Competitive balance and gate revenue sharing in team sports

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Abstract: This paper shows that under reasonable conditions that increasing gate revenue sharing among teams in a sports league will produce a more uneven contest, i.e. reduce competitive balance. This result has significant implications for antitrust authorities and legislators, who have tended to assume that revenue sharing arrangements will necessarily promote competitive balance.

Keywords: Team sports, contest (tournament) theory, competitive balance

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1. Introduction

It is a widely held belief that a sporting competition such as a soccer or baseball league will be more successful the greater the degree of competitive balance among the teams, because the matches will be more uncertain and therefore more entertaining. Owners of teams have consistently used this argument to justify revenue sharing schemes. The idea, they claim, is to equalise resources so that “weak drawing” teams can compete with “strong drawing” teams. This argument has been accepted by competition authorities and legislators in North America, Europe and elsewhere and agreements to share revenues are generally considered to be pro-competitive³.

Critics have pointed out that revenue sharing will also blunt the incentive for profit maximising team owners to compete, since for each team the returns to winning are reduced. This may mean less competition to attract players, lower salaries and a lower standard of competition. In this paper we analyse a specific form of revenue sharing- gate revenue sharing⁴- and show that under reasonable assumptions it will not only blunt incentives but will also produce a more uneven distribution of talent in a league and therefore reduce competitive balance. While this does not imply that all revenue sharing agreements reduce competitive balance, it does suggest that such arrangements require more careful economic analysis.

2. Revenue sharing in contest models

Many commentators on sporting issues, not least the team owners themselves, have drawn attention to the collective action problem facing the members of a league. Suppose that some large market teams have the potential to draw a significant following from a given level of success while smaller teams will draw only a relatively small following. Self interested behaviour, they claim, will cause the large market teams to dominate the competition to the point where it becomes too predictable and demand will fall below the level that maximises joint profits (and consumer interest). One widely advanced solution to this perceived problem is to share revenues and so provide relatively equal opportunities for all the teams.

³ In North America most revenue sharing agreements have not been challenged in the courts. The collective selling of national broadcast rights, which team owners claimed was intended to promote revenue sharing but which was deemed anti-competitive by the courts, has been exempted from antitrust law by Congress (Weiler and Roberts (1998)). In Europe the European Commission has encouraged redistributive agreements aimed at maintaining “solidarity” between the professional and amateur levels of sport, and has also recognised the need to maintain competitive balance. While the specific issue of revenue sharing between members of the same league has not been addressed, it is widely presumed that it would not be deemed anti-competitive (see e.g. European Commission (1999)).

⁴ Gate revenue sharing is most notably practised in the National Football League in the US, where 40% of designated gate revenues go to the visiting team. There is more limited gate sharing in Major League Baseball, and none in NBA (basketball) or NHL (hockey). European soccer leagues have practised various forms of gate sharing in their history. In England visiting teams in League matches received up to 20% of the gate until the early 1980s. Gate sharing is relatively rare in European league soccer, but quite common in most Cup (knock-out) competitions.

Analysis of this problem requires a contest model. A sporting contest is a type of all-pay auction in which the players or teams make bids in the form of effort or investment in talent. A contest success function (see e.g. Skaperdas (1996)) defines each agent's probability of success as a function of that agent's share of the total contribution of all agents to the contest. Here we think of the teams as the agents but instead of competing for a fixed prize teams have their own revenue generating functions which depend on the degree of success of the team. All-pay auctions are typically "perfectly discriminating" so that the highest bidder wins with probability one (see e.g. Hillman and Riley (1979)). A sporting contest is always imperfectly discriminating since the most expensive team or the player who makes most effort can never be certain of winning. The most widely used contest success function in sports, as in a number of other applications, is the logit⁵ (e.g. El-Hodiri and Quirk (1971) and Fort and Quirk (1995)).

