DEPARTMENT OF ECONOMICS

Mirror, mirror, on the wall, who in this land is fairest of all? Revisiting the extended concentration index

Guido Erreygers, Philip Clarke & Tom Van Ourti

UNIVERSITY OF ANTWERP
Faculty of Applied Economics
Stadscampus
Prinsstraat 13, B.213
BE-2000 Antwerpen
Tel. +32 (0)3 265 40 32
Fax +32 (0)3 265 47 99
http://www.ua.ac.be/tew

FACULTY OF APPLIED ECONOMICS

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University of Antwerp, City Campus, Prinsstraat 13, B-2000 Antwerp, Belgium
Research Administration – room B.213
phone: (32) 3 265 40 32
fax: (32) 3 265 47 99
e-mail: joeri.nys@ua.ac.be

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Revisiting the extended concentration index

Guido ERREYGERS, Philip CLARKE, and Tom VAN OURTI

Addresses: Guido Erreygers, Department of Economics, University of Antwerp, City Campus, Prinsstraat 13, 2000 Antwerpen, Belgium; guido.erreygers@ua.ac.be

Philip Clarke, School of Public Health, University of Sydney, Edward Ford Building, NSW 2006 Australia; philipc@health.usyd.edu.au

Tom Van Ourti, Erasmus School of Economics, Erasmus University Rotterdam, PO Box 1738, 3000 DR Rotterdam, The Netherlands; Tinbergen Institute and NETSPAR; vanourti@ese.eur.nl

Abstract:

This paper explores three alternative indices for measuring health inequalities in a way that takes into account attitudes towards inequality. Firstly, we revisit the extended concentration index which has been proposed to generalise the value judgements implicit in the standard concentration index. We then examine two alternative measures which have desirable mirror properties. One of these indices applies symmetric weights which is a property of the standard concentration index. We also examine the bias that arises when all three measures are applied to small samples. We propose a correction for this small sample bias and use Monte Carlo simulations to check whether it works. We empirically compare the different indices for under-five mortality rates in developing countries.

JEL Classification Codes: I19, I32, D63

Keywords: health inequality, socioeconomic inequality, extended concentration index, inequality aversion, small-sample bias

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1. Introduction

Pereira (1998) and more recently Wagstaff (2002) have proposed to extend the concentration index by including a distributional judgement parameter. The extension is seen as a device which makes it possible to formally incorporate attitudes towards inequality into the calculation of the index of socioeconomic inequality of health. It builds on suggestions of Kakwani (1980) developed by Yitzhaki (1983), who shows how a similar extension of the Gini coefficient allows the expression of distributional judgements in the context of income inequality measurement.

While the extended concentration index can be applied to both health and health care, our focus here is on the former. Moreover, we are especially interested in health variables which can be looked at from two points of view: the positive side, where the focus is on 'good health' (e.g. the proportion of children without malnutrition), and the negative side, where the emphasis is on 'ill health' (e.g. the proportion of children with malnutrition). For any good health variable which has a finite upper bound it is in principle possible to define a corresponding ill health variable by calculating the shortfall with regard to the maximum. This twofold character of many health variables introduces an element into the measurement of health inequality which does not occur in the measurement of income inequality. The difference is, however, often overlooked or simply ignored.

For Wagstaff (2002) the extended concentration index can be applied to both (good) health and ill health variables, and there are a growing number of empirical studies following this approach. For example, the index has been used to calculate the degree of socioeconomic inequality in child mortality across both developing countries (Wagstaff 2002) and developed countries (Pereira 1998), in health limitations within eight European countries across time (Hernandez-Quevedo et al. 2006), and in child malnutrition in Nigeria (Uthman, 2009). It has also been applied to health variables such as the proportion of the population being immunized in developing countries (Gaudin and Yazbeck 2006; Meheus and Van Doorslaer 2008).

The purpose of this paper is to explore the extended concentration index and the implications for the measurement of socio-economic inequality. Our initial focus is on understanding the extended concentration index particularly in its application to health and ill health. We then propose and examine two alternative indicators, one of which is based upon a different distributional weighting scheme. The fifth section suggests a solution for biases which arise when individuals might have the same socioeconomic position, or when applying these indicators on samples with a low number of observations. The sixth section contains an empirical illustration and the final section some conclusions.

2. Background

We consider a bounded health variable of the cardinal or ratio-scale type. Any such variable can always be transformed into a standardized variable with a range equal to the interval [0,1]: if x_i is a variable with a range equal to [a,b], then the corresponding standardized variable h_i is defined as: $h_i \equiv (x_i - a)/(b-a)$. From now on we assume that the (standardized) health status of individual i is equal to h_i, a scalar between 0 and 1. Correspondingly, her (standardized) ill-health level is equal to $s_i = 1 - h_i$, which is also a scalar between 0 and 1.

Suppose the population N consists of n individuals, where n is a finite, positive natural number. Let $N = \{1, 2, ..., n\}$, and assume that individuals are ranked according to their socioeconomic position, in ascending order (i.e. individual 1 is the poorest, and individual n the richest person). If individual i is not tied to any other, his rank ρ_i coincides with his number i ($\rho_i = i$); if he is tied to other persons, all persons of the tied group have a rank equal to the average number of the members of this group. The fractional rank R_i of individual i is equal to $(2\rho_i - 1)/(2n)$, and varies between $1/(2n)$ and $1-1/(2n)$ (if there are no ties). The average rank μ_ρ is equal to $(n+1)/2$, and the average fractional rank μ_R equal to $1/2$.

When n becomes very large, the fractional rank can be approximated by a continuous variable p defined over the interval $[0,1]$. The interval $[0, p]$ then represents the $100p\%$ poorest individuals of the population, just as those with fractional ranks $1/(2n), 3/(2n), ..., (2i-1)/(2n)$ represent the $100(i/n)\%$ poorest individuals. The function $h(p)$ expresses the health status of an individual as a function of where this individual is located in the interval $[0,1]$. Clearly, $h(0)$ is the health status of the poorest individual and $h(1)$ that of the richest. The corresponding ill-health function is $s(p) = 1 - h(p)$.

In case of a finite number of individuals, the average health and ill-health status are defined as:

$$\mu_h = \frac{1}{n}\sum_{i=1}^{n} h_i, \quad \mu_s = \frac{1}{n}\sum_{i=1}^{n} s_i \tag{1}$$

For the continuous case we have:

$$\mu_h = \int_0^1 h(p)dp, \quad \mu_s = \int_0^1 s(p)dp \tag{2}$$

It is easy to check that in both cases we have:

$$\mu_h + \mu_s = 1 \tag{3}$$

3. The Extended Concentration Index

3.1. The index defined

Following suggestions by Kakwani (1980) and Yitzhaki (1983), both Pereira (1998) and Wagstaff (2002) have introduced the following Extended Concentration Index:

$$C(h,\nu) = 1 - \frac{\nu}{n\mu_h}\sum_{i=1}^{n}(1-R_i)^{\nu-1}h_i \tag{4}$$

where $\nu \geq 1$ is a parameter expressing the sensitivity to inequality. Expression (4) can be formulated in many equivalent ways. Using the definitions of the previous section, (4) can be transformed into a weighted sum of health:

$$C(h,\nu) = \frac{1}{\mu_h}\frac{1}{n}\sum_{i=1}^{n}\left[1-\nu\left(1-R_i\right)^{\nu-1}\right]h_i \tag{5}$$

Since p corresponds to R_i, the continuous counterpart of (5) is:

$$C(h,\nu) = \frac{1}{\mu_h}\int_0^1 [1-\nu(1-p)^{\nu-1}]h(p)dp \tag{6}$$

The extended concentration index, in a similar fashion to income inequality measures such as the extended Gini index (Yitzhaki 1983), assigns weights to individuals based upon their fractional rank modified by the inequality aversion parameter ν. A way to understand how the extended concentration index is influenced by ν is to focus on the weighting function, which expresses how the weight of a person depends on her fractional rank p and the inequality parameter ν:

$$w(p,\nu) = 1-\nu(1-p)^{\nu-1} \tag{7}$$

Figure 1 plots the weighting function for a range of values of ν.

[Insert Figure 1 somewhere here]

What we will term the *standard* concentration index is simply a special case of (6) with ν being set equal to 2. In this case the weighting function is $w(p,2) = 2p-1$, a linear function of p which goes from -1 to $+1$ as the individual's position increases from the lowest to the highest in the population. Those above the median have positive weights, and those below negative ones. In case all have the same health level, the standard concentration index is zero; a negative $C(h,2)$ indicates that health is concentrated more among the poor than the rich, and a positive one the reverse.

