Moment Condition Failure
Australian Evidence

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Abstract :

Statistical population moments may be finite or infinite. Determining whether certain moments of a population are finite or not based on a finite sample turns out to be a very daunting and difficult task. If one assumes stock returns to behave according the sum stable law, characteristic exponent point estimates of approximately 1.5 are found for Australian stocks. This result is fully in line with previous US findings and implies that the population variance is infinite. Hill-estimates, on the other hand, are above 2 for all stocks, indicating that the second moments do exist. This conflicting result is resolved by showing that the (unconditional) sum stable hypothesis can be rejected firmly. We do this by setting up a simulation experiment, in which we show that combinations of the Hill-estimate and the characteristic exponent produced by the real data are extremely unlikely for sum stables. These results confirm the existence of at least second moments. There is a good chance that third moments exist as well but this calls for further research.

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1. Introduction

Undertaking empirical financial analysis is the cornerstone of most financial and investment research. However, when applying most econometric procedures a number of assumptions are made regarding the existence of certain moments. For example, covariance stationary models require at least the first two moments (mean and variance) to exist, and tests on the second moment like the standard LM test for ARCH, typically require the existence of the first four moments (Loretan and Phillips, 1994). However, for returns of various speculative asset prices, the existence of second and higher moments has been debated since the 1960s. Mandelbrot (1963) was the first to suggest that the observed fat tails could be explained by relying on the generalised central limit theorem (Feller, 1971). This generalised central limit theorem states that the sums of independent and identical random variables with infinite variance converge to a (non-normal) sum stable distribution.

Mandelbrot’s suggestion of infinite second moments really squeezed the heart of finance. It would mean that volatility as measured by a standard deviation is a dangerous sample dependent risk measure that will explode if we take longer and longer time series. Standard practice of the finance profession has been to ignore Mandelbrot’s hypothesis and simply to assume that Mandelbrot’s exploding sequential variance plots could be generated by other reasons such as time varying volatility. The fear of infiniteness of the variance turned up again when GARCH(1,1) models were borderline to Integrated GARCH models (Engle and Bollerslev, 1986).

Fairly recently, however, Jansen and de Vries (1991), Loretan and Phillips (1994) Hiemstra and Jones (1995), and Pagan (1996) have observed empirical evidence that is consistent with the existence of second moments. This research focuses directly on the tail of the empirical distribution, instead of fitting a pre-specified density function to the data. The logic is that the latter method will be unduly influenced by the vast majority of observations lying in the centre of the distribution and therefore not carry any information about the tails. To alleviate this problem, a distinction is made between distributions with exponentially declining tails (like the tails of the normal distribution) and distributions having a tail that declines with a power law (i.e. fat tailed distributions).

Given that fat tail distributions exist in most financial return series (e.g. Bolleslev 1987) researchers *assume* the tails of the distribution to behave according to the Pareto (power) law, i.e. $\Pr(X>x) = k \cdot x^{\alpha}$, with k constant and tail index α. Estimating this tail index α is of interest, and is useful in at least two respects. First, it provides a statistically rigorous way of discriminating between various suggested candidate stochastic processes for modelling the unconditional distribution of financial returns. Although the sum stable distributions and other fat tailed processes such as the Student-*t* distribution and non-integrated ARCH models are not nested, they are *nested in the tail of the distribution*. Jansen and de Vries (1991) and Hols and de Vries (1991) exploit the property that $\alpha < 2$ for stable processes, whereas otherwise α is restricted only to be positive. Finding α's that are statistically significantly higher than 2 hence rules out the sum stable distributions as suitable approximations for unconditional return distributions. At the same time this finding rescues the mean-variance paradigm. Second, under the assumption that the tails are truly from the Pareto type, α can be used to determine the maximum number of moments that exists since only moments up to the value of α exist (Loretan and Phillips, 1994).

Unfortunately, both the estimation of the tail index and the statistical inference lead to various statistical problems.