In the contest literature the main issues have been the amount of effort/investment that can be elicited and the degree to which any rents to be earned in the contest are dissipated by competition. To analyse this issue researchers have mainly focused on symmetric contests⁶. In sports, unlike most other contest situations, demand exists for the contest itself, rather than its consequences. Moreover, most people believe that consumers have a preference for more balanced contests⁷. Hence the sports literature's main concern is with the distribution of effort/investment⁸, which clearly requires a model of an asymmetric contest.

In its most general form we can define the logit contest success function for n contestants as

$$(1) \quad w_i = \frac{h_i(t_i)}{\sum_{j=1}^n h_j(t_j)}$$

where w is the probability of success, t is the effort contribution of the contestant and $h' > 0$ and $h'' \leq 0$ with $h_i(0) = 0$ ⁹. In the context of a team sports league we take w to be the expected percentage of matches won and t the investment in playing talent by the team owner. Given that the probabilities must sum to unity for any contest we have the useful adding up constraint that

⁵ e.g. Loury (1979), Tullock (1980), and Nti (1997). The principal alternative is the probit function explored, inter alia by Lazear and Rosen (1981) and Dixit (1987).

⁶ Dixit (1987) and Baik (1994) are notable exceptions.

⁷ The uncertainty of outcome hypothesis goes back at least as far as Rottenberg (1956) in the sports economics literature and is its most resilient theme. There is a small empirical literature testing the validity of this hypothesis, see for example Schmidt and Berri (2001) and Forrest and Simmons (2002).

⁸ For team sports contests of the type considered in this paper, it is more natural to think of the contribution of each team as an investment in playing talent rather than the effort of the players on the field. We suppose that individual players supply optimal effort. This is consistent with the belief that top players are highly motivated and therefore financial incentives are largely irrelevant to the effort supply decision or that effort is observable so that first best contracts can be written and enforced.

⁹ If $h_i(0) = 0$ for all contestants we define $w_i = 0$ ("no contest").

$$(2) \quad \frac{\partial w_i}{\partial t_i} = - \sum_{\substack{j=1 \\ j \neq i}}^n \frac{\partial w_j}{\partial t_i}$$

In what follows we will adopt a common assumption from that literature, that the h functions are identical for each team (Baik considers asymmetry in the h functions). Symmetry of the h functions implies that if all contestants invested the same amount of resources in trying to win the contest then each contestant would have an equal probability of winning. This amounts to assuming that teams share the same production technology, so a unit of talent is expected to be equally productive of wins, for a given level of team talent, as at any other team. This might seem a controversial assumption, but in markets as competitive as professional team sports where player salaries are counted in the millions of dollars it seems hard to believe that managers do not all adopt best practice.

Assuming symmetric h functions is not the same as assuming the contest itself is symmetric. Even with symmetric h 's teams can have asymmetric objective functions. This might be due to larger revenue generating opportunities (e.g. a larger local market, a greater reputation, a more media friendly team) or a lower cost base (e.g. due to some distinctive capability in team management, closeness to a rich source of talent or a location that was more attractive to players).

We consider a two team model where each team is a profit maximiser and profits consist of gate revenues less the cost of talent investment. The gate revenue function from success (winning) can be asymmetric and the marginal revenue from winning can be constant or decreasing. We also allow that excessive dominance by one team can lead to a fall in revenues for the dominant team as well as the weaker team. Thus the gate revenue function for each team in the contest is defined as

$R_i = R_i(w_i)$ where either $R_i' > 0$ and $R_i'' \leq 0$ for all w_i in $[0,1]$ or there exists a $w_i^* \in [0,1]$ such that if $w_i \geq w_i^*$ then $R_i' < 0$ otherwise $R_i' > 0$, and $R_i'' < 0$ everywhere. We assume that $w^* \geq \frac{1}{2}$ for at least one team, otherwise both teams would prefer to win fewer than 50% of their matches and no equilibrium would exist.

