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Hypergraph-based centrality metrics for maritime container service networks: A worldwide application

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Abstract

Centrality metrics are commonly applied to analyse maritime container service networks, usually modelled as L-graphs (with links representing legs of each service) or P-graphs (with links representing direct port-to-port connections enabled by each service). In fact, maritime container service networks are characterised by routing strategies encompassing multiple alternative routing options between ports – e.g., different sequences of services and/or of transhipment operations in diverse hub ports of call – with an overall transit time depending upon the cumulated frequency of concerned services at loading ports. This resembles exactly the structure of transit networks, modelled usually with a hypergraph-based approach, thus preferable to also represent container service networks. The topology of a hypergraph consists of a dedicated set of links (either in a L- or a P-approach) for each service, and of hyperlinks/waiting links at each port modelling the waiting time as a function of the cumulated frequency of relevant services calling at that port. This allows hypergraphs to account properly for routing strategies in the above sense. Extension of centrality metrics to hypergraphs modelling maritime container services is not straightforward as well and deserves attention. This paper aims to contribute to this topic: theoretical and practical implications of calculation of centrality metrics in hypergraphs are discussed first, by introducing the concepts of HL-graphs and HP-graphs. Then, a new formulation of the betweenness centrality metric consistent with the concept of hyperpath is proposed, leveraging the probability of occurrence of each elemental path in an hyperpath. Finally, an application to a worldwide network of container services related to year 2019 showcases the effectiveness and the easiness of calculation of the new proposed betweenness centrality metric.

Keywords: maritime container networks; centrality metrics; hypergraphs.

1 Background and motivation

Containerization plays a crucial role in worldwide freight transport (UNCTAD, 2020; Lloyd's list, 2020) and is acknowledged as a key driver of globalisation of trade (Levinson, 2008). An important research stream, with many practical implications for stakeholders and policymakers, aims to analyse the structure of container services operated by maritime companies and to measure the consequent connectivity of container terminals/ports (Calatayud et al., 2016). The most consolidated approach consists of modelling the network of container services via a graph, and then applying network analysis (Newman, 2010), specifically centrality metrics consistent with the paradigm of complex networks (Estrada, 2012). Following Freeman (1978), three broad categories of centrality metrics can be defined: degree centrality, betweenness centrality, and closeness centrality; a detailed review is illustrated in Section 3. Centrality metrics are commonly adopted in many transport sectors (Rodrigue et al., 2006), especially for the analysis of airline networks (Paleari et al., 2010) and of transit networks (Wang et al., 2020).

Application of these metrics to a network of maritime container services is straightforward and implies first deciding how to turn the network of maritime services into a graph. In this respect, the literature focuses primarily on synchronic[1] graphs (Cascetta, 2009), with two approaches. The first models the sequence of legs of each container service string, that is each service calling at n ports is represented via n-1 links representing subsequent port-to-port sailing routes. This approach is named “Graph of Direct Links (GDL)” by Tovar et al. (2015), or “L space” by Hu and Zhu (2009); the term “L-graph” will be adopted in the remaining of the paper. The second models all connections between any pairs of ports called by the service, that is each service calling at n ports is represented by $n\cdot(n-1)$ links: this approach is termed “Graph of All Links (GAL)” by Tovar et al. (2015), and “P space” by Hu and Zhu (2009); the term “P-graph” will be adopted in the remaining of the paper. Link directions in the L-graph are consistent with the direction of the concerned service legs, whilst the P-graph is characterised by two-way links, with possible different impedances by direction. An illustrative example of the two types of graph approaches is depicted in Fig. 1; further details are reported by Ducruet et al. (2020).

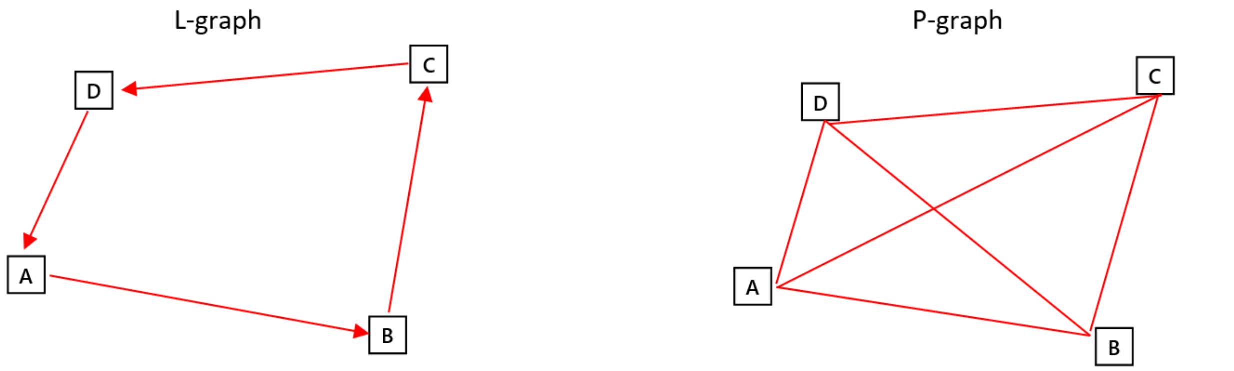


Fig. 1. Illustrative example of an L-graph and a P-graph to model a container service calling at the string of ports A→B→C→D→A (links in the P-graph are two-way links).

The choice of the most appropriate approach depends upon the concerned application. By way of example, an L-graph is preferable in vulnerability/resilience analysis, wherein single critical links of service strings should be identified, whilst a P-graph is preferable in the context of an accessibility analysis, because direct port-to-port linkages are explicitly modelled irrespective of the presence of intermediate port calls. In both approaches, links are associated with relevant impedances (usually a sailing distance) and service attributes (primarily service frequency and capacity), to obtain centrality metrics based on the “strength” of port-to-port linkages: details on how centrality metrics are calculated in L-graphs and P-graphs are reported in Section 4.2. As a side note, non-additive impedances associated with links in L-graphs are turned into additive impedances in P-graphs, see Marzano et al. (2018) for details.

However, both the L-graph and the P-graph exhibit an important limitation. In fact, the inherent

[1] A synchronic graph models liner (e.g., maritime, transit) services without any time dimensions, that is links in a synchronic graph correspond to the physical route of each service: consistently, service frequencies and concerned waiting times are represented as impedances on those links. Conversely, diachronic graphs are characterised also by a time dimension, such that links in a diachronic graph represent a leg of each service run of each liner service. The reader may refer to Cascetta (2009) for details.

structure of maritime container service networks enables routing strategies, rather than focussing only on the shortest path between ports. More precisely, a routing strategy may include multiple paths altogether, with possibly different transhipment options between the ports of origin and of destination, depending jointly upon sailing times, service frequencies and number of transhipments.

In this respect, modelling maritime container service networks is very similar to transit networks modelling, whose literature is very consolidated and spans over forty years (Cascetta, 2009). A key theoretical aspect of transit network modelling is that travellers usually formulate a mixed pre-trip/en-route travel strategy: the pre-trip component includes choice of the onboarding station/bus stop, whilst the en-route component accounts for the fact that multiple transit services/lines might be attractive for the traveller, and that the choice/occurrence of a specific service/line (or a sequence of services/lines with transfer) depends upon their service frequency.

The mathematical/topological vehicle to represent such mixed pre-trip/en-route travel strategy is a hypergraph, introduced by Nguyen and Pallottino (1988) and Spiess and Florian (1989), see also Cascetta (2009) and Ortúzar and Willumsen (2011). A formal definition of hypergraph will be introduced in Section 4; in short, in a hypergraph, each port node is connected, via a hyperlink – that is a link with a single tail and possibly multiple heads – to all services calling at that port. Overall, the topology of a hypergraph consists of a dedicated set of links (either in a L- or a P-approach) for each service, and of hyperlinks/waiting links at each port modelling the waiting time as a function of the cumulated frequency of relevant services calling at that port. This allows hypergraphs to account properly for routing strategies in the above sense. Specifically, a travel/routing strategy between an origin and a destination can be explicitly modelled on a hypergraph by means of an hyperpath, that is a collection of elemental paths, yielding the issue of finding the shortest hyperpath representing the best routing strategy. Overall, resorting to hypergraphs enables a more realistic representation of maritime container service networks: interestingly, Bell et al. (2011) and Bell et al. (2013) proposed the particularisation of the approach by Spiess and Florian (1989) to maritime container service networks, showcasing its viability and theoretical soundness.

This allows hypergraphs to account properly for routing strategies represented by the cumulated presence of multiple alternative routing options between ports, e.g., resorting to different services and/or sequences of transhipment operations in diverse hub ports of call: an illustrative example to clarify the difference between hypergraphs and graphs will be discussed in Section 4.

That said, the research presented in this paper is motivated by the fact that calculation of centrality metrics on hypergraphs is not straightforward, and much less explored in the literature. To the authors' knowledge, only few applications are available in the context of transit networks – see e.g., Anis & Sacco (2020) and Wang et al. (2020), who however rely on proxies of the hypergraph – and there are no applications to container service networks.

This paper aims to fill this gap, by exploring the viability of calculation of centrality metrics in hypergraphs representing container service networks. Theoretical implications and practical calculation of centrality metrics in hypergraphs are discussed, by introducing the concepts of HL-graphs and HP-graphs. A new formulation of the betweenness centrality metric consistent with the concept of hyperpath is also proposed, leveraging the probability of occurrence of each elemental path in a hyperpath. Finally, an application to a worldwide network of container services related to year 2019 showcases the effectiveness and the easiness of calculation of the new proposed betweenness centrality metric.

The paper is organised as follows: Section 2 reviews the literature on centrality metrics applied to maritime container service networks; Section 3 sets the mathematical notation of centrality metrics adopted in the paper; Section 4 introduces the concepts of hypergraph and hyperpath, their application to maritime container service networks, and illustrates how centrality metrics can be applied to hypergraphs; Section 5 showcases the proposed approach with an application to the worldwide maritime container service network, analysing the performance of the proposed metric against existing ones; Section 6 draws conclusions and research prospects.

2 Centrality metrics for maritime container service networks: Literature review

Centrality metrics are amongst the most widely used approaches to characterise transport networks (e.g., structure) and their elements (e. g. importance of nodes), and thus are a common tool to also analyse container service networks: Tian et al. (2007), Wang and Cullinane (2008), Cullinane and Wang (2009), Ducruet et al. (2010a, 2010b), Montes et al. (2012), Cullinane and Wang (2012), Doshi et al. (2012); Gonzalez-Laxe et al. (2012); Freire Seoane et al. (2013),Tran and Haasis (2014) and Tovar et al. (2015) provide, amongst others, comprehensive reviews of earlier papers and showcase the first steps on this research topic. Other relevant contributions are reported in the special issue of Transportation Research Part E: freight and logistics on *Maritime logistics and port connectivity in the globalised economy* (Lee et al., 2016). A recent review by Ducruet (2020) recalls exhaustively how complex networks and concerned metrics/analyses have been applied to analyse maritime networks. Apart from the above papers on centrality metrics applied to maritime networks, it is worth recalling some relevant topics for the purposes of the paper, as synthesized in the following paragraphs.