1. If the tail index is estimated assuming that the true distribution is sum stable, α is *a priori* restricted to be below 2, which makes it impossible to discriminate between alternative stochastic processes based on α.
2. Alternatively, one can assume that the tail of the distribution is Pareto type. Then α can be estimated in an unbounded way by using a Hill-estimator (Hill, 1975; see section 2). Unfortunately, theoretical standard deviations on the Hill-estimator have been derived under the assumptions of independent realisations and Pareto type tails.
    a) Kearns and Pagan (1997) have already shown that GARCH-type dependence causes the theoretical standard errors to be so large that the interesting tests on α being smaller or larger than 2 are virtually always inconclusive. Kearns and

Pagan (1997)'s result again questions the conclusions of Jansen and de Vries (1991) and Hols and de Vries (1991) that second moments doe exist.

b) Moreover, as implicitly shown by Groenendijk, Lucas and de Vries (1995) in their tables 1 and 3, the assumption of Pareto type tails also is not as innocuous as it seems. Departures from this assumption lead to severe biases in the estimation of α. They report that for the Student-t distribution with 30 degrees of freedom the number of existing moments is estimated to be as low as 1.767. Even after applying a bias correction only 5.403 moments were found to exist!

Taken together all this research seems to put us back on square one. Even the existence of the –for finance crucial - second moment is back into question. In this paper, we will estimate the tail index α for a number of Australian large cap stocks. We estimate α based on the Hill-estimator. Our results are consistent with and are complementing the US evidence of Loretan and Phillips (1994) and the UK evidence of Omran (1997). Given the findings of Kearns and Pagan (1997), however, we know that the theoretical standard deviations used by Loretan and Phillips (1994) and Omran (1997) are severely downward biased in the presence of dependency in asset returns. Based on simulated combinations of Hill-estimates and characteristic exponents for truly sum stables, we are able to forcefully rule out the sum stable law as a possible candidate for the unconditional distribution of stock returns. Our research does *not* prove that the second moment is finite beyond any reasonable doubt. It does show that the moment condition failure due to the sum stable hypothesis does not apply. Integrated GARCH process –studied by Kearns and Pagan (1997) - still can be a serious alternative. However, some older research (e.g. Lastrapes, 1989) also casts serious doubts on this hypothesis, since structural breaks often cause integrated GARCH like behaviour. At this present stage it seems fair to conclude that the odds are in favour for the existence of second moments. For the series we studied, the existence of the fourth moment on the other hand is still more problematic. This last result implies that the statistical inference with respect to the variance equation has to be interpreted with extreme carefulness.

The remainder of the paper is organised as follows. In section 2 we explain how the tail index can be estimated based on a Hill-estimator. In section 3 we introduce our data and provide some descriptive statistics. Section 4 reports the results of the Hill estimates. In section 5 we put up a simulation experiment in order to judge the reasonableness of the sum stable hypothesis as approximation of the unconditional distribution of stock returns. We can forcefully reject this hypothesis. Finally, we conclude.

2. Estimating the tail index

Let $\{x_t\}$ be an identically and independently distributed series following a distribution with asymptotic Pareto-type tails. Pareto-type tails imply that

$$\log \Pr(X>x) = k - \alpha \log x$$

or

$$\log x = k' - \gamma \log \Pr(X>x), \qquad (1)$$

where $\gamma = 1/\alpha$.

If we conveniently assume that the N data points have been sorted to produce the order statistics $x_{(1)} > x_{(2)} > \ldots > x_{(N)}$, $\Pr(X>x_{(j)})$ can be estimated by the empirical survival function (i.e. the number of observations larger than $x_{(j)}$ divided by N).

