Wilson lines in transverse-momentum dependent parton distribution functions with spin degrees of freedom

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Received 28 April 2010; received in revised form 2 July 2010; accepted 23 July 2010

Available online 29 July 2010

Abstract

We propose a new framework for transverse-momentum dependent parton distribution functions, based on a generalized conception of gauge invariance which includes into the Wilson lines the Pauli term $\sim F^{\mu\nu}[\gamma_\mu, \gamma_\nu]$. We discuss the relevance of this nonminimal term for unintegrated parton distribution functions, pertaining to spinning particles, and analyze its influence on their renormalization-group properties. It is shown that while the Pauli term preserves the probabilistic interpretation of twist-two distributions—unpolarized and polarized—it gives rise to additional pole contributions to those of twist-three. The anomalous dimension induced this way is a matrix, calling for a careful analysis of evolution effects. Moreover, it turns out that the crosstalk between the Pauli term and the longitudinal and the transverse parts of the gauge fields, accompanying the fermions, induces a constant, but process-dependent, phase which is the same for leading and subleading distribution functions. We include Feynman rules for the calculation with gauge links containing the Pauli term and comment on the phenomenological implications of our approach. © 2010 Elsevier B.V. All rights reserved.

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1. Introduction

Parton distribution functions (PDF)s are the key nonperturbative ingredients of completely inclusive QCD processes, like deeply inelastic scattering (DIS). The process-dependent hard-scattering part of such processes can be calculated order by order in QCD perturbation theory on account of the hard scale of the process $Q^2 \gg \Lambda_{\text{QCD}}^2$. Though the determination of the initial PDF requires the application of nonperturbative methods, its Q^2 evolution is controlled by renormalization-group (RG) evolution equations with anomalous dimensions calculable within perturbative QCD.

This simple picture changes significantly when one considers semi-inclusive processes, like semi-inclusive DIS (SIDIS), or the Drell–Yan (DY) process in hadronic collisions, in which hadrons are detected in the final (initial) state with a sizeable transverse momentum. In that case, one needs information about the generation of the transverse momentum $P_\perp^h$ of the final (initial) hadrons, e.g., by means of the transverse-momentum distribution of the partons. This mechanism is believed to be dominant at small $P_\perp^h \ll Q$, while at large $P_\perp^h \sim Q$, the transverse momentum $P_\perp^h$ is produced by the perturbative gluon exchanges. The second mechanism, as well as the relationship between the two in the intermediate region, are outside the scope of the present work. In any case, integrated PDFs of leading twist are not sufficient to describe semi-inclusive processes. One therefore introduces transverse-momentum dependent (TMD) PDFs which keep track of the intrinsic transverse motion of the partons inside the hadrons and reveal this way fine details about their substructure (pioneering works are [1–4]—see also [5–10] and references cited therein, and [11] for a review). The introduction of TMD PDFs, though intuitively clear and physically appealing, still poses serious challenges. The first problem is related to the TMD factorization: Its status beyond leading twist (and to all orders) is far from being satisfactory at the moment [12–15]. Next, there is a possible *non-universality* of TMD PDFs entailed by the extremely complicated and often process-dependent structure of the gauge links,¹ see Refs. [16–18]. Finally, in the light-cone gauge, *extra divergences* appear that have to be properly treated [4,5,19,20]—in contrast to the integrated case. In the present paper, we focus on the last two issues.

In the integrated case, the parton density $f_{i/h}(x, Q^2)$ describes the probability to find a parton i with longitudinal momentum fraction xP^+ inside hadron h with momentum P , and can be given a gauge-invariant definition in terms of the gauge link (Wilson line) (see, for instance, [21])

$$[\xi^-; 0^- | \Gamma] = \mathcal{P} \exp \left[-ig \int_{0^- | \mathcal{C}}^{\xi^-} dz^\mu A_\mu^a(z) t^a \right] \quad (1)$$

for a contour $\mathcal{C}$ along the light-cone, where the path-ordered exponential A_μ^a refers to the (gluon) gauge field. The renormalization of the integrated PDF obeys the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) [22,23] evolution equation with its integral kernel being related to the anomalous dimension $\Gamma_{i/h} = \Sigma_q \Gamma_q + 2\Gamma_{\text{end}}$, where Γ_{end} is the endpoint anomalous dimension of the integration contour $\mathcal{C}$ in the gauge link (see [24] for a more detailed discussion of this issue and [25–27] for the original derivations and earlier references).

It was pointed out in [16,28,29] that a completely gauge-invariant definition of the TMD PDF in those gauges in which $\mathbf{A}_\perp$ does not vanish at infinity has to include also transverse gauge

¹ These are path-ordered exponentials of the gauge field, needed to render the definition of PDFs gauge invariant.

links. Hence, in the light-cone gauge $A^+ = 0$, applied in conjunction with q^- -independent pole prescriptions (like the advanced, retarded or principal-value prescription) in order to avoid singularities at q^+ in the gluon propagator, the transverse gauge links receive radiative corrections that can be associated with a cusp-like junction point at light-cone infinity [24,30,31]. The emerging cusp anomalous dimension [32] has to be removed, if one aims to recover the results valid in covariant gauges. To this end, a redefinition of the TMD PDF was proposed by two of us [24,30] which involves a soft factor, termed R , consisting of two eikonal lines evaluated along a particular gauge (integration) contour with a jackknifed path segment in the transverse direction (see next section). Note that the introduction of the soft factor can be justified from a different point of view as well—see, for instance, Refs. [17,33,34], where the soft factor was used to take care of rapidity divergences in covariant gauges. The anomalous dimension related to the ultraviolet (UV) divergences (e.g., pole terms in $1/\epsilon$ in dimensional regularization) of the soft factor was found [24,30] to exactly compensate (at the one-loop order) the cusp anomalous dimension of the transverse gauge link, hence, ensuring the independence of the (redefined) TMD PDF from artificial contour-generated anomalous-dimension artifacts.

More recently [35], we have shown that this factorization scheme remains valid also for the Mandelstam–Leibbrandt pole prescription [36,37], which is q^- -dependent. In that case, the UV divergent part of the soft factor reduces to unity, while the transverse gauge link does not give rise to a defect of the anomalous-dimension that has to be compensated. As a result, the TMD PDF has the same anomalous dimension as in covariant gauges, rendering the proposed definition of the TMD PDF gauge and pole-prescription independent.

The basic tenet in the gauge-invariant formulation of hadronic quantities, like TMD PDFs, is to use a gauge link with an exponent which contains only the gauge field A . However, this is only the simplest (or minimal-twist) possibility which pays attention to the fact that color vectors cannot be compared at a distance. Because the gauge potential A_a^μ is spin-blind, one should actually include into the gauge link an additional term proportional to the gluon tensor $F_{\mu\nu}^a$ —called the Pauli term—which can accommodate the direct spin-dependent interaction in accordance to the Lorentz group. This term represents the *minimal* coupling of a spinning particle to an external field and may become important for nontrivial contours, while additional terms of still higher twist are not prohibited but are relatively power-suppressed. Thus, the gauge links will be generalized to take into account the Pauli contribution [38] $\sim F_{\mu\nu}^a S_{\mu\nu}$, where $S_{\mu\nu} = (1/4)[\gamma_\mu, \gamma_\nu]$, normally ignored. This means that in order to accommodate spin-dependent interactions in a manifestly gauge-invariant formalism, one has to include the following path-ordered exponential $\mathcal{P} \exp[-ig \int_0^\infty d\sigma S_{\mu\nu} F_a^{\mu\nu}(u\sigma) t^a]$. The graphic illustration of this concept is depicted in simple contextual terms in Fig. 1 which shows a generic process with gauge links that contain the Pauli term—codified by small rings around the double lines which stand for the conventional gauge links.

It is expected that any effects of such spin-dependent terms should be non-vanishing only in the case of (at least) transverse gauge links off the light-cone. In the integrated collinear PDFs, the Wilson lines are one-dimensional in the sense that the paths of the integration reduce to lightlike rays. Going beyond the fully collinear picture, which is unavoidable in semi-inclusive processes, one must make use of gauge links which involve more complicated integration contours, which have at least one additional—transverse—dimension. Note that in the case of integration paths off the light-cone—used to regularize rapidity divergences [4]—one has, in fact, even more nontrivial contours because they contain, beyond the minus light-cone segments, also plus components. Therefore, in the TMD case, effects related to the spin transfer from the starting point, say, $(0^-, \mathbf{0}_\perp)$, to the terminating point $(\xi^-, \xi_\perp)$ may become (at least, in principle) apparent due to

Fig. 1. Graphical representation of a generic TMD PDF in coordinate space with the spin-dependent Pauli terms included in the gauge links. The (hidden) contour obstruction at transverse and lightlike infinity (“earth” symbol) is represented by $\times$.

the nontrivial structure of the contour. Hence the cross-talk between a pair of quantum fields at distant points off the light-cone will contain spin-dependent Pauli terms which are of higher twist order with respect to the spin-blind ones containing the gauge potential. Our analysis reveals that the inclusion of the Pauli term, although non-visible in the completely unpolarized TMD PDFs, can produce non-vanishing effects in a number of polarized distributions, in particular, those responsible for time-reversal-odd phenomena.²

Adopting this encompassing concept of gauge invariance, questions arise whether the definition of TMD PDFs, we proposed before in Refs. [24,30], has to be modified and whether the inclusion of the Pauli term has phenomenological consequences—as already indicated. The first issue is related to the question whether spin-dependent terms affect the factorization schemes discussed in our previous works, while the second addresses possible changes of the RG, i.e., evolution behavior of TMD PDFs. The present work is devoted to the clarification of these issues.

The rest of the paper is organized as follows. The pivotal Section 2 argues that the correct treatment of spin degrees of freedom in the TMD PDFs necessitates the inclusion into the gauge links of the Pauli term. This contribution describes the interaction between spinning particles and the gauge-field strength and leads to a generalization of Eq. (1). Its implications are worked out in Section 3. The calculation of virtual gauge-field correlators for the leading-twist distributions, as well as for those of subleading twist, is carried out in Section 4, whereas those related to fermions are discussed in Section 5. Section 6 is concerned with the consideration of contributions to the TMD PDFs stemming from real-gluon emission. Finally, in Section 7, we summarize the results and present our conclusions. To go further with QCD calculations with gauge links, which include the Pauli term, we develop a set of Feynman rules and display them in Fig. 5.

2. Inclusion of spin effects

The TMD PDF for an unpolarized/polarized quark of flavor i in an unpolarized/polarized target h following our generalized concept of gauge invariance reads

² We thank A.V. Efremov for important comments on this point.

$$\begin{aligned}
f_{i/h}^\Gamma(x, \mathbf{k}_\perp) &= \frac{1}{2} \text{Tr} \int dk^- \int \frac{d^4\xi}{(2\pi)^4} e^{-ik \cdot \xi} \langle h | \bar{\psi}_i(\xi) \\
&\times [\xi^-, \xi_\perp; \infty^-, \xi_\perp]^\dagger [\infty^-, \xi_\perp; \infty^-, \infty_\perp]^\dagger \cdot \Gamma \\
&\times [\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0) | h \rangle \cdot R
\end{aligned} \quad (2)$$

where Γ denotes one or more γ matrices in correspondence with the particular distribution in question, and the state $|h\rangle$ stands for the appropriate target. In the unpolarized case, we have $|h\rangle = |h(P)\rangle$, with P being the momentum of the initial hadron, whereas for a (transversely) polarized target the state is $|h\rangle = |h(P), S_\perp\rangle$. The “enhanced” gauge links $[\xi_2; \xi_1]$ and the soft factor R will be defined shortly.

