Coexistence of species with different dispersal across landscapes : a critical role of spatial correlation in disturbance

Electronic supplementary material

Coexistence of species with different dispersal across landscapes: a critical role of spatial correlation in disturbance

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Supplementary Figure S1

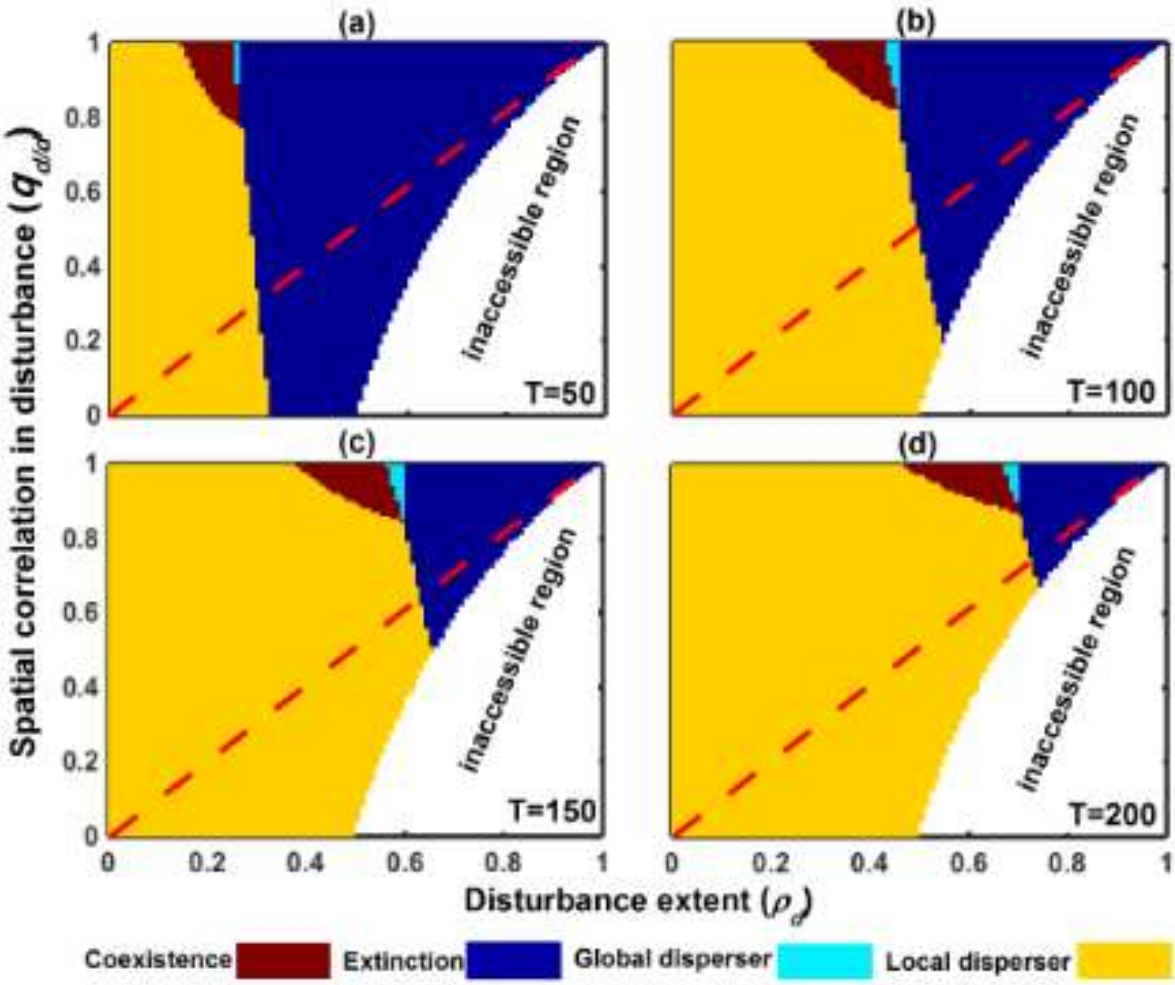

Figure S1. Effect of disturbance periodicity (T) on the competitive outcome of a local disperser interacting with a global disperser (red dashed line at $\rho_d = q_{d/d}$ indicating randomly structured disturbance), while simultaneously varying disturbance extent ρ_d and spatial aggregation $q_{d/d}$. (a) T=50, (b) T=100, (c) T=150 and (d) T=200. Other parameters: see figure 3 except mortality rate m_2=0.014 (global disperser). Inaccessible region: see equation (2).

