DEPARTEMENT BEDRIJFSECONOMIE

BETA ESTIMATION AND UNLEVERING
CALCULATING BETAS FOR BELGIAN LISTED FIRMS

by

Eddy LAVEREN
Eduard DURINCK
Marc DE CEUSTER
Nadine LYBAERT

WORKING PAPER
96-243
December 1996

D/1996/2263/18

BETA ESTIMATION AND UNLEVERING

CALCULATING BETAS FOR BELGIAN LISTED FIRMS

E. Laveren
E. Durinck
M. De Ceuster
N. Lybaert

Abstract

Many attempts have been made in the literature to measure the systematic (business) risk of a security. This paper too examines the proportions of different beta measures. The central research question is whether it makes a significant difference which unlevering procedure to use. Beneath the procedure of unlevering, there is also the option concerning the levered beta to be used as basis. Thus, another objective is to look whether beta-adjustments for thin trading and systematic biases in beta are material, and to examine the impact of unlevering on each. The estimates of beta are obtained by using 133 four-weeks observations for the 58 mostly traded Belgian firms, covering the period 1985-1995. Statistical tests made us conclude that the averages are not significantly different.

I. INTRODUCTION

The modern capital market theory implies that investors are expected to be rewarded only for that part of the risk of a security which cannot be eliminated by diversification. This part of the risk is called systematic, while the diversifiable component is referred to as unsystematic risk. The fundamental concept of systematic risk is central to contemporary theory in finance, and is better known as the "beta" for a security. Modern finance theory suggests that the identification of systematic risk is a necessary prerequisite to solving many types of financial problems. One example is capital budgeting, in which an estimate of systematic risk is required for computing the cost of capital.

Systematic risk is often measured by beta, within the context of the capital asset pricing model (CAPM). Here, beta is estimated with a time-series regression, in which the dependent variable is the stock return and the independent variable is the market return. In other words, this beta is computed on the basis of stock data of firms using debt financing, making it a "levered" beta. The levered betas incorporate the business risk as well as the financial risk. The business risk has to do with the difference between realized and expected cash flows. The financial risk on the other hand is determined by the company's capital structure. It is the result of using debt financing at which fixed interest obligations are attached, so that the stockholder's residual income will be subject to greater fluctuations.

The academic literature is fairly consistent in stating that the capital structure has an impact on the systematic risk of a firm (Ehrhardt, 1994). More specifically, adding debt to the capital structure of a firm will increase the beta of the firm (for empirical and theoretical research concerning this relationship, see Section III). And since different industry groupings typically have different degrees of leverage (for instance, Bradley *et al.*, 1984), the levered betas might give a distorted view of the part of the total risk attached to the proceeds of the stockholders which cannot be diversified by these stockholders. At this point, it must be clear that applications of the beta are only valid for projects that have the same business risk as the current activities of that company. [1]

Therefore, the levered beta needs to be adjusted for the financial risk, so as to obtain a beta which only reflects the systematic business risk associated to the concerning activity. This corrected beta is called the "unlevered" beta. However, unlevering can be applied making use of various procedures. The complexity of the calculations can differ substantially, since some methods require additional data. One of the main differences is the fact whether or not to take into account the tax rate (i.e., the choice Hamada or Laveren). Other choices concern, for instance, which tax rate to consider (i.e., the actual or the statutory tax rate), the valuation of the various components (i.e., market values or book values), and the time period upon which to make valuations (i.e., averages or end of the period values). The central research question in this study is whether it makes a significant difference which unlevering procedure to use.

Beneath the procedure of unlevering, there is also the option concerning the levered beta to be used as basis. To our knowledge, all unlevering published so far concerns the beta obtained by the OLS regression. However, in literature various econometric corrections can be found which should lead to a more accurate measure of systematic risk. Thus, another objective of this paper is to calculate these levered betas, and to examine the impact of unlevering on each.

We start with estimating market model betas. These betas might be biased since the Belgian stock market is a rather thin market. Literature shows that when shares are thinly traded, their beta estimates based on the CAPM model are biased downwards (for an overview see Dimson and Marsh, 1983). Futhermore, since trading frequency is stable over time, this bias will be persistent, and will impart a spurious stability to estimates of beta and other risk measures. Although several adjustments for correcting this bias have been proposed in the literature, we follow Cohen, Hawawini, Mayer, Schwartz and Whitcomb (1983).

The third beta estimator included in this study is the Bayesian estimator proposed by Vasicek (1973). The Bayesian decision theory provides formal procedures which utilize information available prior to sampling, together with the sampling information, to construct estimates which are optimal with respect to the minimization of the expected loss. A prior information which can be used whenever a beta of a security is estimated is the knowledge of the cross-sectional distribution of betas for all the sample firms.

The paper is organized as follows. Section II starts with a theoretical exposition concerning beta estimation. In Section III, some procedures for unlevering as well as estimation problems are discussed. In the following two sections, we describe our sample and next go into the empirical results concerning the estimated betas. Here we show adjustments for a levered beta matter, and the applied procedure of unlevering does make a difference. A summary concludes.

II. THE ESTIMATION OF THE LEVERED BETA

In this section, three levered betas will be discussed: OLS betas, CHMSW betas and Vasicek betas.

A. THE CAPM AND THE MARKET MODEL

The capital asset pricing model – better known as the CAPM – of Treynor (1961), Sharpe (1964), Lintner (1965a, 1965b) and Mossin (1966) is a model of equilibrium securities prices. It assumes (see Copeland and Weston, 1988) that there is only one systematic influence affecting the return on common stock, being the return on the market. Therefore, the only systematic risk that affects the expected return on stock is the sensitivity of a stock to the return on the market. This sensitivity is usually denoted as beta. A β_i of 2, for instance, means that a stock's return is expected to increase (decrease) by 2% when the market increases (decreases) by 1%.

β_i is estimated by examining the historical relationship between a stock and the market. More specifically, the single period CAPM is of the following form: [2]

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f] \qquad (1)$$

where:

$E(R_i)$ = the expected returns on security i
$E(R_m)$ = the expected returns on the market portfolio m
R_f = the risk-free rate
β_i = beta, the systematic risk measure

Equation (1) states that the expected rate of return on a security in excess of the risk-free rate is proportional to the slope coefficient of the regression (being the beta) of that security's rates of return on a market index. This equation simply breaks the return on a stock into two components, that part due to the market and that part independent of the market.

Empirical assessments of the systematic risk can be obtained by applying the market model (Levy and Sarnat, 1984). Thus, the value of security betas is found by fitting a time series regression of the form:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \qquad (2)$$

where:

R_{it} = the return on security i in time period t

R_{mt} = the return on the market portfolio m in time period t

β_i = security i's beta

α_i = the intercept for security i. This is the expected value of the return on a stock i if the return on the market is zero.

ε_{it} = the residual return on security i in period t which is unexplained by the market returns with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}\varepsilon_{jt}) = 0$, $E(\varepsilon_{it}\varepsilon_{it+1}) = 0$. In other words: ε_i is the variability of stock i that is not explained by movements in the market.

There is evidence that historical betas provide useful information about future betas, and can be used as an estimate for it (Luoma *et al.*, 1994).