With regard to other values of the inequality aversion parameter, if we take $\nu = 1$ the weighting function is constant and equal to 0. Therefore the index will always have the value of 0 and so inequalities are not taken into account in the extended index. From now on we assume that $\nu > 1$; the weighting function is then a strictly increasing function of the fractional rank, with some individuals having a negative weight and some a positive. (The cut-off point between positive and negative values can be determined by searching for the individual whose fractional rank p is such that $p = 1-(1/\nu)^{1/(\nu-1)}$.) For values $1 < \nu < 2$ only the individuals at the higher end of the income distribution have positive weights For values $\nu > 2$ also individuals below the median receive positive weights, but those at the bottom of the income distribution have quite large negative weights. As the value of ν increases, gradually more and more individuals have positive weights (which will all tend to 1) and in the end only the poorest individual has a negative (and very large) weight.[1] In the most extreme case when $\nu \to +\infty$, the extended concentration index in equation (6) tends to $\frac{\mu_h - h(0)}{\mu_h}$. So unless we take $\nu = 2$ the weighting scheme is asymmetric, and the more so the higher the value of ν.

[1] For $\nu \to +\infty$, the value of the index based on a finite number of individuals becomes distorted and needs to be adjusted. This will be explained in the section on the small sample bias.

The bounds of the extended concentration index can be derived by assuming that either the poorest or the richest individual is the only one with a positive level of health. Using the continuous version of the index we obtain:

$$1-\nu \leq C(h,\nu) \leq 1 \qquad (8)$$

Except for the case when $\nu = 2$, these bounds are not symmetric. An intuitive interpretation of these bounds is that they provide the weights given to the poorest and the richest person when calculating the extended index. In the standard case these weights are -1 and $+1$, which means that the absolute distance between the two is equal to 2. The choice of a particular value of ν can therefore be made dependent on the desired distance between the two.

3.2. Shortcomings of the index

It is now well-known that the standard concentration index may give conflicting information when applied separately to health and ill health. When comparing two different distributions, it can occur that the distribution with the highest measured degree of health inequality does not show the highest degree of measured ill health inequality (Clarke et al. 2002; Erreygers 2009). In other words, the standard concentration index does not have the mirror property, by which we mean that for a given distribution the value of the health index must always be exactly equal to minus the value of the ill-health index.[2] The violation of the mirror property carries over to the extended concentration index. If the mirror property is considered to be an essential

[2] Within the context of one-dimensional inequality measurement, Lambert and Zheng (2010) have recently shown that the mirror condition is a sufficient *and* necessary condition to ensure that health distributions and the corresponding ill-health distributions are similarly ranked in terms of inequality.

property – as we think it is when the health variable is bounded – then obviously the extended concentration must be abandoned or modified.

A related property concerns the cases in which the chances of having high or low health levels are symmetrically distributed over the rich and the poor. An extreme example of such a symmetric distribution is the one in which only the richest and the poorest individuals have the maximum health level, and all others the minimum level. This is of course a very unequal distribution, but it may be argued that there is no systematic bias in favour of either the rich or the poor, and that the index of socioeconomic inequality should therefore be equal to zero. This is exactly what we find if we use the standard concentration index, but not if we use the extended concentration index with ν different from 2. Again, if the symmetry property is considered to be essential, then we should not use the extended concentration index.

4. Two Alternative Indices

4.1. A General Formulation

When looking for an alternative, we will try to remain as close as possible to the extended concentration index. Let us consider an index of the following type:[3]

$$I(h,\varepsilon) = f(\mu_h,\varepsilon)\int_0^1 w(p,\varepsilon)h(p)dp \qquad (9)$$

where $f(\mu_h,\varepsilon)$ is a normalization function, $w(p,\varepsilon)$ a weighting function, and ε an inequality sensitivity parameter. With regard to the normalization function we assume that it is positive-valued; this implies that there is no level of average health or of the inequality parameter such that $f(\mu_h,\varepsilon)=0$. As far as the weighting function is

[3] This is the continuous version; we introduce the version for a finite number of individuals in the next section.

concerned, we assume that it is continuous and not identically equal to zero (because then the index would always be equal to zero, as in the case $\nu = 1$). We call $f(\mu_h, \varepsilon) w(p, \varepsilon)$ the normalized weighting function. It is immediately clear that (6) is a special case of (9), with $\varepsilon = \nu$, $f(\mu_h, \nu) = 1/\mu_h$ and $w(p, \nu) = 1 - \nu(1-p)^{\nu-1}$.

4.2. The Mirror Property

As we explained above, we think an acceptable index must satisfy the mirror property. For the general form of the index we have just introduced, this means that we must always have $I(h, \varepsilon) = -I(s, \varepsilon)$.

Theorem 1

The index $I(h, \varepsilon)$ satisfies the mirror property if and only if the following two conditions hold: (1) the weights always sum up to zero; (2) $f(\mu_h, \varepsilon) = f(1-\mu_h, \varepsilon)$ for any value of μ_h and any value of ε.

Proof. (i) Necessity. Suppose that the mirror property holds. This implies $f(\mu_h, \varepsilon)\int_0^1 w(p, \varepsilon) h(p) dp = -f(\mu_s, \varepsilon)\int_0^1 w(p, \varepsilon) s(p) dp$; since $\mu_s = 1 - \mu_h$ and $\int_0^1 w(p, \varepsilon) s(p) dp = \int_0^1 w(p, \varepsilon) dp - \int_0^1 w(p, \varepsilon) h(p) dp$, we obtain:

$$[f(\mu_h, \varepsilon) - f(1-\mu_h, \varepsilon)]\int_0^1 w(p, \varepsilon) h(p) dp = -f(1-\mu_h, \varepsilon)\int_0^1 w(p, \varepsilon) dp \qquad (10)$$

This must hold whatever may be the value of μ_h, hence also when $\mu_h = \frac{1}{2} = 1 - \mu_h$. Since we then have $f(\mu_h, \varepsilon) - f(1-\mu_h, \varepsilon) = 0$ and $f(1-\mu_h, \varepsilon) \neq 0$, it follows from (10) that $\int_0^1 w(p, \varepsilon) dp = 0$. If that is true, then this implies that we must either have

$f(\mu_h,\varepsilon)-f(1-\mu_h,\varepsilon)=0$ for any value of μ_h, or $\int_0^1 w(p,\varepsilon)h(p)dp=0$ whatever may be the function $h(p)$. The second possibility would mean that the index $I(h,\varepsilon)$ is zero for any distribution of health over the population, which could happen only if $w(p,\varepsilon)=0$; but that is a case we have excluded. Hence we must have $f(\mu_h,\varepsilon)=f(1-\mu_h,\varepsilon)$ for any value of μ_h.

(ii) Sufficiency. If we have $\int_0^1 w(p,\varepsilon)dp=0$ and $f(\mu_h,\varepsilon)=f(1-\mu_h,\varepsilon)$, then:

$$\begin{aligned} f(\mu_s,\varepsilon)\int_0^1 w(p,\varepsilon)s(p)dp &= f(1-\mu_h,\varepsilon)\left[\int_0^1 w(p,\varepsilon)dp-\int_0^1 w(p,\varepsilon)h(p)dp\right] \\ &= -f(\mu_h,\varepsilon)\int_0^1 w(p,\varepsilon)h(p)dp \end{aligned} \tag{11}$$

and therefore $I(s,\varepsilon)=-I(h,\varepsilon)$, which means that the mirror property holds.■

Theorem 1 shows that the reason why the extended concentration index fails to satisfy the mirror property is that its normalization function does not have the required property. One obvious way to remedy the situation is therefore to modify the normalization function, keeping the weighting function intact.

4.3. The Symmetry Property

A second property we may want to impose is symmetry. This property holds when the value of the index is zero whenever the distribution is symmetric around $\frac{1}{2}$, i.e. whenever $h(\frac{1}{2}-a)=h(\frac{1}{2}+a)$ for any $0\leq a\leq\frac{1}{2}$.

Theorem 2

The index $I(h,\varepsilon)$ satisfies the symmetry property if and only if the weighting function is inversely symmetric around $\frac{1}{2}$, i.e. if and only if we have $w(\frac{1}{2}-a,\varepsilon)=-w(\frac{1}{2}+a,\varepsilon)$ for any $0\leq a\leq\frac{1}{2}$.

Proof. Let us rewrite (9) as $I(h,\varepsilon)=f(\mu_h,\varepsilon)\left[\int_0^{1/2}w(p,\varepsilon)h(p)dp+\int_{1/2}^1 w(p,\varepsilon)h(p)dp\right]$. If $w(\frac{1}{2}-a,\varepsilon)=-w(\frac{1}{2}+a,\varepsilon)$ for any $0\leq a\leq\frac{1}{2}$, then obviously for any symmetric distribution we have $\int_0^{1/2}w(p,\varepsilon)h(p)dp=-\int_{1/2}^1 w(p,\varepsilon)h(p)dp$. Hence the value of the index will be zero, which proves sufficiency. Suppose alternatively that for some $0\leq a^*\leq\frac{1}{2}$ we have $w(\frac{1}{2}-a^*,\varepsilon)\neq -w(\frac{1}{2}+a^*,\varepsilon)$. Since the weighting function is continuous, we can find an interval $[b,c]$ such that $a^*\in[b,c]$ and $0\leq b<c\leq\frac{1}{2}$ for which we have $\int_b^c w(p,\varepsilon)dp\neq-\int_{1-c}^{1-b}w(p,\varepsilon)dp$. Let us then consider the symmetric distribution characterized by $h(p)=1$ for $p\in[b,c]$ and $p\in[1-c,1-b]$, and $h(p)=0$ for all other values of p. Since for this distribution we have $\int_0^{1/2}w(p,\varepsilon)h(p)dp=\int_b^c w(p,\varepsilon)dp$ and $\int_{1/2}^1 w(p,\varepsilon)h(p)dp=\int_{1-c}^{1-b}w(p,\varepsilon)dp$, it follows that $I(h,\varepsilon)\neq 0$. This proves necessity.■

Theorem 2 shows that if we want the symmetry property to hold, then we are obliged to change the weighting function of the extended concentration index. It can be checked that the weighting function of this index has the desired property only if $\nu=2$.

