q}}[u, v]$ as the set of *path trees* of $\mathcal{Q}$ obtained from extending each path $\mathcal{P} \in P_{\mathcal{Q}}[u, v]$ by the non-ring structures in $\mathcal{Q}$ attached to the nodes of $\mathcal{P}$. Finally, for a node set $X \subset V$ we define $\Delta_{\mathcal{Q}}[X]$ as the set of edges with one end in X and the other end in $V[\mathcal{Q}] \setminus X$.

1**-edge-opt** In contrast to purely ring-based models, a 1-edge-opt neighborhood can be defined for the CRTP by considering the feasible removal of an edge $e \in E[\mathcal{Q}]$ followed by the insertion of an edge $e' \notin E[\mathcal{Q}]$ for each ring tree $\mathcal{Q} \subseteq \mathcal{N}$. We first observe that given a ring without type 2 customers, the edge with the highest cost can be deleted and $\mathcal{N}$ is still feasible. Therefore, we assume

that each ring in $\mathcal{N}$ contains a type 2 customer. In the case that e is a ring edge e' is required to *repair* the destroyed ring if possible. The ring-tree-opt neighborhood below will cover this case. Thus let $e = \{u, v\}$ be a non-ring edge of $\mathcal{Q}$ and let w.l.o.g. u be the node on each path from v to d. Then the deletion of e creates two connected components of $\mathcal{Q}$, one containing d and another one that contains v, more precisely a tree $\mathcal{T}_v$. To establish a valid solution we consider the insertion of each re-connecting edge $e' \in \Delta_{\mathcal{Q}}[V[\mathcal{T}_v]]$ subject to adherence to the capacity constraints. In particular, we may create a new (ring)tree by allowing e' to be incident to d.

2-edge-opt The prominent TSP-tailored edge swaps can be applied to each ring in $\mathcal{N}$. In a similar manner ties can be broken by facilitating capacity-feasible recombinations of two distinct ring trees $\mathcal{Q}_1$ and $\mathcal{Q}_2$ as known for the VRP. More specifically, for two ring edges $e = \{u, v\} \in E[\mathcal{Q}_1]$ and $e' = \{w, x\} \in E[\mathcal{Q}_2]$ we consider their replacement by $\{u, w\}$ and $\{v, x\}$ or $\{u, x\}$ and $\{v, w\}$. Figure 2 illustrates such an improvement move. If both edges are incident to d the neighborhood is empty. By allowing $\mathcal{Q}_1 = \mathcal{Q}_2$ and avoiding sub-tours we obtain the mentioned 2-opt for the TSP.

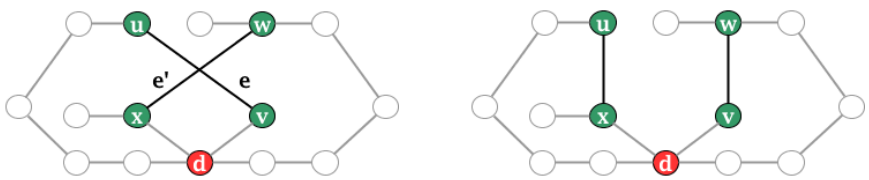

Fig. 2. A 2-edge-opt improvement based on the ring edges $\{u, v\}$ and $\{w, x\}$.

Moreover, we consider the deletion of two non-ring edges followed by the reconnection of the cut-off sub-trees $\mathcal{T}_1 \subseteq \mathcal{Q}_1$ and $\mathcal{T}_2 \subseteq \mathcal{Q}_2$ to other ring trees as depicted in Fig. 3. We hereby partially generalize the 1-edge-opt neighborhood. Since we regard the capacity constraints such a move can have an ejecting effect with respect to attached sub-trees when for instance reconnecting $\mathcal{T}_1$ to $\mathcal{Q}_2$.

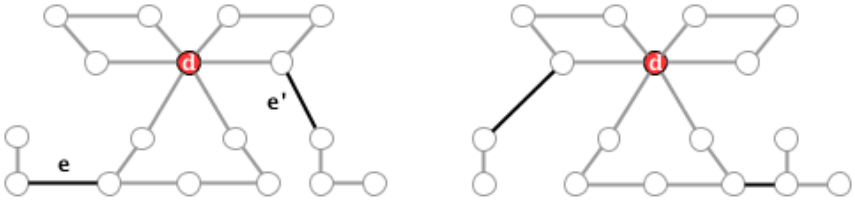

Fig. 3. A 2-edge-opt improvement based on the non-ring edges e and e'.