2.1 Analysis of properties of maritime networks based on complex network approaches

Pioneering papers on this topic focused on how to represent maritime networks with a complex network approach, also exploring their underlying properties. Hu and Zhu (2009) built a worldwide container network based on a Containerisation International database, in the form of both L-graph and P-graph. Links in the network were either unweighted or weighted based only on the number of direct services on those links, i.e., not accounting for capacity or frequency. They found that the worldwide container network is a so-called small-world network, with a hierarchical rich-club structure (Ducruet, 2013), that is with the presence of a small subset of crucial nodes, clear consequence of the prominent presence of hub-and-spoke transhipment (Rodrigue & Notteboom, 2010). Furthermore, they observed the degree centrality to follow a truncated power-law distribution in the L-graph and an exponential decay distribution in the P-graph. Centrality measures were found to be strongly correlated amongst themselves. Similar results on the network structure of worldwide container services are reported by Kaluza et al. (2010).

2.2 Worldwide applications of centrality metrics

Leveraging the mentioned pioneering results, subsequent papers dealt with worldwide container service network analysis, e.g., by Wang and Cullinane (2008), Ducruet and Notteboom (2012), and Angeloudis et al. (2013). Ducruet and Zaidi (2012) explored the role of both container hubs and regional ports, with an application of a *k*-clusters approach to identify relevant sub-networks. Ducruet et al. (2014) applied degree and betweenness metrics to analyse changes in the maritime container networks between 1996, 2006, and 2011. Non-container related applications are available as well, for instance Kosowska-Stamirowska et al. (2016) analysed an historical database of worldwide merchant vessel movements between 1890 and 2000 by means of metrics calculated on an unweighted maritime network. Applications to solid bulkers and oil tankers were provided by Kaluza et al. (2010) and by Cui (2014).

2.3 Non-worldwide applications of centrality metrics

Many studies circumvent the inherent difficulty of worldwide data collection by limiting interest only to most relevant ports and/or trade lanes. Kang et al. (2014) calculated four centrality metrics for top 5 container ports between 2006 and 2011. Mengqiao et al. (2015) applied degree and betweenness centrality to the network of main trade routes between world regions. Wang and Cullinane (2016) applied centrality and betweenness metrics to 39 worldwide ports on a network, weighted by the weekly transportation capacity deployed by top 20 liner shipping companies, with an interesting analysis of correlation between centrality metrics and port throughput. Chen et al. (2015) built a weighted P-graph amongst the first 100 container ports in the world, with the weight represented by flows in TEU between pair of ports. Kutin et al. (2017) collected a dataset of 153 ports from 50 countries, including statistics for more than six thousand maritime routes in 2014. Low et al. (2009), Ducruet et al. (2010b), Tang et al. (2011), and Song et al. (2019) applied degree and betweenness metrics to some Asian ports. Mou et al. (2018) proposed an application to the so-called silk road along the Europe-Far East trade lane. McCalla et al. (2005) focussed on Caribbean ports. Ducruet et al. (2010a) analysed a P-graph network of container services in the Atlantic for years 1996 and 2006 based on Automatic Identification System (AIS) data. Tran and Haasis (2014) proposed an empirical analysis of the container liner shipping network on the East-West corridor for the period 1995-2011. Calatayud et al. (2017) introduced trade strength relationship measures in addition to the centrality metrics recalled above, with an application to port connectivity in America. Notteboom (2010), Kitsos (2014), Elsayeh (2015), and Arvis et al. (2019) focused on within-Mediterranean services. Varan and Cerit (2014) dealt with Turkish ports, whilst Elbayoumi and Dawood (2016) dealt with Middle Eastern ports.

2.4 *Other indicators than centrality metrics suited for maritime networks*

Centrality metrics are not the sole indicators applied to maritime networks. Taylor et al. (2006), Low et al. (2009), and Tang et al. (2011) introduced a port-specific graph metric, termed port connectivity metric. Jiang et al. (2015) developed optimisation-based connectivity port metrics, accounting for sailing time and throughput considerations, from the perspective of a single global carrier. Another very common metric in container shipping is the Liner Shipping Connectivity Index (LSCI) released yearly by UNCTAD since 2004 and described by Hoffman (2005). The LSCI is country-based and considers four main aspects: number of container vessel calls; container vessel carrying capacity; number of shipping companies, liner services and vessels; average and maximal vessel size. Further details are discussed by Fugazza and Hoffman (2017). The relationship between the LSCI and other logistics indicators and trade data was investigated amongst others by Ojala and Hoffman (2010) and by Arvis et al. (2013). Some variants have been also proposed in the literature, such as the Liner Shipping Bilateral Connectivity Index (LSBCI) by UNCTAD (2015), that accounts for pairwise container liner shipping service analysis between countries, with an upper bound threshold on the maximum number of intermediate transhipments. Bartholdi et al. (2016) proposed a new indicator, termed Container Port Connectivity Index (CPCI), leveraging the LSCI and the hub/authority centrality metric. It is also worth mentioning other indicators not directly related to container but rather to the Ro-Ro market, see amongst others the connectivity, costs, and congestion indicators by PORTOPIA (2014), de Langen et al. (2016), and Marzano et al. (2020).

2.5 *Applications informed by centrality metric*

In general, centrality metrics or similar measures can inform also other research activities, see, e.g., Ducruet (2020) for a recent review. By way of example, Lange and Bier (2019) propose a Principal Component Analysis to cluster graph nodes based on a set of graph theory metrics. Variation on the theme include Leicht and Newman (2008) and Kaluza et al. (2010), who applied clustering algorithms at port level, aiming to identify areas of port competition and cooperation. The multiple linkage analysis by Cullinane and Wang (2012) provides insights on the hierarchical configuration of the container port market. Ducruet (2013) and Ducruet (2017) investigated maritime networks considering multiple commodities altogether, finding the distribution of maritime traffics amongst ports to be influenced strongly by the concerned commodity diversity.

A synopsis of the cited literature is reported in Table 1.

Table 1.

Synopsis of the literature review presented in Section 2.

Author	Year	Topic					
		General papers on centrality metrics applied to maritime networks	Analysis of properties of maritime networks based on complex network approaches	Worldwide applications of centrality metrics	Non-worldwide applications of centrality metrics	Other indicators than centrality metrics suited for maritime networks	Applications informed by centrality metrics
McCalla et al.	2005				X		
Hoffman	2005					X	
Taylor et al.	2006					X	
Tian et al.	2007	X					
Wang & Cullinane	2008	X		X			
Leicht & Newman	2008						X
Cullinane & Wang	2009	X					
Hu & Zhu	2009		X				
Low et al.	2009				X	X	
Ducruet et al.	2010 (a)	X			X		
Ducruet et al.	2010 (b)	X			X		
Kaluza et al.	2010		X	X			X
Notteboom	2010				X		
Ojala & Hoffmann	2010					X	
Tang et al.	2011				X	X	
Montes et al.	2012	X					
Cullinane & Wang	2012	X					X
Doshi et al.	2012	X					
Gonzalez-Laxe et al	2012	X					
Ducruet & Notteboom	2012			X			
Ducruet & Zaidi	2012			X			
Freire Seoane et al	2013	X					
Ducruet	2013		X				X
Angeloudis et al.	2013			X			
Arvis et al.	2013					X	
Tran & Haasis	2014	X			X		
Ducruet et al.	2014			X			
Cui	2014		X	X			
Kang et al.	2014				X		
Kitsos	2014				X		
Varan & Cerit	2014				X		
Hoffmann et al.	2014					X	
PORTOPIA 7th FP EU project	2014					X	
Tovar et al.	2015	X					
Mengqiao et al.	2015				X		
Chen et al.	2015				X		
Elsayeh	2015				X		
Jiang et al.	2015					X	
Kosowska-Stamirowska et al.	2016			X			
Wang & Cullinane	2016				X		
Elbayoumi & Dawood	2016				X		
Bartholdi et al.	2016					X	
De Langen et al.	2016					X	
Kutin et al.	2017				X		
Calatayud et al.	2017				X		
Fugazza & *Hoffmann*	2017					X	
Ducruet	2017						X
Mou et al.	2018				X		
Song et al.	2019				X		
Arvis et al.	2019				X		
Lange & Bier	2019						X
Ducruet	2020		X				
Marzano et al.	2020					X	

3 Centrality metrics for maritime container networks: Definitions and notation

Let G≡{N, L} be a directed graph characterised by a set of nodes N and a set of links L; for any $n \in$N, let FWS(n) the forward star of node n, that is the set of links whose tail is n and similarly let BWS(n) the backward star of n, that is the set of links entering node n. Following Freeman (1978), Newman (2010) and Mishra et al. (2012), three main centrality metrics can be defined to quantify the centrality of nodes in G:

- *betweenness centrality* $BC(n)$: it represents the percentage of shortest paths between any other pairs of nodes $i,j \in$N-{n} passing through a given a node n. Letting n_{ij} be the number of (possible multiple) shortest paths between i and j and $n_{ij}(n)$ the number of shortest paths between i and j passing through n, it occurs:

$$BC(n) = \frac{1}{(|N|-1)(|N|-2)} \sum_{i,j \in N-\{n\}} \frac{n_{ij}(n)}{n_{ij}} \quad (1)$$

- *closeness centrality* $CC(n)$. Given a pair of nodes $n,m \in$N, let i_{nm} the impedance of the shortest path between n and m. The closeness centrality, normalised to account for the dimension of the network, is defined by Bavelas (1950) as:

$$CC(n) = \frac{|N|-1}{\sum_m i_{nm}} \quad (2)$$

  More specifically, (2) is termed *outcloseness centrality* because it is based on the shortest paths from n towards all other (reachable) nodes in the networks. Symmetrically, an *incloseness* centrality can be defined by replacing $\sum_m i_{nm}$ with $\sum_n i_{mn}$ in Eq. (2).
- *degree centrality* $DC(n)$: it is expressed by two metrics $DC_{in}(n)$ and $DC_{out}(n)$, named *indegree* and *outdegree* centrality, representing respectively the cardinality of the backward star and of the forward star of a node, such that:

$$DC(n) = DC_{in}(n) + DC_{out}(n) = |\mathrm{BWS}(n)| + |\mathrm{FWS}(n)| \quad (3)$$

  Metric (3) can be also normalised by considering that a node can be connected at most to all other nodes in the network but itself, i.e., $DC(n) \leq 2(|N|-1)$. Clearly, it occurs $DC_{in}(n) \leq |N|-1$ and $DC_{out}(n) \leq |N|-1$.

The above basic definition of centrality metrics inspired also relevant generalisations and extensions, including the following:

- *harmonic centrality* $HC(n)$. As discussed by Boldi and Vigna (2020), the presence of many non-connected pairs of nodes might warp the closeness centrality (2), that can be replaced by a normalised *harmonic centrality* given by:

$$HC(n) = (|N|-1) \sum_m \frac{1}{i_{nm}} \quad (4)$$

  wherein is assumed conventionally $1/i_{nm}$=0 if n and m are not connected.
- *node strength* $NS(n)$. The degree centrality (3) was generalised by Barrat et al. (2004), Newman (2004), and Barthelemy (2011) by replacing the count of ingoing and outgoing links with the sum of corresponding link weights, that is:

$$NS(n) = NS_{in}(n) + NS_{out}(n) = \sum_{l \in BWS(n)} w_l + \sum_{l \in FWS(n)} w_l \quad (5)$$

being w_l a weight of link l, e.g., a link flow or the reciprocal of a link impedance. Opsahl et al. (2010) proposed a further generalised metric depending upon a parameter $\alpha \in [0,1]$, such that $\alpha=0$ yields (3) and $\alpha=1$ yields (5).