The impact of gate revenue sharing on both the quantity of talent hired in the market and competitive balance is a standard problem in the sports literature (see e.g. Quirk and El-Hodiri (1974), Fort and Quirk (1995) and Vrooman (1995)). The principal difference between these models and the model presented here is that they treat the total supply of talent to the league as fixed (since they have in mind a major league where all the best players want to play), whereas here we allow that the supply of talent may be fixed or elastic, the latter case having more in common with the situation in European soccer leagues (see Szymanski (2003) for a more detailed discussion of differences).

For the purposes of our model, we assume that each team retains a fraction $\alpha > \frac{1}{2}$ of income generated from home matches and pays $1 - \alpha$ to their opponents. In line with most

of the existing literature we assume that talent can be hired in the market at a constant marginal cost c . We can write the profit function for either team (here team 1) as:

$$(3) \quad \pi_1 = \alpha R_1[w_1(t_1, t_2)] + (1 - \alpha) R_2[w_2(t_2, t_1)] - ct_1$$

The first order condition for team 1 is:

$$(4) \quad \frac{\partial \pi_1}{\partial t_1} = \alpha R'_1 \frac{\partial w_1}{\partial t_1} + (1 - \alpha) R'_2 \frac{\partial w_2}{\partial t_1} - c = 0$$

(In equation (4) and in what follows the equivalent conditions for team 2 can be obtained by simply switching the subscript 1 for 2, and vice versa). In our analysis of the general case we will assume that the marginal revenues in equilibrium are large enough to ensure that an interior solution to (4) always exists. Note that even if an interior solution exists for $\alpha=1$, gate revenue sharing may introduce the possibility of corner solutions. This possibility is considered using an example below.

Given that in a two team model $\frac{\partial w_1}{\partial t_1} = -\frac{\partial w_2}{\partial t_1}$ (this is the adding up constraint (2)), using the contest success function (1) we obtain

$$(5) \quad \frac{\partial \pi_1}{\partial t_1} = (\alpha R'_1 - (1 - \alpha) R'_2) \left(\frac{w_2 h'_1}{h_1 + h_2} \right) - c = 0$$

Since marginal costs are identical equation (5) says that in equilibrium the marginal revenue from the hiring of talent is equalised across teams. This is not the same as saying the marginal revenue of a win is equalised which is only true if $h'_1 = w_1$ and $h'_2 = w_2$.

We can also see from (5) that if there is no revenue sharing ($\alpha=1$), in equilibrium both teams must choose a quantity of talent such that marginal revenue is positive (otherwise marginal profit is always negative). For $\alpha \in (1/2, 1)$ R'_1 could only be negative and satisfy (5) if R'_2 is also negative. But this cannot be an equilibrium since the teams could raise profits by hiring no talent at all. Hence marginal revenues must be positive for both teams in equilibrium. If we take the difference of the two first order conditions we obtain

$$(6) \quad R'_1 - R'_2 = \frac{c(h_1 + h_2)(w_1 h'_2 - w_2 h'_1)}{w_1 w_2 h'_1 h'_2}$$

From which it is apparent that in equilibrium the team with the higher expected winning record ($w_1 > w_2$) will always have the larger own marginal revenue from winning ($R'_1 > R'_2$). A natural interpretation of this is that the “stronger drawing” dominates competition, which is true in the sense that a team whose support values an additional win more than its rivals for any given win percentage will dominate in equilibrium.

While in many cases this is likely to be true for large city teams competing against small city teams, it is also possible that a small city team could dominate a larger rival city if its fans are sufficiently sensitive to the success of the team¹⁰. However, despite dominating the contest, the stronger team will still have a greater marginal revenue from a win than the weaker team (but not from hiring an additional unit of talent) in equilibrium.

To make further progress we need to simplify the problem. We want to examine the properties of the revenue sharing equilibrium for the most general revenue functions possible, and to do so we restrict our analysis to the simplest and most widely used contest success function by assuming $h(t_1) = t_1$ so that $w_1 = t_1/(t_1 + t_2)$ and $w_2 = 1 - w_1$.