B. ADJUSTING FOR THIN TRADING

The market model implicitly assumes continuous trading over time. However, trading in real markets is always more or less infrequent. This causes problems in the estimation of the systematic risk, since the beta is biased downwards (for an overview see, for instance, Cohen *et al.*, 1983a, or Dimson and Marsh, 1983). Another implicit assumption in applying the market model is that stock markets are informationally efficient in a sense that all stocks react simultaneously and without bias to new market-wide information. Here, the results, especially from small security markets, indicate that the real markets may be less efficient by nature (for a review see, for instance, Hawawini and Michel, 1984).

Thus, stock market data for small markets are generally regarded as less useful for testing theories than US data due to a smaller number of stocks and a more severe problem of thin trading. As mentioned, the reason is, referring to Luoma *et al.* (1994) that on a thin market there are two main sources biasing the beta estimates. First, there are days when the stocks are not traded, which makes the true values of stocks on those days unknown. Second, information does not reach all investors simultaneously and immediately causing lags in price adjustments. Of course, these two sources are interrelated in the real world. A good beta should overcome both of these problems.

In Dimson (1979), Berglund *et al.* (1989) and Luoma *et al.* (1994), several alternatives to correct for the bias in beta estimates produced by thin trading are reviewed. These studies also go into the behaviour of the various beta adjustments in infrequently traded and informationally inefficient stock markets. Berglund *et al.* (1989) concluded that the different procedures to estimate systematic risk did not perform much better than the traditional market model. Luoma *et al.* (1994) show that the level of infrequent trading and inefficiency has a significant impact upon the behaviour of different beta adjustments. According to these authors, the most appropriate estimation method varies for different types of markets and different types of stocks.

Most prominent among these methods is the one originally proposed by Scholes and Williams (1977). This aggregated coefficients method was proposed to deal with infrequent trading in situations in which no information about the exact timing of a price within a day is available. In 1977, Dimson proposed a more general variant to cope with stocks which were less frequently traded than once a day. The variant used in this paper is a generalization of the original Scholes and Williams (1977) estimator suggested by Cohen, Hawawini, Mayer, Schwartz and Whitcomb (henceforth CHMSW) in 1983.

The formula by which the CHMSW-beta is computed is:

$$\beta_i^{CHMSW} = \frac{\beta_i + \sum_{n=1}^{N} \beta_{i+n} + \sum_{n=1}^{N} \beta_{i-n}}{1 + \sum_{n=1}^{N} \beta_{M+n} + \sum_{n=1}^{N} \beta_{M-n}} \qquad (3)$$

where β_i is the OLS beta for the ith stock, β_{i+n} is the β_i relative to the market with an n period lead, β_{i-n} is the β_i relative to the market with an n period lag, β_M is the corresponding coefficient for the market, n is an index, and N is the number of leads and lags. Thus, experiments can be made with various different numbers of leads and lags, for instance, 1, 2, 5 and 10 days. Since we work with four-weeks data (as is mentioned in Section IV), we take one lag, being about a month.

Although we are aware of the fact that by using daily returns more powerful empirical tests are possible, many securities listed on organized exchanges – and the Belgian stock market surely is no exception – are traded only infrequently. Since their reported closing prices typically represent trades prior to the actual close of the trading day, measured returns often deviate from true returns. The resulting nonsynchronization of measured returns across different securities in turn introduces into the market model the econometric problem of errors in variables, making that OLS estimators for almost all securities are both biased and inconsistent (Scholes and Williams, 1977).

C. MEASURING THE TENDENCY OF BETAS TO REGRESS TOWARD ONE

The actual beta in the forecast period tends to be closer to the average beta than is the estimate obtained from historical data (as shown, for instance, by Blume, 1975). A straightforward way to adjust for this tendency is to simply adjust each beta toward the average beta. For example, taking one-half of the historical beta and adding it to one-half of the average beta moves each historical beta halfway toward the average. This technique is widely used (Elton *et al.*, 1994).

It would be desirable not to adjust all stocks by the same amount toward the average but rather to have the adjustment depend on the size of the uncertainty (sampling error) about beta. The larger the sampling error, the greater the chance of large differences from the average, being due to sampling error, and the greater the adjustment. Vasicek (1973) has suggested a scheme that incorporates these properties. As he has shown, it concerns a Bayesian estimation technique.

Bayesian decision theory provides formal procedures which utilize information available prior to sampling, together with the sample information, to construct estimates which are optimal with respect to the minimization of the expected loss. Vasicek (1973) presents a method for generating Bayesian estimates of the regression coefficient of rates of return of a security against those of a market index. The distribution of the regression coefficients across securities is used as the prior distribution in the analysis.

The Vasicek beta (β_i^V) is computed using the following formula:

$$\beta_i^V = \frac{\beta' / s^2_{\beta'} + \beta_i / s^2_{\beta i}}{1 / s^2_{\beta'} + 1 / s^2_{\beta i}} \qquad (4)$$

where, as before, β_i is the OLS beta for the ith stock and $s^2_{\beta i}$ is its estimated variance. β' stands for the unweighted market average of OLS betas, and $s^2_{\beta'}$ is the cross-sectional variance of OLS betas of different stocks. The formula reveals that the larger the variance of the OLS beta estimate, the smaller the weight that the estimate is given as opposed to the weight given to the overall average of betas. Since betas for less frequently traded shares tend to have a higher estimated variance than betas for more frequently traded shares this Bayesian adjustment will adjust betas for less frequently traded shares more towards the overall average than betas for the frequently traded shares.

Thus, as opposed to the previously presented method the Bayesian estimator was not originally proposed to correct for the thin trading bias but to account for the differences in the statistical reliability of beta estimates for different stocks. However, it is very probable that the variation in the reliability of the beta estimates between stocks is correlated with trading frequency. Less frequently traded shares are likely to have less reliable beta estimates than more frequently traded stocks, and thus a Bayesian adjustment probably reduces some of the thin-trading related problems in beta estimation.

Research by Bogue (1972), amongst others, has indicated that the Bayesian adjustment procedure is superior to the OLS method, the weighted regression and the instrumental variable approach, when evaluated in terms of minimum mean square error of predicting future returns. Also the study of Beaver and Manegold (1975) conclude that the Bayesian adjustment removes some of the error. However, they point out that, from a decision-theoretic point of view, this sort of adjustment procedure must be viewed as heuristic for several reasons.

III. ADJUSTING FOR LEVERAGE

As is mentioned in the introduction, the unlevered beta can be seen as an adjusted levered beta, corrected for the effect of debt in the capital structure. The underlying reason for this correction is that the beta of a firm is positively related to its leverage. That there is a theoretical relationship between the systematic risk of a firm's common stock and the firm's financial leverage has been shown, for instance, by Hamada (1969, 1972), Pettit and Westerfield (1972), Bowman (1979) and Hill and Stone (1980).

So as to quantify the linkage between the capital structure and the systematic risk of a firm, consider a firm with debt D and equity E, where these are market values. β_L is the beta of the firm, where the subscript L indicates that the firm has leverage in its capital structure. β_U is the beta that the firm would have if it were unlevered. G is a function of tax and the tax regime (being corporate taxes, personal taxes on equity and personal taxes on debt). The relationship between the levered and the unlevered betas can be defined as:

$$\beta_L = \beta_U \, [\, 1 + (1 - G)\, D/E \,] \tag{5}$$

Clearly, this equation implies that adding debt to the capital structure of a firm will increase the beta of the firm. This positive relation between the beta of a firm and its leverage was also empirically validated by various authors, for instance, by Hamada (1972), Hill and Stone (1980) and Mandelker and Rhee (1984), each using a different type of empirical test. The empirical studies of Beaver, Kettler and Scholes (1970) and Elgers and Murray (1982) provide more evidence on a positive relation at the 1% level of significance. Also Rosenberg and McKibben (1973), Logue and Merville (1972) and Melicher and Rush (1974) included the proportion of financing provided by debt in their multivariate regressions and found it to be significant. In contrast, Thompson (1976), Breen and Lerner (1973) and Lev and Kunitzky (1974) report no significant explanatory value.