- *eigenvector centrality EC*(n). Following Mishra et al. (2012), the degree centrality of a node n can be modified by including the importance of its adjacent nodes, measured through their degree centrality. This induces a circular dependence amongst the degree centralities of all nodes in the graph, mathematically set as the search of eigenvalues of the transformation induced by the adjacency matrix of the graph. A power method can be applied to calculate the eigenvector centrality EC(n) of a node n, usually adopting the largest eigenvalue λ^* as input. For directed graph, separated eigenvector centralities can be calculated with reference only to the tail nodes of links of BWS(n), that is $EC_{in}(n)$, or to the head nodes of links belonging to FWS(n), that is $EC_{out}(n)$. Usually, $EC_{in}(n)$ is called prestige of node n and $EC_{out}(n)$ importance of node n. A recent application of eigenvector centrality to L-graphs is provided by Cheung et al. (2020). Calculation of eigenvector centrality for directed graphs wherein nodes exist such that BWS(n)$\equiv\varnothing$ can be problematic, an issue circumvented by the Katz metric (1953). Further variants and extensions exist as well, e.g., the page rank algorithm by Page et al. (1999).
- *hubs and authorities (HITS algorithm)*. Following Kleinberg (1999), it resembles the concept of eigenvector centrality, by labelling nodes as hubs and authorities: an authority is a node such that the tail nodes of links of its backward star are hubs, and a hub node is such that the head nodes of links of its forward star are authorities. Calculation is performed via a recursive algorithm and, similarly with (3) and (5), links can be either unweighted or weighted.

For the purposes of the paper, it is worth elaborating upon the practical meaning/interpretation of the centrality metrics, when applied to L-graphs or P-graphs: indeed, as illustrated in Section 1, L-graphs and P-graphs are two alternative representations of the same network of maritime container services. In general, metrics based on backward and forward stars – e.g., degree centrality, node strength, eigenvector centrality, hubs, and authorities – are more meaningful on P-graphs rather than on L-graphs: indeed, the former allows considering all port-to-port connections enabled by service strings, whilst the latter includes only incoming and outgoing concerned container service legs.

On the contrary, closeness centrality should not be affected in principle by the type of graph, because expression (2) is based on the impedance of the shortest path between nodes, which does not change between L-graphs and P-graphs, unless when dealing with non-additive impedances, e.g., non-linear freight rates: in this case, a P-graph allows directly embedding those impedances on port-to-port links and thus should be preferred (Marzano et al., 2018). However, there might be the case of multiple service legs with different sailing times between the same pair of ports in an L-graph: in this case, the shortest path algorithm will consider the fastest service between the pair of ports, yielding unrealistic (that is, not corresponding to any real services) shortest sailing times lower than in a P-graph. Thus, it is better to calculate closeness centrality on P-graphs when referring to sailing time as impedance in Eq. (2).

Finally, interpretation of the betweenness centrality differs appreciably by type of graph. In L-graphs, the shortest path between two nodes i and j includes all ports called in sequence by all container services along the shortest path, that is both transhipment ports and intermediate ports of call. In other words, a node n is part of a shortest path between i and j also if n is only an intermediate port of call in one of the services along the shortest path between i and j. By way of example, application of expression (1) yields a betweenness centrality of 0.50 for all nodes in the L-graph in Fig. 1.

In P-graphs, multiple shortest paths between a pair of ports might be detected, just because of how strings of maritime services are modelled. By way of example, ports D and B in the P-graph in

Fig.1 (Section 1) are connected both by a direct (i.e., without other nodes) path given by the link D-B and by an indirect path given by the two legs D-A and A-B. If impedances are additive, both paths are shortest, and application of expression (1) yields a betweenness centrality of 0.25 for all nodes in the P-graph in Fig. 1. This result is not theoretically sound, because it leverages fictitious links created in the P-graph to represent direct connections irrespective of the string of intermediate port calls, and is not desirable, because it corresponds to the same (normalised) betweenness centrality calculated using the L-graph.

The issue can be circumvented by recognising that only the shortest *direct* path(s) between nodes should enter Eq. (1) when applied to P-graphs: in the example in Fig. 1, this would lead to a zero betweenness centrality for each node in the P-graph, which is desirable because no transhipment operations are involved in that example. Non-additive impedances, typically decreasing freight rates by unit of distance, are such that the shortest direct path is the one corresponding to the direct link, thus overcoming this issue; conversely, additive impedances (e.g., sailing times) yield the same transit time for all direct and indirect paths related to a given service. Interestingly, a practical trick to discard non-direct shortest paths when impedances are additive is the following: given a pair of ports o and d, the impedance $i_{l_{od}}$ of the direct link l_{od} should be in principle given by $i_{l_{od}} = \sum_{l \in L_{od}} i_l$, being L_{od} the set of links representing all legs of the service between o and d and i_l the impedance of the generic link l. In the practice, it suffices assuming $i_{l_{od}} = (1 - \varepsilon) \sum_{l \in L_{od}} i_l$, with ε small enough (e.g., in the order of 10^{-4}), to let the direct link between o and d to dominate other non-direct shortest paths.

Notably, referring again to the example of Fig. 1, this yields a zero betweenness centrality for all ports in the P-graph. Overall, this allows embedding in the calculation of the betweenness centrality in P-graphs only the role of transhipment ports, thus providing a more realistic interpretation of Eq. (1).

4 The proposed methodological approach

The review of the literature in Section 2 reveals that, to the authors' knowledge, there are no papers dealing with the calculation of centrality metrics in hypergraphs modelling container service networks, and in general with the application of hypergraphs to real worldwide maritime container service networks, despite their more appealing theoretical soundness (Bell et al., 2011; Bell et al., 2013), so far showcased in very small toy networks.

Therefore, this section illustrates the methodological approach for the extension of centrality metrics to hypergraphs: for this aim, Section 4.1 recalls basic concepts on hypergraphs and their concerned notation, whilst Section 4.2 particularises calculation of centrality metrics on hypergraphs, with a focus on the contribution of the paper.

4.1 Hypergraphs: Concept and notation

As recalled in the introduction, the hyperpath approach in transport networks was developed by Spiess and Florian (1989) to model strategic route choice in transit networks: its extension to maritime container service networks is straightforward. Let P be a set of ports and S a set of maritime container services; each service $s \in S$, characterised by a weekly capacity cap_s and a weekly frequency φ_s, is associated with an ordered string P_s of ports of call. In turn, each port $p \in P$ is associated with a set S_p of services calling at p. The hypergraph includes a single node $p_s \in P_s$ for each called port, and each port $p \in P$ is the tail of a hyperlink whose heads are all nodes p_s with $s \in S_p$. The hypergraph proposed by Spiess and Florian (1989) can be turned into a hyperlink-free graph by introducing appropriate waiting links, as reported by Cascetta (2009). The impedance associated with waiting links is related to the cumulated frequency of all "attractive services" calling at port p, whose set S_{ph} is a function of the overall considered hyperpath h.

That said, the representation of services in a hypergraph may follow the same rationale illustrated in Section 1: that is, each service can be modelled either via an ordered sequence of links representing legs (i.e., a sailing between two subsequent ports in the service string), yielding a L-hypergraph, or by a star of direct port-to-port links (irrespective of the possible presence of intermediate ports of call), yielding a P-hypergraph. The two types of hypergraphs will be termed respectively HL-graph and HP-graph in the remaining of the paper. An illustrative example of an HL-graph is reported in Fig. 2, that highlights how relevant container operations (loading, unloading, waiting in terminal) are modelled in the hypergraph. The main impedance associated with waiting links is related to the cumulated frequency of concerned maritime container services, however additional impedances can be introduced, for instance to account for custom clearance times. Extension to the case of an HP-graph from a P-graph is straightforward, and not reported for the sake of brevity.

service	string of ports
#1 (red)	A → B → C → D → A
#2 (blue)	A → C → D → A
#3 (green)	B → D → B

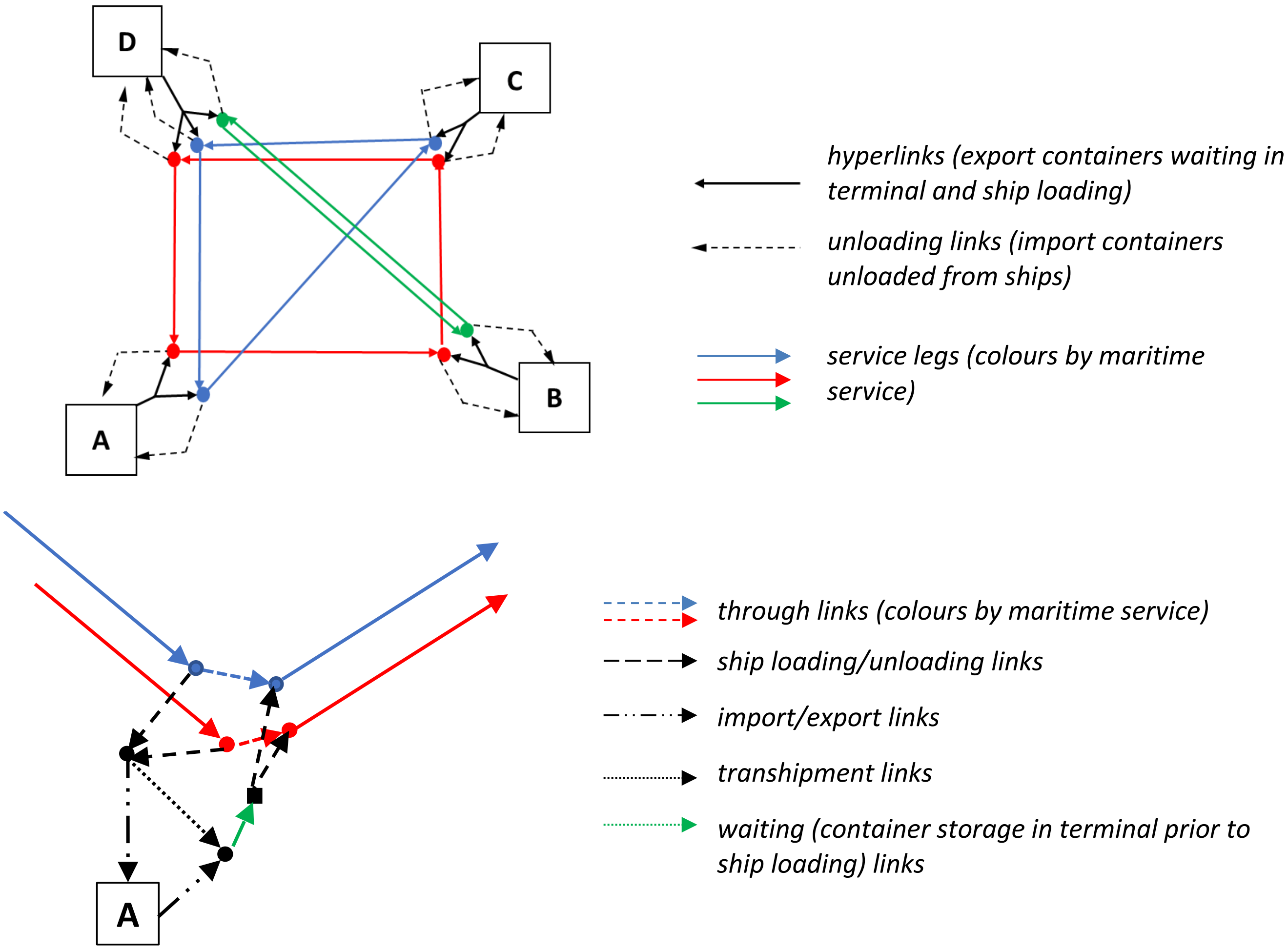


Fig. 2. Illustration of a HL-graph: representation by means of hyperlinks (top) and waiting links (detail for node A, bottom).