Finally, taking into account the removal of an edge e in a ring $\mathcal{R} \subseteq \mathcal{Q}_1$ and a non-ring edge $e' \in E[\mathcal{Q}_2]$ yields the remainder of this neighborhood. Let $\mathcal{T}_2$ be

the sub-tree of $\mathcal{Q}_2$ induced by e' as in the 1-edge-opt neighborhood. The corresponding modification of $\mathcal{Q}_1$ in $\mathcal{N}$ corresponds to the replacement of a e by a path tree obtained from $\mathcal{T}_2$, whereas $\mathcal{Q}_2$ is reduced by $\mathcal{T}_2$. Figure 4 shows such a transformation.

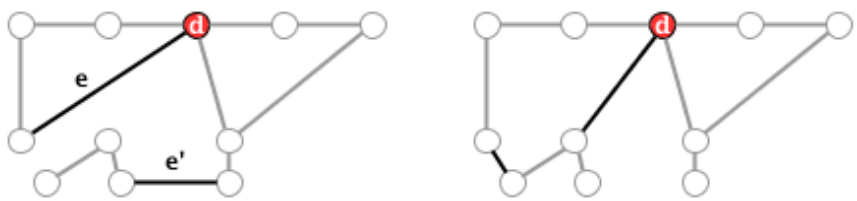

Fig. 4. A 2-edge-opt improvement based on a ring edge e and a non-ring edge e'.

1**-node-opt** We consider moving a single customer node u from its current ring tree $\mathcal{Q}_1$ to a ring tree $\mathcal{Q}_2$. Obviously, the capacity of $\mathcal{Q}_2$ needs to be sufficient when performing such an operation. We ensure the preservation of the ring tree structure after the extraction of u from $\mathcal{Q}_1$ by the incorporation of a MST on the neighbors $N_{\mathcal{Q}_1}[u]$. Note that the degree of the depot has to be limited by m minus the number of current ring trees beside $\mathcal{Q}_1$ to satisfy the ring tree capacity m. Although the *degree constrained minimum spanning tree problem* (DCMSTP) is known to be NP-hard in general this special case can be solved polynomially using a Prim's algorithm in a slightly modified version starting from d. If u is of type 1 it may be inserted into $\mathcal{Q}_2$ either as a leaf or as an intermediate node that splits an edge $\{v, w\}$ into edges $\{v, u\}$ and $\{u, w\}$. Type 2 customers may only be inserted in this edge replacing manner into a ring instead.

2**-node-opt** We consider swapping two customers that are not necessarily in distinct ring trees. This neighborhood can be constructed by intersecting two 1-node-opt spaces.

Steiner-node-opt This neighborhood is inspired by known STP improvement moves and consists of all the feasible solutions obtained after deleting or inserting a single Steiner node. Certainly, a Steiner leaf node can simply be removed, whereas a node with degree 2 can be replaced by an edge connecting both neighbors if this results in an overall cost reduction. For an arbitrary Steiner node $x \in V[\mathcal{Q}]$, the re-connection can be accomplished by a MST on $N_{\mathcal{Q}}[x]$ as for the 1-node-opt neighborhood. Conversely, we also consider the insertion of a Steiner node $x \notin V[\mathcal{N}]$ into $\mathcal{N}$. We take into account the *splitting* of an existing edge $\{u, v\}$ into $\{u, x\}$ and $\{x, v\}$. Moreover, two incident edges $\{u, v\}$ and $\{u, w\}$ with $u \neq d$ can be replaced by the *star configuration* $\{x, u\}$, $\{x, v\}$ and $\{x, w\}$.