4.4. The Generalized Extended Concentration Index

Suppose we are interested only in the mirror property and limit ourselves to a change in the normalization function. The simplest way of ensuring that $f(\mu_h, \varepsilon) = f(1-\mu_h, \varepsilon)$ holds for any value of μ_h is to make the function independent of μ_h. This procedure constitutes also the basis of the corrected version of the standard concentration index proposed by Erreygers (2009) (hereafter the Erreygers index), which is itself closely related to the so-called generalized concentration index. The generalized extended concentration index we propose here is defined as follows:

$$F(h,\nu) = \frac{\nu^{\nu/(\nu-1)}}{\nu-1} \int_0^1 \left[1-\nu(1-p)^{\nu-1}\right] h(p)dp \tag{12}$$

This means that we take $\varepsilon = \nu$ and:

$$f(\mu_h, \nu) = \frac{\nu^{\nu/(\nu-1)}}{\nu-1}, \quad w(p,\nu) = 1-\nu(1-p)^{\nu-1} \tag{13}$$

For $\nu = 2$ we have $f(\mu_h, 2) = 4$ and $w(p,2) = 2p-1$, and we obtain the continuous version of the Erreygers index.

We have already observed that, for a given value of ν, those with fractional ranks below $1-(1/\nu)^{1/(\nu-1)}$ have negative weights and those with fractional above this value positive weights. Since $\int \left[1-\nu(1-p)^{\nu-1}\right] dp = p + (1-p)^{\nu} + C$, the sum of the positive weights is equal to $(\nu-1)\nu^{-\nu/(\nu-1)}$, and those of the negative weights $-(\nu-1)\nu^{-\nu/(\nu-1)}$. This implies that when all individuals with positive weights have maximum health and all individuals with negative weights minimum health, then the index is equal to $+1$; in the opposite case the index is equal to -1. In all other cases the value of the index will be strictly higher than -1 and strictly lower than $+1$. In fact, for given values of ν and μ_h the lower and upper bounds of the index are such that:

$$\frac{\nu^{\nu/(\nu-1)}}{\nu-1}(1-\mu_h)\left[(1-\mu_h)^{\nu-1}-1\right]\leq F(h,\nu)\leq\frac{\nu^{\nu/(\nu-1)}}{\nu-1}\mu_h(1-\mu_h{}^{\nu-1}) \tag{14}$$

(The maximum upper bound of $+1$ can be reached only when $\mu_h=(1/\nu)^{1/(\nu-1)}$, and the minimum lower bound of -1 only when $\mu_h=1-(1/\nu)^{1/(\nu-1)}$). Finally, when $\nu\to+\infty$, the generalized extended index in equation (12) tends to $\mu_h-h(0)$.

4.5. The Symmetric Index

The generalized extended concentration index is based on an asymmetric weighting scheme and therefore does not have the symmetry property. If we want to have both mirror and symmetry, the easiest way to proceed is to design a weighting scheme that is inversely symmetric and monotonously increasing in the fractional rank p (needed to sustain the principle of income-related health transfers (Bleichrodt and van Doorslaer 2006)); and a normalization function that is independent of average health. Hence, if we make the weights of the poorest more negative, we should also make the weights of the richest more positive. The symmetric index we propose here is defined as follows:

$$S(h,\alpha)=(1+\alpha)2^{2(1+\alpha)}\int_0^1\left[(p-\tfrac{1}{2})^2\right]^{\alpha}(2p-1)h(p)dp \tag{15}$$

with $\alpha>-\frac{1}{2}$. This means that we have $\varepsilon=\alpha$ and:

$$f(\mu_h,\alpha)=(1+\alpha)2^{2(1+\alpha)},\quad w(p,\alpha)=\left[(p-\tfrac{1}{2})^2\right]^{\alpha}(2p-1) \tag{16}$$

One can check that for $\alpha=0$ we have $f(\mu_h,0)=4$ and $w(p,0)=2p-1$, which means that for this value the symmetric index coincides with both the generalized extended with $\nu=2$ and the Erreygers index.

The weighting scheme has been devised in such a way that those with fractional ranks above the median always have positive weights, and those below the median always negative weights. As can be seen from Figure 2, which represents the normalized weights[4], by taking $-\frac{1}{2} < \alpha < 0$ we give relatively higher weights to those with a fractional rank close to the median, while by taking $\alpha > 0$ we give relatively higher weights to those at the upper and lower end of income distribution. In the most extreme case ($\alpha \to +\infty$) the symmetric index tends to $h(1) - h(0)$. Moreover, the sum of the normalized positive weights is always equal to 1, which implies that the index is equal to $+1$ when all individuals above the median have maximum health and all individuals below it minimum health; in the opposite case the index is -1. In general, for given values of α and μ_h the bounds of the index are equal to:

$$-2^{2(1+\alpha)}\left\{\left(\tfrac{1}{2}\right)^{2(1+\alpha)} - \left[\left(\tfrac{1}{2}-\mu_h\right)^2\right]^{1+\alpha}\right\} \le S(h,\alpha) \le +2^{2(1+\alpha)}\left\{\left(\tfrac{1}{2}\right)^{2(1+\alpha)} - \left[\left(\tfrac{1}{2}-\mu_h\right)^2\right]^{1+\alpha}\right\} \quad (17)$$

(The maximum bound of $+1$ and the minimum bound of -1 can be reached only when $\mu_h = \frac{1}{2}$.)

[Insert Figure 2 somewhere here]

4.6. Some discussion

In sections 3.2 and 4.3, we have only given some basic intuition for the symmetry condition as the symmetric index and its weighting function were not yet defined at that stage. In this section, we give additional arguments for the symmetry condition – implicitly assuming that the mirror condition is satisfied – and in particular discuss the pivotal role of the individual who occupies the median position.

With respect to the latter, suppose there is a ceteris paribus change in the health level of one person located at position p in the socioeconomic distribution. How should such a change impact upon an index measuring socioeconomic health inequalities? Let

[4] As mentioned earlier, the normalized weights are obtained as a product of the normalization function and the weighting function.

us start at $p=0$ (the poorest individual). Obviously this is a pro-poor change, and we expect the index to become more pro-poor, i.e. decrease. Next, let us increase p and wonder from what value the change becomes pro-rich. If we consider this threshold value p^* to be different from the median, we should opt for the generalized extended concentration index: by choosing the value of the inequality parameter $\nu=\nu^*$, where ν^* is such that $p^* = 1-(1/\nu^*)^{1/(\nu^*-1)}$, we obtain the desired result. If, however, we consider the threshold value to be equal to the median, the symmetric index seems a more appropriate choice.

The threshold value p^* demarcates the group of the poor from the group of the non-poor. We believe that the choice of $p^* = 0.5$ is a reasonable point of departure as 0.5 is the expected location of a person. In other words, the lower half of the population is considered as poor, and the upper half as rich. We do not exclude that another value, say $p^* = 0.25$, might be more appropriate than our *a priori* choice, but without additional information (e.g. on income levels) we think it is very hard to make a case for such an alternative boundary. By construction, rank-dependent inequality measures leave that kind of information out of consideration, and therefore naturally lead us to take $p^* = 0.5$, at least as a starting point.

Another issue concerns how the index of socioeconomic health inequality should react to health transfers at different locations in the distribution. Suppose there is a transfer of health Δ from a person located at position p_j to a person located at position p_i, with $p_j = p_i + d$ and $d > 0$ (i.e. the first person is richer than the second). We can compare the effect of such a transfer for different equidistant individuals. A good measure of where the transfer takes place is given by the number $z = p_j - d/2 = p_i + d/2$, i.e. the location halfway between p_i and p_j. If we believe that the effect of such a transfer should become smaller and smaller as z increases, we have to opt for the generalized extended concentration index with $v > 2$. If, however, we think that the effect should be smaller the closer z lies to the centre (i.e. $p^* = 0.5$), then the symmetric index with $\alpha > 0$ seems more appropriate. The first property is that of sensitivity strictly increasing with poverty, in short 'sensitivity to poverty'; the second that of sensitivity strictly increasing when one moves from the centre towards the extremities of the distribution, in short 'sensitivity to extremity'.