Whatever type of hypergraph (i.e., HL-graph or HP-graph), an hyperpath h between two ports p_1 and p_2 is a collection of paths linking p_1 and p_2, that represent jointly a travel/routing strategy, in the sense defined in Section 1. Let K_h be the set of elemental paths belonging to an hyperpath h

between two ports p_1 and p_2. Each path $k \in K_h$ represents a sequence of (one or more) services, and is associated with a choice probability $\pi[k|h]$ given by the product of the occurrence probability of k at each of the diversion nodes belonging to k.

A diversion node is any ports p in the set P_k of ports belonging to k, and the probability $\pi[k|p]$ of k given p depends on the frequency of all attractive services within h, that is:

$$\pi[k|h] = \prod_{p\in P_k} \pi[k|p,h] = \prod_{p\in P_k} \frac{\varphi_{s(k,p)}}{\sum_{s'\in S_{ph}} \varphi_{s'}} \tag{6}$$

being $s(k,p)$ the service on path k from port p. Eq. (6) allows also defining the impedance i_h of the whole hyperpath h, as the average of the impedances i_k of the paths within K_h weighted with the corresponding probabilities (6), that is:

$$i_h = \sum_{k\in K_h} i_k \cdot \pi[k|h] \tag{7}$$

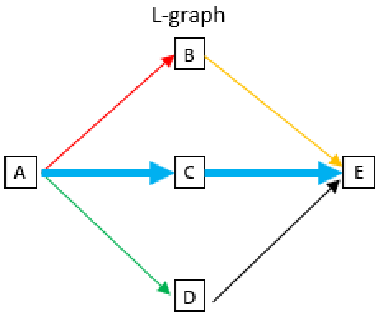
Nguyen and Pallottino (1988) proposed an arborescent shortest path algorithm to find the shortest hyperpath tree in a (not necessarily) transit network, with a backward network exploration from a destination node, setting the basis for several variants and applications. As a result, calculation of shortest hyperpaths and hyperpath-based assignment are very common procedures. It is worth looking how a shortest hyperpath on a HL-graph may differ from the shortest path on the corresponding L-graph, as illustrated in the example in Fig. 3, encompassing five ports connected by means of five container services.

Restricting attention to the pair of ports A (origin) and E (destination), the shortest time path on the L-graph is along the blue container service, with a total time of 9.50 days. Notably, the waiting time is calculated assuming uniform random arrival of export containers at terminal and perfect reliability of services, yielding a waiting time of $1/2\varphi$ for a service of frequency φ. Conversely, looking at the HL-graph – whose topology is not represented in detail as in Fig. 2 for the sake of simplicity – the shortest time hyperpath, calculated applying Eq. (7) to each hyperpath – is the one including paths P1 (red service A→B and then transhipment to the orange service B→E) and P2 (direct blue service A→C→E). Such hyperpath has a total time of 8.125 days and represents the routing strategy of including a less frequent and slightly slower direct service (path P2) and a more frequent and faster sequence of services with transhipment (path P1). For the sake of completeness, expressing frequencies in days, the transit time of the hyperpath H1 comes from Eq. (7) applied to paths P1 and P2, that is $i_{P1}\cdot\pi[P1|H1]+i_{P2}\cdot\pi[P2|H1]=i_{P1}\cdot\varphi_{P1|A}/(\varphi_{P1|A}+\varphi_{P2|A})+i_{P2}\cdot\varphi_{P2|A}/(\varphi_{P1|A}+\varphi_{P2|A})=5\cdot0.5+6\cdot0.5=5.5$ days, whilst the waiting time is given by the cumulated waiting time at port A, given by $0.5/(\varphi_{P1|A}+\varphi_{P2|A})=0.5/(1/7+1/7)=1.75$ days plus the additional waiting time for transhipment at node B with probability $\varphi_{P1|B}/(\varphi_{P1|B}+\varphi_{P2|B})=0.5$, given by $0.5\cdot0.5/\varphi_{P2|B}=0.25/(2/7)=0.875$ days, yielding a total waiting time of 1.750+0.875=2.625 days.

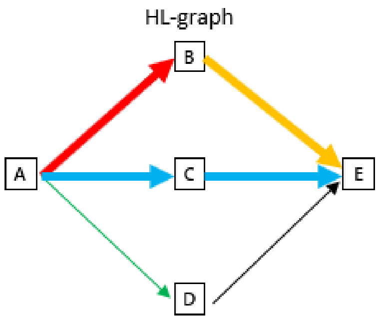
This example showcases that the hypergraph approach represents the characteristics of maritime container service networks more realistically with respect to a classical L- or P-graph approach. In synthesis, the topology of a hyperpath is theoretically sounder to model frequency-based services, such as transit systems and container service networks, because each service is represented with a dedicated set of links (either in a HL- or a HP-graph approach). This allows hypergraphs to account for routing strategies represented by the cumulated presence of multiple alternative routing options between ports, e.g., resorting to different services and/or sequences of transhipment operations in diverse hub ports of call, and to represent explicitly the effect of waiting time depending on the cumulate frequencies of relevant services by means of hyperlinks/waiting links. Such phenomena are in fact ignored in a simple graph, wherein each route is modelled independently of each other. This impacts significantly on the calculation of the impedance between ports, as illustrated in Fig. 2, and on the calculation of centrality metrics, as discussed in Section 4.2.

The next relevant question is how to specify/particularise the centrality metrics on a hypergraph: this aspect is discussed in the next section.

service	string of ports	total transit time [days]	weekly frequency [calls/week]
#1 (red)	A → B	2	1
#2 (blue)	A → C → E	6	1
#3 (green)	A → D	5	1
#4 (orange)	B → E	3	2
#5 (black)	D → E	4	1



path id	sequence of services	sequence of called ports	transit time [days]	waiting time [days]	total time [days]
P1	1 → 4	A → B → E	5	5.25	10.25
P2	**2**	**A → C → E**	**6**	**3.50**	**9.50**
P3	3 → 5	A → D → E	9	7.00	16.00



hyperpath id	underlying paths	transit time [days]	waiting time [days]	total time [days]
H1	**P1, P2**	**5.5**	**2.625**	**8.125**
H2	P1, P3	7.0	4.375	11.375
H3	P2, P3	7.5	3.500	11.000
H4	P1, P2, P3	6.6	2.899	9.499

Fig. 3. Illustrative example of calculation of a shortest path in an L-graph (top) vs. a shortest hyperpath in an HL-graph (bottom, node-based links for the hyperpath not reported for the sake of simplicity)**.**

4.2 Calculation of centrality metrics in hypergraphs for maritime container service networks

The following subsections particularise the metrics introduced in Section 2 to hypergraphs (either HL-graphs or HP-graphs), with a specific emphasis on the proposed new betweenness centrality metric in Section 4.2.1.

4.2.1 Betweenness centrality

Extension of betweenness centrality to hypergraphs has been faced by few authors, see, e.g., Xiao

(2013) and Amato et al. (2018), who proposed the following straightforward extension of Eq. (1):

$$BC(n) = \frac{1}{(|N|-1)(|N|-2)} \sum_{i,j \in N-\{n\}} \frac{nh_{ij}(n)}{nh_{ij}} \quad (8)$$

wherein nh_{ij} is the number of (possible multiple) shortest hyperpaths between i and j and $nh_{ij}(n)$ the number of shortest hyperpaths between i and j passing through n.

This paper deviates from this definition, proposing a new definition of the betweenness centrality, inspired by recognising that (1) and (8) are based on 0/1 values, that is the contribution of each pair of ports $\{i \neq n, j \neq n\}$ to the betweenness centrality of the port n equals 1 if the shortest path/hyperpath between i and j goes through n and 0 otherwise. That said, one might leverage Eq. (6), that is calculating the sum of the occurrence probabilities of the elemental paths in the shortest hyperpath passing through n, namely:

$$BC(n) = \frac{1}{(|N|-1)(|N|-2)} \sum_{i,j \in N-\{n\}} \frac{\sum_{k \in K_{n,h_{ij}}} \pi[k|h_{ij}]}{nh_{ij}} \quad (9)$$

wherein $K_{n,h_{ij}}$ is the set of paths passing through n within hyperpath h_{ij} connecting i and j. In other words, the proposed betweenness centrality is a "fuzzy" metric, in the sense that the contribution of each pair of ports $\{i \neq n, j \neq n\}$ to the betweenness centrality of the port n in Eq. (1) accounts for the probability that the shortest hyperpath between i and j goes through n, that is in between 0 and 1. To the authors' knowledge, (9) has never been applied to container service networks. Interestingly, Eq. (9) can be applied to both HL-graphs and HP-graphs, leading to the same thoughts on the interpretation of the betweenness centrality in L-graphs and P-graphs, illustrated in Section 4.2.

The illustrative example in Fig. 3 allows understanding how the calculation of the betweenness centrality differs when calculated on L-graphs rather than on HL-graphs: in the L-graph case, the sole node with a non-zero betweenness centrality is the port C, whilst in the HL-graph both ports C and B have equal non-zero betweenness centrality. Furthermore, the discussion illustrated at the end of Section 3 on the differences in calculating the betweenness centrality in L- vs. P-graphs can be extended straightforwardly also to HL- vs. HP-graphs, with the same rationale.

From a practical perspective, calculation of (9) might appear cumbersome at a first sight. However, it is useful to recall that a transit all-or-nothing (AoN) assignment[2] of a unit square o-d flow matrix with dimension $|P-1|^2$ to the hypergraph yields all concerned flows for each node n, that is import/export flows (i.e., unloaded/loaded containers directed to/coming from landside), transhipment flows (i.e., containers unloaded from a maritime service and loaded to another maritime service), and through flows (i.e., containers remaining onboard the ship during the call at the port), see the bottom of Fig. 2 for a clarification of concerned links. Since the assigned o-d flow matrix has unit entries, such flows represent the contribution of all o-d pairs with respect to each node n, that is the outer summation in Eq. (9). Intuitively, considering the sum of transhipment flows and through flows for each node n yields the betweenness centrality on an HL-graph, whilst considering only transhipment flows for each node n yields the betweenness centrality on an HP-graph. This can be regarded as a nice feature, because one does not need to build practically an HP-graph, being sufficient to work on concerned link flows coming from the assignment of a unit matrix on the HL-graph, as explained before, to obtain all concerned measures.