Electronic supplementary material

Supplementary Methods

Constructing a closed dynamical system for two dispersers

In this model, we choose ρ_1, ρ_2, $q_{1/1}$, $q_{2/2}$ and $q_{1/2}$ as independent variables so as to construct a closed dynamical system for two competitors. According to the definition of local density [1], we have

$$q_{i/j} = \rho_{ij} / \rho_j \quad (i,j \in \{0, 1, 2\}), \tag{S1}$$

where local density $q_{i/j} \in [0,1]$ is the conditional probability of a neighbouring site for a randomly chosen individual *j* being occupied by an individual *i*, and pair density ρ_{ij} represents the probability when choosing a pair of neighbouring sites at random that one of them is species *i* while another one is species *j*. Based on equation (S1), we can further derive the dynamics of $q_{i/j}$ as

$$\frac{dq_{i/j}}{dt} = -\frac{q_{i/j}}{\rho_j}\frac{d\rho_j}{dt} + \frac{1}{\rho_j}\frac{d\rho_{ij}}{dt}. \tag{S2}$$

Therefore, we obtain

$$\begin{cases} \dfrac{dq_{1/1}}{dt} = -\dfrac{q_{1/1}}{\rho_1}\dfrac{d\rho_1}{dt} + \dfrac{1}{\rho_1}\dfrac{d\rho_{11}}{dt} \\ \dfrac{dq_{2/2}}{dt} = -\dfrac{q_{2/2}}{\rho_2}\dfrac{d\rho_2}{dt} + \dfrac{1}{\rho_2}\dfrac{d\rho_{22}}{dt} \\ \dfrac{dq_{2/1}}{dt} = -\dfrac{q_{2/1}}{\rho_1}\dfrac{d\rho_1}{dt} + \dfrac{1}{\rho_1}\dfrac{d\rho_{12}}{dt} \end{cases}. \tag{S3}$$

For the transition rate of pair density ρ_{11}, we have

$$\frac{d\rho_{11}}{dt} = 2\rho_{10}\left(\frac{\alpha}{z} + \frac{z-1}{z}\alpha \cdot q_{1/01}\right) - 2\rho_{11}\left(m_1 + \frac{\gamma_{11}}{z} + \frac{z-1}{z}\gamma_{11}q_{1/11} + \frac{z-1}{z}\gamma_{12}q_{2/11}\right), \tag{S4}$$

in which the first term on the right hand describes the transition of the pair 0-1 (or 1-0) becoming 1-1 due to local dispersal. The empty site in the pair 0-1 can be colonized by two pathways: local dispersal from the individual 1 in the pair of 0-1 (with the probability 1/z) or the potential presence of individual 1 from other z-1 neighbours of the target 0-site (z=4). The second term is the transition of the pair 1-1 becoming 1-0 (or 0-1) because of species mortality. Besides species intrinsic morality (m_1), the death of a target individual 1 in the pair 1-1 is also influenced by both the already presence of a neighbouring individual 1 in the pair of 1-1 sites and the potential presence of conspecific individual 1 from its remaining z-1 neighbours (i.e. intraspecific competition), as well as the potential presence of heterospecific individual 2 in the z-1 neighbours (i.e. interspecific competition). The parameter $q_{i/jk}$ (e.g. $q_{1/01}$, $q_{1/11}$ and $q_{2/11}$) is the triplet conditional probability for an *i*-site which is another neighbour of a *j*-site in the pair of *j*-*k*. According to the pair approximation principle [1], we have

$$q_{i/jk} \approx q_{i/j} \quad (i,j,k \in \{0, 1, 2\}). \tag{S5}$$

Similar to equation (S4), the change rate of the pair density ρ_{22} can be written as

$$\frac{d\rho_{22}}{dt} = 2\beta\rho_2\rho_{20} - 2\rho_{22}\left(m_2 + \frac{\gamma_{22}}{z} + \frac{z-1}{z}\gamma_{22}q_{2/22} + \frac{z-1}{z}\gamma_{21}q_{1/22}\right), \tag{S6}$$

where the only difference is the first term in which the vacant site in the pair of 0-2 sites becomes occupied by seed random dispersal across the whole landscape.

Following the same principle, we can derive the dynamics of the pair density ρ_{12} as

$$\begin{aligned}\frac{d\rho_{12}}{dt} = {} & \beta\rho_2\rho_{10} + \frac{z-1}{z}\alpha q_{1/02}\rho_{20} - \rho_{12}\left(m_1 + \frac{\gamma_{12}}{z} + \frac{z-1}{z}\gamma_{12}q_{2/12} + \frac{z-1}{z}\gamma_{11}q_{1/12}\right) \\ & - \rho_{12}\left(m_2 + \frac{\gamma_{21}}{z} + \frac{z-1}{z}\gamma_{21}q_{1/21} + \frac{z-1}{z}\gamma_{22}q_{2/21}\right)\end{aligned}. \tag{S7}$$

In order to form the closed system, we should consider the following constraints

$$\begin{cases} \rho_0 + \rho_1 + \rho_2 = 1 \\ q_{i/j} = q_{j/i}\rho_i / \rho_j \\ q_{i/j} = 1 - q_{j/j} - q_{k/j} \end{cases}, \tag{S8}$$

with $i,j,k \in \{0, 1, 2\}$.