In Ehrhardt (1994) references are also made for an approach in case the debt is risky (for instance, models including risky preferred stock or risky tax shields), since (5) assumes that the debt is not risky. A theoretical discussion of the impact of the personal taxation and other market imperfections on debt is also found in Laveren (1990).

The most commonly used model probably is that of Hamada (1972), following the assumptions of MM (1963). The result is the same as (5), except that G is equal to the corporate tax rate. The following relation between β_U and β_L is derived:

$$\beta_U^{*} = \frac{\beta_L}{1 + (1 - t)\, D/E} \tag{6}$$

where:

β_U^{*} = the unlevered beta according to Hamada
β_L = the levered beta
t = the tax rate for the firm's profits
D = the market value for the debt
E = the market value for the common equity

Hamada's unlevering procedure is derived under the assumption that the tax advantages of the interests will be realized – with certainty – as they are presupposed at time period 0. Laveren (1990) makes the proposition to actualize the tax advantages at k_u, being the cost of the equity capital of a firm completely financed with own resources. Or: as MM (1963) states: the rate at which the market capitalizes the expected return (net of tax) of an unlevered company. This discount rate only gives an allowance for the business risk attached to the operational cash flows.

For firms using debt financing, this k_u is equal to the unlevered cost of the equity capital or in other words: the levered cost of the equity capital (k_e) adjusted for the financial risk. [3]

After some transformations, and assuming that the risk associated with the flow (t D) is equal to β_U (and not to zero as is assumed by Hamada, 1972), Laveren (1990) obtains:

$$\beta_U' = \beta_L \frac{E}{V} \qquad (7)$$

where:

β_U' = the unlevered beta according to Laveren
β_L = the levered beta
E = the market value for the common equity
V = the total market value

The main difference between both formulas to unlever is the integration of the paid taxes. Consequently, the question that arises with respect to Hamada's formula is which tax rate to consider. We can either use the statutory tax rate, being around 40% in Belgium, or the actual tax rate. It is expected this last rate will be situated somewhere between the maximum of 40% and 0%, making it equal to Laveren's method. Intuitively it needs little discussion it is better to use the actual tax rate, a suggestion formulated in literature (Ehrhardt, 1994) as well. However, no matter which method is used to calculate this rate, the use of annual reports is required. Due to the difficulties arising when calculating this rate accurately, as well as the fact we are working with non-consolidated annual reports, the numbers might give a distorted view.

Another estimation problem with respect to the components of formula (6) and (7) concerns the fact whether these data have to be based on data obtained at the end of the period, or whether averages over the whole period must be used. In literature, some justification for use of the latter can be found. Vogt (1994), for instance, concludes that firm financial behavior is consistent with the partial stock adjustment model in which firms slowly adjust toward a target financial structure. This finding supports some former research. Consequently, additional information might be included by taking into account averages.

Finally, it needs to be mentioned that in the above formulas, the market values for the common equity and the debt are used. The use of market values is indeed preferable, since rational investors base their decisions on the market value. The book value only concerns historical costs and realized values, and does not incorporate the effect of future events, like expected inflation. The accounting return also is difficult to interpret because of the various methods and the distortion by fiscal reasons or thinking about window-dressing (Levy and Sarnat, 1986).

Although the calculation of E and D looks very simple in concept [4], it is not that obvious to collect the necessary data, especially with respect to total debt. Indeed, the debt does not only consist of creditors and other short-term debt, for which it is less difficult to obtain market values, but also of bonds and other long-term debt. So as, for instance, to calculate the market value of long-term debt on the basis of the submitted annual reports, additional data are needed concerning the terms of repayment, the effectively applied long-term interest rate for each type, and the like. For outsiders it can be very complicated to gather these needed data. Therefore, in a lot of cases, people have to be pleased with working with the book values for the estimation of D. In other words, often there is no alternative but to measure the value of debt by employing book values.

With respect to common equity, the situation is less problematic. It is relatively easy to get an estimate of the market values of equity, in the case when the considered firms are publicly traded. A plausible solution is to multiply the number of shares with stock prices: an alternative we follow in this study, as will become clear in Section V. It is not unlikely that this market value of the common equity shows rather substantial differences with its book value. In our sample, for instance, we find that the former value is on average 2.05 times as high as the latter, with a peak of 8.61 and a minimum of 0.16. Thus, it might be worth the trouble to make calculations twice.

IV. THE GATHERING OF THE DATA

The original sample was composed of all Belgian firms listed on the Brussels Stock Exchange (i.e., the cash market and the forward market) in 1995, excluding financial institutions and insurance companies, as well as AFV-VVPR stocks (i.e., stocks having fiscal advantages). For the available firms, only the most important stock was considered, i.e., in case more than one category of stock for the same firm was listed.

The turnover volume of the firms remaining this way, being 138 in total, are presented in Figure 1. As is clear, most of them only have a small turnover volume. Having in mind the problems of thin trading (see Section II.B.), we decided to concentrate on firms with a minimum yearly turnover of 100 000 000 BEF which is equal to an average daily turnover of 400 000 BEF.

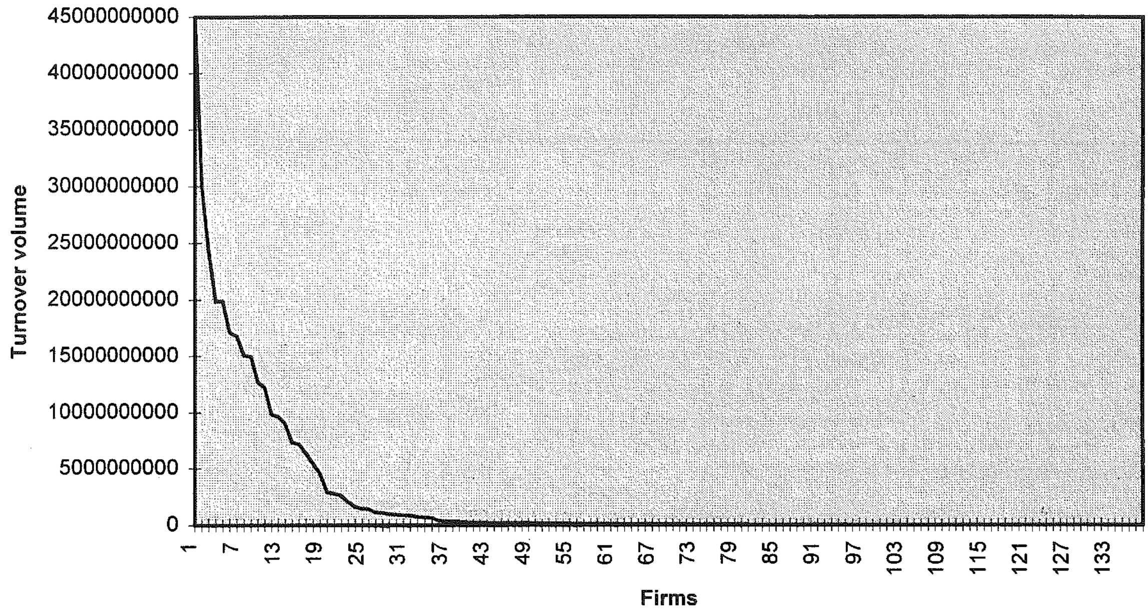

Figure 1: The turnover volume of the Belgian listed firms in 1995

After this limitation, the remaining sample of firms consisted of 62. For all these firms, two kinds of information had to be collected, being the stock prices and the annual reports, so that further limitations could result. Thus, the first kind of data we needed were the stock prices. For the collection of these data, we made use of CRESUS, a commercial software program, containing all Belgian stock information. When data were available on both the cash market and the forward market, we chose to collect the prices of the latter one. [5]