[2] See for details, amongst others, Cascetta (2009) and Ortuzar and Willumsen (2011). AoN transit assignment is a very common feature offered by most of the commercial transport software.

4.2.2 Closeness centrality

Calculation of closeness centrality in hypergraphs leverages a straightforward extension of (2), that is replacing the impedance of the shortest path calculated on the L-graph and on the P-graph with the impedance of the shortest hyperpath calculated on the HL-graph and HP-graph respectively, that is using (7). Again, this calculation is theoretically sounder with respect to P-graph because impedance (7) accounts for the whole underlying routing strategy modelled by the hyperpath. Notably, the impedance of a shortest hyperpath (including also elemental hyperpaths, i.e., with a single path) is lower or at most equal to that of the corresponding shortest path, thus the absolute value of the closeness centrality of a port p in a hypergraph is always equal or greater than on a graph.

4.2.3 Degree centrality

Calculation of degree centrality on HL-graphs and HP-graphs resembles the same calculation on L-graphs and P-graphs respectively because, looking also at Fig. 1 and at the top of Fig. 2, backward stars, and forward stars in L-graphs versus HL-graphs and in P-graphs versus HP-graphs include the same nodes. This means that the degree centrality in a L-graph is the same as in a HL-graph and, similarly, that the degree centrality in a P-graph is the same as in a HP-graph. As a side results, since the degree centrality does not change in a HL-graph with respect to a L-graph, the HL-graph preserves the scale-free property already proved in the literature for L-graphs modelling maritime container service networks (see, e.g., Ducruet, 2013).

5 Application to worldwide maritime container service network

The application of the proposed approach to a worldwide maritime container service network is showcased in the following subsections: Section 5.1 describes the dataset implemented for the analysis; Section 5.2 reports on how concerned graphs have been built; Section 5.3 illustrates key results of the comparison between the proposed approach and the classical methods for the calculation of centrality metrics.

5.1 Data on maritime container services

Building a database of worldwide maritime container services is a time-consuming, cumbersome yet feasible task, thanks to the availability of various data sources, primarily provided by container shipping companies, vessel information/tracking systems, and ports/terminal operators; further port-related information can be collected from additional sources, such as the World Port Index database from the National Geospatial-Intelligence Agency of the United States (2019) and the Marine Traffic website (www.marinetraffic.com). Once collected, data have been harmonised to ensure self-consistency; relevant missing information, especially with reference to service speed, frequency, and transit time, have been calculated using the topological model described in Section 5.2. Concerned information have been organised in a relational database, whose structure is illustrated in Fig. 4. Overall, more than 1500 container services for the year 2019 have been identified, corresponding to some 900 ports of call.

Data quality and coverage have been double-checked based on relevant aggregates, e.g., weekly capacity deployed by trade lane or by port, or overall figures by ocean carrier, with results in general satisfactory. By way of example, the following Table 2 compares the number of ships and the corresponding overall capacity for the top ten ocean carriers provided by our database and by two well-known sources, namely Alphaliner (www.alphaliner.com) and Ship Technology (www.ship-technology.com).

Furthermore, the total number of ships contained in the database is 5762, whilst Alphaliner[3] indicates as per January 1st, 2020, a total containership fleet of 6149 ships, of which 5337 full cellular. Overall, based also on other checks not reported here for the sake of brevity, validation is usually satisfactory for deep sea services, and sufficient for feeder services, except for some world areas with many ports and not consolidated regular services.

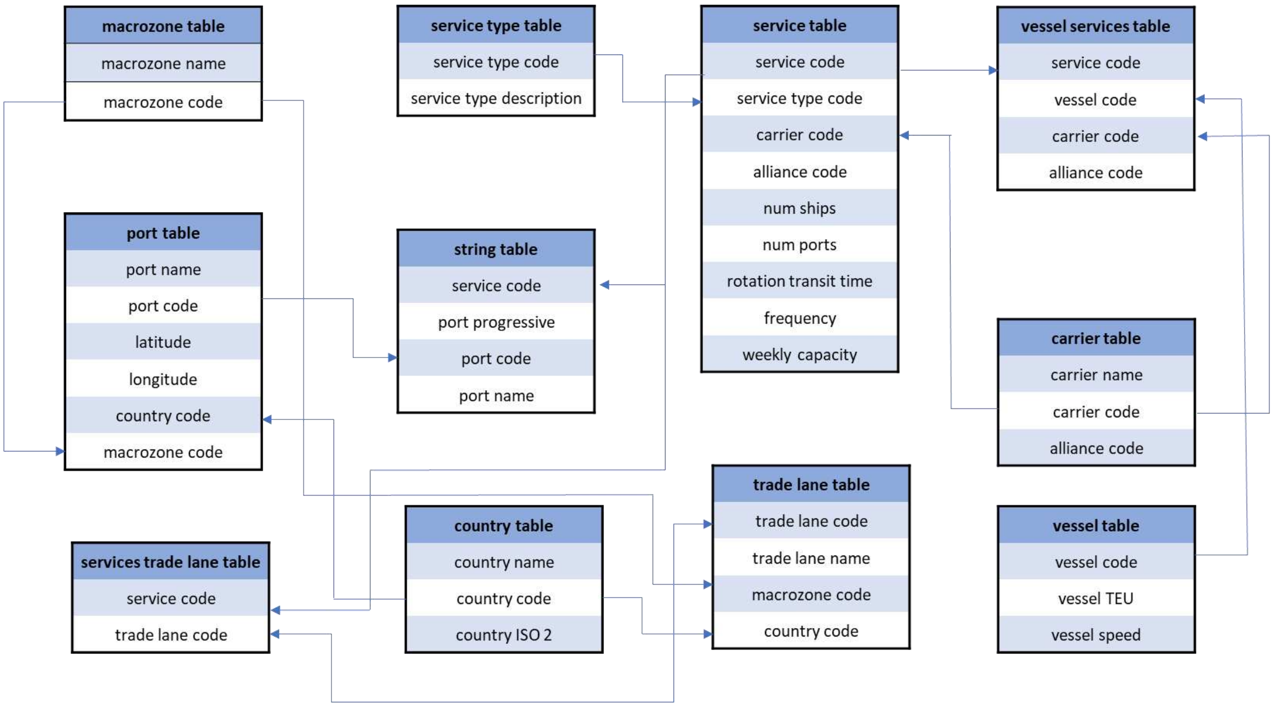


Fig. 4. Structure of the database of worldwide maritime container services.

Table 2.

Number of ships and their capacity for top ten ocean carriers by different sources.

	our database		***Ship Technology***		***Alphaliner***	
Ocean carrier	# ships	capacity [kTEU]	# ships	capacity [kTEU]	# ships	capacity [kTEU]
Maersk	641	3977	708	4100	707	4193
MSC	545	3452	560	3800	565	3766
COSCO *	442	2729	507	3100	481	2938
CMA-CGM	486	2589	502	2700	507	2696
Hapag-Lloyd	234	1644	248	1700	240	1718
ONE	213	1491	224	1500	221	1581
Evergreen Line	200	1194	333	1200	200	1277
OOCL*	97	657	104	734	-	-
HHM	79	426	110	728	63	389
Yang Ming	93	601	95	616	100	647

* *Alphaliner* considers OOCL as part of the COSCO conglomerate, after its acquisition in 2017.

[3] Data obtained from the January 2020 edition of the *Monthly Monitor*, available online on April 14, 2021, at https://www.alphaliner.com/resources/Alphaliner_Monthly_Monitor_Jan_2020.pdf.

5.2 Building L-graphs, P-graphs, and hypergraphs

Whatever type of graph, a raw topological model representing port-to-port sailing routes should be developed first. Unfortunately, no shapefiles or graphs are available to inform this task, different of other transport modes, whose topological models can leverage reliable and up-to-date sources such as the *OpenStreetMap* project. Thus, starting from the geographic coordinates of ports, available from the port table in the dataset illustrated in Fig. 3, links representing sailing routes have been built through the following steps:

- a sufficiently dense grid of points has been drawn on the entire water surface of the Earth, i.e., excluding lands, with an average distance of 0.5 nautical miles between adjacent points;
- each point of the grid has been connected, via direct two-way links, to all surrounding points within a radius of 30 nautical miles; links touching/crossing land have been clearly discarded. Further links have been then added to account for cases (e.g., sailing across straits) wherein the above procedure did not yield satisfactory results;
- each link has been associated with a length calculated as great circle on the Earth;
- shortest distance paths between all pairs of ports have been calculated on the above graph, to select only links effectively used to sail between ports in the dataset. Specifically, recalling the target of the analysis, shortest paths have been calculated twice, once including and once excluding the Panama channel, to account for the presence of new post-Panamax vessels.

The procedure has been implemented by combining a MatLab R2020b code and the GIS features of TransCAD 7.0 by Caliper Inc., and the resulting final topological model includes 406.059 links and 127.811 nodes. Such number of links and nodes is primarily aimed to guarantee that real sailing routes (e.g., observed by ship Automatic Identification Systems) are substantially close to sailing routes represented by piecewise linear curve in the topological model, whose granularity depends by the density of the grid of points and by the number of links created on that grid (first two points of the bullet list above). In this respect, the proposed approach generalises the one by Bunel et al. (2017). In principle, this allows the topological model to model any possible connections between each pair of ports, however the actual number of links sailed by container services in the database illustrated in Section 5.1 is significantly lower, resulting in a substantially lower number of links and nodes practically adopted for the calculation of centrality metrics.

The model has been validated by double-checking a sample of sailing distances between ports calculated with the topological model against available shortest distances from various sources (e.g., www.classic.searoutes.com, www.sea-distances.org), with very satisfactory results. Clearly, this approach assumes implicitly independency of the sailing route of actual weather conditions, that might imply appreciable detours from shortest routes; however, this assumption is acceptable, given the nature and the objectives of the study. The topological model illustrated so far has been used primarily to fill missing information in the dataset illustrated in Section 5.1, e.g. port-to-port distances and sailing times, to be used as elemental bricks for the implementation of concerned graphs, and for calculating centrality metrics.

5.3 Results

This section illustrates the results of the application of the centrality metrics presented in Section 2 and 4.2 to the worldwide maritime container service network described in Section 4, namely betweenness centrality (Section 5.3.1), closeness centrality (Section 5.3.2), and degree centrality (Section 5.3.3).

5.3.1 Betweenness centrality

Betweenness centrality has been calculated on both HL-graphs and HP-graphs with the proposed approach, that is via Eq. (9), and contrasted respectively with classical calculations, that is via Eq.

(1), on L-graphs and P-graphs. Results are visualised on a 45-degree biplot in Fig. 5, that contrasts betweenness centrality on L-graphs and HL-graphs for some top ports worldwide, and with an overall geographic visualization in Fig. 6, that shows the values of betweenness centrality on the HL-graph and their corresponding differences with the L-graph. Notably, colours in Fig. 6 are proportional to the difference in betweenness centrality between L-graph and HL-graphs, whilst the circle size is proportional to the absolute value of the betweenness centrality on the HL-graph. Analogous charts/figures for the comparison of betweenness centrality on P-graphs and HP-graphs are reported in Fig. 7 and Fig. 8, wherein colours are again proportional to the difference in betweenness centrality between P-graph and HP-graph, and the circle size is proportional to the absolute value of the betweenness centrality on the HP-graph.