Equation (1) reveals that one can estimate γ by fitting a straight line into the $\log x$ – $\log \Pr(X>x)$ plane. Any two points $x_{(i)}$ and $x_{(j)}$ can produce an estimate of γ. Obviously, one of them should be –depending on the tail under consideration - the most extreme observation, $x_{(1)}$ or $x_{(N)}$. The other point, $x_{(m)}$, still has to be a point in the tail of the distribution. Unfortunately, no strong arguments exist to determine the start of the tail region unambiguously. Although Dumouchel (1983) suggest that $m < 0.1 \cdot N$ is a practical rule of thumb, common practice varies m between 1% and 10% of the sample size in order to judge the robustness of the estimate.

Several estimators, all based on the intuition behind equation (1), have been suggested. Kearns and Pagan (1997) perform simulation experiments in order to calculate the bias and the efficiency of the Picands (1975), the Hill (1975) and the de

Haan & Resnick (1980) estimator. Bias and efficiency were evaluated under the null hypothesis of sum stable distributed returns on the one hand and under the null hypothesis of integrated GARCH processes on the other hand. Although a small bias remained, under both null hypotheses, the Hill-estimator turned out to be the most efficient estimator.

This estimator reads

$$\hat{\gamma}_{hill} = \left[\frac{1}{m-1} \sum_{i=1}^{m-1} \log x_{(i)} \right] - \log x_{(m)} \qquad \text{and} \qquad \hat{\alpha}_{hill} = 1 / \hat{\gamma}_{hill} \tag{2}$$

However, Kearns and Pagan (1997) at the same time showed that the theoretical standard deviations for the Hill-estimator derived by Hall (1982) and Goldie and Smith (1987) should not be taken too literal and may severely underestimate the true standard errors whenever there is dependence of the integrated GARCH type. Mittnik, Paolella and Rachev (1998) also showed that the small sample performance of the Hill-estimator does not resemble its asymptotic behaviour, even not for samples of 10,000 observations. Taking these findings into account we will not compute the theoretical standard deviations, but instead we will build up an alternative simulation based approach to assess moment condition failure due to an infinite variance problem.

3. Data and descriptive statistics

This study calculates the Hill-estimator based on Australian large caps. We selected 23 actively traded stocks on the Australian Stock Exchange. The data used in this study were taken from Datastream. Given the importance of sample size in the estimation procedures, we required the stocks to have a full history from January 1985 onwards till July 2000. This gives us a sample of 3,891 daily returns. Returns were calculated as continuously compounded returns, i.e. the logarithm of the ratio of two successive prices. Whenever available, we took the return index from Datastream, which is adjusted for stock splits, bonus shares and dividends.

Table 1 summarises some basic descriptive statistics. Average annualised returns vary between –1.4% and 20.4%. Historical annualised volatilities range between 21.4%

and 39.9%. Clearly, the daily returns are not normally distributed as can be inferred from the sky high skewness and kurtosis figures. First order autocorrelation based on the returns, *r*, is small. The autocorrelations on the absolute returns and on the squared returns are more prominent as one would suspect if GARCH type dynamics are present (see Ding, Granger and Engle (1993) for a discussion of the autocorrelations of absolute returns).