An important comment about definition (2) is here in order before we proceed. We started from the “fully unintegrated” correlation function, which depends on *all* four components of the parton’s momentum [13,39]. Thus, the TMD PDF is obtained *after* performing the k^- integration, which formally renders the coordinate ξ^+ equal to zero:

$$\int dk^- e^{-ik \cdot \xi^+} = 2\pi \delta(\xi^+). \quad (3)$$

However, one must be careful: This operation may produce additional divergences because, carrying it out, all quantum fields involved (quarks and gluons) will be defined on the light ray $\xi^+ = 0$. This means that the plus light-cone coordinates of the product of two quantum fields will always coincide. To avoid this, we will regularize this singularity in what follows by taking into account that a particle, once created at the point $\xi^+ = 0$, will be reabsorbed (destroyed) with the *same* probability at (potentially very distant) points $0 \pm \Delta/2$, where $\Delta \sim 1/p^- \sim p^+/2M^2$ is the uncertainty of determining a point along the plus direction. In other words, we have to sum (average) over all indistinguishable possibilities in order to get the correct answer in the quantum mechanical sense. For instance, the regularized two-gluon correlator is written as

$$\begin{aligned}
&\frac{1}{T} \int_{-\Delta/2}^{\Delta/2} dt \langle \mathcal{A}^\mu(0^+, \xi^-, \xi_\perp) \mathcal{A}^\nu(t, \xi'^-, \xi'_\perp) \rangle_0 \\
&\stackrel{\Delta \rightarrow 0}{=} \frac{-iC_F}{T} \int \frac{d^4q}{(2\pi)^4} e^{-iq^+(\xi^- - \xi'^-) + iq_\perp \cdot (\xi_\perp - \xi'_\perp)} 2\pi \delta(q^-) D^{\mu\nu}(q),
\end{aligned} \quad (4)$$

whereas without regularization, the corresponding term $\sim \int dq^- D^{\mu\nu}(q^+, q^-, \mathbf{q}_\perp)$ would face unphysical UV divergences.

The constant $T \sim 1/p^+$ (so to say the “length” of the plus ray) will drop out from all final results, provided a suitable parametrization of the vectors along the contour integral is adopted. This is crucial for the enhanced gauge link which includes the Pauli term, since the latter is not reparameterization invariant—in contrast to the usual gauge link. Therefore, we make use of the following reparameterization of the (initially dimensionless) constant vectors that define the motion along the line integral:

$$n_\mu^+ \rightarrow u_\mu^* = p^- n_\mu^+, \quad n_\mu^- \rightarrow u_\mu = p^+ n_\mu^-, \quad l_\perp \rightarrow p^+ l_\perp, \quad (5)$$

which implies boosts in the collinear directions. Note that the plus-component of the momentum, p^+ , is large in our kinematics and is the only mass scale entering the above reparameterization. Thus, the uncertainty of determining a position along the plus ray in Eq. (4) is very large, namely,

$$\Delta \sim \frac{1}{u^*} = \frac{1}{p^-},$$

while it is very small along the minus or the transverse directions:

$$\Lambda \sim \frac{1}{u} \sim \frac{1}{|\mathbf{l}_\perp|} \sim \frac{1}{p^+}.$$

We can now define the enhanced lightlike gauge link along the x^- direction:

$$[\![\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]\!] = \mathcal{P} \exp \left[-ig \int_0^\infty d\sigma u_\mu A_a^\mu(u\sigma) t^a - ig \int_0^\infty d\sigma S_{\mu\nu} F_a^{\mu\nu}(u\sigma) t^a \right]. \quad (6)$$

An analogous definition holds for the x^+ direction by making the replacement $u \rightarrow u^*$. On the other hand, the enhanced transverse gauge link is given by

$$[\![\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp]\!] = \mathcal{P} \exp \left[-ig \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\mathbf{l}\tau) t^a - ig \int_0^\infty d\tau S_{\mu\nu} F_a^{\mu\nu}(\mathbf{l}\tau) t^a \right], \quad (7)$$

where the two-dimensional vector $\mathbf{l} \equiv \mathbf{l}_\perp$ drops out from all final results, and the Lorentz generators for the spin are defined by $S_{\mu\nu} = (1/4)[\gamma_\mu, \gamma_\nu]$. Note that the path ordering, denoted by $\mathcal{P}$ in the compound expressions above, means

$$\begin{aligned} \mathcal{P}[\dots] = & 1 + \int_0^\infty d\tau_1 \mathcal{P} \exp \left(\int_{\tau_1}^\infty d\tau u \cdot \mathcal{A} \right) g S \cdot \mathcal{F}(u\tau_1) \mathcal{P} \exp \left(\int_0^{\tau_1} d\tau u \cdot \mathcal{A} \right) \\ & + \int_0^\infty d\tau_2 \int_0^{\tau_2} d\tau_1 \mathcal{P} \exp \left(\int_{\tau_2}^\infty d\tau u \cdot \mathcal{A} \right) \cdot g S \cdot \mathcal{F}(u\tau_2) \mathcal{P} \exp \left(\int_{\tau_1}^{\tau_2} d\tau u \cdot \mathcal{A} \right) \\ & \times g S \cdot \mathcal{F}(u\tau_1) \cdot \mathcal{P} \exp \left(\int_0^{\tau_1} d\tau u \cdot \mathcal{A} \right) + \dots, \end{aligned} \quad (8)$$

where we have used the following convenient abbreviations: $\mathcal{A} = \sum_a A_a t^a$, $u \cdot \mathcal{A} = \sum_\mu u_\mu A^\mu$, $S \cdot \mathcal{F} = \sum_{\mu,\nu} S_{\mu\nu} F^{\mu\nu}$, and the path ordering inside Eq. (8) is the usual one. It becomes obvious that the enhanced gauge links, defined above, and the standard ones fulfil the same gauge transformations.

The soft factor R in Eq. (8)—introduced in [24,30] with the aim to remove the defect of the anomalous dimension of the TMD PDF—may, in principle, be upgraded to include the tensor term as well. This amounts to the following expression

$$\begin{aligned} R(p^+, n^- | \xi^-, \xi_\perp) = & \text{Tr} \langle 0 | \mathcal{P} \exp \left[ig \int_{C_{\text{cusp}}} ds \dot{\xi} \cdot \mathcal{A}(\xi) + ig \int_{C_{\text{cusp}}} ds S \cdot \mathcal{F}(\xi) \right] \\ & \times \bar{\mathcal{P}} \exp \left[-ig \int_{C_{\text{cusp}}} ds \dot{\xi} \cdot \mathcal{A}(\xi + \zeta) - ig \int_{C_{\text{cusp}}} ds S \cdot \mathcal{F}(\xi + \zeta) \right] | 0 \rangle, \end{aligned} \quad (9)$$

where $\dot{\xi}(s) = d\xi/ds$, $\bar{\mathcal{P}}$ denotes anti-path ordering, and the integration contour C_{cusp} is the same as that employed in [24,30] (see Fig. 2 for an illustration). Note in this context that the soft

Fig. 2. Integration contour associated with the soft factor R in Eq. (9). The cusp angle χ is explicitly indicated.

factor was introduced before (without the Pauli term) in Refs. [33,34] with the purpose to control rapidity divergences of non-lightlike Wilson lines in covariant gauges. The soft factors in both approaches are multiplicative renormalization eikonal factors, though in [33,34] the contribution from the gauge link at infinity is not considered owing to the use of a covariant gauge.

3. Influence of the Pauli term

Before we focus our attention to the specific implications of the Pauli term, let us first summarize the key features of the proposed scheme. The usefulness of Eq. (2) derives from the fact that by virtue of the soft factor R all gauge-dependent anomalous-dimension artifacts, potentially contributing to the TMD PDF, are absent *ab initio* [24,30] so that, integrating over the transverse momenta, one obtains a PDF which is controlled by the DGLAP evolution equation [20,24] with the usual anomalous dimension. Moreover, to this definition all pole prescriptions adopted to evaluate the gluon propagator in the light-cone gauge are fungible [35].

To study the effects of the spin-dependent terms, induced by the inclusion of the Pauli contribution, it suffices to take them into account only in the fermionic part of Eq. (2), leaving the soft factor unmodified. The justification of this treatment is based on the fact that the structure of the soft factor is practically prescribed by the RG properties of the *unsubtracted* TMD PDF, as shown in detail in Refs. [24,30,35]. To be more specific, it was found there that the particular contour C_{cusp} in the soft factor, depicted in Fig. 2, pertains to the cusp-like UV singularities of the fermionic part of Eq. (2). Another argument of retaining the original form of the soft factor unchanged is provided by the requirement that it should be boost invariant (see, e.g., Ref. [10]). Given that the Pauli term is not invariant under scale transformations, we refrain from including it into the soft factor in the present investigation. From the calculational point of view, the above argument is related to the fact that, in the absence of any Lorentz structure, the spin-field interaction cannot produce nontrivial results for integration paths without self-intersections—this will be considered elsewhere.

Carrying out the k^- integration in Eq. (2) and leaving out the soft factor R , one obtains the following *unsubtracted* TMD PDF

$$f_{i/q}^{\Gamma}(x, \mathbf{k}_{\perp}) = \frac{1}{2} \text{Tr} \int \frac{d\xi^- d^2 \xi_{\perp}}{2\pi (2\pi)^2} \times \exp(-ik^+ \xi^- + i\mathbf{k}_{\perp} \cdot \xi_{\perp}) \langle p, s | \bar{\psi}_i(\xi^-, \xi_{\perp}) [\xi^-, \xi_{\perp}; \infty^-, \xi_{\perp}]^{\dagger}$$

$$\begin{aligned} & \times \llbracket \infty^-, \xi_\perp; \infty^-, \infty_\perp \rrbracket^\dagger \Gamma \llbracket \infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp \rrbracket \llbracket \infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp \rrbracket \\ & \times \psi_i(0^-, \mathbf{0}_\perp) |p, s\rangle. \end{aligned} \quad (10)$$

For our concrete calculations to follow, we consider matrix elements between quark states having momentum p and spin s : $|p, s\rangle$. Using the fermionic density matrix ($\hat{p} \equiv p \cdot \gamma$)

$$u(k) \otimes \bar{u}(k) = \frac{1}{2} (\hat{k} + m)(1 + \gamma_5 \hat{s}), \quad s^2 = -1, \quad (11)$$

with the spin vector $s^\mu = (s^+, s^-, s_\perp)$ being given by [39]

$$s^\mu = \lambda \left(\frac{k^+}{m}, \frac{\mathbf{k}_\perp^2 - m^2}{2mk^+}, \frac{\mathbf{k}_\perp}{m} \right) + \left(0^+, \frac{\mathbf{k}_\perp \cdot \mathbf{s}_\perp}{k^+}, s_\perp \right), \quad (12)$$

one obtains in the tree-approximation (indicated by the subscript 0 for α_s^0)

$$f_{(0)}^\Gamma(x, \mathbf{k}_\perp) = \frac{1}{2} \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s}) \Gamma] \delta(p^+ - xp^+) \delta^{(2)}(\mathbf{k}_\perp). \quad (13)$$

In particular, for the unpolarized TMD PDF with $\Gamma = \gamma^+$, one has at leading twist two the expression

$$\begin{aligned} f_1^{(0)} & \equiv f_{(0)}^{\gamma^+}(x, \mathbf{k}_\perp) = \frac{1}{2} \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s}) \gamma^+] \delta(p^+ - xp^+) \delta^{(2)}(\mathbf{k}_\perp) \\ & = \delta(1 - x) \delta^{(2)}(\mathbf{k}_\perp). \end{aligned} \quad (14)$$

On the other hand, the helicity and the transversity distributions are given, respectively, by

$$\begin{aligned} f_{(0)}^{\gamma^+ \gamma_5}(x, \mathbf{k}_\perp) & = \delta(1 - x) \delta^{(2)}(\mathbf{k}_\perp) \lambda, \\ f_{(0)}^{i\sigma^{i+} \gamma_5}(x, \mathbf{k}_\perp) & = \delta(1 - x) \delta^{(2)}(\mathbf{k}_\perp) s_\perp^i, \end{aligned} \quad (15)$$

where λ denotes the helicity and $s_\perp^i$ the transverse spin of the parton quark i .