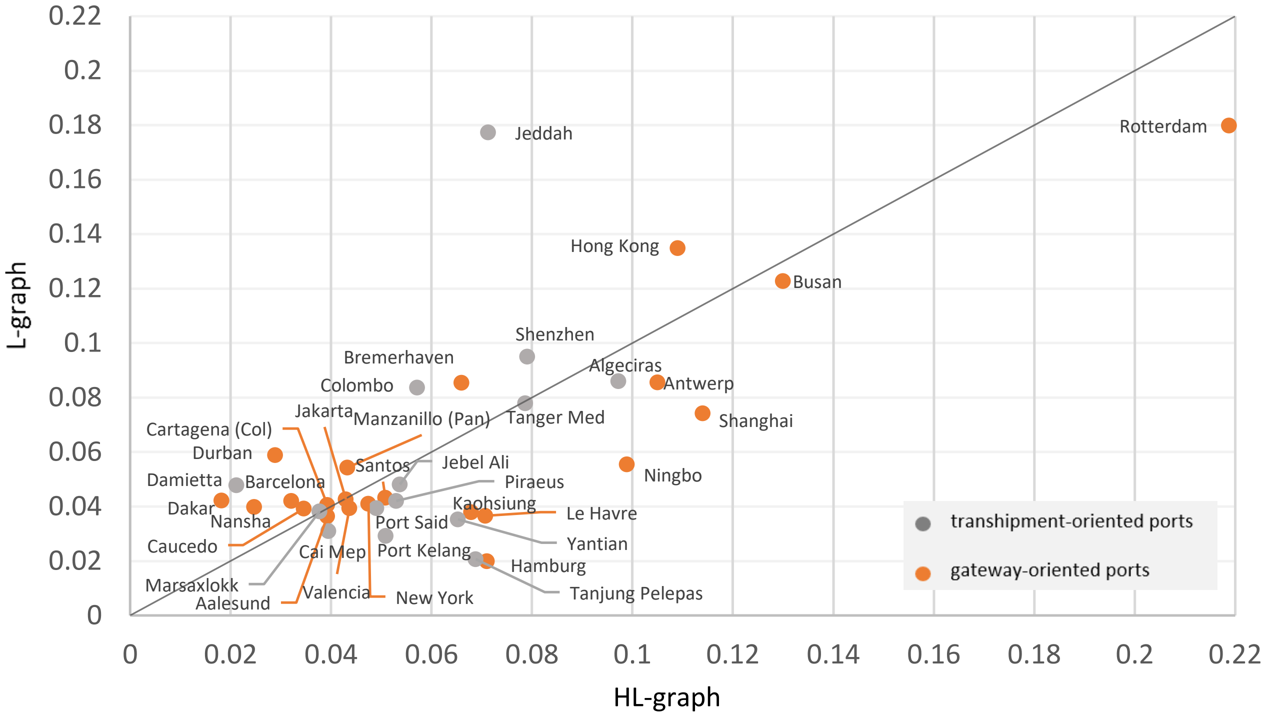


Fig. 5. Betweenness centrality in HL-graphs vs. L-graphs: 45-degree biplot for relevant ports worldwide (Singapore not shown to enhance diagram readability, see related values in Tab. 2)

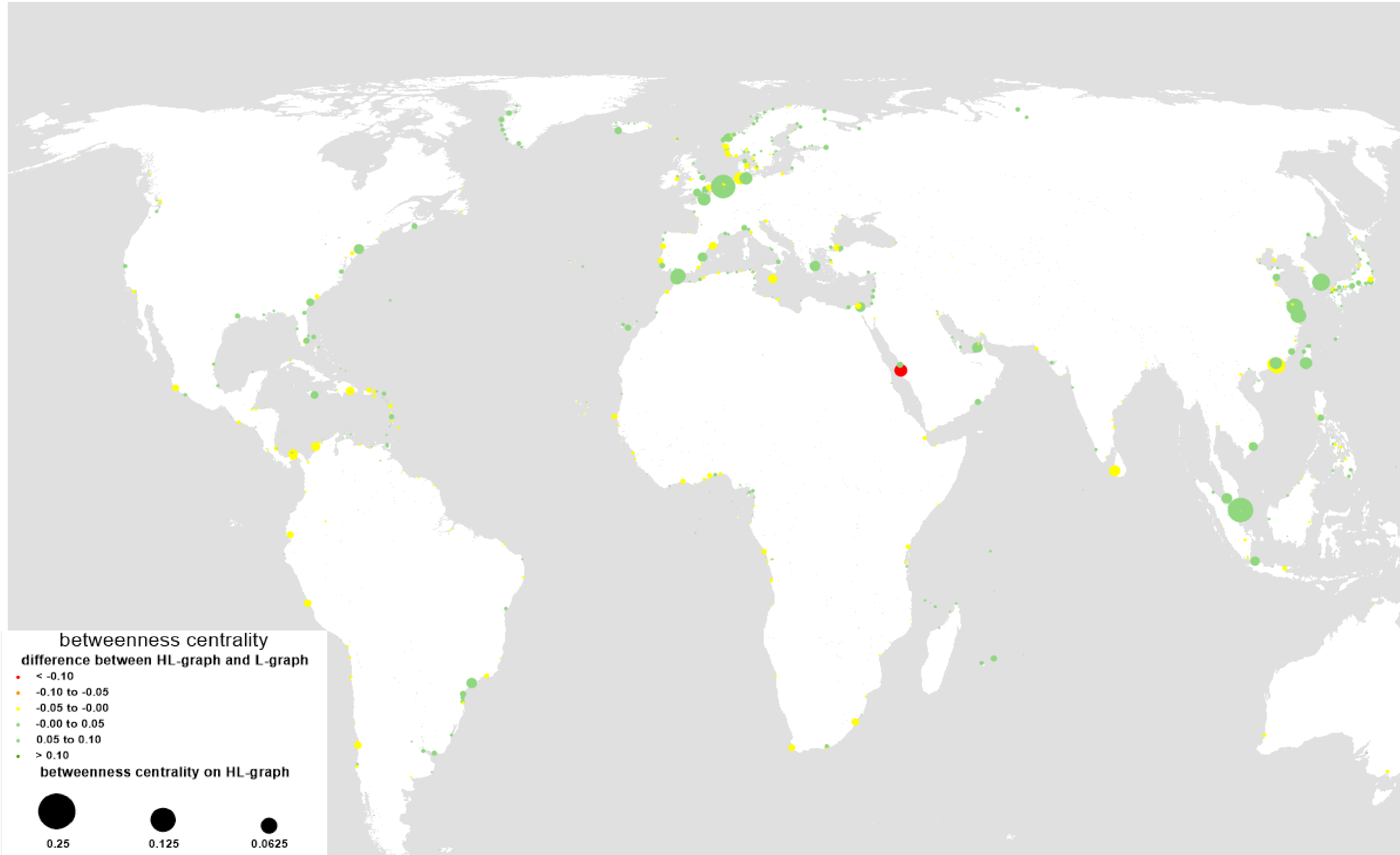


Fig. 6. Geographical representation of betweenness centrality on HL-graphs (size of port points) and differences with betweenness centrality on L-graphs (scale of colours).

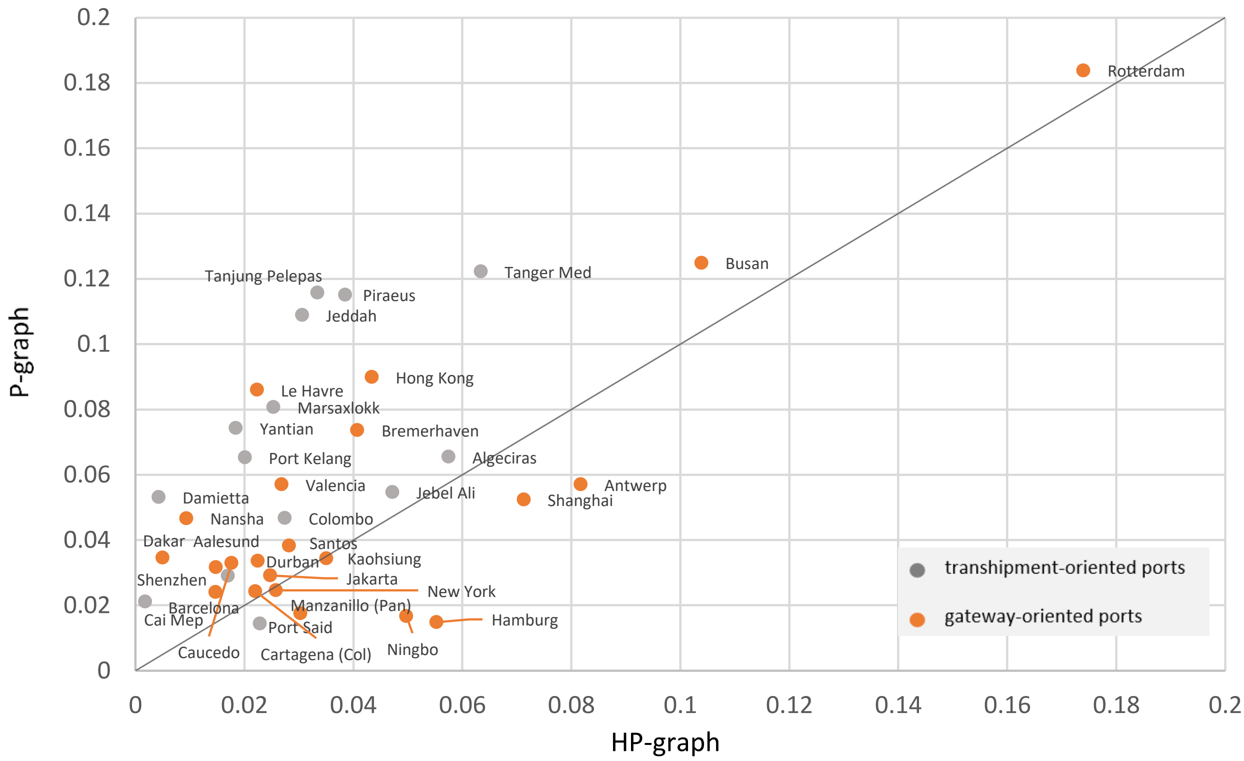


Fig. 7. Betweenness centrality in HP-graphs vs. P-graphs: 45-degree biplot for relevant ports worldwide (Singapore not shown to enhance diagram readability, see related values in Tab. 2).