We opted to use monthly data. Though since only a quotation is available at the end of the week [6], we preferred four-week data. A shortcoming of CRESUS is that when a certain stock is not traded on a Friday, no price is given at all, and the last price available is the one of the Friday(s) before, and not the one of the day before. Since data can be missing for a few weeks this way, we feared a distortion of our data, and we looked up the missing data in the "Financieel Economische Tijd". [7]

So as to compute the stock return of a security in a given time period, data also had to be collected in CRESUS with respect to the cash dividends of the firms and the adjustment coefficients (giving a correction for stock splits and bonuses). [8] Here, data were complete, and no manual additions had to be made. Extreme values of four-week returns were not eliminated or windsorized. [9] As market index we used the spot return of the Belgian market which contains all Belgian stocks on the spot market from the first day they are listed, weighted by the stock capital, and taking into account all net dividends. [10]

Having these data, the levered betas could be computed according to (2), (3) and (4). Now, it only remained to unlever these betas. So far, all market data possibly usable for this purpose were already available. Though, as mentioned at the end of Section III, the use of book values is an alternative, often unavoidable route. Thus, so as to possess accounting data for unlevering the beta (and for an extension of this research as well), we needed to collect the annual reports. For gathering these accounting data we made use of the M&M software, making the data of the "Balanscentrale" of the National Bank of Belgium available.

The first annual reports that are available in electronic form date from 1985. Since also CRESUS started up its activities relatively recently, and the first date available here is 26 April 1985, we started our sample with these dates. The last annual reports which were available at the time of research were those of 1994. Thus, we looked for stock data until May-June 1995. So ending the sample at 09 June 1995, and having an interval of four weeks, this led us to a total of 133 observations.

According to the literature, five years (being 60 monthly observations) provide reasonably stable parameter estimates (Elgers and Murray, 1982). Thus, our time span allows two distinct periods, and the continuation of this study works with two subperiods. Subperiod 1 represents the first 66 observations (being the period 26 April 1985 / 20 April 1990) and subperiod 2 the next 67 observations (being the period 18 May 1990 / 09 June 1995).

After these data were collected, we had to eliminate four firms. Three firms (being Quick, Desimpel and Creyf's) were skipped because they were listed on the Brussels Stock Exchange too late to be considered for a whole subperiod, and one firm (being Sibeka) was skipped because it was no longer listed. [11] The 58 firms finally making up the sample can be found in Appendix 1, column (a). In column (b), the sector to which they are classified by the Brussels Stock Exchange is reported.

V. THE RESULTS

A. THE ESTIMATION OF THE LEVERED BETA

Our β_i - compared with other sources

So as to compute the levered OLS beta β_i (see (2)) for the two subperiods, 2 times 58 multiple regressions had to be carried out. A look in the individual results show that all these regressions are significant. In 96 regressions, we even find an F-value with a significance of 0.0001. Thus, we can conclude that the chance of not having to reject the null hypothesis – that the stock returns are independent of the market return – is very minimal. Also the values of R^2 show that the variability explained is not unimportant. In subperiod 1, this measure has a mean of 0.404, and shows a miniumum of 0.059 and a maximum of 0.722. There is a significant correlation of 0.56 with the values in subperiod 2. Here the range of R^2 is situated between 0.078 and 0.695, while the mean amounts 0.364.

In subperiod 1 the resulting betas have a mean of 0.927, with a minimum of 0.187 and a maximum of 2.479. In subperiod 2 these values are 1.052, 0.330 and 2.211. Details by firm for the most recent subperiod are given in Appendix 1, column (c).

Having these betas, it is interesting to compare the numbers with the ones computed by others. Indeed, in Belgium the beta is computed and published by different institutions, especially banks. Probably the best known are those of Budget Week, of the Generale Bank and of Degroof. They all compute the beta in the same way. More specifically, the beta of a stock is estimated with an OLS regression of the monthly stock return on the monthly Brussels Spot Return Index. The data sample used hereby is five years.

Although we use the same time period, the same market index and the same technique, a small difference with our β_i is to be expected anyway, since we use the four-week period. Nevertheless, we collected the available data, and for 36 firms we have betas from the four sources, including ours. The correlations with respect to subperiod 2 are given in Table 1. [12]

Table 1: The Pearson correlation between β_i as computed by four sources (N=36)

Subperiod 2	our β_i	Degroof	BudgetWeek	Generale Bank
our β_i	1.00000			
Degroof	***0.90079***	1.00000		
BudgetWeek	***0.92828***	***0.97343***	1.00000	
Generale Bank	***0.78441***	***0.72847***	***0.73699***	1.00000

p = 0.0005

Out of these numbers, it can be concluded that the betas of Degroof and BudgetWeek are very much alike, and that the difference – as was expected – with our β_i is a little bit. Though surprising is the somewhat greater distortion of these three betas with the one as calculated by the Generale Bank. Knowing that a certain firm is listed on the cash and the forward market, and has different categories of stocks (amongst which fiscal advantages stocks), a plausible explanation could be found here. Another filling in of missing data could be another reason, as well as the trimming or windsorizing of the collected data. Anyhow, all correlations are very significant.

Besides a correlation between the absolute values, it is also important to have an idea about the ranking of the firms by the four sources. For this purpose, the Spearman Rank Correlation is given in Table 2. According to these results, the hypothesis of zero correlation in the population between the variable can be rejected at the 0.0005 level of significance. Thus, the 36 firms of the sample seem to be ordered in the same way by the four sources. Again, a somewhat greater distortion by the Generale Bank can be noticed.

Table 2: The Spearman Rank correlation between β_i as computed by four sources (N=36)

Subperiod 2	our β_i	Degroof	BudgetWeek	Generale Bank
our β_i	1.00000			
Degroof	***0.89575***	1.00000		
BudgetWeek	***0.93668***	***0.96859***	1.00000	
Generale Bank	***0.75830***	***0.70347***	***0.74131***	1.00000

p = 0.0005

Our β_i – compared with the other two levered betas

The betas obtained until now can be used so as to make an adjustment for thin trading and a Bayesian adjustment, or, stated differently, to compute β_i^{CHMSW} (see (5)) and β_i^{V} (see (6)). In Table 3, data concerning some size variables are presented for the two subperiods.