To continue this kind of calculation beyond the tree level, we have to expand the product of the enhanced gauge links and retain all terms contributing up to $\mathcal{O}(g^2)$. That is, employing the light-cone gauge $A^+ = (A \cdot n^-) = 0$, we have to evaluate

$$\begin{aligned} & \llbracket \infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp \rrbracket \cdot \llbracket \infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp \rrbracket_{A^+=0} \\ & = 1 - ig(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) - g^2(\mathcal{U}_4 + \mathcal{U}_5 + \dots + \mathcal{U}_{10}), \end{aligned} \quad (16)$$

where the individual contributions entering this equation are compiled in Table 1.³ In these expressions we used for the sake of convenience the longitudinal vector $u_\mu = p^+ n_\mu^-$ which has units of mass instead of the dimensionless light-cone vector n_μ^- , cf. Eq. (5).

The entries in Table 1 call for some comments and explanations. First, the fermion fields in the definition of the TMD PDF given by Eq. (10) are Heisenberg field operators, meaning that we have to use

$$\begin{aligned} \psi_i(\xi) & = e^{-ig \int d\eta \bar{\psi} \hat{A} \psi} \psi_i^{\text{free}}(\xi), \\ \left[\int d\eta \bar{\psi} \hat{A} \psi \right] & \equiv \int d^4\eta \bar{\psi}(\eta) \gamma_\mu \psi(\eta) A^\mu(\eta). \end{aligned} \quad (17)$$

³ The nonlinear part of the gluon tensor does not contribute in the considered order of the coupling.

Table 1

Individual virtual-gluon contributions appearing in the evaluation of the product of the gauge links in Eq. (16) up to the order $\mathcal{O}(g^2)$.

Symbols	Expressions	Fig. 3	Value
$\mathcal{U}_1$	$\int_0^\infty d\tau \, l \cdot \mathcal{A}(l\tau)$	(a)	$\neq 0$, [24]
$\mathcal{U}_2$	$\int_0^\infty d\tau \, S \cdot \mathcal{F}(u\tau)$	(b)	$\neq 0$, see text
$\mathcal{U}_3$	$\int_0^\infty d\tau \, S \cdot \mathcal{F}(l\tau)$	—	0, Eq. (21)
$\mathcal{U}_4$	$\int_0^\infty d\tau \int_0^\tau d\sigma \, (l \cdot \mathcal{A}(l\tau))(l \cdot \mathcal{A}(l\sigma))$	—	0, [24]
$\mathcal{U}_5$	$\int_0^\infty d\tau \int_0^\tau d\sigma \, (l \cdot \mathcal{A}(l\tau))(S \cdot \mathcal{F}(l\sigma))$	—	0, Eq. (21)
$\mathcal{U}_6$	$\int_0^\infty d\tau \int_0^\tau d\sigma \, (S \cdot \mathcal{F}(l\tau))(l \cdot \mathcal{A}(l\sigma))$	—	0, Eq. (21)
$\mathcal{U}_7$	$\int_0^\infty d\tau \int_0^\tau d\sigma \, (S \cdot \mathcal{F}(u\tau))(S \cdot \mathcal{F}(u\sigma))$	(c)	0, see text
$\mathcal{U}_8$	$\int_0^\infty d\tau \int_0^\tau d\sigma \, (S \cdot \mathcal{F}(l\tau))(S \cdot \mathcal{F}(l\sigma))$	—	0, Eq. (21)
$\mathcal{U}_9$	$\int_0^\infty d\tau \int_0^\infty d\sigma \, (l \cdot \mathcal{A}(l\tau))(S \cdot \mathcal{F}(u\sigma))$	(d)	$\neq 0$, see text
$\mathcal{U}_{10}$	$\int_0^\infty d\tau \int_0^\infty d\sigma \, (S \cdot \mathcal{F}(l\tau))(S \cdot \mathcal{F}(u\sigma))$	—	0, Eq. (21)

Therefore, the $\mathcal{O}(g)$ contributions $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3$ should be contracted with the quark–gluon interaction terms $[\int d\eta \, \bar{\psi} \hat{A} \psi]$, originating from the Heisenberg fields (17), in order to give rise to the one-gluon exchange graphs (a) and (b), which are of $\mathcal{O}(g^2)$. Second, all virtual-gluon terms $N_i \equiv \langle \mathcal{U}_i \rangle$ with $i = 1, \dots, 10$ produce contributions of the following generic form⁴

$$\begin{aligned} & \sim \int \frac{d\xi^- d^2 \xi_\perp}{2\pi(2\pi)^2} \exp(-i(p^+ - k^+)\xi^- + i\mathbf{k}_\perp \cdot \xi_\perp) \frac{1}{2} \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s}) \Gamma(N)] \\ & = \delta(p^+ - xp^+) \delta^{(2)}(\mathbf{k}_\perp) \frac{1}{2} \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s}) \Gamma(N)]. \end{aligned} \quad (18)$$

Their Hermitean conjugated (mirror) counterparts contribute terms of the form

$$\sim \delta(p^+ - xp^+) \delta^{(2)}(\mathbf{k}_\perp) \frac{1}{2} \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s}) \langle N \rangle^\dagger \Gamma]. \quad (19)$$

The Dirac structure of the quantities $\langle \mathcal{U}_i \rangle$ is nontrivial owing to the spin-dependent terms from the gauge links—which we will show explicitly below. On the other hand, the contributions of the real-gluon exchanges, stemming from the contractions of the gauge fields belonging to different planes in the ξ -space, will be considered in Section 6.

It is obvious that an analogous expansion has to be carried out in Eq. (10) also for the product of the gauge links $[\xi^-, \xi_\perp; \infty^-, \xi_\perp]^\dagger \cdot [\infty^-, \xi_\perp; \infty^-, \infty_\perp]^\dagger$. Let us emphasize that the various contributions of the Pauli term, evaluated along the n^- -lightlike direction, do not vanish completely when one employs the light-cone gauge—as opposed to the standard $\sim dx_\mu A^\mu$ term. Moreover, it was shown in [16,24,28–30,35] that the transverse gauge field in the axial gauge at light-cone infinity is given by a total derivative, viz.,

$$A^i(\infty^-, z^+, \mathbf{z}_\perp) = -\frac{1}{2} g \int \frac{dq^+ dq^-}{(2\pi)^2} \frac{e^{-iq^+ \infty^- - iq^- z^+}}{[q^+]} 2\pi \delta(q^-) \nabla^i \varphi(\mathbf{z}_\perp), \quad (20)$$

whereas the longitudinal components are equal to zero. Thus, the field-strength tensor on the transverse segment vanishes:

⁴ The bra-ket notation used will be explained in the next section.

$$F_a^{\mu\nu}(\infty^-, 0^+, \xi_\perp) = 0. \quad (21)$$

Therefore, expanding (16), only the terms with *longitudinal* spin-dependent contributions survive, while those with $\mathcal{F}(l\tau)$ (or $\mathcal{F}(l\sigma)$) cancel out. Nevertheless, we verify the vanishing of these terms by explicit calculation in the next section. Hence, by virtue of Eq. (20), expression (16) reduces to

$$\begin{aligned} & \llbracket \infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp \rrbracket \cdot \llbracket \infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp \rrbracket_{A^+=0} \\ &= 1 - ig \int_0^\infty d\tau l \cdot \mathcal{A}(l\tau) - ig \int_0^\infty d\tau S \cdot \mathcal{F}(u\tau) - g^2 \int_0^\infty d\tau \int_0^\tau d\sigma (l \cdot \mathcal{A}(l\tau))(l \cdot \mathcal{A}(l\sigma)) \\ & \quad - g^2 \int_0^\infty d\tau \int_0^\infty d\sigma (l \cdot \mathcal{A}(l\tau))(S \cdot \mathcal{F}(u\sigma)) + g^2 \int_0^\infty d\tau \int_0^\tau d\sigma (S \cdot \mathcal{F}(u\tau))(S \cdot \mathcal{F}(u\sigma)) \\ & \quad + O(g^3). \end{aligned} \quad (22)$$

4. Calculation of (virtual) gauge-field correlators

We are now able to calculate the spin-dependent contributions in Eq. (10), which we will do up to the g^2 -order level. Using light-cone coordinates (also in the transverse direction), the Pauli term reads

$$S \cdot \mathcal{F} \equiv S_{\mu\nu} \mathcal{F}^{\mu\nu} = 2S_{+-} \mathcal{F}^{+-} + 2S_{+i} \mathcal{F}^{+i} + 2S_{-i} \mathcal{F}^{-i} + S_{ij} \mathcal{F}^{ij}. \quad (23)$$

Imposing the light-cone gauge $\mathcal{A}^+ = 0$, we obtain the following non-zero components of the field-strength tensor:

$$\begin{aligned} \mathcal{F}^{+-} &= \partial^+ \mathcal{A}^-, & \mathcal{F}^{+i} &= \partial^+ \mathcal{A}^i, \\ \mathcal{F}^{-i} &= \partial^- \mathcal{A}^i - \partial^i \mathcal{A}^-, & \mathcal{F}^{ij} &= \partial^i \mathcal{A}^j - \partial^j \mathcal{A}^i. \end{aligned} \quad (24)$$

We proceed with the explicit calculation of the *virtual* gluon exchanges in Eq. (10), relegating the inclusion of real-gluon contributions to Section 6. The reason is that only the former are UV divergent and give rise to anomalous dimensions, while the latter contribute only UV-finite terms. To systematize the calculation of the various contributing correlators, we appeal to Table 1 in conjunction with Fig. 3. There are two different types of contributions: those terms in Eq. (16) which are proportional to g^2 stem from the evaluation of correlators between the standard gauge links and the enhanced ones. In Fig. 3 the latter are denoted by double lines with a ring attached to them in order to indicate the Pauli contribution which encodes spin effects. The standard gauge links are represented by simple double lines. The other contributions to Eq. (16), which are proportional to g , i.e., the terms $\mathcal{U}_1$, $\mathcal{U}_2$, and $\mathcal{U}_3$ in Table 1, have to be contracted with the gauge fields generated by the Heisenberg fermion operators, cf. (17), retaining again those terms which contribute to $\mathcal{O}(g^2)$. It is understood that each of these terms has to be averaged over the fluctuations of the gauge field via a functional integration. This is done with the aid of Eq. (27) using in what follows Dirac's bra-ket notation $\langle \dots \rangle$.⁵

The term $\langle \mathcal{U}_1 \rangle$ —graph (a) in Fig. 3—reduces in the considered order of the coupling to what one obtains with the standard gauge links; it has been computed in our previous work in Ref. [24].

⁵ Strictly speaking, one should write $\langle \dots \rangle_A$.

Fig. 3. Feynman graphs contributing to the quark-in-a-quark TMD PDF. Double lines denote standard gauge links, while those supplemented with a ring represent enhanced gauge links with tensor-field (Pauli) contributions. Fermions and gluons are shown as solid and curly lines, respectively. Graphs (a), (b), (c), and (d) describe virtual gluon corrections; graphs (e), (f), and (g) represent real-gluon exchanges across the cut (vertical dashed line). The conjugated (“mirror”) graphs are not shown for the sake of brevity.