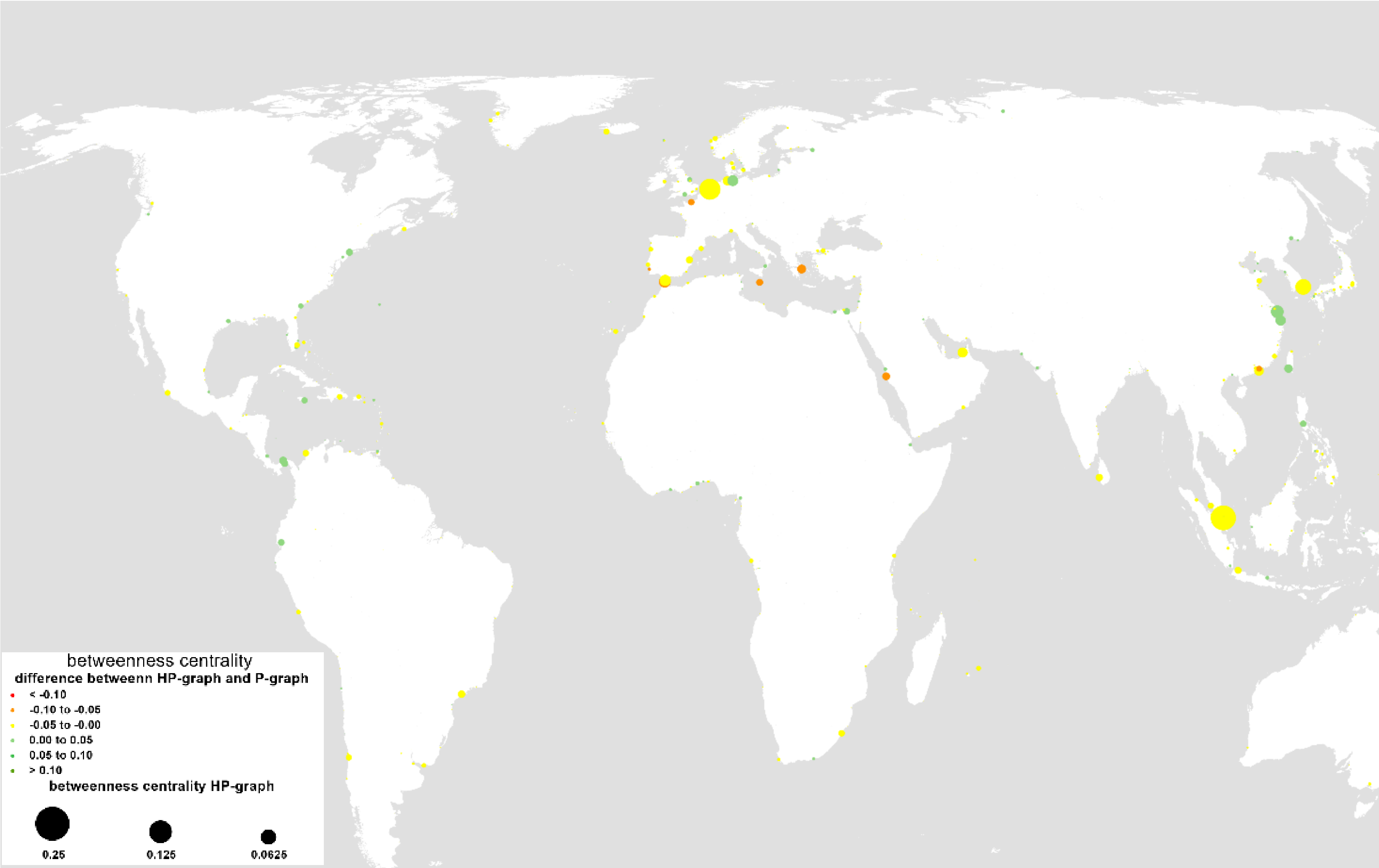


Fig. 8. Geographical representation of betweenness centrality on HP-graphs (size of port points) and differences with betweenness centrality on P-graphs (scale of colours).

The main phenomenon driving the interpretation of the results is the less deterministic behaviour of the proposed hypergraph-based betweenness centrality given by Eq. (9) against the classical method given by Eq. (1), as already highlighted in the example in Fig. 3. A shortest path approach is expected to polarise betweenness centrality results on a fewer number of ports, mainly transhipment hubs, whilst a hyperpath-based approach tends to recognise the role of a larger number of ports in the context of the routing strategies modelled by hyperpaths. This effect is magnified in the presence of a denser/more connected network of maritime services calling at the port under analysis, that enables a wider range of routing strategies again effectively represented by hyperpaths and not captured by standard shortest path-based centrality metrics. This yields a significant betweenness centrality also to transhipment ports not only in obliged sailing route waypoints (e.g., Malacca strait, Suez channel, Gibraltar strait) but also at the centre of highly connected networks of within-region feeder services (e.g., within Far-East or in the Mediterranean). As a side note, hypergraphs allow accounting also for the effects of the cumulated frequency of services – as formally expressed by Eqs. (6) and (7) – that impact on the cumulated transfer waiting time at transhipment nodes, thus providing a theoretically sounder representation of containerised maritime connections.

A first practical real-world effect of the above emerges by looking at the differences in the port ranking by betweenness centrality using the various approaches, coming from a visual inspection of Fig. 5 to Fig. 8 and summarised for the sake of clarity in Table 3 for the top 10 ports.

Table 3.

Top 10 ports by betweenness centrality using the different approaches contrasted in the paper.

rank	L-graph		HL-graph		P-graph		HP-graph	
	port	BC	port	BC	port	BC	port	BC
1	Singapore	0.311	Singapore	0.339	Singapore	0.270	Singapore	0.233
2	Rotterdam	0.180	Rotterdam	0.219	Rotterdam	0.184	Rotterdam	0.174
3	Jeddah	0.177	Busan	0.130	Busan	0.125	Busan	0.104
4	Hong Kong	0.135	Shanghai	0.114	TangerMed	0.122	Antwerp	0.082
5	Busan	0.123	Hong Kong	0.109	Tanjung Pelepas	0.116	Shanghai	0.071
6	Shenzhen	0.095	Antwerp	0.105	Piraeus	0.115	TangerMed	0.063
7	Algeciras	0.086	Ningbo	0.099	Jeddah	0.109	Algeciras	0.057
8	Antwerp	0.086	Algeciras	0.097	Hong Kong	0.090	Hamburg	0.055
9	Bremerhaven	0.085	Shenzhen	0.079	Le Havre	0.086	Ningbo	0.050
10	Colombo	0.084	TangerMed	0.079	Marsaxlokk	0.081	Jebel Ali	0.047

Interestingly, seven out of the top 10 ports in both the L-graph and the HL-graph betweenness centrality rankings coincide (Singapore, Rotterdam, Hong Kong, Busan, Algeciras, Antwerp, Shenzhen), whilst the L-graph assigns a higher betweenness centrality to Jeddah, Bremerhaven, and Colombo, replaced in the HL-graph ranking by Shanghai, Ningbo, and TangerMed. This is likely explained because in particular Jeddah and Colombo are usual ports of call along the Europe-Far East trade lane, properly detected with a shortest path approach on the L-graph, whilst Shanghai, Ningbo and TangerMed are key hubs of dense networks of deep-sea and feeder services, respectively in the Far East for the first two ports and in the Mediterranean for the third port, thus enabling a variety of routing strategies better highlighted by the proposed hyperpath-based approach.

Furthermore, a comparison between L-graph and P-graph rankings highlights that important transhipment ports (Tanjung Pelepas, Piraeus, Marsaxlokk) are detected only in the P-graph approach, as expected based on the discussion in Section 3. However, this comes at the price of discarding other important transhipment ports (e.g., Algeciras), as well as key regional gateway ports (e.g., Antwerp), such that only five ports are present in both the top-10 rankings on the L-graph and on the P-graph. On the contrary, the HL-graph approach already recognises relevant transhipment ports (e.g., TangerMed) and moving towards the top-10 HP-graph ranking preserves 8 over 10 top ports, with an appreciable inherent greater stability of the rankings. In addition, the ranking in the HP-graph can be regarded as sounder because it treats equally ports serving both as regional transhipment hubs/gateways (e.g., Antwerp, Hamburg, Shanghai) and pure transhipment ports (e.g., TangerMed and Algeciras). This is likely due to the capability of HP-graph to compensate the tendency of P-graphs to assign a higher ranking to transhipment ports by the already mentioned magnification of routing strategies.

A look at Northern Range ports and Chinese Far-East ports in Fig. 6 and Fig. 8 showcases that L-graphs and P-graphs tend to polarise betweenness centrality towards fewer (or even a single) ports along the shortest paths within the same geographical region, whilst HL-graphs and HP-graphs provides a betweenness centrality more spread across ports, as a positive consequence of the more realistic representation of routing strategies through hyperpaths. A similar effect can be observed along the Malacca strait, wherein the betweenness centrality of other important ports than Singapore, e.g., Port Kelang (12.3 MTEU in 2018, of which 7.8 MTEU transhipped) and Tanjung Pelepas (9.0 MTEU in 2018 with 8.2 MTEU transhipped), is larger in the HL-graph rather in the L-graph.

Differences between the L-/P-graph and HL-/HP-graph presented so far are not merely analytical, because they might support relevant policy implications, e.g., in terms of strategies for ports to enhance their betweenness centrality. In this respect, the betweenness centrality on HL-graphs and HP-graphs suggests that setting up a resilient and effective network of within-region feeder services towards key regional transhipment hub is as important as to attract direct calls of deep-sea services. In other words, a L-/P-graph policy viewpoint would recommend increasing the betweenness centrality by resorting as much as possible to the attraction of direct calls of transhipment services, whilst a LP-/HP-graph viewpoint would assign also key relevance to strengthening the density of the network of services calling at that port, thus resorting also to feeder linkages as an effective policy to improve betweenness centrality.

5.3.2 *Closeness centrality*

Closeness centrality is analysed for the proposed approach (Section 4.2.2) and for the classical approach (equation #2) with the same rationale underlying Section 5.3.1. For the sake of brevity, results are reported only for the P-graph and HP-graph approaches, again in terms of a 45-degree biplot in Fig. 9 for some top ports worldwide and of a geographical representation of closeness centrality in the HP-graph and concerned differences with the P-graph (Fig. 10).

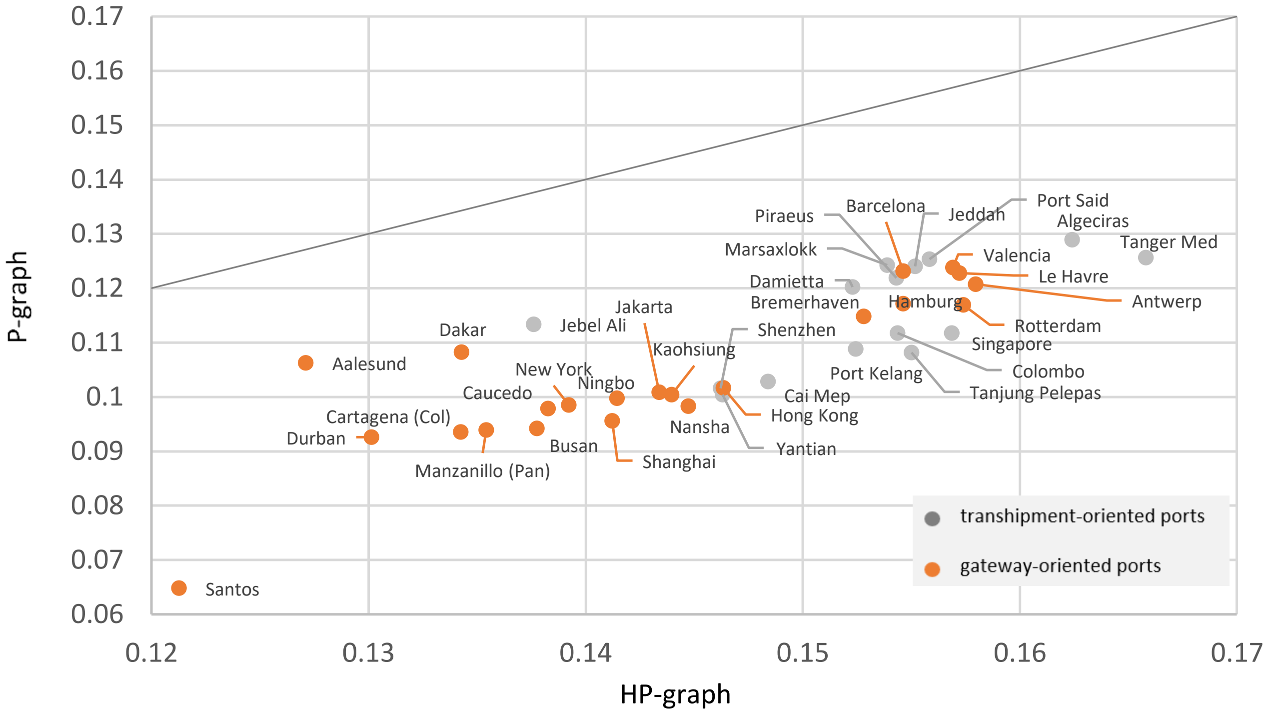


Fig. 9. Closeness centrality in P-graphs vs. HP-graphs: 45-degree biplot for relevant ports worldwide (non-symmetric axes limits to enhance plot readability).