Table 1: Descriptive statistics for 23 Australian large caps

	Average return	Standard Deviation	Skewness	Kurtosis	AC(r)	AC(|r|)	AC(r²)
National Australia Bank	18.9%	22.5%	-1.24	20.63	0.08	0.21	0.14
Lend Lease Corporation	16.6%	25.1%	-4.52	21.17	0.07	0.21	0.05
Amcor	11.1%	24.6%	-4.62	123.88	0.03	0.25	0.08
CSR	7.6%	26.4%	-1.12	27.04	-0.00	0.28	0.21
Brambles Industry	20.4%	25.8%	-3.95	110.08	0.03	0.25	0.08
Coles Myer	12.9%	22.8%	-1.71	30.49	0.02	0.26	0.24
Pioneer International	12.0%	28.8%	0.06	30.64	0.07	0.19	0.08
Fosters Brewing Group	11.3%	30.4%	-1.02	24.92	0.05	0.24	0.11
Rio Tinto	13.4%	29.2%	-2.17	54.13	0.13	0.25	0.07
Broken Hill Proprietary	13.6%	23.3%	-0.37	14.69	0.07	0.17	0.05
Coca-Cola Amatil	11.8%	30.2%	-1.59	37.66	0.07	0.32	0.19
Southcorp	14.4%	27.6%	-0.14	12.20	0.00	0.22	0.25
Comalco	12.4%	33.5%	-1.27	28.31	0.08	0.19	0.15
General Pr. Tst.	9.5%	21.4%	-2.99	72.99	-0.07	0.18	0.05
Santos	4.4%	26.9%	-1.02	18.84	0.04	0.19	0.06
Australian Gas and Light	14.4%	28.9%	-0.60	15.19	0.05	0.22	0.24
QBE Insurance Group	17.9%	27.1%	-1.12	27.41	0.03	0.12	0.04
MIM	-1.4%	39.9%	-1.79	43.15	0.06	0.21	0.03
North	8.9%	39.6%	-2.82	84.86	0.04	0.20	0.05
Westpac Banking	7.5%	25.1%	-1.45	27.13	0.09	0.19	0.10
Westfield Holdings	19.5%	33.9%	-4.61	110.68	0.01	0.19	0.03
Woodside Petroleum	14.8%	34.0%	-1.84	46.85	0.01	0.27	0.10
WMC	7.0%	32.4%	-1.74	42.65	0.11	0.20	0.04

Averages and standard deviations are annualised figures and are based on a daily data set starting in January 1985 and ending in July 2000 (3,891 returns). Annualisation was based on 250 trading days and the $\sqrt{250}$ rule for the standard deviation.

4. Results of the Hill-estimator

Before applying the Hill-estimator, Loretan and Phillips (1994) demeaned their series by first regressing the data on five weekday dummies and in a second step on twelve monthly dummies to eliminate potential day of the week and seasonal effects. Third, they passed the residuals from these regressions through an AR filter in order to remove any linear serial correlation. We also pre-withened the series by performing step (1) and step (3) of the Loretan-Phillips procedure. This procedure did not in any material way affect the Hill-estimates. Hence we report the results based on the continuous returns themselves.

Table 2: Hill-estimates of the tail index

Company / *m*	40	80	120	160	200	240	280	320	360	400
National Australia Bank	3.03	2.97	2.83	2.91	2.79	2.59	2.51	2.32	2.23	2.14
Lend Lease Corporation	2.84	3.20	2.86	2.81	2.74	2.65	2.66	2.55	2.48	2.37
Amcor	3.49	3.27	2.86	2.68	2.52	2.51	2.35	2.19	2.18	2.13
CSR	2.86	2.91	2.80	2.90	2.77	2.70	2.62	2.50	2.35	2.24
Brambles Industry	2.75	2.73	2.67	2.97	2.83	2.71	2.56	2.52	2.39	2.27
Coles Myer	2.60	2.88	2.80	2.79	2.88	2.59	2.55	2.32	2.31	2.19
Pioneer International	3.09	3.42	3.31	2.91	2.73	2.53	2.49	2.43	2.36	2.31
Fosters Brewing Group	2.76	2.62	2.78	2.69	2.61	2.68	2.56	2.57	2.47	2.23
Rio Tinto	2.64	2.93	2.89	2.94	2.73	2.73	2.56	2.49	2.37	2.38
Broken Hill Proprietary	4.28	3.98	3.46	3.42	3.25	3.07	2.77	2.66	2.48	2.43
Coco-Cola Amatil	3.25	3.25	2.79	2.58	2.40	2.30	2.23	2.20	2.11	2.01
Southcorp	3.23	2.93	2.68	2.65	2.58	2.52	2.58	2.53	2.57	2.40
Comalco	3.48	3.15	3.10	2.72	2.65	2.56	2.55	2.34	2.35	2.29
General Pr. Tst.	2.98	3.31	3.18	3.08	2.80	2.91	3.02	2.65	2.41	2.27
Santos	2.92	3.20	3.14	3.12	3.21	3.13	3.03	2.86	2.64	2.53
Australian Gas and Light	2.21	2.37	2.41	2.46	2.51	2.49	2.56	2.56	2.55	2.36
QBE Insurance Group	2.69	2.81	2.85	2.64	2.49	2.37	2.13	1.99	1.94	1.91
MIM	2.92	3.27	3.41	3.41	3.25	3.14	2.89	2.86	2.63	2.59
North	2.55	3.08	3.13	2.81	2.87	2.67	2.43	2.39	2.35	2.30
Westpac Banking	3.61	3.53	3.41	3.14	2.97	2.90	2.77	2.69	2.44	2.38
Westfield Holdings	2.24	2.67	2.58	2.34	2.15	2.15	2.17	2.07	2.06	1.97
Woodside Petroleum	2.30	2.73	2.57	2.45	2.40	2.39	2.32	2.29	2.39	2.28
WMC	2.78	3.20	3.47	3.37	3.33	3.22	3.08	2.81	2.72	2.57