Combining equations (1a,b) and (S3-S8), we obtain

$$\begin{aligned} \frac{dq_{1/1}}{dt} = & -q_{1/1}\left[\alpha\left(1-q_{1/1}-q_{2/1}\right)+m_1+\frac{\gamma_{11}}{2}+\frac{1}{2}\gamma_{11}q_{1/1}+\frac{1}{2}\gamma_{12}q_{2/1}\right] \\ & +\left(1-q_{1/1}-q_{2/1}\right)\left(\frac{\alpha}{2}+\frac{3}{2}\alpha\rho_1\frac{1-q_{1/1}-q_{2/1}}{1-\rho_1-\rho_2}\right) \end{aligned}, \tag{S9}$$

$$\begin{aligned} \frac{dq_{2/2}}{dt} = & -q_{2/2}\left[\beta\left(1-\rho_1-\rho_2\right)+m_2+\frac{\gamma_{22}}{2}+\frac{1}{2}\gamma_{22}q_{2/2}+\frac{1}{2}\gamma_{21}q_{2/1}\rho_1/\rho_2\right], \\ & +2\beta\rho_2\left(1-\rho_1 q_{2/1}/\rho_2-q_{2/2}\right) \end{aligned} \tag{S10}$$

$$\begin{aligned} \frac{dq_{2/1}}{dt} = & -q_{2/1}\Bigg[\alpha\left(1-q_{1/1}-q_{2/1}\right)-\frac{1}{4}\left(\gamma_{11}q_{1/1}+\gamma_{12}q_{2/1}\right) \\ & +m_2+\frac{\gamma_{21}+\gamma_{12}}{4}+\frac{3}{4}(\gamma_{21}\rho_1 q_{2/1}/\rho_2+\gamma_{22}q_{2/2})\Bigg] \\ & +\beta\rho_2\left(1-q_{1/1}-q_{2/1}\right)+\frac{3}{4}\alpha\rho_2\frac{1-q_{1/1}-q_{2/1}}{1-\rho_1-\rho_2}\left(1-q_{2/2}-\rho_1 q_{2/1}/\rho_2\right) \end{aligned}. \tag{S11}$$

Therefore, equations (1a,b) and (S9-S11) with five variables (ρ_1, ρ_2, $q_{1/1}$, $q_{2/2}$ and $q_{1/2}$) form a closed mathematical system for two competitors with different dispersal modes in a homogeneous landscape.

Similar to equation (3), when the periodic pulse disturbance (occurring at $t = nT,\ n \in N$) is incorporated into the PA model, we can derive the dynamics of species clumping by combining equations (8) and (S9-S11)

$$
\left\{
\begin{aligned}
\frac{dq_{1/1}(t)}{dt} &= -q_{1/1}(t)\left[\alpha\left(1-q_{1/1}(t)-q_{2/1}(t)\right)+m_1+\frac{\gamma_{11}}{2}+\frac{1}{2}\gamma_{11}q_{1/1}(t)+\frac{1}{2}\gamma_{12}q_{2/1}(t)\right] \\
&\quad +\left(1-q_{1/1}(t)-q_{2/1}(t)\right)\left(\frac{\alpha}{2}+\frac{3}{2}\alpha\rho_1(t)\frac{1-q_{1/1}(t)-q_{2/1}(t)}{1-\rho_1(t)-\rho_2(t)}\right) \\
\frac{dq_{2/2}(t)}{dt} &= -q_{2/2}(t)\left[\beta\left(1-\rho_1(t)-\rho_2(t)\right)+m_2+\frac{\gamma_{22}}{2}+\frac{1}{2}\gamma_{22}q_{2/2}(t)+\frac{1}{2}\gamma_{21}q_{2/1}(t)\rho_1(t)/\rho_2(t)\right] \\
&\quad +2\beta\rho_2(t)\left(1-\rho_1(t)q_{2/1}(t)/\rho_2(t)-q_{2/2}(t)\right) \\
\frac{dq_{2/1}(t)}{dt} &= -q_{2/1}\left[\alpha\left(1-q_{1/1}(t)-q_{2/1}(t)\right)-\frac{1}{4}\left(\gamma_{11}q_{1/1}(t)+\gamma_{12}q_{2/1}(t)\right)+m_2+\frac{\gamma_{21}+\gamma_{12}}{4}\right. \\
&\quad \left.+\frac{3}{4}\left(\gamma_{21}\rho_1(t)q_{2/1}(t)/\rho_2(t)+\gamma_{22}q_{2/2}(t)\right)\right]+\beta\rho_2(t)\left(1-q_{1/1}(t)-q_{2/1}(t)\right) \\
&\quad +\frac{3}{4}\alpha\rho_2(t)\frac{1-q_{1/1}(t)-q_{2/1}(t)}{1-\rho_1(t)-\rho_2(t)}\left(1-q_{2/2}(t)-\rho_1(t)q_{2/1}(t)/\rho_2(t)\right)
\end{aligned}
\right\} \quad t \neq nT,\ n \in N.
\tag{S12}
$$