When the issue is the measurement of one-dimensional inequality, for instance of incomes, we believe 'sensitivity to poverty' is the appropriate concept. But it can be questioned whether the same concept is also the most appropriate one in the case of two-dimensional inequality, for instance of health in relation to socioeconomic rank. In the latter case, we are not measuring the inequality of health as such, but the degree of association between the distribution of health and the socioeconomic ranking. The measure of this degree of association should take into account the whole spectrum of possibilities, and not privilege inequality in one dimension over inequality in the other. By making the measure more sensitive to one end of the income spectrum ('the poor') than to the other ('the rich'), we run the risk of reducing or even neglecting part of the existing inequality. Why should a person with a low income rank but a high health level count more than a person with a high income rank but a low health level? While the symmetric index does not address this issue directly, it expresses the idea that what is happening at the extremities of the income distribution, whether it be at the high end or the low end, should carry more weight than what is happening in the middle.

Due to its exclusive focus on income poverty, the generalized extended concentration index may lead to counterintuitive results. Consider a health distribution in which the poorest 10% of the population have maximum health, the richest 20% also, and all the rest minimum health. Since there are twice as much rich persons in good health than poor persons, we believe few people would doubt that health is distributed rather strongly in favour of the rich, and therefore we expect a positive value of the index. Yet, the generalized extended concentration index will be *negative* for any value of ν (approximately) higher than 3.33 (the value of the generalized concentration index is in this example equal to $\frac{\nu^{\nu/(\nu-1)}}{\nu-1}\left[(0.9)^{\nu}-(0.2)^{\nu}-0.7\right]$). By contrast, the symmetric index will always be positive.

The explanation of the divergence lies in the way in which the two indices treat different combinations of ranks and health levels. Low health levels always have a small contribution to the value of the generalized extended concentration index (positive in case of a high rank and negative for low ranks); and this also holds for the symmetric index. But things are different for high health levels. In case of the generalized extended concentration index, these lead to a moderate positive contribution for high ranks, and a very large negative contribution for low ranks; while there is no such difference (apart from the sign of the contribution) for the symmetric index, i.e. there is a large positive contribution for high ranks; and a large negative contribution for low ranks.

5. Adjusting for biases that arise when using finite samples

In empirical work one works with finite samples, which means that p is not a continuous variable and that a discrete version of the formulas has to be used. The value of p is usually approximated by the fractional rank R_i (Lerman and Yitzhaki 1989), and the integral replaced by a summation divided by *n*. The fractional rank version of the extended concentration index is given by (5) and is here repeated:

$$C^{R}(h,\nu)=\frac{1}{\mu_h}\frac{1}{n}\sum_{i=1}^{n}\left[1-\nu\left(1-R_i\right)^{\nu-1}\right]h_i \tag{18}$$

while those of the other two indices are:

$$F^R(h,\nu) = \frac{\nu^{\nu/(\nu-1)}}{\nu-1}\frac{1}{n}\sum_{i=1}^{n}\left[1-\nu(1-R_i)^{\nu-1}\right]h_i \qquad (19)$$

$$S^R(h,\alpha) = (1+\alpha)2^{2(1+\alpha)}\frac{1}{n}\sum_{i=1}^{n}\left[(R_i-\tfrac{1}{2})^2\right]^{\alpha}(2R_i-1)h_i \qquad (20)$$

(The superscript *R* is added to indicate that these expressions are based on the fractional ranks R_i.) The normalization functions are the same as for the continuous versions, and the fractional rank weighting functions are respectively $w^R(R_i,\nu) = \frac{1}{n}\left[1-\nu(1-R_i)^{\nu-1}\right]$ and $w^R(R_i,\alpha) = \frac{1}{n}\left[(R_i-\tfrac{1}{2})^2\right]^{\alpha}(2R_i-1)$.

In general it can be said that the value of the fractional rank indices will be very close to the value of the continuous indices for high values of the number of persons *n*, and that the degree of approximation increases with *n*. However, for relatively small values of *n*, the deviation between the two indices may be substantial. The magnitude of this 'small-sample bias' is distribution-specific and will be larger for values of the inequality sensitivity parameters ν or α that are relatively further away from 2 and 0.

One of the remarkable things, though, is that the small-sample bias also shows up for the extended and generalized extended indices in the case of an equal distribution of health. If everyone has the same health level, there is no socioeconomic inequality and the index must be zero. Nevertheless, the fractional rank versions of the extended and generalized extended index will have positive/negative values when ν is larger/smaller than 2 and falsely indicate that the distribution is pro-rich/pro-poor. This small-sample bias is illustrated in Table 1 for different values of *n* and ν for the case where all individuals have health level 0.5. The reason for the small-sample bias is that when we replace p by R_i, the sum of the weights becomes slightly positive (negative) when $\nu > 2$ ($\nu < 2$), whereas they should be equal to zero.

[Insert Table 1 somewhere here]

An additional (but not illustrated in Table 1) reason for the small-sample bias in case of the generalized extended index *and* the symmetric index is that when $\nu \neq 2$ and $\alpha \neq 0$ the normalized positive weights do not add up to 1 (a similar property holds for the normalized negative weights).

Our solution to this small-sample bias is based on the idea that the individual weights should be adjusted in such a way that they are equal to the corresponding continuous weights. We assume that individual i in a sample of n individuals corresponds to the interval $\left[\frac{i-1}{n}, \frac{i}{n}\right]$ in the continuous population [0,1]. Given the continuous weighting function $w(p,\varepsilon)$, the corresponding 'small-sample corrected' weights $w^S(i,\varepsilon)$ are therefore defined as:

$$w^S(i,\varepsilon) = \int_{(i-1)/n}^{i/n} w(p,\varepsilon)dp \tag{21}$$

The small-sample adjusted version of our general family of indices becomes:

$$I^S(h,\varepsilon) = f(\mu_h,\varepsilon)\sum_{i=1}^{n} w^S(i,\varepsilon)h_i \tag{22}$$

(The superscript S indicates that this expression refers to the small-sample adjusted version.) To illustrate what the adjustment implies, we have to distinguish the case in which there are no ties in the income ranks from the one in which there are.

5.1. Small-sample adjustment in the absence of ties

When there are no ties, the fractional rank R_i of individual i is equal to $(2i-1)/2n$. Without small-sample adjustment, the weight of individual i in the extended and generalized extended concentration indices would therefore be equal to

$w^R(R_i,\nu)=\frac{1}{n}\left[1-\nu\left(1-\frac{2i-1}{2n}\right)^{\nu-1}\right]$. By contrast, on the basis of (21) the small-sample adjusted weight is equal to:

$$w^S(i,\nu)=\frac{1}{n}-\left[\left(1-\frac{i-1}{n}\right)^{\nu}-\left(1-\frac{i}{n}\right)^{\nu}\right] \tag{23}$$

It can be checked that the adjusted and non-adjusted weights coincide only when $\nu=2$. The small-sample adjusted versions of the extended and generalized extended concentration indices can be written as:

$$C^S(h,\nu)=\frac{1}{\mu_h}\sum_{i=1}^{n}w^S(i,\nu)h_i \tag{24}$$

$$F^S(h,\nu)=\frac{\nu^{\nu/(\nu-1)}}{\nu-1}\sum_{i=1}^{n}w^S(i,\nu)h_i \tag{25}$$

Likewise, in the case of the symmetric index the non-adjusted weight would be $w^R(R_i,\alpha)=\left[\left(\frac{2i-n-1}{2n}\right)^2\right]^{\alpha}\left(\frac{2i-n-1}{n^2}\right)$, whereas the adjusted weight is equal to:

$$w^S(i,\alpha)=\frac{\left[\left(\frac{i}{n}-\frac{1}{2}\right)^2\right]^{1+\alpha}-\left[\left(\frac{i-1}{n}-\frac{1}{2}\right)^2\right]^{1+\alpha}}{(1+\alpha)} \tag{26}$$

Again, these only coincide when $\alpha=0$, and the small-sample adjusted version of the symmetric index can be written as:

$$S^S(h,\alpha)=(1+\alpha)2^{2(1+\alpha)}\sum_{i=1}^{n}w^S(i,\alpha)h_i \tag{27}$$

5.2. Small-sample adjustment in the presence of ties

When there are ties, the fractional ranks of the tied individuals are based on the average rank within the group to which they belong, which creates an additional source of bias.

The latter source of bias is identical to the so-called bias from grouping which arises when data are grouped into categories or ranges (e.g. income quintiles). Wagstaff suggested to correct for the bias due to grouping by subtracting the excess value of the sum of the weights from the value of the fractional rank index (Wagstaff 2002, Appendix A.2; O'Donnell et al. 2008, equation 9.6).[5] We follow an alternative approach that generalizes the idea that the adjusted weights should be equal to the corresponding continuous weights. In addition, it addresses the small-sample *and* the bias due to grouping.

To treat this case properly, we have to introduce some additional notation. Suppose there are K groups in the population, denoted as $1,2,\ldots,K$, with 1 referring to the poorest group, 2 to the second poorest, etc. The number of people in group J is equal to n_J. Let us define the total number of people in all groups up to and including J, i.e. in the groups $1,2,\ldots,J$, as I_J, which means that we have $I_J = n_1 + n_2 + \ldots + n_J$. By convention, we take $I_0 = 0$. The average health level in group J is denoted as h_J.