Our principal interest is in the impact of a change in the revenue sharing parameter α on the level of competitive balance. This can be measured by the ratio of winning percentages, which now also equals the ratio of talent units hired by each team, i.e. $w_1/w_2 = t_1 / t_2$, and so

$$(7) \quad \frac{\partial \left(\frac{w_1}{w_2} \right)}{\partial \alpha} = \frac{t_2 \frac{dt_1}{d\alpha} - t_1 \frac{dt_2}{d\alpha}}{t_2^2} = \frac{1}{t_2} \left(\frac{dt_1}{d\alpha} - \frac{w_1}{w_2} \frac{dt_2}{d\alpha} \right)$$

Proposition

Assuming that a stable solution to the teams' optimisation problem exists, increased revenue sharing

- (a) Causes competitive balance to deteriorate.
- (b) Reduces the number of units of talent hired by each team for a given wage rate per unit of talent.

Proof

See appendix 1.

What is the intuition for these results? The result that revenue sharing reduces talent investment is long established and follows simply from the dulling of incentives to win. However, part (a) also shows that the dulling effect is greater for the weak drawing team. This is because a higher probability of own success leads to a greater loss of gate revenue generated by the opponent for the weak drawing team. It is worth noting that this result depends on the logit formulation which ensures that the marginal impact on the dominant team's winning probability of an increase in the investment of the weaker team is greater than the marginal impact on the weaker team's winning probability of an increase in the investment of the dominant team. If the contest success function were specified such that

¹⁰ In practice the phenomenon is seldom observed.

there were increasing returns to talent investment, even over a relatively limited range, this result might no longer hold¹¹.

It is also important to note that part (b) is derived under the condition where the marginal cost of talent (the wage rate) is treated as a parameter, i.e. there is no price adjustment in the labour market. This is appropriate if the supply of talent is perfectly elastic. However, if we assume that the supply of talent is fixed then increased revenue sharing will create an excess supply of talent and its marginal cost must fall to restore labour market equilibrium. Since competitive balance must worsen the dominant team must increase the number of units of talent hired while the weaker team reduces its demand (but by less than in the case where marginal cost is fixed). Total investment in talent falls. To illustrate this possibility and the possibility of corner solution it is useful to look at an example.

3. An Example: $R_1 = \sigma w_1$, $R_2 = w_2$, $\sigma > 1$

Here marginal revenue is a positive constant and $\sigma > 1$ implies that team 1 is the strong drawing team.

$$\frac{w_1}{w_2} = \frac{\alpha\sigma - (1 - \alpha)}{\alpha - \sigma(1 - \alpha)}$$

and

$$\frac{\partial \left(\frac{w_1}{w_2} \right)}{\partial \alpha} = \left[\frac{(1 - \sigma)(1 + \sigma)}{(\alpha - (1 - \alpha)\sigma)^2} \right] < 0$$

Note also that the expected win percentages are

$$w_1 = \frac{\alpha(\sigma + 1) - 1}{(2\alpha - 1)(1 + \sigma)} \quad \text{and} \quad w_2 = \frac{\alpha - \sigma(1 - \alpha)}{(2\alpha - 1)(1 + \sigma)}$$

So that an interior solution requires $\sigma < \alpha/(1 - \alpha)$. Note that this is satisfied no matter how great the inequality in drawing power when there is no revenue sharing, but can never be satisfied if there is equal revenue sharing. Because revenue sharing has a negative effect on competitive balance, there is a narrower and narrower range of initial inequality that will sustain an interior solution as the degree of sharing increases.

¹¹ We are grateful to the editor for this point. One example of such a function might involve increasing for low levels of success but then decreasing returns at high levels of success.

If $\sigma > \alpha/(1-\alpha)$ then the weaker drawing team's profit function is decreasing in its own success. This implies an equilibrium where the weak team invests in zero units of talent while the strong team invests in the smallest number of talent units possible and wins 100% of its matches. Once this corner solution is reached, increased revenue sharing has no effect on the equilibrium.