Estimates are based on daily returns computed from January 1985 through July 2000 (3,891 observations). Estimates for the tail index are based on equation (2).

As choosing *m* remains a delicate issue, we varied *m* between 40 and 400 with steps of 40. This *m* values are proportional to those studied by Loretan and Phillips (1994) and Omran (1997). The point estimates in Table 2 are almost always more than 2 and usually less than 4, ranging from 1.91 to 4.28. Based on this evidence, it would be tempting to discard the sum stable distributions as a good description of the unconditional distribution of stock returns. Unfortunately, theoretical standard deviations are not very useful given the GARCH-type dependence present in the time series (see Table 1). On the other hand, the point estimates indicate that the existence of fourth moments probably is doubtful since almost all Hill estimates are below 4. If this is truly so, this result casts doubt on the validity of ARCH-tests and other tests on second moments that require the existence of four moments. We are not so pessimistic on this account because Hill-estimates only provide an accurate estimate of the existing number of moments under the assumption of truly Pareto tails. For other distributions the Hill estimate may severely be downward biased. To illustrate this we simulated 1000 runs from a Student-t distribution with 5 degrees of freedom and 2000 observations. From this experiment we estimated the average the number of existing moments to be 4.2. Decreasing the degrees of freedom to 4 resulted in an average number of 3.65. These averages are severe downward biased estimates of the existing number of moments. Hence they prevent us to make strong further conclusions with respect to higher moments.

Recognising the flaws in the applicability of the theoretical standard deviations hence puts us –together with all the previous authors - in a very weak position. We cannot draw strong conclusions on the upper bound of the existing moments due to severe downward biases in the Hill-estimates. Even worse, the sum stable hypothesis - implying infinite variances – cannot be rejected in a statistically safe and sound way based on Hill-estimates and their standard errors.

5. Discarding the sum stable hypothesis

Sum stable distributions are usually parameterised in terms of their log-characteristic function since for most sum stables the density function has no analytical closed-form expression. Stable distributions are determined by four parameters (McCulloch, 1996). First, the location parameter δ has the potential to shift the distribution to the left (negative values) or right (positive values). Second, the positive scale parameter γ expands or contracts the distribution about δ. Third, a skewness parameter β, whose absolute value is constrained to be less or equal to one, indicates the symmetry of the distribution. Finally, the characteristic exponent α^* (positive and less than or equal to 2) governs the tail behaviour of the distribution and should be equal to the tail index α. Therefore, if stock returns were truly sum stable, we can estimate the tail index also through direct estimation of the characteristic exponent of the distribution.
In order to estimate the parameters of the sum stable distribution, we used the McCulloch (1986) procedure that was also used by Ghose and Kroner (1995). Akgiray and Lamoureux (1989) and Bates and McLaughlin (undated) both show that McCulloch's (1986) technique provides good all round estimators for the parameters of the sum stable distributions (with $\alpha > 0.6$). Table 3 reports the estimated α^*s and the other distribution parameters for our sample of Australian stocks. We estimated