It is obvious that the four mentioned values are very similar, so it is expected that the correlation between the three levered betas will be significant. This correlation between the three measures is given in Table 4. The results demonstrate that our expectation is proved to be true, and all correlations are very significant. [13]

Table 3: The mean, standard deviation, minimum and maximum of the three different levered betas (N=58)

Subperiod 1	Mean	Std. Dev.	Minimum	Maximum
β_i β_i^{CHMSW} β_i^{V}	0.927 0.923 0.916	0.325 0.343 0.278	0.187 0.229 0.205	2.479 2.686 2.038
Subperiod 2	**Mean**	**Std.Dev.**	**Minimum**	**Maximum**
β_i β_i^{CHMSW} β_i^{V}	1.052 1.097 1.044	0.408 0.408 0.383	0.330 0.362 0.334	2.211 2.498 1.941

Table 4: The correlation between the three different levered betas (N=58)

		β_i	β_i^{CHMSW}	β_i^{V}
Subperiod 1	β_i β_i^{CHMSW} β_i^{V}	1.00000 ***0.91060*** ***0.99210***	 1.00000 ***0.88866***	 1.00000
Subperiod 2	β_I β_i^{CHMSW} β_i^{V}	1.00000 ***0.90821*** ***0.99707***	 1.00000 ***0.89740***	 1.00000

p = 0.0005

B. ADJUSTING FOR LEVERAGE

Unlevering – a closer look at the applied procedures

In this study, unlevering will be done by the method of Hamada (see (6)) as well as by the method of Laveren (see (7)). As was mentioned in Section III, different options must be taken herewith. A first choice concerns the tax rate to be considered. Although the use of the actual tax rate is preferable, data restrictions make us suspect they might not be very realistic. Consequently, the statutory tax rate is integrated. Another choice to be made concerns the time period for the components under study. Given the findings in literature, we opt to use the averages over the concerned subperiod, and this for any number.

A last question that arises is whether to use market values or book values. Although it is preferable to use market values, this is difficult with respect to debt. Therefore, we postulate to use the book value of debt as a proxy for the market value. [14] Market values can be used with respect to the total owner's equity. This value is computed by multiplying the number of shares with the share prices as on 31 December. [15] In this option, the market value for total liabilities is composed of the sum of the market value for common equity plus the book value for debt. [16] This hybrid definition of financial leverage is found in Bowman (1980) to perform as well as financial leverage ratios based solely on market values, in terms of contemporary association with estimates of beta. Given the differences between the market value and the book value of the common equity, we also take the option to use but the book values, for equity as well as for debt. [17]

Summarized, we will unlever the beta according to four methods: Hamada/Laveren and market values/book values, making use of the statutory tax rate and of averages over the subperiod. Not only the procedure of unlevering might differ, but also the levered beta to be unlevered. In accordance to former research, we start the procedure of unlevering on β_i and present these results. Since the high numbers of Table 4 make us presume that after unlevering the correlations will stay significant, we will test this presumption afterwards.

Unlevering – the expectations for an impact

The main problem of this study is to find out whether the different unlevering procedures have a significant different impact. As mentioned in Section III, the procedure of unlevering is based on those results in literature that concluded that the beta of a firm is in fact positively related to its leverage. Therefore, before starting calculations, we want a closer look with respect to this plausible relation in our sample. For this purpose, we calculated the financial leverage [18] in two ways, once completely based on book values and once partly based on market values. The results concerning the correlation with β_i are given in Table 5.

From these results we can conclude that in subperiod 2, β_i shows a positive and significant correlation with the financial leverage, computed by using book values and market values. In subperiod 1, results are not analogous. Here we find no significant correlation at the 0.10 level of significance.

Nonetheless, it can be expected that the adjustments for the effect of debt in the capital structure will result in more accurate measures of systematic business risk. And since our sample is composed of firms situated in various sectors, we have additional reason to believe unlevering will be worth the trouble.

Table 5: The correlation between β_i and the financial leverage (N=58)

		β_i	fin.lev. book	fin.lev. market
Subperiod 1	β_i	1.00000		
	fin.lev. book	-0.00581	1.00000	
	fin.lev. market	-0.16293	***0.75256***	1.00000
Subperiod 2	β_i	1.00000		
	fin.lev. book	*0.23766*	1.00000	
	fin.lev. market	**0.26879**	***0.74034***	1.00000

p = 0.0005; **p = 0.025**; *p = 0.05*

Unlevering – results

The impact on the mean and three other size variables of β_i by the four presumed unlevering procedures is given in Table 6. To have a more accurate interpretation concerning this impact, a T-test is performed so as to examine whether the means significantly differ. These results are presented in Table 7.

All together we can conclude that no very significant differences arise no matter which unlevering procedure is used. At a significance level of 0.025 or less, we could not even reject the null hypothesis that the two methods compared lead to the same mean. At a lower level of significance, however, we can state that the first method leads to a higher mean than the second method.

The first two comparisons concern the unlevering of β_i according to market values/book values. Out of Table 6 we concluded the financial leverage based on market values and on book values showed a significant, although not perfect correlation. Analogous we find a slightly different impact, being that unlevering by market values leads to a higher mean of β_i than unlevering by book values. This difference is absolutely normal, since β_i is computed on basis of market values, and greater differences with book values can arise.

The last two comparisons are with respect to the unlevering of β_i according to Hamada/Laveren. Here results are more significant, showing that in any case the unlevering by Hamada leads to a higher mean of β_i than unlevering by Laveren. The fact that the elimination of the statutory tax rate leads to a slightly greater impact was to be expected too, since Hamada integrates an additional constant variable in the denominator, reducing its value.

Table 6: The mean, standard deviation, minimum and maximum of β_i and of the four unlevered β_i's (N=58)

Subperiod 1	Mean	Std. Dev.	Minimum	Maximum
β_i	0.927	0.325	0.187	2.479
β_i unlevered by				
- Hamada, market	0.663	0.343	0.061	2.145
- Hamada, book	0.578	0.309	0.109	2.006
- Laveren, market	0.596	0.327	0.046	1.976
- Laveren, book	0.493	0.294	0.071	1.778
Subperiod 2	**Mean**	**Std.Dev.**	**Minimum**	**Maximum**
β_i	1.052	0.408	0.330	2.211
β_i unlevered by				
- Hamada, market	0.718	0.314	0.093	1.307
- Hamada, book	0.670	0.306	0.052	1.323
- Laveren, market	0.644	0.287	0.083	1.223
- Laveren, book	0.589	0.282	0.094	1.203

The correlation of β_i with its unlevered values can be found in Table 8. These results confirm the former findings. More specifically it is clear that, when referring to absolute values, the greatest differences are caused by the use of Laveren/book values whereas the least impact is brought about when Hamada/market values is used. In Appendix 1, column (d), the results per firm by using this first method is given for the second subperiod.

Also notice that Table 5 made us conclude that the correlation between beta and the leverage was more significant in the second subperiod. Analogous, out of these correlation coefficients, we find indeed that a greater absolute impact is caused by each unlevering method separately in this subperiod. However, with respect to the impact of the various unlevering procedures (given in Table 7), the results were analogous and not more outspoken for this period.