Following the same reasoning as before, the adjusted weight of group J for the extended and generalized extended concentration indices turns out to be equal to:

$$w^S(J,\nu) = \frac{n_J}{n} - \left[\left(1 - \frac{I_{J-1}}{n}\right)^{\nu} - \left(1 - \frac{I_J}{n}\right)^{\nu} \right] \tag{28}$$

The small-sample *and* grouped bias adjusted versions of both indices can therefore be written as:

$$C^S(h,\nu) = \frac{1}{\mu_h} \sum_{J=1}^{K} w^S(J,\nu) h_J \tag{29}$$

[5] An improved correction for the bias due to grouping in the standard concentration index (and the Gini index) has recently been proposed by Clarke and Van Ourti (2010) and Van Ourti and Clarke (forthcoming).

$$F^S(h,\nu) = \frac{\nu^{\nu/(\nu-1)}}{\nu-1}\sum_{J=1}^{K} w^S(J,\nu)h_J \tag{30}$$

Likewise, the adjusted weight of group *J* for the symmetric index is equal to:

$$w^S(J,\alpha) = \frac{\left[\left(\frac{I_J}{n}-\frac{1}{2}\right)^2\right]^{(1+\alpha)} - \left[\left(\frac{I_{J-1}}{n}-\frac{1}{2}\right)^2\right]^{(1+\alpha)}}{(1+\alpha)} \tag{31}$$

and the small-sample and grouped bias adjusted version of it can be written as:

$$S^S(h,\alpha) = (1+\alpha)2^{2(1+\alpha)}\sum_{J=1}^{K} w^S(J,\alpha)h_J \tag{32}$$

5.3. Monte Carlo evidence on the small-sample adjustment

The previous sections have revealed that the application of the extended, generalized extended and symmetric indices to finite samples can lead to a small-sample bias (the bias of grouping in case of ties in the fractional ranks is illustrated in the next section). It is easy to check that the small-sample adjusted weights in equation (23) will remove all small-sample bias from the extended and generalized extended indices when all individuals have the same health level. For all other distributions, we only know that the bias of the three indices is distribution-specific, but its magnitude, how it varies with the number of observations, the parameters ν and α, and the shape of the distribution is unknown. Similarly, it is not a priori clear how much of the bias is removed by applying the small-sample adjusted weights in equations (23) and (26).

In order to increase our understanding of the small-sample bias, we have performed Monte Carlo simulations using four versions of the beta distribution, i.e.:

(a) beta(0.5;0.5), a bimodal distribution whose density function is symmetric around the median with higher spikes at the extreme values of 0 and 1;

(b) beta(1;1), corresponding to the uniform distribution;

(c) beta(3;25), a right-skewed distribution corresponding to the typical shape of a distribution of ill-health; and

(d) beta(25;3), a left-skewed distribution corresponding to a typical health distribution.

Since the indices under consideration focus on the association between health and the income rank, we also considered two (extreme) scenarios to assign ranks to health levels in each Monte Carlo draw, i.e. a 'random ranking' scenario where assignment is random (all three indices should equal zero in the population without having a perfectly equal distribution), and a 'maximal association ranking' scenario where the income rank is identical to the health rank (the case where socioeconomic inequality is maximal).[6] The results are presented in Table A1 in the appendix. The three indices (with and without small-sample adjustment using equations (28)-(32) were computed for the parameter values $\nu = 1.5; 2; 4; 6; 100$ and $\alpha = -0.25; 0; 1; 2; 49$, and next compared to the population value of these indices. We choose different parameter values for α in order to reflect a wide range of distributional concerns; the values of ν were derived from $\nu = 2(1+\alpha)$.[7]

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[6] This boils down to one-dimensional inequality measurement and thus informs on the small-sample bias of the extended Gini (Yitzhaki 1983), the generalized extended Gini and the symmetric version of the Gini.

[7] The idea behind $\nu = 2(1+\alpha)$ is to look at the maximum power to which p is raised in the weighting functions, which seems a plausible basis for choosing values of ν and α that are comparable. For $\nu = 2$ and $\alpha = 0$, both weighting functions are linear functions of p. For $\nu = 3$ and $\alpha = 0.5$ they are quadratic functions, viz. $w(p,\nu) = -3p^2 + 6p - 2$ and $w(p,\alpha) = \pm(2p^2 - 2p + 0.5)$. For $\nu = 4$ and $\alpha = 1$, the functions are cubic, etc.

The Monte Carlo simulations show that the magnitude of the small-sample bias can be large and that it is more important under the 'maximal association' scenario (due to the population values of the indices being non-zero), especially when $n < 50$. We also find that the small-sample adjustments reduce the small-sample bias, although there are a few cases where the point estimates of the indices did not improve. The small-sample adjusted versions remove all bias under the 'random ranking' scenario, confirming the results in Table 1, while the (overall good) performance under the 'maximal association ranking' scenario varies with the values of n and ν. While our Monte Carlo simulations show that one should not worry about small-sample bias when there are 50 or more observations *without ties in the ranking variable*, it is ultimately the number and the relative share of the groups with the same value for the ranking variable that matter. For example, we will illustrate in the next section that some widely used microdata do not manage to pass this seemingly innocent requirement of 50 observations.

6. An empirical application

In this section we will illustrate some of the measurement issues concerning each of the three previously developed inequality measures using real data. All calculations[8] are based on data collected from the Demographic and Health Surveys (DHS) which involve a range of measures regarding health (and ill health) and use of types of health care collected over 40 developing countries between 1996 and 2004. The surveys range in size from around 2,500 to over 30,000 individuals. In this study we focus on under-five mortality which has previously been used by Wagstaff (2002) to illustrate the extended concentration index. The key characteristics of the surveys and the proportion

[8] The STATA-programs used to calculate the three indices can be obtained from the authors upon request.

of children dying under five years of age by country are reported in Table 2. Since this variable is binary and only takes 0 and 1, there is no need to standardize before applying these indices. The DHS has also been used in other studies of health inequalities such as Van De Poel et al. (2007).

[Insert Table 2 somewhere here]

It deserves to be stressed that the socio-economic variable on the basis of which ranks are assigned, is constructed using principal component analysis by combining information on a set of household assets and living conditions into one indicator (Filmer and Pritchett 2001). In some countries, such as Haiti, a high proportion (around 38%) of households have the same value for the constructed socioeconomic variable, which results in ties of the socio-economic rank. Therefore, we have used equations (28)-(32) to address the biases.[9] (We explore what impact this bias can have on the ranking of countries in the DHS below.)

Before applying the indices to all countries it is useful to examine two countries (Niger and The Philippines) in detail in order to understand how and why the indices vary for different levels of ν and α. The distribution of under-five mortality across socio-economic deciles in these two countries is presented in Figure 3a-b. In Niger

[9] It is worth noting that equations (28)-(32) are sufficiently general to allow for small-sample bias, ties in the fractional income ranks *and* differences in (ex-post) sampling probabilities. In survey design such differences are usually counterbalanced by using so-called 'sampling weights'. If we denote the sampling weight of individual *i* by w_i, the only adjustments to equations (28)-(32) are that: (a) the number of people in group *J* is now equal to $n_J = \sum_{i \in J} w_i$; (b) the number of observations equals the sum of the sampling weights, i.e. $n = \sum_{i \in N} w_i$; (c) the average health level in group *J* now equals $h_J = \frac{1}{n_J} \sum_{i \in J} w_i h_i$; and (d) the average health of the population equals $\mu_h = \frac{1}{n} \sum_{i \in N} w_i h_i$.

under-five mortality rates are very high, with around 33% of children in households in the lowest socio-economic decile dying before the age of five, compared to around 20% in the highest decile. In The Philippines the rates of mortality range from 8.9% in the lowest to 2.1% in the highest decile.

Figure 3c presents the values of the three indices for Niger, calculated on the basis of individual level data for a range of levels of inequality aversion with the formula $\nu = 2(1+\alpha)$ being used to produce comparable values of ν and α. The extended concentration index attains its maximum negative value (i.e. reaches its minimum value in a mathematical sense) when $\nu = 2$; for higher levels of inequality aversion the index increasingly focuses on individuals with a lower socio-economic rank and becomes less negative (i.e. increases in value in a mathematical sense). The generalized extended concentration index reaches its maximum negative value for $\nu = 1.5$ and then declines in absolute magnitude.[10] Interestingly, for $\nu = 10$ the values of both indices are small but positive, which is due to the non-monotonic nature of the distribution of under-five mortality. The symmetric index displays the opposite pattern across values of α: the index becomes more negative as more weight is given to the extremes of the distribution.

Figure 3d displays the same information for The Philippines. Here the extended concentration index sharply decreases in value when the level of inequality aversion increases, since this has the effect of magnifying relative differences between lower socio-economic groups. By contrast, the generalised extended concentration index remains fairly constant (e.g. when $\nu = 2$ the index is –0.046, and when $\nu = 10$ it is

[10] When comparing these two indices, it is important to keep in mind that the bounds of the extended concentration index are much wider and asymmetric; its lower bound is $1-\nu$.