the characteristic exponents under two alternative assumptions. In the first part of the table all parameters are freely estimated, whereas in the second part of the table a symmetry restriction (β=0) is imposed. Without any formal statistical testing needed, one can observe that the difference between the two estimations for the characteristic exponents is very marginal. All point estimates are in the range 1.1-1.7 with estimates around 1.5 being the most representative estimate. This results are fully in line with the evidence of Fama (1965), Blattberg and Gonedes (1974), Fielitz and Rozelle 1983, …).

Although the results of table 3 are fully in line with the published US results, the implied conclusion of finding infinite variances still is (undesirable and) in contradiction with the point estimates of the Hill-estimator in table 2 (that where also in line the findings in the literature). Recall that the tail index nests various models in the tail of the distribution. The Hill-estimator and the McCulloch estimator are two estimators that should produce the same “alpha” under the assumption that the observations are independent and have true pareto type tails. The discrepancy we document is not unique to Australian stock returns. Ghose and Kroner (1995, Table 4) document similar discrepancies for currencies (GBP, DEM, CHF, JPY), stocks (S&P500) and commodities (soyabeans, live cattle and live hogs).

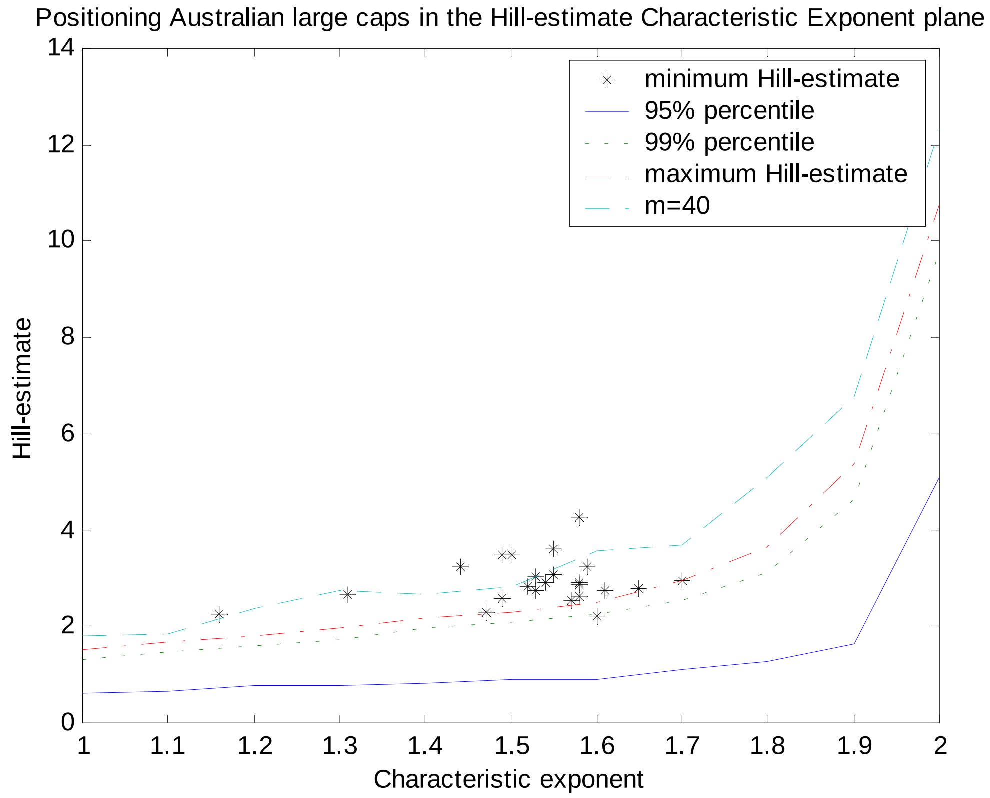
Table 3: Estimates of the parameters of the sum stable distribution