Table 7: The impact of the different unlevering procedures (N=58)

The two unlevering procedures being compared	Results	
Hamada, market versus Hamada, book		
subperiod 1	t = 1.38 °	d.f. = 115
subperiod 2	t = 0.82	d.f. = 116
Laveren, market versus Laveren, book		
subperiod 1	t = 1.77 *	d.f. = 115
subperiod 2	t = 1.04	d.f. = 116
Hamada, market versus Laveren, market		
subperiod 1	t = 1.06	d.f. = 116
subperiod 2	t = 1.31 *	d.f. = 115
Hamada, book versus Laveren, book		
subperiod 1	t = 1.51 *	d.f. = 116
subperiod 2	t = 1.48 *	d.f. = 115

*: p = 0.05; °: p = 0.10

Table 8: The correlation of β_i with the four unlevered β_i's (N=58)

	The correlation of β_i in subperiod 1 with	The correlation of β_i in subperiod 2 with
β_i unlevered by		
- Hamada, market values	***0.80651***	***0.61076***
- Hamada, book values	***0.74523***	***0.60850***
- Laveren, market values	***0.78123***	***0.58307***
- Laveren, book values	***0.67059***	***0.58517***

p = 0.0005

The unlevering – compared for the three levered betas

As mentioned before, we applied all these unlevering calculations with respect to the OLS β_i. The main inducement for this was that we presumed correlations would remain high with the unlevered β_i^{CHMSW} and with the unlevered β_i^{V}. Out of Table 9, we can conclude this presumption is right. The table only shows the results concerning subperiod 1, though the results for subperiod 2 have the same high correlations. With reference to Table 3, we can mention that the high correlation between the three levered betas is even improved when these variables are unlevered.

Table 9: The correlation between the three different unlevered betas (N= 58)

Subperiod 1		β_i	β_i^{CHMSW}	β_i^V
unlevered by Hamada, market values	β_i β_i^{CHMSW} β_i^V	1.00000 ***0.95664*** ***0.99270***	 1.00000 ***0.94216***	 1.00000
unlevered by Hamada, book values	β_i β_i^{CHMSW} β_i^V	1.00000 ***0.97281*** ***0.99232***	 1.00000 ***0.96032***	 1.00000
unlevered by Laveren, market values	β_i β_i^{CHMSW} β_i^V	1.00000 ***0.96306*** ***0.99296***	 1.00000 ***0.94983***	 1.00000
unlevered by Laveren, book values	β_i β_i^{CHMSW} β_i^V	1.00000 ***0.97980*** ***0.99284***	 1.00000 ***0.96862***	 1.00000

p = 0.0005

Consistency between the two subperiods

Notwithstanding the opposite observation concerning the relation between β_i and the financial leverage (see Table 5) for the two subperiods, we find that all the results show a certain consistency. Stated differently: all conclusions derived for 1985-1990 are applicable for 1990-1995. A question we can ask here is: "Does this also implicate that the magnitude of the beta stays the same?" So as to obtain an answer, correlations for the two subperiods are examined. Out of these results we can conclude that the correlation for any levered beta amounts around 0.33. Since more fluctuations can be assumed in the leverage [19] than in the risk associated to a given activity, it can be expected that the correlation for unlevered betas is higher. And indeed, we find that the correlation for any unlevered beta amounts between 0.55 and 0.70. [20]

The low correlation for the levered beta made us look at the variability of β_i for the firm. Therefore we shifted our periods 8 times 4-weeks. In other words: first we looked at observation 1 to 66, second we looked at observation 11 to 76, ... and at last we looked at observation 71 to 132 (remember we had 133 four-week observations). This way we found that the firms of the sample had rather small standard variations: for 13 of them this variable was lower than 0.10, and for an additional 28 firms it was situated between 0.10 and 0.20. Three peaks were found, being 0.49, 0.50 and 0.63.

CONCLUSION

The beta of a security – being the measurement of its systematic risk – is one of the basic concepts of modern capital market theory, and considerable attention has been devoted to its measurement. Many attempts have been made in literature to adjust the levered OLS beta, as well as to unlever this beta. This paper too examines the properties of different beta measures. More specifically, estimates of beta are obtained by using 133 four-weeks observations for the 58 mostly traded Belgian firms, covering the period 1985-1995.

The first research question that is presumed hereby is whether beta-adjustments for thin trading and systematic biases in beta are material. Both corrections of the OLS beta were suggested in literature, so as to obtain a more accurate measure of the systematic risk of a stock. However, when comparing the estimates produced by OLS with the estimation method proposed by Cohen, Hawawini, Mayer, Schwartz and Whitcomb (1983), as well as with betas obtained after the Bayesian adjustment according to Vasicek (1973), we find the measures are significantly correlated. Only for few firms, a significant difference is to notice. Thus, although the Belgian stock market is said to be a thin market, it does not make that much difference to implement an adjustment for the measurement of beta. It even looks less necessary to make the Vasicek adjustment.

Two remarks are worth mentioning here. The first remark concerns the selection biases in our sample. Attention must be paid to the fact that this study only examines the mostly traded firms. Any extension of the sample might probably lead to more important adjustments for thin trading. Also we only selected firms being listed during the whole period under study, meaning that firms having failed in the meantime are not integrated (for instance, Sibeka, as mentioned in Section III). Nonetheless, results showed that in spite of this selection criteria which is operating in favor of selecting big and successful firms, the average riskiness of the sample firms is approximately 1, which would be the value obtained if the beta of all firms on the Brussels Stock Exchange had been computed.

The second remark concerns the interpretation of the results. Although the different beta measures are significantly correlated, this does not mean the small differences are not important. Indeed, even small changes in a parameter may not be neglected, since they can lead to a completely different decision, for instance, with respect to asset allocation.

The reason for the interest in an accurate measure for systematic risk is the search for an accurate measure of systematic business risk. Since capital structure has an impact on the systematic risk of a firm, it is important indeed to eliminate this financial risk. Various procedures are plausible to unlever, and the second research question presumed in this study is whether different unlevering procedures do matter. For this purpose, we applied four unlevering procedures, being Hamada/market values, Hamada/book values, Laveren/market values and Laveren/book values. Hereby it was opted to integrate the statutory tax rate, as well as to make use of averages over the period. Once again we concluded that no significant differences arise when making use of these procedures. Only at the 0.10 level we found that the method of Hamada and the use of market values leads to higher means.

The question can be posed whether the differences in the impact of different unlevering procedures might have been more significant if other choices had been made with respect to the tax rate or the valuation of the components. Additional research made clear that, without exception, the mean of the unlevered β_i was higher when using the statutory tax rate (versus the actual tax rate [21]) and when using the end of the period values (versus the averages over the period). However, statistical tests made us conclude that the averages were not significantly different either. This finding is rather surprising, given the high differences between the statutory and the actual tax rate. Anyhow, it easens decisions to be made in future concerning the various estimation problems.

Lastly, it needs to be mentioned that the various unlevering procedures were also applied with respect to the adjusted systematic risk. An extremely high correlation was obtained with the unlevered unadjusted systematic risk. Therefore, we are inclined to conclude that, if unlevering is applied, it is not that relevant to make adjustment on the levered beta before. However, as mentioned before, even small differences in the beta might be important, since they can have a substantial impact on decisions to be made. To find out this precise impact, additional research is required.