–0.043), whereas the symmetric index also tends to decrease in value but much less sharply.

[Insert Figure 3 somewhere here]

Table A.2 presents results with regard to under-five mortality for all countries for which data were available: it lists the mean and the values of the extended, generalized extended and symmetric indices for comparable values of ν and α. For both versions of the extended concentration index the values of ν range from 1.5 to 6, and for the symmetric index the values of α range from –0.25 to 2.

With regard to the mirror condition, mortality when expressed as a proportion is bounded and so the inequalities for under-five survival can easily be calculated from the information supplied in Table A.2.[11] Table 3 presents the ranking of countries in terms of shortfall (mortality) and attainment (survival) generated by the extended concentration index. For mortality, rank 1 indicates the lowest degree of 'pro-poor' inequality (i.e. the country with the highest value of $C(s,\nu)$) and rank n the highest degree of 'pro-poor' inequality, while for survival rank 1 indicates the lowest degree of 'pro-rich' inequality (i.e. the country with the lowest value of $C(h,\nu)$) and rank n the highest degree of 'pro-rich' inequality. It is clear that there are considerable changes in the rankings of countries when inequality is measured in terms of mortality or of survival. Niger, for example, is ranked between 10th and 5th when inequality is measured in terms of mortality, but between 41st and 12th when measured in terms of survival.

[Insert Table 3 somewhere here]

[11] The extended concentration index for survival is simply: $C(h,\nu) = -\frac{\mu_s}{(1-\mu_s)} C(s,\nu)$.

Table 4 presents the Spearman rank correlation coefficients of the country rankings according to the generalized extended concentration index and the symmetric index. The ranks have been calculated using the index values reported in Table A2. The main diagonal represents the rank correlation coefficients between the indices for corresponding values of the two aversion parameters. For $\nu = 2$ and $\alpha = 0$ the values of these indices coincide, so they are perfectly correlated. As the values of ν and α increase, the correlation between these indices declines; e.g. when $\nu = 6$ and $\alpha = 2$, the correlation is 0.770, which indicates an increasing divergence in the ranking across countries.

[Insert Table 4 somewhere here]

Finally, Table 5 provides summary measures for the impact of using the small-sample and grouped biased adjusted weighting schemes. To calculate these statistics both the fractional rank versions of the indices and the adjusted ones were applied and the rankings compared. The use of the adjusted weighting functions does have a significant impact on the ranking of countries particularly for higher values of ν. So for example when $\nu = 6$ there are re-rankings when using the adjusted versions of the extended concentration index involving 24 countries, in which the maximum change in ranking for a single country is 13 places and the total number of re-rankings across all countries is 86. The adjustment appears to be even more important for the generalized and symmetric indices. We thus find that the small-sample bias and the bias due to grouping is an important issue that is all too often overlooked or neglected in empirical work.

[Insert Table 5 somewhere here]

7. Conclusions

This paper deals with ways to incorporate attitudes towards inequality into the measurement of socioeconomic health inequalities. We started by revisiting the extended concentration index that was proposed by Pereira (1998) and Wagstaff (2002). Similarly to the standard concentration index, we found that it does not satisfy the 'mirror condition', i.e. the requirement that socioeconomic inequality in health attainments should 'mirror' socioeconomic inequality in health shortfalls (or ill-health). A re-normalization – the generalized extended concentration index – does satisfy the 'mirror' condition and thus rescues the possibility to express one's 'aversion to inequality' when measuring socioeconomic health inequalities.

Nevertheless, the generalized index does not satisfy a second requirement which may be considered relevant: that of measuring no systematic bias in favour of either the rich or the poor when the chances of having high or low health levels happen to be symmetrically distributed over the rich and the poor. This new requirement led us to propose a new index of socioeconomic health inequality – the symmetric index – which is based on a distributional weighting scheme that is increasing in the income rank *and* symmetric around the median income rank.

Due to the properties of both weighting schemes, the estimators of these three indices have a small-sample bias. We suggested a solution for this bias based on the idea that the weighting function calculated from finite samples should correspond to the weighting scheme in the population. Using Monte Carlo simulations we showed that this solution has great appeal when applied to distributions where health levels are unequally distributed, but not associated with the income ranks. When the latter association is maximal, we find that our small-sample adjustment removes part of the bias, but its performance depends on the number of observations and one's aversion to inequality.

The empirical section illustrated that a country's rank according to the degree of inequality of a common ill-health measure – the under-five mortality rate – can vary significantly, both between the three indices and for different values of the aversion parameters. Unlike what is observed for the extended Gini coefficient, increasing the degree of inequality aversion (i.e. adopting higher values for ν and α) does not always lead to increasing pro-rich inequality. These indices can rise or fall in value and can even switch sign whenever measures of health (or ill-health) are not monotonically changing across socio-economic groups. In the empirical analyses, this issue appears to be most relevant for the extended and generalized extended concentration indices. Hence more caution needs to be exercised when applying different aversion parameters than in the case of the extended Gini coefficient. Finally, while we have examined the measurement properties of three indices that apply different weighting schemes to measure socio-economic health inequalities, we can still say very little about the preferences of society regarding the distribution of health across income. Developing a plausible range of weighting schemes which can be employed in empirical work to reliably inform policy analysis remains an important challenge

Appendix

[Insert Table A1 somewhere here]

[Insert Table A2 somewhere here]

[Insert Table A3 somewhere here]

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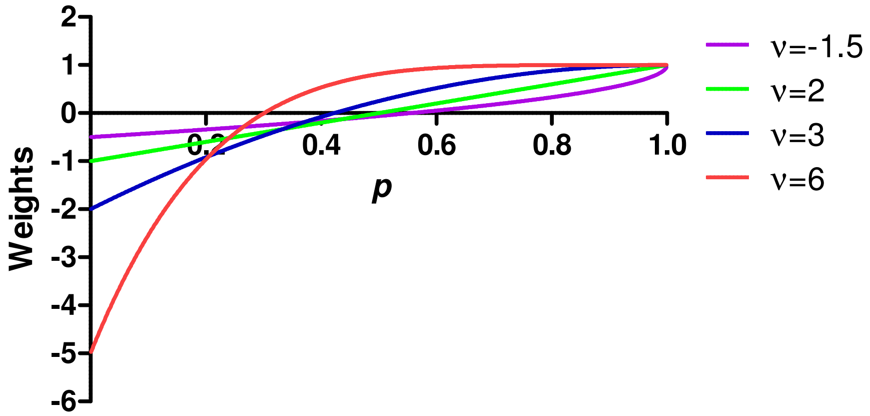
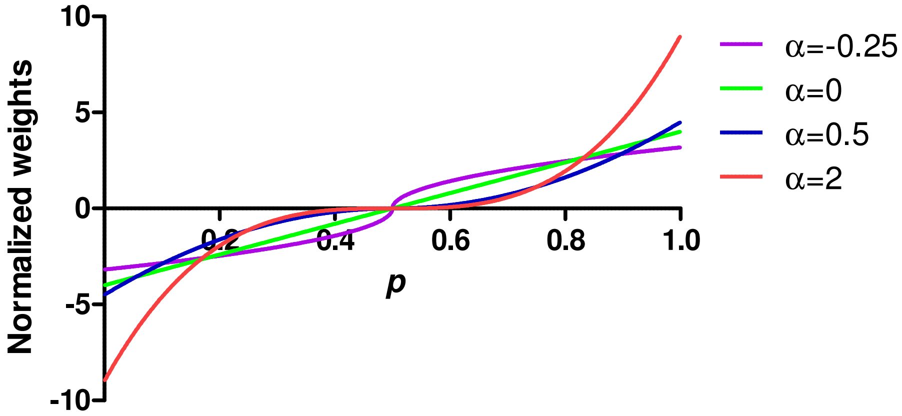
Figure 1: The weighting function of the extended concentration index



Note: The weights $w(p,\nu)$ are calculated from equation (7).

Figure 2: The normalized weights of the symmetric index



Note: The weights are calculated from the product of the normalization function and the weighting function in equation (16).