Ring-tree-opt This advanced neighborhood contains the solutions obtained by the rearrangement of the tree structure induced by two specifically situated mandatory ring nodes. Let $\mathcal{T} \in T_{\mathcal{Q}}(u, v)$ be a path tree in a ring tree $\mathcal{Q} \in \mathcal{N}$ for $\{u, v\} \subseteq U_2 \cup \{d\}$ such that $V[\mathcal{T}] \setminus \{u, v\}$ does not contain type 2 customers or the distributor. Then we can build a DCMSTP on the nodes of $\mathcal{T}$. As in

previous neighborhoods a single degree constraint applies when $d \in \{u, v\}$ to avoid the installation of more additional ring trees than allowed. An improving solution in this neighborhood connects u and v by a path tree of less cost as illustrated in Fig. 5. This neighborhood is also valid for nodes u and v such that $V[T_{\mathcal{Q}}(u, v)] \cap U_2 = \emptyset$ and therefore, in particular applicable when $\mathcal{Q}$ is a tree.

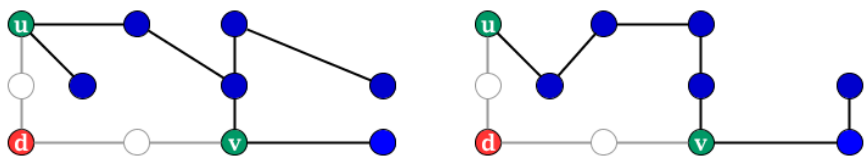

Fig. 5. A minimum spanning tree based improvement in a ring-tree-opt.

Ring-tree-split-opt This neighborhood contains solutions that can be obtained by *splitting* a ring tree $\mathcal{Q} \subseteq \mathcal{N}$ into two separate ring trees. This presumes enough capacity in $\mathcal{N}$ to install an additional ring tree. Basically, we try to repair a single ring edge removal by the feasible insertion of two new ring-closing edges. As in the ring-tree-opt search let $\mathcal{T}$ be a path tree for two distinct nodes u and v in $V[\mathcal{Q}] \cap (U_2 \cup \{d\})$ with $V[\mathcal{T}] \setminus \{u, v\} \cap \{d\} \cup U_2 = \emptyset$. Then we consider the removal of each ring edge $e \in E[\mathcal{T}]$ followed by the insertion of two edges $\{d, w\}$ and $\{d, x\}$ for $\{w, x\} \subseteq V[\mathcal{T}]$ as shown in Fig. 6. If $u = d$ then $\mathcal{Q}$ splits into a tree and a ring tree, whereas the splitting of a pure tree $\mathcal{Q}$ is contained in the 1-edge-opt neighborhood.

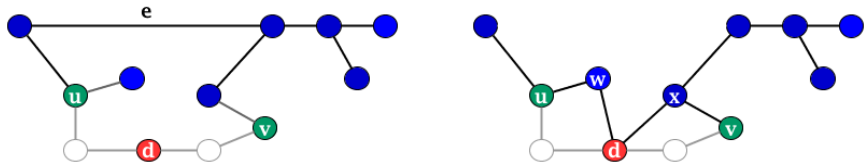

Fig. 6. An improving solution in the ring-tree-split neighborhood.

Ejection-chain-opt Extracting a customer node u_1 from a ring tree $\mathcal{Q}_1$ and inserting it into a ring tree $\mathcal{Q}_2$ might be cost saving but not feasible because $\mathcal{Q}_2$ is capacity tight, i.e. $\mathcal{Q}_2$ contains q customers. However, the ejection of a customer u_2 in $\mathcal{Q}_2$ and its insertion into a ring tree $\mathcal{Q}_3$ can facilitate the move. In this ejection-chain-opt neighborhood we consider all these double node moves for distinct ring trees $\mathcal{Q}_1$, $\mathcal{Q}_3$ and $\mathcal{Q}_3$. Not that if $\mathcal{Q}_3 = \mathcal{Q}_1$ then it corresponds to the 2-nodes-opt neighborhood.