Term $\langle \mathcal{U}_2 \rangle$ —corresponding to graph (b) in the same figure—will be worked out below, whereas term $\langle \mathcal{U}_3 \rangle$ vanishes by virtue of Eq. (21). For the same reason, also the contributions termed $\langle \mathcal{U}_5 \rangle$, $\langle \mathcal{U}_6 \rangle$, $\langle \mathcal{U}_8 \rangle$, and $\langle \mathcal{U}_{10} \rangle$ vanish as well. Moreover, it is proved in a few lines that $\langle \mathcal{U}_5 \rangle + \langle \mathcal{U}_6 \rangle = 0$. Term $\langle \mathcal{U}_4 \rangle$ was computed in [24] and was found to vanish, while the term $\langle \mathcal{U}_7 \rangle$, which represents the longitudinal self-energy contribution of the Pauli term (graph (c) in Fig. 3), will be computed further below; it amounts again to a vanishing contribution. Hence, the only remaining terms giving non-zero contributions are $\langle \mathcal{U}_2 \rangle$ and $\langle \mathcal{U}_9 \rangle$. The first one stems from the interaction of the longitudinal gauge field, produced by the fermion, with the Pauli term along the enhanced longitudinal link—graph (b) in Fig. 3—while the second one, represented by graph (d), describes the cross talk between the transverse gauge potential of the standard gauge link and the longitudinal part of the Pauli term (enhanced gauge link). Its calculation will be carried out below. Recall that the analogous cross talk between the longitudinal parts of the Pauli term and the standard gauge link vanishes because of Eq. (21).

Having sketched the general computational framework, let us now turn a spotlight on the calculation of the various terms, starting with $\langle \mathcal{U}_4 \rangle$, while the Fermion-induced terms $\langle \mathcal{U}_1 \rangle$, $\langle \mathcal{U}_2 \rangle$, and $\langle \mathcal{U}_3 \rangle$ will be picked up in Section 5. The term $\langle \mathcal{U}_4 \rangle$ represents the self-energy of the usual transverse gauge link and vanishes in the light-cone gauge [24]. This can be seen from the following equation

$$\int_0^\infty d\tau \, l \cdot \mathcal{A}(\infty^-, 0^+, l\tau) = \int \frac{dq^+}{2\pi} \frac{d^2 q_\perp}{(2\pi)^2} l \cdot \tilde{\mathcal{A}}(q) \frac{ie^{-iq^+ \infty^-}}{q \cdot l + i0}, \quad (25)$$

where we have used the Fourier transformation of the gauge field

$$\mathcal{A}_\mu(z) = \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot z} \tilde{\mathcal{A}}_\mu(q), \quad (26)$$

working in an ω -dimensional momentum space ($\omega = 4 - 2\epsilon$). Employing this expression in Eq. (25) and the gluon correlator in the light-cone gauge

$$\langle \mathcal{A}_\mu(q) \mathcal{A}_\nu(q') \rangle = (-i) C_F (2\pi)^4 \delta^{(4)}(q + q') D_{\mu\nu}(q), \quad (27)$$

in which the regularized free gluon propagator appears [cf. Eq. (4)],

$$D_{\mu\nu}(q) = \frac{2\pi\delta(q^-)}{q^2 + i0} \left(g_{\mu\nu} - \frac{q_\mu n_\nu^- + q_\nu n_\mu^-}{[q^+]} \right), \quad (28)$$

we get

$$\langle \mathcal{U}_4 \rangle = i C_F l_\perp^2 \int \frac{dq^+}{2\pi} \int_0^\infty d\tau \int_0^\tau d\sigma \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{e^{-iq_\perp \cdot l_\perp (\tau - \sigma)}}{q_\perp^2 + \lambda^2}. \quad (29)$$

One notes that the gluon propagator bears a pole-prescription dependence, codified by the symbol $[q^+]$, whereas its non-zero parts are given by

$$D_{i-} = -\frac{1}{q^2 + i0} \frac{q^i}{[q^+]}, \quad D_{ij} = i \frac{\delta_{ij}}{q^2 + i0}. \quad (30)$$

Despite the pole-prescription dependence of the gluon propagator, expression (29) does not depend on the pole prescription, because only the Feynman term of the gluon propagator contributes. Moreover, inspection of the last term in this equation reveals that it will be canceled by its mirror contribution anyway, i.e., finally,

$$\langle \mathcal{U}_4 \rangle = 0. \quad (31)$$

Note that this cancelation occurs in any case: polarized or unpolarized because there is no Dirac structure in this term.

The next two terms $\langle \mathcal{U}_5 \rangle$ and $\langle \mathcal{U}_6 \rangle$, which contain expressions of the sort $S \cdot \mathcal{F}(l_\perp \tau)$, can be treated in unison. To evaluate them we make use of the derivative of the transverse gauge field, viz.,

$$\mathcal{A}^i(l_\perp \sigma) = \int \frac{d^4 q}{(2\pi)^4} e^{-iq^+ \infty^- + iq_\perp \cdot l_\perp \sigma} \tilde{\mathcal{A}}^i(q). \quad (32)$$

For the transverse gauge strength at light-cone infinity, one has

$$\begin{aligned} \mathcal{F}^{+-}(\xi_\perp) &= \partial^+ \mathcal{A}^-(\xi_\perp) = 0, \\ \mathcal{F}^{-i}(\xi_\perp) &= -\partial^i \mathcal{A}^-(\xi_\perp), \\ \mathcal{F}^{ij}(\xi_\perp) &= \partial^i \mathcal{A}^j(\xi_\perp) - \partial^j \mathcal{A}^i(\xi_\perp), \end{aligned} \quad (33)$$

implying for the Pauli term in the transverse direction

$$S \cdot \mathcal{F}(l_\perp \sigma) = 2i \int \frac{d^4 q}{(2\pi)^4} e^{-iq^+ \infty^- + iq_\perp \cdot l_\perp \sigma} q_\perp^i [-S^{+i} \tilde{\mathcal{A}}^-(q) + S^{ij} \tilde{\mathcal{A}}^j(q)]. \quad (34)$$

Then we find

$$\begin{aligned} \langle \mathcal{U}_5 \rangle &= -2C_F \frac{1}{T} \int_0^\infty d\tau \int_0^\tau d\sigma \int \frac{dq^+}{2\pi} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp \cdot l_\perp (\tau - \sigma)} \frac{1}{q_\perp^2 + \lambda^2} \\ &\quad \times \left[-S^{+i} \frac{q_\perp^i (q_\perp \cdot l_\perp)}{[q^+]} + S^{ij} q_\perp^i l_\perp^j \right]. \end{aligned} \quad (35)$$

An analogous calculation for the term $\langle \mathcal{U}_6 \rangle$ yields

$$\langle \mathcal{U}_6 \rangle = -\langle \mathcal{U}_5 \rangle, \quad (36)$$

confirming that these two contributions cancel each other.

Going forth, we can now compute the longitudinal self-energy spin-dependent (Pauli) contribution [graph (c) in Fig. 3]

$$\langle \mathcal{U}_7 \rangle = \int_0^\infty d\tau \int_0^\tau d\sigma (S \cdot \mathcal{F}(u\tau))(S \cdot \mathcal{F}(u\sigma)) \quad (37)$$

using

$$S \cdot \mathcal{F}(u^- \tau) = 2i \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot u\tau} q^+ [S^{+-} \tilde{\mathcal{A}}^-(q) + S^{-i} \tilde{\mathcal{A}}^i(q)] \quad (38)$$

and employing the regularization embodied in Eq. (4) to obtain

$$\begin{aligned} \langle \mathcal{U}_7 \rangle = & -4i C_F \frac{1}{T} \int_0^\infty d\tau \int_0^\tau d\sigma \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot u(\tau-\sigma)} (q^+)^2 2\pi \delta(q^-) \frac{1}{q^2 - \lambda^2 + i0} \\ & \times \left[S^{+-} S^{+-} \frac{2q^-}{[q^+]} + S^{-i} S^{+-} \frac{2q_\perp^i}{[q^+]} - S^{-i} S^{-j} g^{ij} \right]. \end{aligned} \quad (39)$$

It is easy to see that all three terms in the square bracket give vanishing results: The first term is zero because of $\delta(q^-)$. The second term gives also zero due to the oddness of the transverse integral, while the last one vanishes by virtue of

$$S^{-i} S^{-i} = \frac{1}{4} (\gamma^- \gamma^i)^2 = 0$$

(recall that $S_{\mu\nu} = (1/4)[\gamma_\mu, \gamma_\nu]$). Therefore, we finally get

$$\langle \mathcal{U}_7 \rangle = 0. \quad (40)$$

We consider now the term $\langle \mathcal{U}_8 \rangle$ in more detail and prove that it vanishes. As we already mentioned in connection with Table 1, this term, which represents the self-interaction of the transverse gauge links with the Pauli terms at light-cone infinity, vanishes by virtue of the particular form of the transverse gauge field in the light-cone gauge (see Eq. (20) and the discussion below). Here, we give a more detailed derivation of this result. By definition, this term reads

$$\langle \mathcal{U}_8 \rangle = \int_0^\infty d\tau \int_0^\tau d\sigma (S \cdot \mathcal{F}(l\tau))(S \cdot \mathcal{F}(l\sigma)). \quad (41)$$

Therefore, we have

$$\begin{aligned} \langle \mathcal{U}_8 \rangle = & -4i C_F \frac{1}{T} \int_0^\infty d\tau \int_0^\tau d\sigma \int \frac{d^\omega q}{(2\pi)^\omega} q_\perp^i q_\perp^j e^{-iq_\perp \cdot l_\perp(\tau-\sigma)} \frac{2\pi \delta(q^-)}{q^2 - \lambda^2 + i0} \\ & \times \left[S^{+i} S^{+j} \frac{2q^-}{[q^+]} - S^{ik} S^{+j} \frac{2q_\perp^k}{[q^+]} - S^{ik} S^{jl} g^{kl} \right]. \end{aligned} \quad (42)$$

The term proportional to q^- vanishes by virtue of the delta-function. The second one is equal to zero because $S^{ik} q_{\perp}^i q_{\perp}^k = 0$. Taking into account that the Dirac structure of the last term can be rewritten as

$$S^{ik} S^{jk} q_{\perp}^i q_{\perp}^j = -\frac{1}{8} \gamma^j \gamma^i q_{\perp}^i q_{\perp}^j = \frac{1}{8} q_{\perp}^2, \quad (43)$$

one obtains

$$\begin{aligned} \langle \mathcal{U}_8 \rangle &= -4i C_F \frac{1}{T} \int_0^{\infty} d\tau \int_0^{\tau} d\sigma \int \frac{dq^+}{2\pi} \int \frac{d^{\omega-2} q}{(2\pi)^{\omega-2}} e^{-q_{\perp} \cdot l_{\perp} (\tau - \sigma)} \frac{q_{\perp}^2}{q_{\perp}^2 + \lambda^2} \\ &= -4i C_F \frac{1}{T} \int \frac{dq^+}{2\pi} \int_0^{\infty} d\tau \int_0^{\tau} d\sigma \frac{\delta^{(\omega-2)}(l_{\perp})}{|\tau - \sigma|^{\omega-2}} \\ &= 0 \end{aligned} \quad (44)$$

in agreement with the result presented in Table 1.