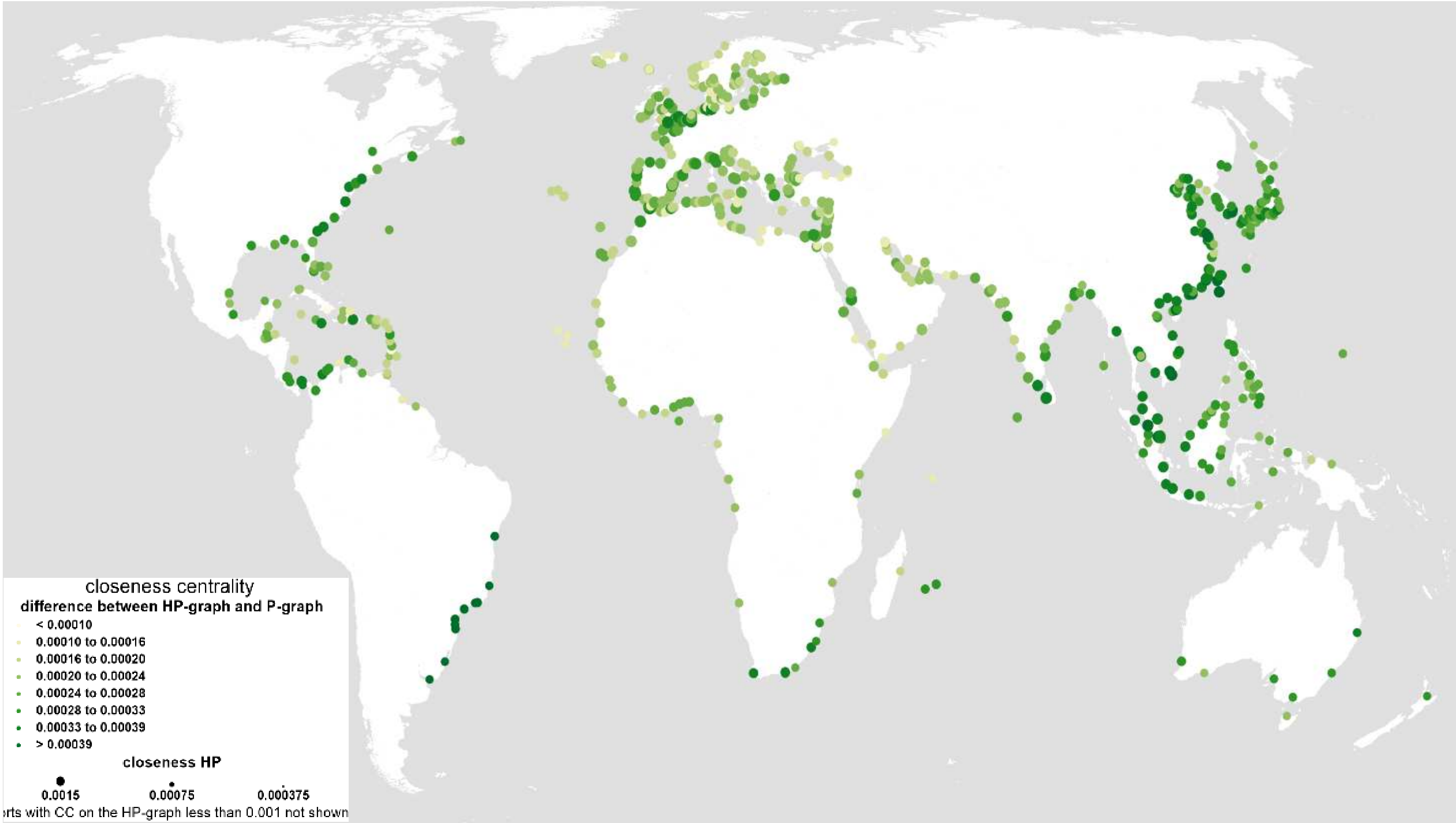


Fig. 10. Geographical representation of closeness centrality on HP-graphs (size of port points) and differences with closeness centrality on P-graphs (scale of colours). Ports with a closeness centrality lower than 0.001 not shown.

Results should be interpreted first by recalling that the closeness centrality of ports in the HP-graph should be larger, or at least equal, than in the P-graph, as already explained in Section 4.2.2. In addition, the magnitude of the difference in the closeness centrality between HP-graph and P-graph for a given port depends intuitively upon the number of elemental paths in each hyperpath connecting that port with each other port in the network. Again, ports at the centre of denser (i.e., highly connected) maritime service networks are expected to exhibit a more appreciable increase of closeness centrality when moving from P-graphs to HP-graphs. Looking at Fig. 10, this happens primarily for ports in the Far East, also significantly for ports in the East coast of North and South America, and lesser for Euro-Mediterranean ports, consistent with the results presented in Fig. 8, wherein the area with the most significant reductions in betweenness centrality moving from the P-graph to the HP-graph is the Euro-Mediterranean region.

This result can be regarded also in the light of the policy implication already discussed in Section 5.3.1: being called by direct deep-sea services is not necessarily the sole development strategy for a port terminal to enhance the maritime accessibility of the economies within its concerned catchment area, if this does not impact significantly on transit times and freight rates.

5.3.3 *Degree centrality (node strength)*

Recalling the discussion in Section 4.2.3, calculation of degree centrality is not affected by the type of underlying graph. Thus, just for the sake of completeness, the following Table 4 reports the degree centrality for the top 20 ports, calculated only for the P-/HP-graph. Specifically, degree centrality is calculated in terms of node strength via Eq. (5), considering the weekly capacity as link weight, and has been normalised with respect to the absolute maximal value calculated for the port of Singapore.

Table 4.

Top 30 ports by degree centrality in P-graph and HP-graph.

Port	*P- and HP- graph*	*rank*
Shanghai	1	1
Singapore	0.988	2
Busan	0.797	3
Hong Kong	0.773	4
Ningbo	0.74	5
Kaohsiung	0.503	6
Shenzhen	0.497	7
Port Kelang	0.486	8
Qingdao	0.445	9
Antwerp	0.443	10
Rotterdam	0.417	11
Yokohama	0.374	12
Tokyo	0.314	13
Yantian	0.311	14
Tanjung Pelepas	0.303	15
Xiamen	0.3	16
Kobe	0.298	17
Hamburg	0.279	18
Nagoya	0.278	19

Xingang	0.271	20

In general, results confirm that the degree centrality/node strength is more simplistic than the other metrics analysed in the paper, mainly because it does not detect the type of connections properly – e.g., multiple feeder links may count less than a single direct service in the node strength – and because it does not consider in any way which port-to-port linkages are enabled by the services calling at that port.

6 Conclusions and research prospects

This paper has explored the extension of centrality metrics – namely, betweenness centrality, degree centrality, and closeness centrality – to hypergraph-based representations of maritime container service networks. Achieving small yet distinct improvements over the state-of-the-art.

First, HL-graphs and HP-graphs have been introduced by combining the definition of hyperpath with the well-known concepts of L-graphs and P-graphs, already extensively applied in the literature. Then, the calculation of centrality metrics in HL-graphs and HP-graphs has been discussed, and a new betweenness centrality measure not found in the literature has been presented in the form of Eq. (9), that can be easily calculated by means of very simple all-or-nothing transit assignment procedures, also embedded in commercial software, clearly however at the price of a slightly improved computational burden with respect to classical centrality measures on L-graphs. Finally, an application to a worldwide maritime container service network has been presented, by contrasting the results of calculation of various centrality metrics on different types of graphs.

Overall, the theoretical analysis of the proposed approach and the analysis of the results of its applications to a worldwide maritime container service network allowed highlighting its main features. Hypergraph-based calculations (that is, on HL-/HP-graphs) exhibit a less deterministic behaviour with respect to shortest path-based calculations (that is, on L-/P-graphs), being based on routing strategies that might include multiple paths between a port of origin and a port of destination; this phenomenon is further magnified in presence of a denser and more connected network of maritime services calling at the port under analysis. As a side note, hypergraphs allow accounting also for the effects of the cumulated frequency of services – as formally expressed by Eqs. (6) and (7) – that impact on the transfer waiting time at transhipment nodes, thus providing a theoretically sounder representation of containerised maritime connections.

In terms of centrality metrics, this yields first a different evaluation of the betweenness centrality of ports: by way of example, the ranking in the HP-graph treats equally ports serving both as regional transhipment hubs and gateways (e.g., Antwerp, Hamburg, Shanghai), and pure transhipment ports (e.g., TangerMed and Algeciras), which can be regarded as conceptually sounder. Similar results can be observed in the calculation of the closeness centrality, which is by proof equal or larger in HL-/HP-graphs with respect to L-/P-graphs. Finally, degree centrality does not vary across approaches.

Working on HL-/HP-graphs accounts for the effects of routing strategies and for the presence of denser/connected network of maritime services, yielding noteworthy differences in terms of port rankings by concerned centrality metrics with respect to L-/P-graphs, that capture mainly the effect of direct calls of deep-sea services on shortest maritime routes. An interesting policy implication has been also presented: a L-/P-graph policy viewpoint would recommend increasing the betweenness centrality by resorting as much as possible to the attraction of direct calls of transhipment services, whilst a LP-/HP-graph viewpoint would assign also key relevance to strengthening the density of the network of services calling at that port, thus resorting also to feeder linkages as an effective policy to improve betweenness centrality.

In this respect, a noteworthy research prospect, currently under way for the inherent difficulty in a comprehensive data collection, deals with the investigation of the linkage between centrality metrics in HL/HP-graphs and container terminal throughput – differentiated by import/export and transhipment – to analyse possible correlation patterns. More in general, another research prospect delas with the capability of the proposed approach to detect better global maritime container trade patterns.

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