4 A multi-start local search heuristic

Our heuristic is based on the iterated exploration of the introduced CRTP neighborhoods. We apply the corresponding local ring tree searches (LQSs) in

a multi-start fashion on a set of start solutions obtained from different initial constructions. For a CRTP instance P, let $\Sigma(P)$ be the procedure that returns a solution pool based on the strategies that we briefly summarize in the following. On the one hand we apply *cluster first*, *route second* techniques as in [4] to solve the VRP obtained after temporarily declaring all customers type 2. Different cluster distance metrics (e.g. min/max/avg cluster node distance) give rise to multiple solutions that are added to the pool. Then we conversely focus on the design of (partial) rings or (partial) trees based on the computation of MSTs and the construction of *nearest first* TSP routes. We combine these partial networks on the different sets of customers and turn them into a feasible solution by a correction mechanism that repeatedly applies moves similar to the ones described in our local search neighborhoods. Our overall algorithm applies the local searches on each of the solutions in $\Sigma(P)$ in a best-fit fashion until no enhancement can be found. The order in which the different neighborhoods are explored corresponds to the increasing potential structural impact. The resulting *multi-start CRTP heuristic* is given in 1.

```
Input CRTP P;
foreach N' ∈ Σ(P) do
    z ← ∞;
    while c(N') < z do
        z ← c(N');
        LQS(N', Ring-tree-opt);
        LQS(N', 1-edge-opt);
        LQS(N', 2-edge-opt);
        LQS(N', 1-node-opt);
        LQS(N', 2-node-opt);
        LQS(N', Steiner-node-opt);
        LQS(N', Ring-tree-split-opt);
        LQS(N', Ring-tree-join-opt);
        LQS(N', Ejection-chain-opt);
    end
    if c(N') < c(N) then N ← N';
end
return N;
```

We implemented the algorithm in c++ tested on an Intel i7-3667U 2.00 GHz processor unit for the 225 small to medium size instances[1] used in [5]. The type 1 customers in these TSPLIB based instances with $|V| \in \{26, 51, 76, 101\}$ are randomly assigned according to a rate $r_1 \in \{0, 0.25, 0.5, 0.75, 1\}$. Various combinations of m and q with an average customer capacity slack $(mq-|U|)/mq$ of 14% make them capacity tight. The computational results are given in Appendix 1. The run times of the heuristic procedure never exceeded 25 seconds.

[1] The instances can be obtained from the author.

5 Conclusions

We introduced advanced multi-edge and multi-node exchange neighborhood structures for the CRTP. They partially generalize existing concepts for prominent tree and ring based combinatorial optimization problems. We presented suitable models to explore these neighborhoods efficiently and a heuristic framework to turn these techniques into an efficient heuristic. Using this diversifying multi-start algorithm we are able to obtain optimal results in many cases for a set of small and medium sized literature instances. The average gap to known lower bounds is 3.8%. We suggest this first heuristic approach for the CRTP as a reference for related models and further algorithms.

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Appendix

Table 1 shows the computational results where the first 4 columns indicate the CRTP instance, the type 1 customer rate r_1 with respect to the total number of customers, the number of nodes $|V|$ and customers $|U|$. The network cost $c(\mathcal{N})$ is then given along with the relative gaps $\Delta_{lb} = [c_{lb}(\mathcal{N}) - c(\mathcal{N})]/c(\mathcal{N})$ and $\Delta_{ub} = [c(\mathcal{N}) - c_{ub}(\mathcal{N})]/c(\mathcal{N})$ to the lower bound $c_{lb}(\mathcal{N})$ and the upper bound $c_{ub}(\mathcal{N})$ obtained by the exact method in [5]. We do not intend to compete with the branch & cut algorithm but rather give an idea of the solution quality obtained by the heuristic. Since we initialized the exact method with the heuristic solution and use the local search techniques along the branch & bound $\Delta_{ub} \geq 0$ holds.

Table 1. Heuristic results for CRTP instances from [5] with type 1 customer rates $r_1 \in \{0, 0.25, 0.5, 0.75, 1\}$ compared to bounds obtained by a branch & cut algorithm.