We can also derive the investment in talent by each team:

$$t_1 = \frac{[\alpha(1+\sigma)-1]^2[\alpha(1+\sigma)-\sigma]}{c[(2\alpha-1)(1+\sigma)]^2} \quad \text{and} \quad t_2 = \frac{[\alpha(1+\sigma)-1][\alpha(1+\sigma)-\sigma]^2}{c[(2\alpha-1)(1+\sigma)]^2}$$

so that total demand for talent in the market is

$$t_1 + t_2 = \frac{\sigma - \alpha(1-\alpha)(1+\sigma)^2}{c(2\alpha-1)(1+\sigma)}$$

This implies the further restriction that $\sigma/(1+\sigma)^2 > \alpha(1-\alpha)$ in order for total demand to be positive. Once again, the constraint is not binding when $\alpha = 1$ but becomes increasingly restrictive as α approaches $1/2$.

If the total supply of talent is fixed at some value T , then there exists a market clearing marginal cost of talent (which we can call the wage rate per unit of talent) that ensures that supply equals demand. Given that $T = t_1 + t_2$ in equilibrium we obtain the usual inverse relationship between the wage rate and the demand for talent, but changes in the wage rate will not affect the talent shares of the two teams (marginal cost does not enter the expression for the win ratios). We can also show that total expenditure on talent will be increasing in α (revenue sharing diminishes total investment).

$$\frac{d(cT)}{d\alpha} = 1 + \sigma - \frac{2(\sigma - \alpha(1-\alpha)(1+\sigma))}{(2\alpha-1)^2(1+\sigma)}$$

which is positive for α and σ within their permissible ranges.

4. Conclusions

This paper has shown that under reasonable assumptions gate revenue sharing will not only reduce total investment in talent by teams in a league but also diminishes the degree of competitive balance. This has important implications for competition authorities and legislators who have generally taken a permissive view of revenue sharing schemes on the grounds that they favour competitive balance.

The main result of this paper contrasts sharply with the well known “invariance principle” which states that gate revenue sharing will have no effect on competitive balance (Quirk and El-Hodiri (1974), Fort and Quirk (1995), Vrooman (1995))¹². As the example of the previous section shows, this is not the result of assuming that the supply of talent is fixed. In the original paper of Quirk and El-Hodiri it seems to emerge as a consequence of assuming that each team maximises profit with respect to the talent choices of all other firms, so that the final allocation of talent ensures joint profit maximisation¹³ (in contrast to the model here, which is noncooperative and whose equilibrium is not joint profit maximising). In the models of Quirk and Fort and Vrooman the result stems from a different assumption about the derivative of the contest success function (see Appendix 2).

While gate revenue sharing is known in team sports, it is by no means the only sharing mechanism used. Pool revenue sharing, adopted in 1997 Major League Baseball, where teams contribute a fixed percentage of revenues which is then redistributed according to another formula is sometimes equivalent to gate sharing, but more work remains to be done to analyse the properties of different pooling schemes. Another form of revenue sharing that deserves detailed consideration is the division of revenue generated from the collective sale of broadcast rights.

In North America this is distributed on the basis of equal shares for all teams. However in European soccer leagues the distribution formula (when collective sale is permitted) typically entails a prize like element¹⁴. Prizes can in principle create balanced contests by evening up the ex ante incentives to invest. Moldovanu and Sela (2001) consider the distribution of prizes in the context of an all-pay auction (a perfectly discriminating contest) while Szymanski and Valletti (2002) have looked at this issue for the case of an imperfectly discriminating contest. The optimal distribution of prizes in asymmetric contests is an important area of research with applications wider than the sports literature.

¹² Several writers have identified exceptions to this rule, notably Atkinson et al (1988), Marburger (1997), Hoehn and Szymanski (1999) and Késenne (2000).

¹³ Quirk and El-Hodiri (1974) p.62.

¹⁴ For example, in the English Premier League 25% of the money distributed to clubs is awarded on the basis of league rank.