Figure 3: Average mortality by decile and index values for two selected countries

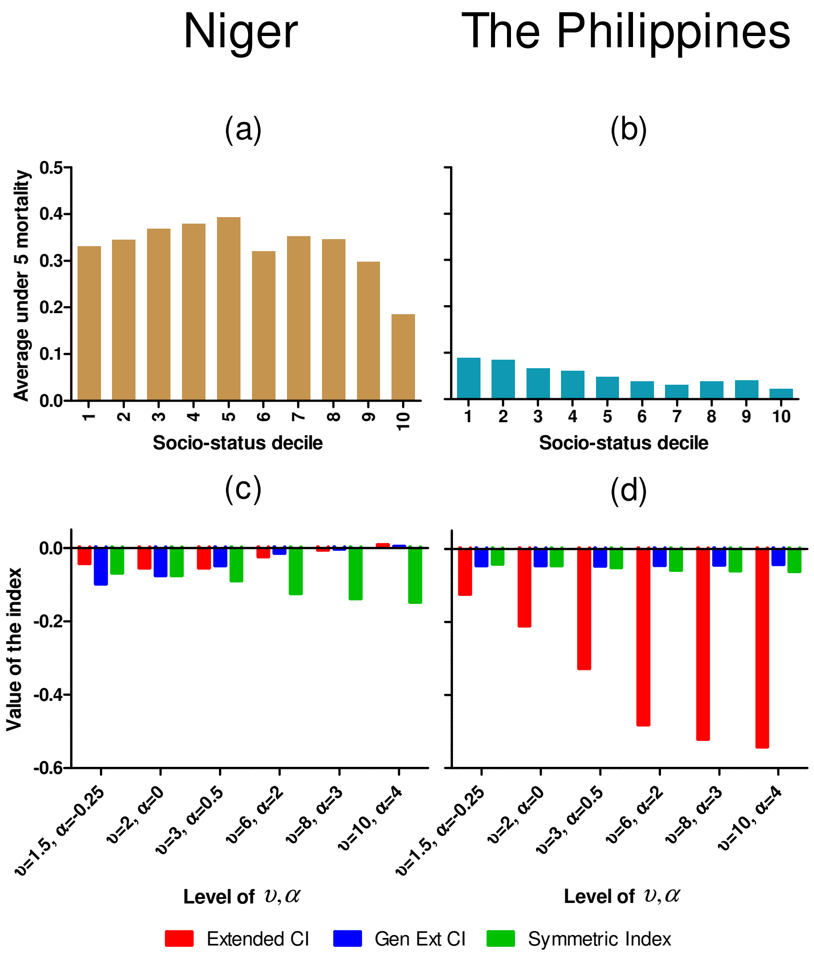


Table 1: The small-sample bias of the extended and generalized extended concentration indices when all individuals have the health level 0.5

number of	extended concentration index					generalized concentration index				
observations	v=1,5	v=2	v=3	v=6	v=100	v=1,5	v=2	v=3	v=6	v=100
population	0	0	0	0	0	0	0	0	0	0
1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
100	0,000	0,000	0,000	0,000	0,040	0,000	0,000	0,000	0,000	0,021
75	0,000	0,000	0,000	0,000	0,070	-0,001	0,000	0,000	0,000	0,037
50	0,000	0,000	0,000	0,001	0,148	-0,001	0,000	0,000	0,000	0,079
25	-0,001	0,000	0,000	0,002	0,450	-0,002	0,000	0,001	0,002	0,238
10	-0,003	0,000	0,003	0,013	0,938	-0,009	0,000	0,003	0,011	0,496
5	-0,007	0,000	0,010	0,049	0,999	-0,023	0,000	0,013	0,042	0,529
4	-0,010	0,000	0,016	0,076	1,000	-0,032	0,000	0,020	0,066	0,529
3	-0,014	0,000	0,028	0,134	1,000	-0,048	0,000	0,036	0,115	0,529
2	-0,025	0,000	0,063	0,285	1,000	-0,083	0,000	0,081	0,245	0,529

Table 2: Summary of survey characteristics and mean under-five mortality

Country	Year	No. individuals	Socio-economic information		U5 Mortality
			No. groups	Size largest group	Mean
Bangladesh	2004	15388	269	20%	0.134
Benin	2001	14121	1905	3%	0.182
Bolivia	2003	26089	591	6%	0.128
Brazil	1996	10222	446	9%	0.097
Burkina Faso	2003	30458	1536	1%	0.204
Cameroon	2004	19933	1706	3%	0.155
CAR	1994	9151	189	19%	0.157
Chad	2004	15151	1283	2%	0.193
Colombia	2005	28885	5524	1%	0.042
Comoros	1996	4070	59	42%	0.126
Cote D'Ivoire	1994	13967	139	14%	0.152
Dominican	2002	21925	4497	6%	0.066
Egypt	2000	26793	3355	1%	0.099
Ghana	2003	10268	170	18%	0.128
Guatemala	1998	12593	605	4%	0.085
Haiti	2000	17348	141	38%	0.162
Indonesia	2002	34068	954	5%	0.085
Kazakhstan	1999	2477	770	3%	0.099
Kenya	2003	14105	2972	4%	0.119
Kyrgyzstan	1997	2744	667	1%	0.115
Madagascar	1997	12523	872	3%	0.183
Malawi	2000	29310	1431	1%	0.089
Mali	2001	36960	270	21%	0.253
Morocco	2003	15349	2985	5%	0.079
Mozambique	2003	13102	384	6%	0.227
Namibia	2000	9215	1062	2%	0.064
Nepal	2001	16075	66	42%	0.167
Nicaragua	2001	17024	866	9%	0.066
Niger	1998	18625	791	5%	0.324
Nigeria	2003	15808	1629	2%	0.232
Pakistan	1990	17587	553	9%	0.119
Peru	2000	34081	4393	6%	0.103
Senegal	1997	21143	152	22%	0.163
Tanzania	2004	21910	1346	2%	0.146
Philippines	2003	16422	1121	6%	0.055
Togo	1998	14942	228	8%	0.171
Turkey	1998	7509	718	7%	0.093
Uganda	2001	17496	2324	1%	0.164
Uzbekistan	1996	2950	875	1%	0.081
Vietnam	2002	2494	1064	1%	0.055
Zambia	2001	5988	69	35%	0.170
Zimbabwe	1999	8636	137	14%	0.073

Table 3: Country rankings of the extended concentration index in terms of shortfall ($C(s,\nu)$) and attainment ($C(h,\nu)$)

	Inequalities in mortality C(s,v)				Inequalities in survival C(h,v)			
Value ν	1.5	2	3	6	1.5	2	3	6
Bangladesh	7	7	8	11	10	8	9	13
Benin	20	21	21	18	36	36	36	31
Bolivia	32	31	29	27	34	34	35	32
Brazil	29	34	38	39	17	24	28	34
Burkina Faso	5	5	6	4	19	14	11	6
Cameroon	35	35	35	37	39	39	40	41
CAR	17	14	14	17	30	25	20	23
Chad	2	2	2	3	2	2	1	2
Colombia	30	30	31	33	4	7	7	9
Comoros	9	11	12	13	11	12	14	15
Cote D'Ivoire	18	19	22	25	31	31	33	37
Dominican	39	41	41	41	14	15	19	24
Egypt	27	28	33	36	21	23	25	30
Ghana	4	4	4	7	6	4	5	7
Guatemala	22	20	19	19	9	9	8	11
Haiti	8	9	10	9	18	18	16	16
Indonesia	40	39	39	40	26	28	30	36
Kazakhstan	31	29	27	32	28	27	24	29
Kenya	23	26	26	28	24	26	27	28
Kyrgyzstan	33	32	30	23	33	33	32	19
Madagascar	21	23	23	24	37	38	39	39
Malawi	6	6	5	6	5	3	4	4
Mali	12	10	9	12	38	35	34	33
Morocco	36	36	36	35	20	20	21	20
Mozambique	3	3	3	2	8	5	3	1
Namibia	16	17	13	8	3	6	6	5
Nepal	11	12	15	16	22	21	22	22
Nicaragua	28	27	28	31	7	10	13	17
Niger	10	8	7	5	41	40	37	12
Nigeria	34	33	32	29	42	42	42	42
Pakistan	19	18	16	14	25	22	18	18
Peru	37	37	37	34	32	32	31	26
Senegal	38	40	40	38	40	41	41	40
Tanzania	13	13	11	10	23	19	15	14
Philippines	42	42	42	42	16	17	23	25
Togo	15	16	17	20	29	29	26	27
Turkey	24	24	25	30	15	16	17	21
Uganda	14	15	18	21	27	30	29	35
Uzbekistan	1	1	1	1	1	1	2	3
Vietnam	41	38	34	22	13	13	12	8
Zambia	26	25	24	26	35	37	38	38
Zimbabwe	25	22	20	15	12	11	10	10

Table 4: Spearman rank correlation coefficients between the country rankings of the generalized extended concentration index and of the symmetric index

	Symmetric Index			
Gen. Ext CI	-0.25	0	0.5	2
1.5	0.961	0.984	0.978	0.923
2	0.986	1.000	0.977	0.906
3	0.972	0.968	0.929	0.840
6	0.827	0.826	0.821	0.770

Table 5 Impact of small-sample adjustment on the country rankings by index

Statistics by index	Value of v/α 1.5/-0.25	2/0	3/0.5	6/2
Extended CI(s,v)				
Total number of countries re-ranked	4	0	8	24
Maximum change in ranking	2	0	4	13
Total number of re-rankings	6	0	16	86
Generalized extended CI				
Total number of countries re-ranked	11	0	20	38
Maximum change in ranking	3	0	7	20
Total number of re-rankings	14	0	38	136
Symmetric Index				
Total number of countries re-ranked	14	0	27	37
Maximum change in ranking	3	0	9	29
Total number of re-rankings	18	0	60	152