Consider next the mixed term $\langle \mathcal{U}_9 \rangle$, which expresses the correlation between the longitudinal Pauli term and the transverse gauge link (graph (d) in Fig. 3), viz.,

$$\langle \mathcal{U}_9 \rangle = \int_0^{\infty} d\tau \int_0^{\infty} d\sigma (l \cdot \mathcal{A}(l\tau)) (S \cdot \mathcal{F}(u\sigma)). \quad (45)$$

With the help of Eqs. (26), (38), and (27), we obtain

$$\begin{aligned} \langle \mathcal{U}_9 \rangle &= 2C_F \mu^{2\epsilon} \frac{1}{T} \int_0^{\infty} d\tau \int_0^{\infty} d\sigma \int \frac{d^{\omega} q}{(2\pi)^{\omega}} e^{iq^+ \infty - iq_{\perp} \cdot l_{\perp} \sigma - iq^+ u \tau} 2\pi \delta(q^-) q^+ \\ &\quad \times [S^{+-} l^i D^{i-}(q) + S^{-j} l^j D^{ij}(q)], \end{aligned} \quad (46)$$

which can be recast in the form

$$\begin{aligned} \langle \mathcal{U}_9 \rangle &= 2C_F \mu^{2\epsilon} \frac{1}{T} \int_0^{\infty} d\tau \int_0^{\infty} d\sigma \int \frac{d^{\omega} q}{(2\pi)^{\omega}} e^{iq^+ \infty - iq_{\perp} \cdot l_{\perp} \sigma - iq^+ u \tau} \frac{2\pi \delta(q^-)}{q^2 + i0} \\ &\quad \times [S^{+-} (q \cdot l) + S^{-i} l^i q^+] \end{aligned} \quad (47)$$

using Eq. (30). Observe the important fact that the dependence on the pole prescription disappeared in the above equation on account of $q^+/[q^+] = 1$, cf. Eq. (46). As a result, this equation is valid for the advanced, retarded, and principal value prescriptions, as well as for the Mandelstam–Leibbrandt pole prescription, though it is not obvious that it holds true in general (see, e.g., Refs. [40–44]).

The τ and σ integrations in Eq. (47) can be performed explicitly:

$$\int_0^{\infty} d\tau e^{-iq^+ u \tau} = \frac{-i}{q^+ u - i0}, \quad (48)$$

$$\int_0^{\infty} d\sigma e^{-iq_{\perp} \cdot l_{\perp} \sigma} = \frac{-i}{q \cdot l - i0}. \quad (49)$$

Making use of the following relation, which stems from the structure of the transverse gauge field at infinity [24,28,29],

$$\frac{e^{iq^+u\infty^-}}{q^+u-i0} = \frac{2\pi i}{u} \delta(q^+), \quad (50)$$

we get

$$\langle \mathcal{U}_9 \rangle = 2i C_F \mu^{2\epsilon} \frac{1}{Tu} S^{+-} \int \frac{d^{\omega-2} q}{(2\pi)^{\omega-2}} \frac{1}{q_\perp^2 + \lambda^2 - i0}, \quad (51)$$

where the “gluon mass” λ^2 was introduced in order to take care of infrared singularities in the gluon propagator. Taking into account that $Tu = 1$ and performing the $q_\perp$ integral

$$\int \frac{d^{\omega-2} q}{(2\pi)^{\omega-2}} \frac{1}{q_\perp^2 + \lambda^2 - i0} = \frac{i}{4\pi} \left(\frac{4\pi}{\lambda^2} \right)^\epsilon \Gamma(\epsilon), \quad (52)$$

we arrive at the following final result

$$\langle \mathcal{U}_9 \rangle = -\frac{1}{8\pi} C_F [\gamma^+, \gamma^-] \Gamma(\epsilon) \left(4\pi \frac{\mu^2}{\lambda^2} \right)^\epsilon \quad (53)$$

that gives rise to a UV divergence.

Its conjugated contribution, corresponding to the product of the gauge links $[\infty^-, \xi_\perp; \xi^-, \xi_\perp]$ $[\infty^-, \infty_\perp; \infty^-, \xi_\perp]$, amounts to the same expression (53), i.e.,

$$\langle \mathcal{U}_9 \rangle^\dagger = \langle \mathcal{U}_9 \rangle. \quad (54)$$

But there is a crucial difference: Now the Dirac matrix Γ in the definition of the TMD PDF stands on the *right side* of this expression—cf. Eq. (19). Because the Dirac structure of Eq. (53) is nontrivial, this will lead to different results. For instance, we get (using obvious abbreviations)

$$\begin{aligned} \text{(a)} \quad \Gamma_{\text{unpol.}} &= \gamma^+: \quad \Gamma_{\text{unpol.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-] \Gamma_{\text{unpol.}}, \\ \text{(b)} \quad \Gamma_{\text{helic.}} &= \gamma^+ \gamma^5: \quad \Gamma_{\text{helic.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-] \Gamma_{\text{helic.}}, \\ \text{(c)} \quad \Gamma_{\text{trans.}} &= i\sigma^{i+} \gamma^5: \quad \Gamma_{\text{trans.}}[\gamma^+, \gamma^-] = -[\gamma^+, \gamma^-] \Gamma_{\text{trans.}}. \end{aligned} \quad (55)$$

From the set of these equations we conclude that after taking into account the conjugated (mirror) contributions, all the leading-twist two functions mutually cancel by virtue of the relation

$$[\gamma^+, \gamma^-] \Gamma_{\text{tw-2}} = -\Gamma_{\text{tw-2}}[\gamma^+, \gamma^-] = 2\Gamma_{\text{tw-2}}. \quad (56)$$

This important property permits the probabilistic interpretation of twist-two TMD PDFs, because in every term $\bar{\psi} \Gamma \psi$, which behaves like a vector under z -boosts, the pole contribution entailed by the correlation between the transverse gauge link and the Pauli term along the longitudinal direction disappears.

Remarkably, higher-twist distribution functions (e.g., twist three), behave differently, the reason being that they are characterized by a different Dirac structure that remains invariant under z -boosts. For example, one has for $\Gamma_{\text{tw-3}} = \gamma^i$

$$[\gamma^+, \gamma^-] \Gamma_{\text{tw-3}} = \Gamma_{\text{tw-3}}[\gamma^+, \gamma^-], \quad (57)$$

so that the mutually conjugated contributions add to each other to give the net result

$$\Gamma_{\text{tw-3}}\langle\mathcal{U}_9\rangle + \langle\mathcal{U}_9\rangle^\dagger\Gamma_{\text{tw-3}} = -\frac{C_F}{4\pi}[\gamma^+, \gamma^-]\Gamma(\epsilon)\left(4\pi\frac{\mu^2}{\lambda^2}\right)^\epsilon \quad (58)$$

making it apparent that the pole contribution in that case is not vanishing.

The last point that has to be verified is that the term $\langle\mathcal{U}_{10}\rangle$ in Table 1 vanishes. We shall do that without assuming the special form of the gauge field at infinity given by Eq. (20). Making use of the explicit form of the gluon propagator (cf. (28)), one obtains

$$\begin{aligned} \langle\mathcal{U}_{10}\rangle = & -4iC_F\frac{1}{T}\int_0^\infty d\tau\int_0^\infty d\sigma\int\frac{d^\omega q}{(2\pi)^\omega}e^{iq^+(\infty-u\sigma)-iq_\perp\cdot l_\perp\tau}\frac{2\pi\delta(q^-)q^+}{q^2-\lambda^2+i0} \\ & \times\left[-S^{+i}S^{+-}\frac{2q_\perp^i q^-}{[q^+]}-S^{+i}S^{-j}\frac{q_\perp^i q_\perp^j}{[q^+]}+S^{ij}S^{+-}\frac{q_\perp^i q_\perp^j}{[q^+]}+S^{ij}S^{-j}q_\perp^i\right]. \end{aligned} \quad (59)$$

The first term equals zero due to the delta-function $\delta(q^-)$, while the third one vanishes by virtue of the antisymmetric–symmetric convolution $S^{ij}q_\perp^i q_\perp^j = 0$. Performing the longitudinal line integral and taking into account Eq. (50), that renders the last term vanishing as well, we reduce the above expression to

$$\langle\mathcal{U}_{10}\rangle = -4iC_F\frac{1}{T}S^{+i}S^{-j}\int_0^\infty d\tau\int\frac{d^{\omega-2}q}{(2\pi)^{\omega-2}}e^{iq_\perp\cdot l_\perp\tau}\frac{q_\perp^i q_\perp^j}{q_\perp^2+\lambda^2-i0}. \quad (60)$$

Given that

$$S^{+i}S^{-j}q_\perp^i q_\perp^j = \frac{1}{4}\gamma^+\gamma^-q_\perp^2, \quad (61)$$

we find after some standard calculations

$$\langle\mathcal{U}_{10}\rangle = iC_F\frac{1}{T}\gamma^+\gamma^-\int_0^\infty d\tau\int\frac{d^{\omega-2}q}{(2\pi)^{\omega-2}}\frac{q_\perp^2 e^{iq_\perp\cdot l_\perp\tau}}{q_\perp^2+\lambda^2-i0} = 0. \quad (62)$$

We thus conclude that the vanishing of the last term in Table 1 can be proved even without additional constraints on the gauge field at light-cone infinity, like Eq. (20).

5. Fermion (virtual) contributions

We will consider now the terms $\langle\mathcal{U}_1\rangle$ and $\langle\mathcal{U}_2\rangle$, pertaining to graphs (a) and (b) in Fig. 3, and also prove that the term $\langle\mathcal{U}_3\rangle$ gives zero contribution. These are the $\mathcal{O}(g)$ terms in the expansion (22) and have to be coupled to the fermion lines retaining their contributions up to the order g^2 . Term I has been considered in Ref. [24] and we will borrow the result from there:

$$\langle\mathcal{U}_1\rangle = C_F 2\pi i C_\infty \int \frac{d^\omega q}{(2\pi)^\omega} \frac{1}{(p-q)^2+i0} \frac{\delta(q^+)}{q^2-\lambda^2+i0}, \quad (63)$$

where the numerical factor $C_\infty = \{0; -1; -1/2\}$ corresponds to different choices of the imposed pole-prescription in the light-cone gluon propagator (see Refs. [24,35]). The UV-singularity produced by this contribution, notably,

$$\langle\mathcal{U}_1\rangle^{\text{UV}} = -\alpha_s C_F \frac{1}{\epsilon} i C_\infty$$

just cancels the prescription-dependent term in the UV-divergent part of the fermion self-energy graph in the light-cone gauge, bearing no relation to the spin-dependent part in question.

The first novel contribution, ensuing from the Pauli term, is represented by the term $\langle \mathcal{U}_2 \rangle$ (graph (b) in Fig. 3) making use of the notation we already employed in Eq. (16). Hence the Pauli term with the tensor gauge field in the longitudinal direction becomes

$$S \cdot \mathcal{F}(u\tau) = 2i \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot u\tau} q^+ [S^{+-} \tilde{A}^-(q) + S^{-i} \tilde{A}^i(q)]. \quad (64)$$

It stems from the interaction of the quark with the spin-dependent part of the longitudinal gauge link in the Pauli term. Consider first its longitudinal component, which we termed $\langle \mathcal{U}_2^- \rangle$, whereas for the transverse one we will use the notation $\langle \mathcal{U}_2^\perp \rangle$. Then, we have

$$\begin{aligned} \langle \mathcal{U}_2^- \rangle &= -2C_F \frac{1}{T} \int_0^\infty d\tau \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot u\tau} 2\pi \delta(q^-) q^+ \frac{1}{(p-q)^2 + i0} \frac{1}{q^2 - \lambda^2 + i0} \frac{1}{[q^+]} \\ &\quad \times [S^{+-} (\hat{p} - \hat{q}) \gamma^+ 2q^- + S^{-i} (\hat{p} - \hat{q}) \gamma^+ q_\perp^i]. \end{aligned} \quad (65)$$