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Appendix 1: Proof of Proposition

To prove part (a) it is sufficient to show (from (7)) that $\frac{dt_1}{d\alpha} - \frac{w_1}{w_2} \frac{dt_2}{d\alpha} < 0$ if team 1 is dominant team when $\alpha = 1$ (recall that $R'_1 > R'_2$ in equilibrium is a sufficient condition for team 1 to be dominant).

To sign (7) we need to conduct a comparative statics exercise on the decision making of the two teams. Totally differentiating the first order conditions (5) for each team we can write

$$(A1) \quad \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} dt_1 \\ dt_2 \end{pmatrix} = \begin{pmatrix} -\pi_{1\alpha} \\ -\pi_{2\alpha} \end{pmatrix} d\alpha$$

where

$$(A2) \quad \begin{aligned} \pi_{11} &= \frac{\partial^2 \pi_1}{\partial t_1^2} = (\alpha R'_1 - (1-\alpha)R'_2) \left(\frac{-2w_2}{(t_1+t_2)^2} \right) + (\alpha R''_1 + (1-\alpha)R''_2) \left(\frac{w_2^2}{(t_1+t_2)^2} \right) \\ \pi_{12} &= \frac{\partial^2 \pi_1}{\partial t_1 \partial t_2} = (\alpha R'_1 - (1-\alpha)R'_2) \left(\frac{w_1 - w_2}{(t_1+t_2)^2} \right) - (\alpha R''_1 + (1-\alpha)R''_2) \left(\frac{w_1 w_2}{(t_1+t_2)^2} \right) \\ \pi_{22} &= \frac{\partial^2 \pi_2}{\partial t_2^2} = (\alpha R'_2 - (1-\alpha)R'_1) \left(\frac{-2w_1}{(t_1+t_2)^2} \right) + (\alpha R''_2 + (1-\alpha)R''_1) \left(\frac{w_1^2}{(t_1+t_2)^2} \right) \\ \pi_{21} &= \frac{\partial^2 \pi_2}{\partial t_1 \partial t_2} = (\alpha R'_2 - (1-\alpha)R'_1) \left(\frac{w_2 - w_1}{(t_1+t_2)^2} \right) - (\alpha R''_2 + (1-\alpha)R''_1) \left(\frac{w_1 w_2}{(t_1+t_2)^2} \right) \\ \pi_{1\alpha} &= \frac{\partial^2 \pi_1}{\partial t_1 \partial \alpha} = (R'_1 + R'_2) \frac{w_2}{t_1 + t_2} \\ \pi_{2\alpha} &= \frac{\partial^2 \pi_2}{\partial t_2 \partial \alpha} = (R'_1 + R'_2) \frac{w_1}{t_1 + t_2} \end{aligned}$$

Each of these expressions can be simplified by observing that for team 1 in equilibrium

$$(\alpha R'_1 - (1-\alpha)R'_2) = \frac{c(t_1+t_2)}{w_2}$$

(and using the equivalent expression for team 2). Applying Cramer's Rule to (A1) we can write

$$(A3) \quad \frac{dt_1}{d\alpha} = \frac{\pi_{12}\pi_{2\alpha} - \pi_{22}\pi_{1\alpha}}{\pi_{11}\pi_{22} - \pi_{12}\pi_{21}} \quad \text{and} \quad \frac{dt_2}{d\alpha} = \frac{\pi_{21}\pi_{1\alpha} - \pi_{11}\pi_{2\alpha}}{\pi_{11}\pi_{22} - \pi_{12}\pi_{21}}$$

We assume that the denominators of (A3) are positive, which is a standard stability condition in the literature (see e.g. Dixit (1986)). It is clear from (A2) that the second order conditions π_{11} and π_{22} are negative given our assumptions about revenues. A sufficient condition for stability therefore is that the expressions π_{12} and π_{21} , which are the slopes of the reaction functions, have opposite signs. It also clear from (A2) that $\pi_{12} > 0$ if team 1 is the stronger team (the reaction function of the dominant team slopes upward). However, the sign of π_{21} is ambiguous. If we assume it to be negative (as, for example, it must be if marginal revenue is constant) then stability is guaranteed. Stability is also ensured under the less restrictive assumption that $\pi_{11}\pi_{22} > \pi_{12}\pi_{21}$ ¹⁵.