NOTES

1. For instance, the cost of capital at which a firm discounts the expected future cash flows of an investment project is often related to two kinds of risk premiums: a premium for the business risk and one for the financial risk. Everybody agrees that as the business risk of a firm changes, that firm's cost of capital also changes. The influence of the financial risk or the capital structure on the cost of capital, however, is not so clear.
2. Notwithstanding there have been a number of articles which provided both theoretical and empirical criticism of the CAPM (for an overview see Elton *et al.*, 1994), this model has provided a theoretically appealing base for a scientific study of the relationship between risk and return. The model, and its empirically testable variant the market model, have been extensively covered in the literature. We will not develop the theory again here. The reader who desires a thorough treatment of the foundation and development of the model is referred to the seminal works or the excellent reviews in Beaver (1972) and Jensen (1972).
3. k_u can be determined by applying the CAP model, more specifically by integrating β_U in the model.
4. A plausible definition is given in Laveren (1990). He states that the market value of the common equity is derived by capitalizing the dividends received by the stockholders at the return they require on their resources in the firm. The market value of the debt is derived by capitalizing the interest allowances which the creditors receive at the cost of the debt.
5. Of course if only prices were available on the cash market, or if these were available further back in time, these ones were collected.
6. Only for the last two years daily quotations are given by CRESUS.
7. Although we spend a lot of time this way (for instance, many firms changed their name during the period under review), we were convinced this would lead to a more accurate data bank, and we hoped that this could be compensated by more accurate results.

8. This makes that R_{it} is computed as [(stock $price_t$ / $coefficient_t$) - stock $price_{t-1}$ + $dividend_t$] / stock $price_{t-1}$.
9. Indeed, contrary to what intuition might suggest, it was not uncommon at all to find stock increases or decreases of more than 30% during a four-week period. We even found a peak of 138% (namely for the Generale Bank in February 1988, cfr. the attempt to take-over by de Benedetti). Also the stock crash of November 1987 is responsible for a lot of fluctuations. However, since this was "reality", we decided not to skip or windsorize any of these observations.
10. Out of an empirical test, Elgers and Murray (1982) found that the stability of beta over time is quite sensitive to the market index employed. So they suggest researchers to develop empirical results using a variety of market indices. However, we limit ourselves to the one mentioned. It is also the index other institutions use so as to calculate the beta on the Belgian market.
11. As could be expected from a commercial program, CRESUS only gives data about currently listed firms.
12. The numbers in Table 1 concern those firms that are common for the four sources. Of course, this number is less than those common in our sample and each other at the time. Actually, the correlation between our data and Degroof (having 42 firms in common) is 0.90027, where Degroof shows 9 times a beta which is at least 0.2 lower, and only 1 time a beta which is at least 0.2 higher. The correlation between our data and BudgetWeek (having 53 firms in common) is 0.91584, and here we noticed 8 times a difference of at least 0.2 more, and only 3 times a difference of at least 0.2 less. The correlation between our data and Generale Bank (having 41 firms in common) is 0.85676. In this smallest subgroup, the most differences of at least 0.2 were found, namely 7 times upward and 7 times downward.
13. In fact, there are only few firms for which we find a substantial absolute difference between its different levered betas.
14. More specifically, we use the numbers in the annual reports under code 16 (i.e., provisions and postponed taxes) and under code 17/49 (i.e., amounts payable) for measuring the total debt.
15. The stock prices were looked up in CRESUS or, in case it was not available there, in the "Financieel Economische Tijd". The number of shares were obtained in the annual reports of the firm under code 8702 (i.e., registered number of shares) plus code 8703 (i.e., bearer number of shares). These data are most accurate, since they give the sum of the common and the preference stocks. On the contrary, CRESUS only informs about the first ones.
16. We are aware of the fact that the information of the annual reports is only reflected in the stock prices at the time of its submission, being about 5 months after the end of the fiscal year, meaning that in fact two measures concerning a different time period are linked here.
17. In this case, we use the numbers in the annual reports under code 10/15 (i.e., capital and reserves) for measuring the total equity, together with the mentioned numbers under code 16 + 17/49 for measuring the total debt. Here, the numbers under code 10/49 can be used for measuring the book value for total liabilities.
18. The financial leverage, being the ratio debt / total liabilities, varied between 0.010 and 0.932. Using book values, the mean was 0.467 and 0.420 for both subperiods. Using market values, the mean was 0.369 and 0.366.
19. The correlation between the two subperiods with respect to the financial leverage based on book values was 0.818, the one based on market values showed a correlation of 0.783.
20. Here we noticed that – no matter which method of unlevering was used – the correlation between the two subperiods was greater for β_i^{CHMSW} than for β_i or β_i^V.
21. These actual tax rates were estimated by dividing the actual taxes paid by the taxable income, numbers we find in the annual reports under code 67/77 (i.e., income taxes), respectively

under code 70/66 (i.e., profit/loss for the period before taxes). So as to eliminate the effect of adjustments of income taxes and write-back of tax provisions, we tried the formula as proposed by Ooghe and Van Wymeersch (1988). They suggest to divide code 9134 (i.e., income tax of the current period) by the sum of code 70/67 (i.e., profit/loss for the period) and code 9134. However, the same values were obtained this way.

APPENDIX

	Firm (a)	Sector (b)	β_i (1985-1990) (c)	β_i /Lav/book (d)
1	Tessenderlo	Chimie	1.668	0.975
2	Solvay	Chimie	1.189	0.693
3	Tractebel	Electricité et gaz	0.933	0.578
4	U.C.B.	Chimie	0.881	0.419
5	Union Minière	Non-ferreux	1.375	0.528
6	Telinfo	Metal Electro	1.025	0.454
7	Ter Beke	Alimentation	0.609	0.210
8	Tubize-Fin	Chimie	1.028	0.991
9	Wagons-Lits	Divers Services	0.782	0.382
10	Walibi	Divers Services	0.872	0.373
11	Bekaert	Metal Electro	1.627	0.769
12	Franki	Construction	1.275	0.324
13	Gen. Belg	Portefeuille	1.409	1.143
14	Gevaert	Portefeuille	0.949	0.732
15	GIB	Grande Distribution	0.977	0.364
16	Barco	Metal Electro	1.797	0.826
17	C.B.R.	Construction	1.364	0.823
18	Cobepa	Portefeuille	0.994	0.768
19	Cock. Sambre	Siderurgie	1.423	0.697
20	Colruyt	Grande Distribution	0.666	0.150
21	Deceuninck	Construction	1.393	0.701
22	Delhaize-Le Lion	Grande Distribution	0.913	0.296
23	Electrabel	Electricité et gaz	0.492	0.194
24	Electrafina	Portefeuille	1.179	1.056
25	Electrorail	Portefeuille	1.995	0.980
26	Finoutremer	Portefeuille	1.164	1.027
27	CMB	Divers Services	1.457	0.242
28	Clabecq	Siderurgie	2.211	0.589
29	Mosane (ex-Cofilim)	Portefeuille	0.540	0.482
30	Ackermans	Portefeuille	0.831	0.588
31	Almanij	Portefeuille	0.864	0.462
32	C.F.E.	Construction	1.043	0.094
33	Immobel	Immobilier	0.963	0.548
34	D'Ieteren	Divers Services	1.092	0.414
35	Asturienne	Divers Industries	1.334	1.068
36	Fabr. Fer Charl .	Siderurgie	0.833	0.505