$$
\left\{
\begin{aligned}
q_{1/1}(t^+) &= q_{1/1}(t)(1-2\rho_d+\rho_d q_{d/d})/(1-\rho_d) \\
q_{2/2}(t^+) &= q_{2/2}(t)(1-2\rho_d+\rho_d q_{d/d})/(1-\rho_d) \\
q_{2/1}(t^+) &= q_{2/1}(t)(1-2\rho_d+\rho_d q_{d/d})/(1-\rho_d)
\end{aligned}
\right\} \quad t = nT,\ n \in N
$$

Consequently, equations (3) and (S12) construct a dynamical system of two competing species in the face of spatially correlated periodic disturbance.

References

1. Matsuda H, Ogita N, Sasaki A, Satō K. 1992 Statistical mechanics of population. *Prog. Theor. Phys.* **88**, 1035–1049. (doi: 10.1143/ptp/88.6.1035)

Supplementary Model Analysis

A. Model analysis of mean-field approximation model (MFA)

According to the mean-field approximation with $q_{i/j} \approx \rho_i$, equation (1) can be rewritten as

$$\begin{cases} \dfrac{d\rho_1}{dt} = \alpha\rho_1\left(1-\rho_1-\rho_2\right)-\left(m_1+\gamma_{11}\rho_1+\gamma_{12}\rho_2\right)\rho_1 \\ \dfrac{d\rho_2}{dt} = \beta\rho_2\left(1-\rho_1-\rho_2\right)-\left(m_2+\gamma_{22}\rho_2+\gamma_{21}\rho_1\right)\rho_2 \end{cases}. \tag{S13}$$

In equation (S13), the semi-trivial equilibrium is given by $E_1 = (0, \frac{\beta-m_2}{\beta+\gamma_{22}})$, which exists and is non-trivial if and only if $\beta > m_2$, and $E_2 = (\frac{\alpha-m_1}{\alpha+\gamma_{11}}, 0)$, which exists and is nontrivial if and only if $\alpha > m_1$. There also exists a unique positive equilibrium $E^* = (\rho_1^*, \rho_2^*)$ with

$$\begin{aligned} \rho_1^* &= \frac{\left(\alpha-m_1\right)\left(\beta+\gamma_{22}\right)-\left(\beta-m_2\right)\left(\alpha+\gamma_{12}\right)}{\left(\alpha+\gamma_{11}\right)\left(\beta+\gamma_{22}\right)-\left(\beta+\gamma_{21}\right)\left(\alpha+\gamma_{12}\right)} \\ \rho_2^* &= \frac{\left(\beta-m_2\right)\left(\alpha+\gamma_{11}\right)-\left(\alpha-m_1\right)\left(\beta+\gamma_{21}\right)}{\left(\alpha+\gamma_{11}\right)\left(\beta+\gamma_{22}\right)-\left(\beta+\gamma_{21}\right)\left(\alpha+\gamma_{12}\right)} \end{aligned},$$

if and only if $\dfrac{\left(\beta+\gamma_{22}\right)}{\left(\alpha+\gamma_{12}\right)} > \dfrac{\left(\beta-m_2\right)}{\left(\alpha-m_1\right)} > \dfrac{\left(\beta+\gamma_{21}\right)}{\left(\alpha+\gamma_{11}\right)}$.

We now investigate the stability of the semi-trivial equilibria by separately linearizing the system (S13) near the equilibria E_1 and E_2, thus yielding the Jacobian matrices

$$\begin{aligned} J\left(E_1\right) &= \begin{bmatrix} \alpha\left(1-2\rho_1-\rho_2\right)-\left(m_1+2\gamma_{11}\rho_1+\gamma_{12}\rho_2\right) & -\alpha\rho_1-\gamma_{12}\rho_1 \\ -\beta\rho_2-\gamma_{21}\rho_2 & \beta\left(1-\rho_1-2\rho_2\right)-\left(m_2+2\gamma_{22}\rho_2+\gamma_{21}\rho_1\right) \end{bmatrix} \\ &= \begin{bmatrix} \alpha\left(1-\rho_2\right)-\left(m_1+\gamma_{12}\rho_2\right) & 0 \\ * & \beta\rho_2\left(1-2\rho_2\right)-\left(m_2+2\gamma_{22}\rho_2\right)\rho_2 \end{bmatrix} \end{aligned}$$