	alpha	beta	gamma	delta	alpha	gamma	delta
National Australia Bank	1.537	0.052	0.007	0.001	1.537	0.007	0.000
Lend LeaseCorporation	1.521	0.135	0.008	0.001	1.523	0.008	0.000
Amcor	1.485	0.041	0.007	0.000	1.486	0.007	0.000
CSR	1.577	-0.020	0.009	0.000	1.577	0.009	0.000
Brambles Industry	1.607	0.153	0.008	0.001	1.608	0.008	0.000
Coles Myer	1.493	0.057	0.007	0.000	1.494	0.007	0.000
Pioneer International	1.549	0.071	0.009	0.001	1.549	0.009	0.000
Fosters Brewing Group	1.531	0.082	0.009	0.001	1.532	0.009	0.000
Rio Tinto	1.579	0.162	0.009	0.001	1.580	0.009	0.000
Broken Hill Proprietary	1.579	0.082	0.008	0.000	1.579	0.008	0.000
Coca-Cola Amatil	1.438	0.047	0.009	0.000	1.439	0.009	0.000
Southcorp	1.586	0.160	0.009	0.001	1.587	0.009	0.000
Comalco	1.502	0.080	0.010	0.001	1.503	0.010	0.000
General Pr. Tst.	1.700	-0.056	0.007	0.000	1.700	0.007	0.000
Santos	1.541	0.028	0.009	0.000	1.542	0.009	0.000
Australian Gas and Light	1.597	0.139	0.009	0.001	1.598	0.009	0.000
QBE Insurance Group	1.308	0.041	0.007	0.000	1.310	0.007	0.000
MIM	1.581	0.025	0.013	0.000	1.581	0.013	0.000
North	1.576	0.015	0.012	0.000	1.576	0.012	0.000
Westpac Banking	1.553	0.040	0.008	0.000	1.553	0.008	0.000
Westfield Holdings	1.158	0.059	0.006	0.001	1.162	0.006	0.000
Woodside Petroleum	1.470	0.127	0.009	0.001	1.473	0.009	0.000
WMC	1.645	0.107	0.011	0.001	1.645	0.011	0.000
Average	1.53	0.07	0.01	0.00	1.53	0.01	0.00
Standard deviation	0.02	0.01	0.00	0.00	0.02	0.00	0.00

Estimates are based on 3, 891 daily returns (January 1985 to July 2000) using the McCulloch-technique. The second to fifth columns contain unrestricted estimates, the last three columns contain estimates with a symmetry restriction imposed. H. McCulloch kindly provided the GAUSS-code.