37	Brederode	Portefeuille	1.145	0.848
38	Obourg-Fin	Chimie	0.775	0.544
39	Recticel	Chimie	1.560	0.704
40	Glaverbel	Divers Industries	1.530	0.382
41	G.B.L.	Portefeuille	1.195	0.884
42	Monceau-Zolder	Portefeuille	0.531	0.427
43	Nat. Portef.	Portefeuille	0.330	0.280
44	Petrofina	Petrole	0.950	0.575
45	Sait Radioh.	Metal Electro	1.020	0.487
46	UCO	Divers Industries	1.080	0.865
47	Powerfin	Electricité et gaz	0.641	0.551
48	Sidro	Petrole	0.578	0.529
49	Sofina	Portefeuille	1.104	0.944
50	BMT	Metal Electro	0.788	0.304
51	Cie Bois Sauvage	Portefeuille	0.563	0.371
52	Ibel	Portefeuille	0.878	0.731
53	Sipef	Tropicales	0.895	0.413
54	Cofinimmo	Immobilier	0.362	0.253
55	Belcofi	Portefeuille	1.599	1.203
56	Spector	Divers Services	0.502	0.253
57	Terca	Construction	0.398	0.216
58	Sun Intern.	Divers Services	1.459	0.974

REFERENCES

Beaver, W.H., "The Behavior of Security Prices and Its Implications for Accounting Research (Methods)"; Accounting Review (Supplement to Volume XLVII 1972)

Beaver, W.H., Kettler, P. and Scholes, M., "The Association between Market Determined and Accounting Determined Risk Measures", Accounting Review (October 1970), 654-682

Beaver, W.H. and Manegold, J., "The Association between Market-Determined and Accounting-Determined Measures of Systematic Risk: Some Further Evidence", Journal of Financial and Quantitative Analysis (June 1975), 231-284

Blann, J. and Balachandran, B.V., "An Empirical Test of the Statistical Association of Market Risk and Financial Accounting Allocation", Journal of Business Finance and Accounting (Spring 1988), 101-113

Blume, M.E., "Betas and their Regression Tendencies", Journal of Finance (June 1975), 785-795

Bogue, M., "The Behavior and Estimation of Systematic Risk", Graduate School of Business, Stanford, unpublished (1972)

Bowman, R.G., "The Theoretical Relationship Between Systematic Risk and Financial Variables", Journal of Finance (June 1979), 617-630

Bowman, R.G., "The Importance of a Market-Value Measurement of Debt in Assessing Leverage", Journal of Accounting Research (Spring 1980), 242-254

Bradley, M., Jarrell, G.A. and Kim, E.H., "On the Existence of an Optimal Capital Structure: Theory and Evidence, Journal of Finance (July 1984), 857-880

Cohen, K., Hawawini, G., Mayer, S., Schwartz, R. and Whitcomb, D., "Estimating and Adjusting for the Intervalling Effect Bias in Beta", Management Science (29 1983a), 135-148

Cohen, K., Hawawini, G., Mayer, S., Schwartz, R. and Whitcomb, D., "Friction in the Trading Process and the Estimation of Systematic Risk," Journal of Financial Economics (August 1983b), 263-278

Copeland, T.E. and Weston, J.F., "Financial Theory and Corporate Policy", Addison Wesley Publishing Co (1988)

Dimson, E., "Risk Measurement When Shares are Subject to Infrequent Trading", Journal of Financial Economics (June 1979), 197-226

Dimson, E. and Marsh, P.R., "The Stability of UK Risk Measures and The Problem of Thin Trading", Journal of Finance (June 1983), 753-783

Ehrhardt, M.C., "The Search for Value: Measuring the Company's Cost of Capital", Boston: Harvard Business School Press (1994)

Elgers, P. and Murray, D., "The Impact of the Choice of Market Index on the Empirical Evaluation of Accounting Risk Measures", Accounting Review (April 1982), 358-375

Elton, E.J., Gruber, M.J. and Mei, J., "Cost of Capital Using Arbitrage Pricing Theory: A Case Study of Nine New York Utilities", Financial Markets, Institutions and Instruments (3 1994), 47-73

Hamada, R.S., "Portfolio Analysis, Market Equilibrium and Corporation Finance", Journal of Finance (March 1969), 13-32

Hamada, R.S., "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks", Journal of Finance (May 1972), 435-452

Hawawini, G. and Michel, P. (eds.), "Stock Market Efficiency", Garland (1984)

Hill, N.C. and Stone, B.K., "Accounting Betas, Systematic Operating Risk, and Financial Leverage: A Risk-Composition Approach to the Determinants of Systematic Risk", Journal of Financial and Quantitative Analysis (September 1980), pp. 594-638

Jensen, M, "Capital Markets: Theory and Evidence", The Bell Journal of Economics and Management Science (Autumn 1972)

Laveren, E., "Interacties tussen investerings- en financieringsbeslissingen. Impact en verwerking van deze interacties in de investeringsanalyse", Doctoraal proefschrift, UFSIA (1990)

Lev, B. and Kunitzky, S., "On the Association between Smoothing Measures and the Risk of Common Stocks", Accounting Review (April 1974), 259-270

Levy, H. and Sarnat, M, "Capital Investment and Financial Decisions, Jerusalem: Prentice Hall Int. (1986)

Lintner, J., "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", The Review of Economics and Statistics (February 1965), 13-37

Lintner, J., "Security Prices, Risk, and Maximal Gains from Diversification", Journal of Finance (December 1965), 587-616

Logue, D.E. and Merville, L.J., "Financial Policy and Market Expectations", Financial Management (Summer 1972), 37-44

Luoma, M., Martikainen, T., Perttunen, J. and Pynnönen, S., "Different Beta Estimation Techniques in Infrequently Traded and Inefficient Stock Markets", Omega (5 1994), 471-476

Mandelker, G.N. and Rhee, S.G., "The Impact of the Degrees of Operating and Financial Leverage on Systematic Risk of Common Stock", Journal of Financial and Quantitative Analysis (1 1984), 45-57

Melicher, R.W. and Rush, D.F., "Systematic Risk, Financial Data, and Bond Rating Relationships in a Regulated Industry Environment", Journal of Finance (May 1974), 537-544

Modigliani, F. and Miller, M., "The Cost of Capital, Corporate Finance and the Theory of Investment: A Correction", American Economic Review (June 1963), 433-443

Mossin, J., "Equilibrium in a Capital Asset Market", Econometrica (October 1966), 768-782

Ooghe, H. and Van Wymeersch, C., "Financiële analyse van ondernemingen. Theorie en toepassing op de jaarrrekening", Kluwer (1994)

Pettit, R. and Westerfield, R., "A Model of Capital Asset Risk", Journal of Financial and Quantitative Analysis (March 1972)

Rosenberg, B. and McKibben, W., "The Prediction of Systematic and Specific Risk in Common Stocks", Journal of Financial and Quantitative Analysis (March 1973), 317-334

Scholes, M. and Williams, J., "Estimating Betas from Nonsynchronous Data", Journal of Financial Economics (December 1977), 309-327

Sharpe, W.F., "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk" Journal of Finance (September 1964), 425-442

Thompson, D.J., "Sources of Systematic Risk in Common Stocks", Journal of Business (April 1976), 173-188

Treynor, J.L., "Toward a Theory of Market Value of Risky Assets", Unpublished Memorandum (1961)

Vasicek, O., "A Note on Using Cross-sectional Information in Bayesian Estimation of Security Betas", Journal of Finance (December 1973), 1233-1239

Vogt, S.C., "The Role of Internal Financial Sources in Firm Financing and Investment Decisions", Review of Financial Economics (Fall 1994), 1-24