$$=\begin{bmatrix} \alpha\left(1-\frac{\beta-m_2}{\beta+\gamma_{22}}\right)-\left(m_1+\gamma_{12}\frac{\beta-m_2}{\beta+\gamma_{22}}\right) & 0 \\ * & \beta\left(1-2\frac{\beta-m_2}{\beta+\gamma_{22}}\right)-\left(m_2+2\gamma_{22}\frac{\beta-m_2}{\beta+\gamma_{22}}\right) \end{bmatrix}$$

and

$$J\left(E_2\right)=\begin{bmatrix} \alpha\left(1-2\rho_1\right)-\left(m_1+2\gamma_{11}\rho_1\right) & * \\ 0 & \beta\left(1-\rho_1\right)-\left(m_2+\gamma_{21}\rho_1\right) \end{bmatrix}$$

$$=\begin{bmatrix} \alpha\left(1-2\frac{\alpha-m_1}{\alpha+\gamma_{11}}\right)-\left(m_1+2\gamma_{11}\frac{\alpha-m_1}{\alpha+\gamma_{11}}\right) & * \\ 0 & \beta\left(1-\frac{\alpha-m_1}{\alpha+\gamma_{11}}\right)-\left(m_2+\gamma_{21}\frac{\alpha-m_1}{\alpha+\gamma_{11}}\right) \end{bmatrix}.$$

It is seen that E_1 is locally asymptotically stable if $\frac{\alpha-m_1}{\alpha+\gamma_{12}}<\frac{\beta-m_2}{\beta+\gamma_{22}}$, while E_2 is locally asymptotically stable if $\frac{\beta-m_2}{\beta+\gamma_{21}}<\frac{\alpha-m_1}{\alpha+\gamma_{11}}$. But if the converse inequalities hold, then the respective equilibria are unstable.

To investigate the stability of the unique positive equilibrium $E^*=(\rho_1^*,\rho_2^*)$, we assume that the existence condition holds. Linearizing the system (S13) around the equilibrium E^* yields the Jacobian matrix

$$J\left(E^*\right)=\begin{bmatrix} -\alpha\rho_1-\gamma_{11}\rho_1 & -\alpha\rho_1-\gamma_{12}\rho_1 \\ -\beta\rho_2-\gamma_{21}\rho_2 & -\beta\rho_2-\gamma_{22}\rho_2 \end{bmatrix}.$$

If $\frac{(\beta+\gamma_{22})}{(\alpha+\gamma_{12})}>\frac{(\beta+\gamma_{21})}{(\alpha+\gamma_{11})}$, E^* is locally asymptotically stable.

Therefore, we can summarize the analytical results of the MFA model without considering disturbance as follows (see Table S1).

Table S1 The local stability analysis of MFA model

Equilibria	Existence	Local stability
$E_1 = (0, \frac{\beta - m_2}{\beta + \gamma_{22}})$	$\beta > m_2$	$\frac{\beta + \gamma_{22}}{\alpha + \gamma_{12}} < \frac{\beta - m_2}{\alpha - m_1}$
$E_2 = (\frac{\alpha - m_1}{\alpha + \gamma_{11}}, 0)$	$\alpha > m_1$	$\frac{\beta - m_2}{\alpha - m_1} < \frac{\beta + \gamma_{21}}{\alpha + \gamma_{11}}$
$E^* = (\rho_1^*, \rho_2^*)$	$\frac{\beta+\gamma_{22}}{\alpha+\gamma_{12}} > \frac{\beta-m_2}{\alpha-m_1} > \frac{\beta+\gamma_{21}}{\alpha+\gamma_{11}}$ and $\beta > m_2$ and $\alpha > m_1$	$\frac{\beta + \gamma_{22}}{\alpha + \gamma_{12}} > \frac{\beta + \gamma_{21}}{\alpha + \gamma_{11}}$

B. Model analysis of pair approximation (PA) model

The pair-approximation model allows us to derive some analytical conditions for successful invasion. We first summarize the PA system as follows:

$$\begin{cases} \dfrac{d\rho_1}{dt} = \alpha\rho_1\left(1-q_{1/1}-q_{2/1}\right)-\left(m_1+\gamma_{11}q_{1/1}+\gamma_{12}q_{2/1}\right)\rho_1 \\ \dfrac{d\rho_2}{dt} = \beta\rho_2\left(1-\rho_1-\rho_2\right)-\left(m_2+\gamma_{22}q_{2/2}+\gamma_{21}q_{1/2}\right)\rho_2 \\ \dfrac{dq_{1/1}}{dt} = \dfrac{\alpha}{2}q_{0/1}\left(1+3q_{1/0}\right)-q_{1/1}\left[\alpha\left(1-q_{1/1}-q_{2/1}\right)+m_1+\dfrac{\gamma_{11}}{2}+\dfrac{\gamma_{11}}{2}q_{1/1}+\dfrac{\gamma_{12}}{2}q_{2/1}\right] \\ \dfrac{dq_{2/2}}{dt} = 2\beta\rho_{20}-q_{2/2}\left[\beta\left(1-\rho_1-\rho_2\right)+m_2+\dfrac{\gamma_{22}}{2}+\dfrac{\gamma_{22}}{2}q_{2/2}+\dfrac{\gamma_{21}}{2}q_{1/2}\right] \\ \dfrac{dq_{1/2}}{dt} = -q_{1/2}\left[m_1+\beta\left(1-\rho_1-\rho_2\right)\right]-\dfrac{q_{1/2}}{4}\left[3\left(\gamma_{11}q_{1/1}+\gamma_{12}q_{2/1}\right)+\gamma_{12}+\gamma_{21}-\gamma_{22}q_{2/2}-\gamma_{21}q_{1/2}\right] \\ \qquad +\beta\rho_{10}+\dfrac{3}{4}\alpha q_{0/2}q_{1/0} \end{cases}, \quad \text{(S14)}$$

in which $q_{i/j} = \rho_i \cdot q_{j/i} / \rho_j \ (i, j = 0,\ 1,\ 2)$.

Here we consider the case in which the population is near the equilibrium where only species 1 (or species 2) exists, and explore whether the rare species 2 (or species 1) can increase in the PA system.

Case I: Invasion by species 2 to a population dominated by species 1

We first calculate the equilibrium global and local densities of species 1 with local dispersal by neglecting the rare invasive species 2 ($\rho_2 = 0, q_{2/1} = 0$), consequently simplifying equation (S14) as

$$\begin{cases} \dfrac{d\rho_1}{dt} = \alpha\rho_1\left(1-q_{1/1}\right) - \rho_1\left(m_1 + \gamma_{11}q_{1/1}\right) \\ \dfrac{dq_{1/1}}{dt} = \dfrac{\alpha}{2}q_{0/1}\left(1+3q_{1/0}\right) - q_{1/1}\left[\alpha\left(1-q_{1/1}\right) + m_1 + \dfrac{\gamma_{11}}{2}(1+q_{1/1})\right] \end{cases}. \tag{S15}$$

Therefore, we obtain the non-trivial equilibrium E^*=($\rho_1^*, q_{1/1}^*$) with

$$\begin{cases} \rho_1^* = \dfrac{3\alpha^2 - m_1\left(4\alpha+\gamma_{11}\right)}{\left(3\alpha - m_1\right)\left(\alpha+\gamma_{11}\right)} \\ q_{1/1}^* = \dfrac{\alpha - m_1}{\alpha+\gamma_{11}} \end{cases}.$$

As a result, E^* exists and is locally asymptotically stable if and only if $\alpha > m_1$ and $3\alpha\left(\alpha - m_1\right)/\left(\alpha+\gamma_{11}\right) > m_1$.

Next, combined with $\rho_2 = 0$ (so $\rho_{20} = 0$) and E^*=($\rho_1^*, q_{1/1}^*$), and if we abbreviate $x = q_{2/2}$ and $y = q_{1/2}$, equation (S14) become

$$\begin{cases} \dfrac{dx}{dt} = -2m_2x - \dfrac{x}{2}\left(\gamma_{22} + 3\gamma_{22}x + 3\gamma_{21}y\right) - x\left[\dfrac{3\alpha\beta\left(m_1+\gamma_{11}\right)}{\left(3\alpha-m_1\right)\left(\alpha+\gamma_{11}\right)} - \left(m_2 + \gamma_{22}x + \gamma_{21}y\right)\right] \\ \dfrac{dy}{dt} = \beta\rho_{10}^* + \dfrac{3\alpha}{4}\left(1-x-y\right)q_{1/0}^* - \beta\left(1-\rho_1\right)y - m_1y - \dfrac{y}{4}\left(3\gamma_{11}\dfrac{\alpha-m_1}{\alpha+\gamma_{11}} + \gamma_{12} + \gamma_{21} - \gamma_{22}x - \gamma_{21}y\right) \end{cases},$$

where $q_{1/0}^* = \dfrac{\rho_1^*}{1-\rho_1^*}\left(1-q_{1/1}^*\right) = q_{1/1}^* - \dfrac{m_1}{3\alpha}$ and $\rho_{10}^* = \rho_0^*q_{1/0}^* = (1-\rho_1^*)q_{1/0}^*$.