P	r_1	$\lvert V\rvert$	$\lvert U\rvert$	Δ_{lb}	$c(\mathcal{N})$	Δ_{ub}	P	r_1	$\lvert V\rvert$	$\lvert U\rvert$	Δ_{lb}	$c(\mathcal{N})$	Δ_{ub}	P	r_1	$\lvert V\rvert$	$\lvert U\rvert$	Δ_{lb}	$c(\mathcal{N})$	Δ_{ub}
1	1	26	12	**0**	**157**	**0**	16	1		37	**0**	**304**	**0**	31	1		75	**-1**	478	**1**
	0.75			**-2,3**	215	**2,3**		0.75			-6,6	375	0		0.75			-6,4	551	0
	0.5			**0**	**227**	**0**		0.5			-3,7	378	0,5		0.5			-4,9	564	0
	0.25			**0**	**236**	**0**		0.25			**-0,3**	380	**0,3**		0.25			-3,7	573	0
	0			**0**	**242**	**0**		0			**-0,3**	381	**0,3**		0			**-2,1**	584	**2,1**
2	1			**-0,6**	164	**0,6**	17	1			**-0,3**	309	**0,3**	32	1			**-2,4**	494	**2,4**
	0.75			**0**	**207**	**0**		0.75			**-1,6**	369	**1,6**		0.75			-7,4	573	0
	0.5			**0**	**240**	**0**		0.5			-3,8	399	0		0.5			-9,9	612	0
	0.25			**0**	**249**	**0**		0.25			-2,2	404	0		0.25			-5,6	618	0
	0			**0**	**251**	**0**		0			**-1,9**	418	**1,9**		0			-4,1	626	0
3	1			**-1,7**	173	**1,7**	18	1			**0**	**314**	**0**	33	1			**-1,4**	495	**1,4**
	0.75			**-0,8**	244	**0,8**		0.75			-8,6	408	0		0.75			-12	623	0
	0.5			**0**	**251**	**0**		0.5			-7	431	0		0.5			-7,2	623	0
	0.25			**0**	**279**	**0**		0.25			-5,3	436	0		0.25			-8,8	656	0
	0			**0**	**279**	**0**		0			-3,1	452	0		0			-6,4	674	0
4	1		18	**0**	**207**	**0**	19	1		50	**-0,3**	377	**0,3**	34	1	101	25	**-1,8**	282	**1,8**
	0.75			**0**	**256**	**0**		0.75			-4,7	436	2,1		0.75			**-4**	327	**4**
	0.5			**0**	**274**	**0**		0.5			-2,9	447	0,7		0.5			-4,3	353	0
	0.25			**0**	**292**	**0**		0.25			**-0,7**	454	**0,7**		0.25			-1,4	363	0,6
	0			**-1,3**	305	**1,3**		0			**-2,3**	473	**2,3**		0			**0**	**366**	**0**
5	1			**-1,4**	220	**1,4**	20	1			**-0,5**	386	**0,5**	35	1			**-1,4**	293	**1,4**
	0.75			**0**	**285**	**0**		0.75			-7,7	458	0		0.75			-6,2	367	0
	0.5			**-1,6**	318	**1,6**		0.5			-9,1	493	0		0.5			-9,3	405	0
	0.25			**0**	**334**	**0**		0.25			-6,4	502	0		0.25			-8,2	416	0
	0			**0**	**339**	**0**		0			**-3,9**	513	**3,9**		0			-2,9	425	0
6	1			**-1,7**	231	**1,7**	21	1			**-0,5**	392	**0,5**	36	1			-33,1	299	0
	0.75			**0**	**278**	**0**		0.75			-10,7	501	2		0.75			-8,1	393	0
	0.5			**0**	**336**	**0**		0.5			-9,1	526	0		0.5			-6,5	403	0
	0.25			**0**	**361**	**0**		0.25			-5,5	525	0		0.25			-5,1	429	0
	0			**0**	**375**	**0**		0			-4,4	541	2		0			-3,8	452	0
7	1		25	**-1,2**	248	**1,2**	22	1	76	18	**0**	**214**	**0**	37	1		50	-26	411	0
	0.75			**0**	**294**	**0**		0.75			**0**	**272**	**0**		0.75			-7,4	492	0
	0.5			**0**	**313**	**0**		0.5			-9,6	318	0		0.5			-5,4	499	0
	0.25			**0**	**327**	**0**		0.25			-6,4	318	0		0.25			-4,3	503	0
	0			**0**	**328**	**0**		0			**0**	**332**	**0**		0			-6	523	0