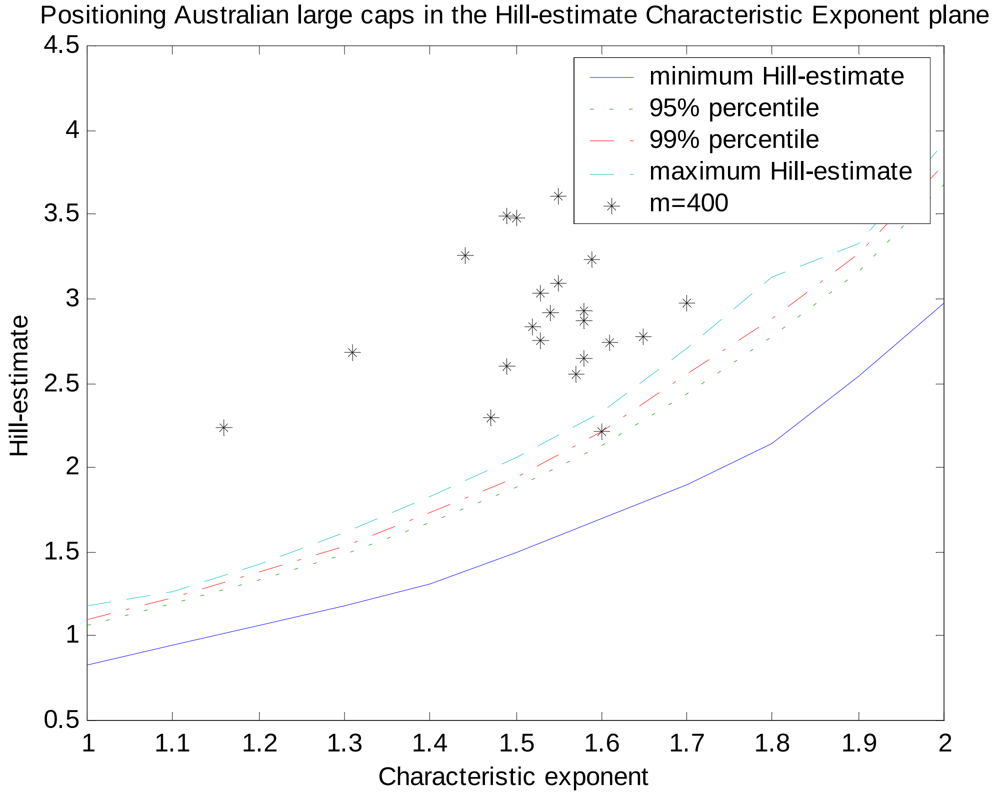
In order to discard the sum stables without relying on the theoretical standard deviations on the Hill-estimator, we have put up a simulation experiment. Table 3 reveals that the estimates of β can be set to approximately zero. The same applies for the location parameter (δ). The scale parameter (γ) is on average 0.01. Given these inputs, we generated sum stable distributions - using the MATLAB-algorithm provided by H. McCulloch on the Mathworks-website - with characteristic exponents varying between 1 and 2 with a step size of 0.1. For each characteristic exponent we generated 1,000 series of 3,891 observations. For each series the Hill-estimator was determined and the empirical distribution of the Hill-estimators was computed. Figures 1 and 2 plot for each simulated characteristic exponent the minimum and maximum Hill-estimates together with their simulated 95% and 99% confidence interval. Hence, the lowest (highest) line connects the minimum (maximum) Hill estimates that were generated over 1000 runs for characteristic exponents varying from 1 to 2. The 95% and 99% bounds were also plotted to give an idea about the likelihood of Hill-estimate – Characteristic exponent combinations. In Figure 1, *m* was set to 40, whereas Figure 2 was based upon *m* equal to 400. We have chosen these m values because they represent the upper and lower cut off rates used to define the tails (see also Table 2). This graph has been produced for all other m values reported in Table 2 but with the same qualitative result.

Figure 1



Finally, the 23 Australian large caps were positioned in the Hill-estimate – Characteristic exponent plane. The Hill-estimates used are the same as those reported in Table 2 whereas the characteristic exponents were the ones estimated in Table 3. If stock returns would follow the sum stable law, we would expect to find the dots scattered for 95% between the minimum and 95% line. For m=40 we notice that only 1 stock touches the (one-sided) 95% confidence interval. Most stocks are situated between the 95% and the 99% line. Several stocks position even above the maximum Hill-characteristic exponent line. For Figure 2 (m=2) the results are even more dramatic. All but one stock position above the simulated maximum Hill-characteristic exponent line. Both figures show that it is extremely unlikely that the real stock returns are being characterised by an unconditional sum stable law.

Figure 2



6. Conclusion

If one assumes stock returns to behave according the sum stable law, point estimates for the tail index of approximately 1.5 will turn up, implying the second moment does not exist. This result is fully in line with previous US findings. Hill-estimates for the tail index, on the other hand, are rendering for all stocks point estimates above 2, indicating that the second moment is existing. The existence of higher moments is unfortunately in doubt. However, the sum stable hypothesis can be rejected firmly since combinations of the Hill-estimate and the characteristic exponent produced by the real data are extremely unlikely for sum stables.

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