Taking into account that $(\gamma^- \gamma^- = 0, p_\perp^i = 0)$

$$S^{-i} (\hat{p} - \hat{q}) \gamma^+ q_\perp^i = S^{-i} [\gamma^- \gamma^+ (p^+ - q^+) q_\perp^i + \gamma^j \gamma^+ q_\perp^j q_\perp^i] = -\frac{1}{2} \gamma^- \gamma^+ \mathbf{q}_\perp^2, \quad (66)$$

the pole-prescription-dependent term, containing $1/[q^+]$, cancels out and we get

$$\langle \mathcal{U}_2^- \rangle = C_F \gamma^- \gamma^+ \frac{1}{T} \int_0^\infty d\tau \int \frac{d^{\omega-2} \mathbf{q}_\perp}{(2\pi)^{\omega-2}} \frac{dq^+}{2\pi} e^{-iq \cdot u\tau} \frac{1}{p^2 - \mathbf{q}_\perp^2 + i0}. \quad (67)$$

Using the fact that $uT = 1$, we finally obtain

$$\langle \mathcal{U}_2^- \rangle = \frac{1}{2} C_F \gamma^- \gamma^+ \int \frac{d^{\omega-2} \mathbf{q}_\perp}{(2\pi)^{\omega-2}} \frac{1}{p^2 - \mathbf{q}_\perp^2 + i0}. \quad (68)$$

Turning our attention to the conjugated contribution, we find out that the ordering of the Dirac matrices has changed:

$$\begin{aligned} \langle \mathcal{U}_2^- \rangle^\dagger &= -2C_F \frac{1}{T} \int_0^\infty d\tau \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot u\tau} 2\pi \delta(q^-) q^+ \frac{1}{(p-q)^2 + i0} \frac{1}{q^2 - \lambda^2 + i0} \frac{1}{[q^+]} \\ &\quad \times [-\gamma^+ (\hat{p} - \hat{q}) S^{+-} 2q^- - \gamma^+ (\hat{p} - \hat{q}) S^{-i} q_\perp^i]. \end{aligned} \quad (69)$$

Therefore, one has

$$\langle \mathcal{U}_2^- \rangle^\dagger = \frac{1}{2} C_F \gamma^+ \gamma^- \int \frac{d^{\omega-2} \mathbf{q}_\perp}{(2\pi)^{\omega-2}} \frac{1}{p^2 - \mathbf{q}_\perp^2 - i0}. \quad (70)$$

Thus, combining these terms, we have for the leading-twist distribution the following final result

$$\Gamma_{\text{tw-2}} \langle \mathcal{U}_2^- \rangle + \langle \mathcal{U}_2^- \rangle^\dagger \Gamma_{\text{tw-2}} = (-i2\pi) C_F \Gamma_{\text{tw-2}} \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} \delta(\mathbf{q}_\perp^2 - p^2) = \frac{i}{2} C_F \Gamma_{\text{tw-2}}. \quad (71)$$

This contribution is UV finite and can be given a physical interpretation. Indeed, recalling Eq. (16), we can express the above result in the form of a constant phase

$$e^{-i\delta} \approx 1 - \frac{i}{2} g^2 C_F, \quad (72)$$

inherited to the TMD PDF by the Pauli term along the longitudinal gauge link. It is worth noting that this finding is valid not only for the unpolarized case with $\Gamma_{\text{tw-2}} = \gamma^+$, but also for the polarized case ($\Gamma_{\text{tw-2}} = \gamma^+ \gamma^5$ or $\Gamma_{\text{tw-2}} = i\sigma^{i+} \gamma^5$). This is because the (leading) twist-two distribution functions are vectors under boosts along the z -direction and their generic Dirac structure has the property

$$\Gamma_{\text{tw-2}} \gamma^- \gamma^+ = \gamma^+ \gamma^- \Gamma_{\text{tw-2}} = 2\Gamma_{\text{tw-2}}. \quad (73)$$

It is precisely this property that gave rise to the constant phase δ and its cause can be traced to the correlation of the longitudinal gauge field, concomitant to the fermions, with the spin-dependent part of the longitudinal gauge link. This phase could, in principle, be absorbed into the soft factor R (cf. (9)). However, this cannot be done in a universal way because the phase sign depends on the direction of the longitudinal gauge link, i.e., on the specific deformation of the integration contour via the $i\epsilon$ prescription. Inverting this direction, the phase factor (72) changes its sign and becomes

$$e^{-i\delta} \rightarrow e^{+i\delta}.$$

Therefore, the phases appearing in the SIDIS and the DY process turn out to have opposite signs:

$$\delta_{\text{SIDIS}} = -\delta_{\text{DY}}. \quad (74)$$

These UV features do not persist for the twist-three distributions. Indeed, their Dirac structures are invariant under z -boosts and behave like scalars, i.e.,

$$\gamma^- \gamma^+ \Gamma_{\text{tw-3}} = \Gamma_{\text{tw-3}} \gamma^- \gamma^+, \quad \Gamma_{\text{tw-3}} \gamma^+ \gamma^- = \gamma^+ \gamma^- \Gamma_{\text{tw-3}}. \quad (75)$$

As a result, the analogous expression to (71) now reads

$$\Gamma_{\text{tw-3}} \langle \mathcal{U}_2^- \rangle + \langle \mathcal{U}_2^- \rangle^\dagger \Gamma_{\text{tw-3}} = -\frac{1}{2} C_F \left\{ \frac{1}{4\pi} [\gamma^+, \gamma^-] \left(4\pi \frac{\mu^2}{p^2} \right)^\epsilon \Gamma(\epsilon) - \frac{i}{2} \right\} \Gamma_{\text{tw-3}}. \quad (76)$$

This quantity is UV divergent, meaning that the Pauli spin-dependent term will contribute to the anomalous dimension of the twist-three TMD PDF.

We focus now on the interaction of the longitudinal spin-dependent gauge link and the transverse part of the gauge field originating from the fermions, namely, the term $\langle \mathcal{U}_2^\perp \rangle$:

$$\begin{aligned} \langle \mathcal{U}_2^\perp \rangle = & -2C_F \frac{1}{T} \int_0^\infty d\tau \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot u\tau} 2\pi \delta(q^-) q^+ \frac{1}{(p-q)^2 + i0} \frac{1}{q^2 - \lambda^2 + i0} \frac{1}{[q^+]} \\ & \times [-S^{+-}(\hat{p} - \hat{q}) \gamma^j q_\perp^j + S^{-i}(\hat{p} - \hat{q}) \gamma^j q^+ g^{ij}]. \end{aligned} \quad (77)$$

Making use of the following simplifications of the terms with Dirac matrices, i.e.,

$$\begin{aligned} S^{+-}(\hat{p} - \hat{q}) \gamma^j q_\perp^j &= S^{+-} [\gamma^-(p^+ - q^+) + \gamma^+(p^- - q^-) + \gamma^k q^k] \gamma^j q_\perp^j \\ &\rightarrow -\frac{1}{4} [\gamma^+, \gamma^-] q_\perp^2, \end{aligned} \quad (78)$$

and

$$S^{-i}(\hat{p} - \hat{q})\gamma^i = S^{-i}[\gamma^-(p^+ - q^+) + \gamma^+(p^- - q^-) + \gamma^k q^k]\gamma^i \rightarrow 0, \quad (79)$$

and proceeding along similar lines of thought as in the previous case, we find

$$\langle \mathcal{U}_2^\perp \rangle = -\frac{1}{4}C_F[\gamma^+, \gamma^-] \int \frac{d^{\omega-2}q_\perp}{(2\pi)^{\omega-2}} \frac{1}{p^2 - q_\perp^2 + i0} \quad (80)$$

and

$$\langle \mathcal{U}_2^\perp \rangle^\dagger = -\frac{1}{4}C_F[\gamma^+, \gamma^-] \int \frac{d^{\omega-2}q_\perp}{(2\pi)^{\omega-2}} \frac{1}{p^2 - q_\perp^2 - i0}, \quad (81)$$

so that

$$\Gamma_{\text{tw-2}}\langle \mathcal{U}_2^\perp \rangle + \langle \mathcal{U}_2^\perp \rangle^\dagger \Gamma_{\text{tw-2}} = -\frac{i}{4}C_F \Gamma_{\text{tw-2}}. \quad (82)$$

The remarks which we have made in connection with $\langle \mathcal{U}_2^- \rangle$ apply equally well to Eq. (82). The computed phase is acquired through the interaction of the Pauli term along the longitudinal link with the transverse part of the gauge field accompanying the fermions and has to be added to the phase originating from the analogous interaction between the Pauli term and the longitudinal gauge field associated to the fermions.⁶

Thus, the full phase, ensuing from the interaction of the fermion fields with the spin-dependent (Pauli) term in the gauge links—crosstalk diagram (b) in Fig. 3—is given according to $\langle \mathcal{U}_2 \rangle$ by

$$e^{-i\delta} \approx 1 - i \frac{g^2}{4\pi} C_F \pi, \quad (83)$$

where we have again taken into account Eq. (16). As we have already noted, this phase flips sign when the direction of the longitudinal link is reversed. As regards the twist-three distribution, we obtain

$$\Gamma_{\text{tw-3}}\langle \mathcal{U}_2^\perp \rangle + \langle \mathcal{U}_2^\perp \rangle^\dagger \Gamma_{\text{tw-3}} = \frac{1}{4}C_F \frac{2}{4\pi}[\gamma^+, \gamma^-] \left(4\pi \frac{\mu^2}{p^2} \right)^\epsilon \Gamma(\epsilon) \Gamma_{\text{tw-3}}. \quad (84)$$

Comparison with Eq. (76) reveals that, taking their UV divergent parts together, their total contribution disappears leaving behind only a constant phase which, moreover, coincides with the one found for the twist-two distributions: $\delta_{\text{tw-2}} = \delta_{\text{tw-3}} = \alpha_s C_F \pi$.

We complete our discussion of the fermion virtual contributions by considering the term $\langle \mathcal{U}_3 \rangle$ in Table 1, which describes the cross talk of the gauge field surrounding the fermions with the transverse spin-dependent gauge link. The discussion proceeds along similar lines as that of the previous term. Likewise, $\langle \mathcal{U}_3 \rangle$ consists of two terms: $\langle \mathcal{U}_3^- \rangle$ and $\langle \mathcal{U}_3^+ \rangle$. Consider first the contribution pertaining to the longitudinal gluons emanating from the quark fields:

$$\begin{aligned} \langle \mathcal{U}_3^- \rangle = & -2C_F \frac{1}{T} \int_0^\infty d\tau \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot \infty^- + iq_\perp \cdot l_\perp \tau} \frac{2\pi \delta(q^-)}{(p-q)^2 + i0} \frac{1}{q^2 - \lambda^2 + i0} \\ & \times \left[-S^{+i}(\hat{p} - \hat{q})\gamma^+ \frac{2q^- q_\perp^i}{[q^+]} + S^{ij}(\hat{p} - \hat{q})\gamma^+ \frac{q_\perp^i q_\perp^j}{[q^+]} \right]. \end{aligned} \quad (85)$$

⁶ The appearance of an imaginary contribution in the cusp anomalous dimension (unrelated to the Pauli term) was already discussed by Korchemsky and Radyushkin in [32].

This contribution vanishes because the first term equals zero by virtue of the delta-function $\delta(q^-)$, while the second one also reduces to zero due to the convolution $S^{ij}q_\perp^i q_\perp^j$.