We can conclude that the sign of (7) depends on the sign of the weighted difference of the numerators of (A3), that is

$$(A4) \quad \text{sgn}\left(\frac{dt_1}{d\alpha} - \frac{w_1}{w_2} \frac{dt_2}{d\alpha}\right) = \text{sgn}\left[\pi_{2\alpha}(\pi_{12} + \frac{w_1}{w_2}\pi_{11}) - \pi_{1\alpha}(\pi_{22} + \frac{w_1}{w_2}\pi_{21})\right]$$

After some manipulation we can reduce the RHS of (A4) to

$$(A5) \quad (R'_1 + R'_2) \left(\frac{w_1 - w_2}{t_1 + t_2} \right) \left(\frac{-c}{w_2(t_1 + t_2)} \right)$$

This expression is negative if $w_1 > w_2$. Hence win percentage of the dominant team increases as revenue sharing increases and competitive balance deteriorates. This proves part (a). Part (b) follows quite simply: from (A2) and (A3) it is clear that $dt_1/d\alpha > 0$ (given that $\pi_{12} > 0$). But if increased revenue sharing causes the dominant team to reduce its investment and competitive balance worsens then it must also be the case that $dt_2/d\alpha > 0$. QED

¹⁵ Atkinson et al (1988) use the result that $\frac{\partial t_i}{\partial \alpha} > 0$ (p33, fn. 14) to conduct their comparative static exercise.

This seems to rest on a confusion of the partial derivative, which must indeed be positive from inspection of (A2), and the total derivative which is required for the comparative statics and can only be signed using the second order conditions and the cross partials, which are neglected by Atkinson et al. This would not be a problem if marginal revenues were constants, but in fact they assume total revenues to be strictly concave (p.29)).

Appendix 2: Derivation of the invariance principle in Fort and Quirk (1995) and Vrooman (1995).

Given the contest success function $w_1 = t_1/(t_1 + t_2)$, its derivative (in (4)) can be written as

$$(A6) \quad \frac{\partial w_1}{\partial t_1} = \frac{t_1 + t_2 - t_1 \left(1 + \frac{dt_2}{dt_1}\right)}{(t_1 + t_2)^2}$$

In this paper it is assumed that $dt_2/dt_1 = 0$, which we interpret to be the usual Nash conjecture, since this derivative appears in the first order condition of the team owner's objective function (5). However, Fort and Quirk and Vrooman assume that $dt_2/dt_1 = -1$, which has a similar consequence as assuming joint profit maximising conjectures in a standard oligopoly model (see e.g. Vives (1999)). They account for this assumption by arguing that the supply of talent is fixed, and that therefore a one unit increase in talent hired at one team necessarily leads to a one unit reduction of talent at another team. As a consequence of their assumption we can rewrite (7) as

$$(A7) \quad \frac{\partial \pi_1}{\partial t_1} = \left(\alpha R'_1 - (1 - \alpha) R'_2 \right) \left(\frac{1}{t_1 + t_2} \right) - c = 0$$

Given that the win percentage derivatives are identical for each team, these cancel out when taking the ratio of the two first order conditions and so we are left with the equilibrium condition:

$$(A8) \quad \alpha R'_1 - (1 - \alpha) R'_2 = \alpha R'_2 - (1 - \alpha) R'_1 \Rightarrow R'_1 = R'_2$$

Thus team marginal revenues are equalised for any value of α , and we obtain the invariance result that the extent of gate revenue sharing has no impact on the equilibrium distribution of playing talent. As specified, this model cannot have a Nash equilibrium since by assumption t_1 and t_2 are fixed (when $dt_2/dt_1 = -1$ the choice of one team automatically constrains the other in a two team model). It might be possible to develop a model that incorporates the fixed supply constraint into the objective function while treating wages or investment as the choice variable (rather than talent). However, the implications of such a model for the invariance principle are not clear.