This autonomous dynamical system has a single non-negative equilibrium ($0 \le x \le 1; 0 \le y \le 1$) that is stable and obtained by letting $\frac{dx}{dt}=0$ and $\frac{dy}{dt}=0$, so

$$\begin{cases} -2m_2x-\frac{x}{2}\left(\gamma_{22}+3\gamma_{22}x+3\gamma_{21}y\right)=x\left[\frac{3\alpha\beta\left(m_1+\gamma_{11}\right)}{\left(3\alpha-m_1\right)\left(\alpha+\gamma_{11}\right)}-\left(m_2+\gamma_{22}x+\gamma_{21}y\right)\right] \\ 4\beta\rho_{10}^{*}+3\alpha q_{1/0}^{*}\left(1-y\right)=4\beta\left(1-\rho_1^{*}\right)y+\left(4m_1+3\gamma_{11}q_{1/1}^{*}+\gamma_{12}+\gamma_{21}-\gamma_{21}y\right)y \end{cases}. \quad \text{(S16)}$$

Thus, we can obtain x=0, while the analytical solution of $y \in [0,1]$ can be achieved from

$$\gamma_{21}y^2-\left[4\beta\left(1-\rho_1^{*}\right)+\gamma_{12}+\gamma_{21}+3\alpha\right]y+4\beta\rho_{10}^{*}+3\alpha q_{1/0}^{*}=0 .$$

If invasive rare species 2 (seed global dispersal) with very small population ($\rho_2 \approx 0$) is introduced into the system (S15), and if $\frac{d\rho_2}{dt}/\rho_2>0$, i.e. the discriminant

$$\omega=\beta\left(1-\rho_1^{*}\right)-\left(m_2+\gamma_{22}q_{2/2}+\gamma_{21}q_{1/2}\right)=\beta\left(1-\rho_1^{*}\right)-\left(m_2+\gamma_{22}x+\gamma_{21}y\right)>0,$$

then the successful invasion by species 2 (with very small population) is possible, otherwise ($\omega<0$) it will be extinct and excluded by resident species 1.

Case II: Invasion by species 1 to a population dominated by species 2

Similarly, we first calculate the equilibrium densities of species 2 ($\rho_2^* \in [0,1]$ and $q_{2/2}^* = \rho_{22}^* / \rho_2^* \in [0,1]$) with seed global dispersal by neglecting the rare invasive species 1 ($\rho_1 = 0, q_{1/2} = 0$), consequently obtaining the simplified version of equations (1) and (S6)

$$\begin{cases} \dfrac{d\rho_2}{dt} = \beta\rho_2\left(1-\rho_2\right) - \left(m_2 + \gamma_{22}q_{2/2}\right)\rho_2 \\ \dfrac{d\rho_{22}}{dt} = 2\beta\rho_2\rho_{20} - 2\rho_{22}\left(m_2 + \dfrac{\gamma_{22}}{z} + \dfrac{z-1}{z}\gamma_{22}q_{2/22}\right) \end{cases}. \tag{S17}$$

Subsequently, we derive the non-trivial equilibrium E^*=($\rho_2^*, q_{2/2}^*$) by letting $\dfrac{d\rho_2}{dt} = 0$ and $\dfrac{d\rho_{22}}{dt} = 0$, i.e.,

$$\begin{cases} \beta\left(1-\rho_2^*\right) = m_2 + \gamma_{22}q_{2/2}^* \\ \beta\rho_2^*\left(1-q_{2/2}^*\right) = m_2 q_{2/2}^* + \dfrac{q_{2/2}^*}{4}\left(\gamma_{22} + 3\gamma_{22}q_{2/2}^*\right) \end{cases},$$

obtaining

$$\begin{cases} \rho_2^* = \dfrac{\beta - m_2 - \gamma_{22}q_{2/2}^*}{\beta} \\ q_{2/2}^* = \dfrac{5}{2} + \dfrac{2\beta}{\gamma_{22}} - \dfrac{2}{\gamma_{22}} \cdot \sqrt{\left(\dfrac{5\gamma_{22}}{4} + \beta\right)^2 - \gamma_{22}\left(\beta - m_2\right)} \end{cases}.$$

As a result, E^* exists and is locally asymptotically stable if and only if $\beta > m_2 + \gamma_{22}q_{2/2}^*$, $\left(\dfrac{5}{4}\gamma_{22} + \beta\right)^2 > \gamma_{22}\left(\beta - m_2\right)$ and $\dfrac{5\gamma_{22}}{4} + \beta > \sqrt{\left(\dfrac{5\gamma_{22}}{4} + \beta\right)^2 - \gamma_{22}\left(\beta - m_2\right)}$.