Continuing with the contribution from the transverse gluons produced by the quark field, we write

$$\begin{aligned} \langle \mathcal{U}_3^\perp \rangle = & -2C_F \frac{1}{T} \int_0^\infty d\tau \int \frac{d^\omega q}{(2\pi)^\omega} e^{-iq \cdot \infty^- + iq_\perp \cdot l_\perp \tau} \frac{2\pi \delta(q^-)}{(p-q)^2 + i0} \frac{1}{q^2 - \lambda^2 + i0} \\ & \times \left[-S^{+i}(\hat{p} - \hat{q})\gamma^k \frac{2q_\perp^i q_\perp^k}{[q^+]} + S^{ij}(\hat{p} - \hat{q})\gamma^j q_\perp^i \right]. \end{aligned} \quad (86)$$

After performing the following transformations of the Dirac matrices

$$S^{+i}(\hat{p} - \hat{q})\gamma^k \rightarrow S^{+i}\gamma^- \gamma^k (p^+ - q^+) - S^{+i}q_\perp^k, \quad (87)$$

and making use of Eq. (20), one can recast Eq. (86) into the form

$$\begin{aligned} \langle \mathcal{U}_3^\perp \rangle = & -4C_F \frac{1}{T} C_\infty \left(\frac{1}{i4\pi} \right)^{1-\epsilon} \gamma^+ \int_0^\infty d\tau \int \frac{d^{\omega-2} q}{(2\pi)^{\omega-2}} e^{iq_\perp \cdot l_\perp \tau} \frac{\gamma^- p^+ - \gamma^i q_\perp^i}{q_\perp^2 - p^2 + i0} \\ = & 4C_F \frac{1}{T} \frac{C_\infty}{i4\pi} \gamma^+ \int_0^\infty d\tau e^{i\tau(p^2 - i0)} \left[\gamma^- p^+ \tau^{\epsilon-1/2} \sqrt{\frac{i\pi}{l_\perp^2}} + i \frac{\gamma^i l_\perp^i}{l_\perp^2} \tau^{\epsilon-1} \right]. \end{aligned} \quad (88)$$

Taking into account that $|l_\perp| \sim p^+$ and $T \sim p^+$, one sees that both terms in the square bracket are power suppressed. The first one, which is of $\mathcal{O}(|p|/p^+)$ does not diverge; the second is of $\mathcal{O}(p^2/(p^+)^2)$ and is logarithmically divergent. In any case, both terms can be left out because we are only interested in the leading-twist contributions. Therefore, for our analysis the correlation between the transverse part of the Pauli term in the enhanced gauge link and the transverse gauge field produced by the fermion can be ignored.

6. Real-gluon contributions

So far, we have presented the results of the calculation of the virtual gluon graphs, which (potentially) contribute to the UV-singularities of the TMD PDFs. Now let us turn to the real-gluon graphs, (e), (f), (g) in Fig. 3. The formal computation of these contributions is quite similar to that we already performed for the evaluation of the virtual graphs (b), (c), (d) in the same figure.

The main differences are:

- (i) The discontinuity goes now across the gluon propagator, so that one has to replace it with the cut one. Then, in the light-cone gauge, we have⁷

$$\begin{aligned} D^{\mu\nu} = & \frac{i}{q^2 - \lambda^2 + i0} \left(-g^{\mu\nu} + \frac{q^\mu n^{-\nu} + q^\nu n^{-\mu}}{[q^+]_{\text{PV}}} \right) \rightarrow \\ \text{Disc} D^{\mu\nu}(q) = & 2\pi\theta(q^+) \delta(q^2 - \lambda^2) \left(-g^{\mu\nu} + \frac{q^\mu n^{-\nu} + q^\nu n^{-\mu}}{[q^+]_{\text{PV}}} \right). \end{aligned} \quad (89)$$

⁷ We omit here the discussion of the Mandelstam–Leibbrandt pole prescription [36,37], making the tacit assumption that the regularization of the $[q^+]$ pole is q^- -independent. This will allow us to avoid an additional term in the cut propagator.

Table 2

Individual real-gluon contributions corresponding to the diagrams (e), (f), (g) in Fig. 3 and retaining terms up to order $\mathcal{O}(g^2)$.

Symbols	Expressions	Fig. 3
$\mathcal{U}_{11}$	$\int_0^\infty d\tau \int_0^\infty d\sigma (S \cdot \mathcal{F}(u\tau)) \Gamma(S \cdot \mathcal{F}(u\sigma + \xi^-; \xi_\perp))$	(e)
$\mathcal{U}_{12}$	$\int_0^\infty d\tau \int_0^\infty d\sigma (\mathbf{l} \cdot \mathcal{A}(\mathbf{l}\tau)) \Gamma(S \cdot \mathcal{F}(u\sigma + \xi^-; \xi_\perp))$	(f)
$\mathcal{U}_{13}$	$\int_0^\infty d\sigma \Gamma(S \cdot \mathcal{F}(u\sigma + \xi^-; \xi_\perp))$	(g)

- (ii) The Dirac structures, abbreviated by Γ , stand now between Dirac matrices from the Pauli terms on different sides of the cut.
- (iii) The momentum delta-functions involve, apart from the “external” momenta p^+ and $(k^+ = xp^+, \mathbf{k}_\perp)$, also the “internal” loop momentum q^μ .

The real-gluon contributions are listed in Table 2 using analogous notations to those in Table 1.

We start with the graph describing the interaction of two spin-dependent gauge links, as depicted in Fig. 3(e):

$$\begin{aligned} \langle \mathcal{U}_{11} \rangle &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{i(p^+ - k^+)\xi^- - i\mathbf{k}_\perp \cdot \xi_\perp} \\ &\times \int_0^\infty d\tau \int_0^\infty d\sigma (S \cdot \mathcal{F}(u\tau)) \Gamma(S \cdot \mathcal{F}(u\sigma + \xi^-; \xi_\perp)). \end{aligned} \quad (90)$$

Making use of the cut propagator (89), and taking into account that the line integrals go along different paths so that they have not to be ordered, one has

$$\begin{aligned} \langle \mathcal{U}_{11} \rangle &= -4C_F \frac{1}{T} \int_0^\infty d\tau \int_0^\infty d\sigma \\ &\times \int \frac{d^4q}{(2\pi)^4} e^{-i(q^+ \cdot u - i0)\tau + i(q^+ u + i0)\sigma} \delta((1-x)p^+) \delta^{(2)}(\mathbf{k}_\perp - \mathbf{q}_\perp) \\ &\times (q^+)^2 2\pi \delta(q^-) \theta(q^+) \delta(q^2 - \lambda^2) \\ &\times \left[S^{+-} \Gamma S^{+-} \frac{2q^-}{[q^+]} + (S^{-i} \Gamma S^{+-} + S^{+-} \Gamma S^{-i}) \frac{q_\perp^i}{[q^+]} - S^{-i} \Gamma S^{-j} g^{ij} \right], \end{aligned} \quad (91)$$

noting that the dimensional regularization becomes redundant in this case because all momentum integrals are finite due to the delta-functions.

The d^4q integration becomes trivial and hence we get

$$\langle \mathcal{U}_{11} \rangle \sim \left[(S^{-i} \Gamma S^{+-} + S^{+-} \Gamma S^{-i}) \frac{\mathbf{k}_\perp^i}{|1-x|p^+} - S^{-i} \Gamma S^{-i} \right]. \quad (92)$$

We can now make use of the following formulas

$$S^{-i} \Gamma S^{+-} + S^{+-} \Gamma S^{-i} = \frac{1}{2} \gamma^i, \quad S^{-i} \Gamma S^{-i} = \frac{1}{2} \gamma^- \quad (93)$$

(assuming, for instance, $\Gamma = \gamma^+$) and perform an averaging with the help of (11) to find

$$\begin{aligned}\text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s})\gamma^i] &= 0, \\ \text{Tr}[(\hat{p} + m)(1 + \gamma_5 \hat{s})\gamma^-] &= 2p^-.\end{aligned}\quad (94)$$

From this expression one may conclude that—within the given kinematics—the term $\langle \mathcal{U}_{11} \rangle$ is power suppressed.

The next term represents the interaction between the longitudinal spin-dependent gauge link and the transverse gauge link at infinity—Fig. 3, graph (f). It reads

$$\langle \mathcal{U}_{12} \rangle = \int_0^\infty d\tau \int_0^\infty d\sigma (\mathbf{l} \cdot \mathcal{A}(\mathbf{l}\tau)) \Gamma(S \cdot \mathcal{F}(u\sigma + \xi^-; \xi_\perp)). \quad (95)$$

The further evaluation is analogous to that in Eq. (46), giving the result

$$\begin{aligned}\langle \mathcal{U}_{12} \rangle &= C_F \frac{1}{T} \int_0^\infty d\tau \int_0^\infty d\sigma \int \frac{d^4 q}{(2\pi)^4} \delta((1-x)p^+ - q^+) \delta^{(2)}(\mathbf{k}_\perp - \mathbf{q}_\perp) 2\pi \delta(q^-) q^+ \\ &\quad \times e^{-iq^+ \infty - iq_\perp \cdot \mathbf{l}_\perp \sigma - iq^+ u \tau} 2\pi \delta(q^2 - \lambda^2) \theta(q^+) \Gamma[S^{+-} \mathbf{l} \cdot \mathbf{q} + S^{-j} l^j q^+].\end{aligned}\quad (96)$$

The last term in the square bracket disappears due to the delta-function $\delta(q^+)$, cf. Eq. (50). Thus, one gets for the sum of diagram Fig. 3(f) and its mirror counterpart a vanishing contribution:

$$\langle \mathcal{U}_{12} \rangle + \langle \mathcal{U}_{12} \rangle^\dagger \sim \Gamma[\gamma^+, \gamma^-] + [\gamma^+, \gamma^-] \Gamma = 0. \quad (97)$$

We complete our task by calculating the contribution associated with diagram (g) in Fig. 3, which stems from the interaction of the Pauli term with the fermion field. Because it is of $\mathcal{O}(g)$, it has to be contracted with the gluon field in the quark–gluon interaction term $[\bar{\psi} \hat{A} \psi]$ in order to contribute at order g^2 . The combined contributions graph (g) and its conjugate) of the longitudinal part are determined by the following combinations of Dirac matrices:

$$\langle \mathcal{U}_{13}^- \rangle + \langle \mathcal{U}_{13}^- \rangle^\dagger \sim S^{-i} \Gamma(\hat{p} - \hat{q}) \gamma^+ - \gamma^+ (\hat{p} - \hat{q}) \Gamma S^{-i}. \quad (98)$$

Analogously, we find for the transverse part

$$\begin{aligned}\langle \mathcal{U}_{13}^\perp \rangle + \langle \mathcal{U}_{13}^\perp \rangle^\dagger \\ \sim S^{-i} \Gamma(\hat{p} - \hat{q}) \gamma^i - \gamma^i (\hat{p} - \hat{q}) \Gamma S^{-i} - S^{+-} \Gamma(\hat{p} - \hat{q}) \gamma^j + \gamma^j (\hat{p} - \hat{q}) \Gamma S^{+-}.\end{aligned}\quad (99)$$

After trivial manipulations with the Dirac matrices in the equations above, we finally arrive at

$$\langle \mathcal{U}_{13}^- \rangle + \langle \mathcal{U}_{13}^- \rangle^\dagger \sim x p^+ (\gamma^- \Gamma + \Gamma \gamma^-) \gamma^i \quad (100)$$

and

$$\langle \mathcal{U}_{13}^\perp \rangle + \langle \mathcal{U}_{13}^\perp \rangle^\dagger \sim -x p^+ [(\gamma^- \Gamma + \Gamma \gamma^-) \gamma^i + 2\gamma^- \Gamma \gamma^-]. \quad (101)$$

From these results we conclude that these terms mutually cancel up to a power-suppressed correction.

The main message from the computation of the real-gluon graphs containing spin-dependent terms is that they do not contribute to the TMD PDF in the leading-twist order. All physically important effects have their roots in the contributions of virtual-gluon exchanges. Exactly those diagrams are responsible for time-reversal-odd effects in more sophisticated models as we shall argue in the discussion to follow.