8	1			**-5,6**	267	**5,6**	23	1			**-0,9**	235	**0,9**	38	1			-25,9	420	0
	0.75			**-1,3**	315	**1,3**		0.75			-3,1	312	1		0.75			-4,2	480	0
	0.5			**0**	**345**	**0**		0.5			**0**	**336**	**0**		0.5			-6,4	517	0
	0.25			**0**	**357**	**0**		0.25			-2,9	369	0		0.25			-5,7	531	0
	0			**0**	**362**	**0**		0			**-1**	390	**1**		0			-3	537	0
9	1			**-3,1**	262	**3,1**	24	1			**0**	**259**	**0**	39	1			-6,3	443	0
	0.75			**-0,9**	322	**0,9**		0.75			**0**	**325**	**0**		0.75			-5,1	505	0
	0.5			**-0,8**	372	**0,8**		0.5			-3,8	379	0		0.5			-6,3	527	0
	0.25			**-0,3**	379	**0,3**		0.25			**0**	**397**	**0**		0.25			-7,6	564	0
	0			**-0,3**	397	**0,3**		0			**-0,7**	451	**0,7**		0			-4,1	574	0
10	1	51	12	**0**	**156**	**0**	25	1		37	**0**	**320**	**0**	40	1		75	-10,2	516	0
	0.75			**-2**	196	**2**		0.75			-6,8	390	0		0.75			-6,9	594	0
	0.5			**0**	**215**	**0**		0.5			-8,1	402	0		0.5			-6	592	0
	0.25			**0**	**222**	**0**		0.25			-3	403	0		0.25			-4,3	612	0
	0			**0**	**242**	**0**		0			**-1**	413	**1**		0			-3,7	622	0
11	1			**-2,5**	163	**2,5**	26	1			**-3**	336	**3**	41	1			-13,7	519	0
	0.75			**0**	**209**	**0**		0.75			-4,8	402	0		0.75			-6,2	595	0
	0.5			**0**	**230**	**0**		0.5			-10,1	455	0		0.5			-4,2	607	0
	0.25			**0**	**238**	**0**		0.25			-9,2	460	0		0.25			-3,1	619	0
	0			**0**	**251**	**0**		0			-3	458	0		0			-2,8	642	0
12	1			**-1,2**	172	**1,2**	27	1			**-0,9**	343	**0,9**	42	1			**-1,3**	529	**1,3**
	0.75			**0**	**203**	**0**		0.75			-8,7	446	0		0.75			-10,6	653	0
	0.5			**0**	**251**	**0**		0.5			-9,9	473	0		0.5			-7,3	645	0
	0.25			**0**	**278**	**0**		0.25			-10,9	497	0		0.25			-7,1	670	0
	0			**0**	**279**	**0**		0			-5,6	506	0		0			-5,8	689	0
13	1		25	**-1,2**	248	**1,2**	28	1		56	**-3**	395	**3**	43	1		100	**0**	**555**	**0**
	0.75			-4	305	1		0.75			-7,6	462	0		0.75			-6,2	652	0
	0.5			**0**	**312**	**0**		0.5			-8,1	477	0		0.5			-5,6	660	0
	0.25			**0**	**322**	**0**		0.25			-3,1	472	0,4		0.25			-2,3	656	1,1
	0			**0**	**328**	**0**		0			-4,1	495	3		0			**-2,9**	683	**2,9**
14	1			**-5,6**	267	**5,6**	29	1			**-3,2**	402	**3,2**	44	1			**-0,7**	568	**0,7**
	0.75			**-5,3**	321	**5,3**		0.75			-9,7	488	0		0.75			-5,9	663	0
	0.5			-3,1	352	0		0.5			-10,4	520	0		0.5			-7	690	0
	0.25			**0**	**357**	**0**		0.25			-8	532	0		0.25			-4,3	691	1,2
	0			**0**	**362**	**0**		0			-5,7	543	1,5		0			-2,3	700	0
15	1			**-3,1**	262	**3,1**	30	1			**-3,6**	414	**3,6**	45	1			**-1**	576	**1**
	0.75			-2,9	339	1,2		0.75			-11,9	533	0		0.75			-9,5	695	0
	0.5			-4,6	372	0,5		0.5			-10,9	554	0		0.5			-6	717	0
	0.25			-5,4	387	0		0.25			-9	558	0		0.25			-5,6	730	0
	0			**-1,8**	397	**1,8**		0			-2,7	561	1,2		0			-4,6	743	0