Next, combined with $\rho_1 = 0$ (thus $\rho_{10} = 0$) and E^*=($\rho_2^*, q_{2/2}^*$), and if we abbreviate $x = (\alpha + \gamma_{11}) q_{1/1}$ and $y = (\alpha + \gamma_{12}) q_{2/1}$, we have $q_{1/1} = x / (\alpha + \gamma_{11})$ and $q_{2/1} = y / (\alpha + \gamma_{12})$. As a consequence, we can change the equations (S9) and (S11) as

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\alpha}{2}\left(1 - \dfrac{y}{\alpha + \gamma_{12}}\right) - \dfrac{x}{\alpha + \gamma_{11}}\left[\dfrac{3\alpha}{2} + m_1 + \dfrac{\gamma_{11}}{2} - \left(\alpha - \dfrac{\gamma_{11}}{2}\right)\dfrac{x}{\alpha + \gamma_{11}} - \left(\alpha - \dfrac{\gamma_{12}}{2}\right)\dfrac{y}{\alpha + \gamma_{12}}\right] \\ \dfrac{dy}{dt} = \left(\beta\rho_2 + \dfrac{3}{4}\alpha q_{2/0}\right)\left(1 - \dfrac{x}{\alpha + \gamma_{11}} - \dfrac{y}{\alpha + \gamma_{12}}\right) - \dfrac{y}{\alpha + \gamma_{12}}\left[\alpha + m_2 + \dfrac{\gamma_{12}}{4} + \dfrac{\gamma_{21}}{4} + \dfrac{3\gamma_{22}}{4} q_{2/2}\right. \\ \qquad \left. - \left(\alpha + \dfrac{\gamma_{11}}{4}\right)\dfrac{x}{\alpha + \gamma_{11}} - \left(\alpha + \dfrac{\gamma_{12}}{4}\right)\dfrac{y}{\alpha + \gamma_{12}}\right] \end{cases} .$$

This autonomous dynamical system has a single non-negative equilibrium ($x \in [0, \alpha + \gamma_{11}]$ and $y \in [0, \alpha + \gamma_{12}]$ and $\dfrac{x}{\alpha + \gamma_{11}} + \dfrac{y}{\alpha + \gamma_{12}} < 1$) that is stable and obtained by letting $\dfrac{dx}{dt} = 0$ and $\dfrac{dy}{dt} = 0$, so the analytical solutions of x and y can be obtained from

$$\begin{cases} \dfrac{\alpha}{2}\left(1 - \dfrac{y}{\alpha + \gamma_{12}}\right) = \dfrac{x}{\alpha + \gamma_{11}}\left[\dfrac{3\alpha}{2} + m_1 + \dfrac{\gamma_{11}}{2} - \left(\alpha - \dfrac{\gamma_{11}}{2}\right)\dfrac{x}{\alpha + \gamma_{11}} - \left(\alpha - \dfrac{\gamma_{12}}{2}\right)\dfrac{y}{\alpha + \gamma_{12}}\right], \\ \dfrac{y}{\alpha + \gamma_{12}}\left[\alpha + m_2 + \dfrac{\gamma_{12}}{4} + \dfrac{\gamma_{21}}{4} - \left(\alpha + \dfrac{\gamma_{11}}{4}\right)\dfrac{x}{\alpha + \gamma_{11}} + \dfrac{3\gamma_{22}}{4} q_{2/2} - \left(\alpha + \dfrac{\gamma_{12}}{4}\right)\dfrac{y}{\alpha + \gamma_{12}}\right] = \left(\beta\rho_2 + \dfrac{3\alpha}{4} q_{2/0}\right)\left(1 - \dfrac{x}{\alpha + \gamma_{11}} - \dfrac{y}{\alpha + \gamma_{12}}\right). \end{cases}$$

If invasive rare species 1 (local dispersal) with very small population ($\rho_1 \approx 0$) is introduced into the system (S17), and if $\dfrac{d\rho_1}{dt} / \rho_1$>0, i.e., the discriminant (Δ>0)

$$\begin{aligned} \Delta &= \alpha\left(1 - q_{1/1} - q_{2/1}\right) - \left(m_1 + \gamma_{11} q_{1/1} + \gamma_{12} q_{2/1}\right) \\ &= \alpha - m_1 - (\alpha + \gamma_{11}) q_{1/1} - (\alpha + \gamma_{12}) q_{2/1} = \alpha - m_1 - x - y > 0 \end{aligned},$$

then the successful invasion by species 1 (with very small population) is possible, otherwise (i.e., $\Delta < 0$) it will be extinct and excluded by resident species 2.

Finally, we summarize *Case I* and *Case II* in Table S2.

Table S2 Invasibility analysis of PA model with two competing species

Δ / ω	$\Delta > 0$	$\Delta < 0$
$\omega > 0$	Coexistence	Species 2 (excluding species 1 with initial small density)
$\omega < 0$	Species 1 (excluding species 2 with initial small density)	Initial population density determining competitive outcome