7. Summary and conclusions

In this work we have presented a new gauge-invariant scheme for TMD PDFs which takes into account in the gauge links (Wilson lines) the Pauli term. This term describes the communication of spin degrees of freedom with the gauge field via the gauge-field strength and provides a rendering geared to calculational purposes of TMD PDFs for spinning partons. The key features of our approach can be summarized as follows:

- The spin-dependent Pauli term, incorporated in the TMD PDFs as integral part of the gauge links, does not affect their UV-singular behavior in leading-twist order. Therefore, the structure of the gauge links in the soft factor, introduced before in [24,30] with the aim to define TMD PDFs in terms of gauge-invariant matrix elements with standard renormalization properties does not need to be changed. This proves the usefulness of the subtraction method, proposed in [33,34], which provides a tool to deal with rapidity divergences that cannot be controlled by dimensional regularization. In fact, in our present analysis (and also in [24, 30]) we employed a soft renormalization factor in order to cure overlapping UV and rapidity divergences and compensate this way the associated one-loop cusp anomalous dimension.
- However, the Pauli term contributes to the UV-divergences of the imaginary parts of the cut diagrams with virtual gluon exchanges. Though these effects cancel in the final result of the considered distribution of a quark in a quark, they signalize that, within a more realistic context involving quark models with spectators [45,46], the spin-dependent terms may contribute to the interference diagrams, where these imaginary parts become crucial.
- By contrast, we found that the UV singularities of the higher-twist TMD PDFs (starting at twist three) are affected by the spin-dependent terms receiving contributions to their anomalous dimensions, which now become a matrix (see Eq. (58)). This is caused by an incomplete cancelation of UV divergences related to the fact that the z -boost induced by the Pauli term along the longitudinal link—pointing in one direction—is not counteracted by the conjugate contribution—pointing in the opposite direction. The net result is that only boosts and rotations around the transverse directions are left over and these give rise to a constant phase (see next item). Thus, to remedy the definition of such TMD PDFs as densities, one has to compensate these divergences by introducing the Pauli term also into the soft factor.
- An important consequence of the presence of the Pauli term in the gauge links is that it gives rise to a phase entanglement, attributable to the interaction of this spin-dependent term with the companion gauge field of the fermion—diagram (b) in Fig. 3.⁸ In technical jargon, the Pauli term along the longitudinal link generates z boosts, canceled by the conjugate link, and rotations along and around the transverse x and y directions, while the analogous term in the transverse gauge link produces boosts and rotations only along and around the transverse directions. The rotations of the longitudinal and the transverse link combine to produce a constant phase. It turns out that this phase correlation is the same for the leading twist-two and the subleading twist-three TMD PDFs, multiplying each of them as a whole. This means that absorbing this phase into the soft factor for the leading distribution, the corresponding phase of the subleading functions is also removed, even though, as we explained in the previous item, these latter functions may lack a density interpretation. However, the Pauli-term-induced phase is not universal because it depends on the direction of the longitu-

⁸ No such phase is induced by diagram (d), the reason being that this diagram does not involve a fermion line.

Fig. 4. Diagrams with a Pauli spin-dependent term, which may contribute to the time-reversal-odd phenomena. The initial/final-state boundary is denoted by a long vertical dashed line.

Fig. 5. The Feynman rules for the calculation of the one-gluon-exchange graphs shown in Fig. 4 in the light-cone gauge and using enhanced gauge links that include the Pauli term. Rules are given for both sides of the final-state cut (long vertical dashed line). Vertical double lines represent the transverse gauge links at light-cone infinity, while the horizontal ones with arrowed rings are the spin-dependent light-like gauge links with the Pauli terms.

dinal gauge link. Reversing the direction of the gauge link, the phase flips its sign. Hence, it contributes with the opposite sign to the DY process relative to a SIDIS situation. This breakdown of universality indicates that the soft renormalization factor R does not fully decouple from the spin effects. For this to be the case, one would have to include into the definition of R spin-dependent terms (cf. Eq. (9)) and evaluate it along a topologically nontrivial contour (work in progress).

- To facilitate calculations with enhanced gauge links, we derive Feynman rules for both the left-hand side and the right-hand side of the final-state cut and display them in Fig. 5. The spin-dependent gauge-link propagator and vertices in the light-cone gauge are displayed in terms of double lines with arrowed rings around them. These Feynman rules may be viewed as supplementing the set of Feynman rules given in [4] for covariant gauges. Pay attention that the propagator of the transverse gauge link contains (in addition to the standard term originating from the line integration in momentum space) a numerical factor C_∞ , which encodes the dependence on the pole-prescription—see Eq. (20). The cancelation of this dependence due to the soft factor is discussed in detail in our previous works [24,30].

Let us now close our discussion by commenting upon possible consequences of the spin-dependent terms for models with spectators. The time-reversal-odd TMD PDFs, like the Sivers or the Boer–Mulders function, which are responsible for observable single-spin asymmetries (SSA)s, can be calculated by means of the graphs presented in Fig. 4—see for a recent analysis in [47]. Such SSAs emerge as the result of the interference of the contributions of type (a) and (b) with their counterparts which bear no gluon exchanges. For instance, within the MIT bag model, non-vanishing time-reversal-odd TMD PDFs appear due to the interplay of the effects of the quark wave functions in the one-gluon-interference diagrams (see, e.g., Refs. [48–51]). Therefore, in view of our results, one may conclude that the imaginary contributions (taken without their conjugated “mirror” counterparts), which derive from the spin-dependent gauge links, can affect the time-reversal-odd TMD PDFs even at the leading-twist level. For instance, the Sivers function of a quark having a flavor α is given by

$$f_{1T}^{\perp\alpha}(x, \mathbf{k}_\perp) \sim [f_\alpha^{\gamma^+}(x, \mathbf{k}_\perp; S_\perp) - f_\alpha^{\gamma^+}(x, \mathbf{k}_\perp; -S_\perp)], \quad (102)$$

so that it is defined by the sum of the imaginary parts of the diagrams (a) and (b) in Fig. 4. This important finding and its phenomenological implications deserve further exploration and verification.

Acknowledgements

We thank Anatoly Efremov for useful discussions and remarks. This work was supported in part by the Heisenberg–Landau Program, Grants 2009 and 2010, and the INFN. A.I.K. thanks the DAAD for a research stipend at Bochum University in the academic year 2009. I.O.Ch. is grateful to Professor Maxim Polyakov for the hospitality extended to him during a visit to Bochum University, during which the major part of this work was done, and the BMBF under Grant 06BO9012 for financial support.

References

- [1] D.E. Soper, Phys. Rev. D 15 (1977) 1141.
- [2] D.E. Soper, Phys. Rev. Lett. 43 (1979) 1847.

- [3] J.C. Collins, D.E. Soper, Nucl. Phys. B 193 (1981) 381;
J.C. Collins, D.E. Soper, Nucl. Phys. B 213 (1983) 545 (Erratum).
- [4] J.C. Collins, D.E. Soper, Nucl. Phys. B 194 (1982) 445.
- [5] J.C. Collins, Acta Phys. Pol. B 34 (2003) 3103.
- [6] A. Bacchetta, U. D'Alesio, M. Diehl, C.A. Miller, Phys. Rev. D 70 (2004) 117504.
- [7] A.V. Belitsky, A.V. Radyushkin, Phys. Rep. 418 (2005) 1.
- [8] U. D'Alesio, F. Murgia, Prog. Part. Nucl. Phys. 61 (2008) 394.
- [9] P. Schweitzer, T. Teckentrup, A. Metz, Phys. Rev. D 81 (2010) 094019.
- [10] A. Bacchetta, D. Boer, M. Diehl, P.J. Mulders, JHEP 0808 (2008) 023.
- [11] J.C. Collins, D.E. Soper, G. Sterman, Adv. Ser. Direct. High Energy Phys. 5 (1988) 1.
- [12] X. Ji, J. Ma, F. Yuan, Phys. Rev. D 71 (2005) 034005.
- [13] J.C. Collins, T.C. Rogers, A.M. Stasto, Phys. Rev. D 77 (2008) 085009.
- [14] J. Collins, J.W. Qiu, Phys. Rev. D 75 (2007) 114014.
- [15] T.C. Rogers, P.J. Mulders, Phys. Rev. D 81 (2010) 094006.
- [16] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201.
- [17] J.C. Collins, A. Metz, Phys. Rev. Lett. 93 (2004) 252001.
- [18] C.J. Bomhof, P.J. Mulders, Nucl. Phys. B 795 (2008) 409.
- [19] J. Collins, PoS LC2008 (2008) 028.
- [20] I.O. Cherednikov, N.G. Stefanis, arXiv:0911.1031 [hep-ph].
- [21] A.V. Efremov, A.V. Radyushkin, Riv. Nuovo Cimento 3N2 (1980) 1.
- [22] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438, Yad. Fiz. 15 (1972) 781;
V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 675, Yad. Fiz. 15 (1972) 1218;
L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94, Yad. Fiz. 20 (1974) 181;
Y.L. Dokshitzer, JETP 46 (1977) 641, Zh. Eksp. Teor. Fiz. 73 (1977) 1216.
- [23] G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298.
- [24] I.O. Cherednikov, N.G. Stefanis, Nucl. Phys. B 802 (2008) 146.
- [25] N.S. Craigie, H. Dorn, Nucl. Phys. B 185 (1981) 204.
- [26] S. Aoyama, Nucl. Phys. B 194 (1982) 513.
- [27] N.G. Stefanis, Nuovo Cimento A 83 (1984) 205.
- [28] X. Ji, F. Yuan, Phys. Lett. B 543 (2002) 66.
- [29] A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B 656 (2003) 165.
- [30] I.O. Cherednikov, N.G. Stefanis, Phys. Rev. D 77 (2008) 094001.
- [31] N.G. Stefanis, I.O. Cherednikov, Mod. Phys. Lett. A 24 (2009) 2913.
- [32] G.P. Korchemsky, A.V. Radyushkin, Nucl. Phys. B 283 (1987) 342.
- [33] J.C. Collins, F. Hautmann, Phys. Lett. B 472 (2000) 129.
- [34] F. Hautmann, Phys. Lett. B 655 (2007) 26.
- [35] I.O. Cherednikov, N.G. Stefanis, Phys. Rev. D 80 (2009) 054008.
- [36] S. Mandelstam, Nucl. Phys. B 213 (1983) 149.
- [37] G. Leibbrandt, Phys. Rev. D 29 (1984) 1699.
- [38] N.G. Stefanis, Diploma thesis, Heidelberg University, 1979, unpublished.
- [39] R.D. Tangerman, P.J. Mulders, Phys. Rev. D 51 (1995) 3357.
- [40] D.M. Capper, J.J. Dulwich, M.J. Litvak, Nucl. Phys. B 241 (1984) 463.
- [41] G. Leibbrandt, S.L. Nyeo, Phys. Lett. B 140 (1984) 417.
- [42] A. Bassetto, I.A. Korchemskaya, G.P. Korchemsky, G. Nardelli, Nucl. Phys. B 408 (1993) 62.
- [43] A. Bassetto, Nucl. Phys. Proc. Suppl. 51C (1996) 281.
- [44] A. Bassetto, G. Heinrich, Z. Kunszt, W. Vogelsang, Phys. Rev. D 58 (1998) 094020.
- [45] L.P. Gamberg, G.R. Goldstein, M. Schlegel, Phys. Rev. D 77 (2008) 094016.
- [46] A. Bacchetta, F. Conti, M. Radici, Phys. Rev. D 78 (2008) 074010.
- [47] B. Pasquini, F. Yuan, Phys. Rev. D 81 (2010) 114013.
- [48] F. Yuan, Phys. Lett. B 575 (2003) 45.
- [49] I.O. Cherednikov, U. D'Alesio, N.I. Kochelev, F. Murgia, Phys. Lett. B 642 (2006) 39.
- [50] H. Avakian, A.V. Efremov, P. Schweitzer, O.V. Teryaev, F. Yuan, P. Zavada, Mod. Phys. Lett. A 24 (2009) 2995.
- [51] H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan, Phys. Rev. D 81 (2010) 074035.