Table A1: Small-sample bias by index

PANEL A: RANDOM RANKING SCENARIO

beta(0,5;0,5)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001
n=100	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,039	0,021	-0,002	-0,001	0,000	-0,002
n=75	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,001	0,072	0,038	0,004	0,002	0,001	0,005
n=50	0,000	-0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,001	0,152	0,080	0,003	0,004	0,002	0,005
n=25	0,001	0,001	0,003	0,001	0,004	0,003	0,002	0,004	0,004	0,002	0,004	0,004	0,004	0,004	0,004	0,003	0,004	0,004	0,007	0,006	0,005	0,005	0,004	0,005	0,450	0,238	0,001	0,000	0,000	0,005
n=10	-0,002	-0,007	0,001	0,001	0,002	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,004	0,005	0,002	0,001	0,001	0,002	0,015	0,012	0,003	0,002	0,002	0,003	0,938	0,495	0,000	0,006	0,002	0,003
n=5	-0,006	-0,020	0,003	0,001	0,004	0,003	0,002	0,003	0,003	0,002	0,003	0,003	0,013	0,016	0,004	0,003	0,003	0,004	0,054	0,046	0,004	0,005	0,003	0,005	0,999	0,529	0,000	0,007	0,003	0,005
n=4	-0,009	-0,031	0,002	0,000	0,001	0,002	0,001	0,002	0,002	0,001	0,002	0,002	0,018	0,023	0,001	0,002	0,003	0,002	0,081	0,069	0,001	0,005	0,003	0,001	1,000	0,531	0,000	0,007	0,002	0,001
n=3	-0,014	-0,049	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	0,026	0,034	-0,001	-0,001	-0,002	-0,001	0,131	0,113	-0,001	-0,002	-0,002	-0,001	1,000	0,528	0,000	-0,003	-0,001	-0,001
n=2	-0,025	-0,087	-0,005	-0,001	-0,004	-0,005	-0,002	-0,005	-0,005	-0,002	-0,005	-0,005	0,060	0,077	-0,003	-0,002	-0,004	-0,005	0,283	0,242	-0,001	-0,003	-0,004	-0,005	1,000	0,528	0,000	-0,003	-0,002	-0,005

beta(1;1)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=100	0,000	-0,001	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,002	-0,002	-0,001	-0,002	0,038	0,020	-0,002	-0,002	-0,001	-0,002
n=75	0,000	-0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,001	0,001	0,001	0,001	0,001	0,069	0,036	-0,001	-0,001	-0,001	-0,002
n=50	-0,001	-0,002	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,002	-0,001	-0,001	-0,002	-0,001	-0,001	-0,002	-0,002	-0,002	-0,002	0,145	0,077	-0,003	-0,004	-0,002	-0,006
n=25	0,000	-0,001	0,001	0,000	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,002	0,001	0,001	0,001	0,001	0,004	0,004	0,002	0,002	0,002	0,002	0,454	0,240	0,001	0,007	0,004	0,004
n=10	-0,003	-0,010	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	0,002	0,002	-0,002	-0,001	-0,001	-0,002	0,011	0,009	-0,003	-0,002	-0,001	-0,003	0,937	0,496	0,000	-0,005	-0,003	-0,004
n=5	-0,005	-0,020	0,003	0,001	0,003	0,003	0,002	0,003	0,003	0,002	0,003	0,003	0,013	0,016	0,003	0,003	0,003	0,003	0,053	0,044	0,003	0,004	0,002	0,003	0,999	0,529	0,000	0,004	0,001	0,003
n=4	-0,010	-0,033	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	-0,001	0,014	0,019	-0,002	-0,002	-0,002	-0,002	0,073	0,063	-0,002	-0,004	-0,002	-0,003	1,000	0,528	0,000	-0,006	-0,002	-0,003
n=3	-0,016	-0,052	-0,004	-0,002	-0,004	-0,004	-0,003	-0,004	-0,004	-0,003	-0,004	-0,004	0,024	0,033	-0,004	-0,004	-0,004	-0,004	0,130	0,112	-0,002	-0,004	-0,003	-0,004	1,000	0,529	0,000	-0,004	-0,002	-0,004
n=2	-0,024	-0,082	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,064	0,082	0,001	0,002	0,001	0,001	0,287	0,245	0,000	0,003	0,001	0,001	1,000	0,528	0,000	0,003	0,001	0,001

beta(3;25)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=100	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,040	0,005	0,000	0,000	0,000	0,000
n=75	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,070	0,008	0,000	0,001	0,000	0,000
n=50	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,001	0,000	0,000	0,148	0,017	0,000	-0,001	0,000	0,000
n=25	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,003	0,000	0,000	0,001	0,000	0,000	0,452	0,051	0,000	0,003	0,000	0,001
n=10	-0,003	-0,002	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,002	0,001	0,000	-0,001	0,000	0,000	0,012	0,002	-0,001	-0,001	0,000	-0,001	0,938	0,106	0,000	-0,003	0,000	-0,001
n=5	-0,007	-0,005	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,008	0,002	0,000	-0,002	-0,001	0,000	0,046	0,008	0,000	-0,004	-0,001	0,000	0,999	0,113	0,000	-0,005	-0,001	0,000
n=4	-0,008	-0,006	0,001	0,001	0,001	0,001	0,002	0,001	0,001	0,002	0,001	0,001	0,018	0,005	0,001	0,002	0,001	0,001	0,079	0,014	0,001	0,003	0,000	0,001	1,000	0,113	0,000	0,003	0,000	0,001
n=3	-0,015	-0,010	0,000	0,000	0,000	0,000	-0,001	0,000	0,000	-0,001	0,000	0,000	0,028	0,008	0,000	0,000	0,000	0,000	0,134	0,025	0,000	0,001	0,000	0,000	1,000	0,113	0,000	0,002	0,000	0,000
n=2	-0,024	-0,018	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,063	0,018	0,000	0,000	0,000	0,000	0,286	0,053	0,000	0,001	0,000	0,000	1,000	0,114	0,000	0,001	0,000	0,000

beta(25;3)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=100	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,040	0,038	0,000	0,000	0,000	0,000
n=75	0,000	-0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,070	0,066	0,000	0,000	0,000	0,000
n=50	0,000	-0,002	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,001	0,000	0,000	0,000	0,000	0,148	0,140	0,000	0,000	0,000	0,001
n=25	-0,001	-0,004	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,002	0,003	0,000	0,000	0,000	0,000	0,450	0,425	0,000	0,000	0,000	0,000
n=10	-0,003	-0,015	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,003	0,006	0,000	0,000	0,000	0,000	0,013	0,019	0,000	0,000	0,000	0,000	0,938	0,886	0,000	0,000	0,000	0,000
n=5	-0,007	-0,042	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,010	0,023	-0,001	0,000	0,000	-0,001	0,049	0,075	0,000	0,000	0,000	0,000	0,999	0,944	0,000	0,000	0,000	0,000
n=4	-0,009	-0,057	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,016	0,036	0,000	0,000	0,000	0,000	0,076	0,117	0,000	0,000	0,000	0,000	1,000	0,945	0,000	0,000	0,000	0,000
n=3	-0,014	-0,085	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,028	0,064	0,000	0,000	0,000	0,000	0,133	0,205	0,000	0,000	0,000	0,000	1,000	0,945	0,000	0,000	0,000	0,000
n=2	-0,024	-0,147	0,001	0,000	0,001	0,001	0,000	0,001	0,001	0,000	0,001	0,001	0,063	0,146	0,000	0,000	0,001	0,001	0,285	0,437	0,000	0,000	0,001	0,001	1,000	0,945	0,000	0,000	0,000	0,001

Note: Population: by definition the small sample bias is zero in the population; $C^R(v)$, $F^R(v)$, $S^R(\alpha)$, $C^S(v)$, $F^S(v)$, $S^S(\alpha)$: denote the small-sample bias of the extended (C), the generalized extended (F) and the symmetric index (S), with superscripts R and S referring to the fractional rank and the small-sample adjusted versions. The Monte Carlo simulations are implemented by taking 20,000 independent samples of size n. The population value of the indices are approximated by taking the average of 20,000 samples of size 200,000; and the small-sample bias is calculated as the difference between the latter population values and the average values of the fractional rank versions of the indices (equations (18), (19) and (20)) or the small-sample adjusted versions (equations (29), (30) and (32)).

Table A1: Small-sample bias in the extended, generalized extended and symmetric indices (ctd.)

PANEL B: MAXIMAL ASSOCIATION RANKING SCENARIO

beta(0,5;0,5)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,002	0,000	0,000	0,000
n=100	-0,001	-0,008	-0,008	-0,001	-0,008	-0,008	-0,002	-0,008	-0,008	-0,002	-0,008	-0,008	-0,004	-0,008	-0,008	-0,004	-0,008	-0,008	-0,006	-0,006	-0,006	-0,006	-0,006	-0,006	-0,001	0,000	-0,149	-0,001	0,000	-0,001
n=75	-0,002	-0,011	-0,011	-0,001	-0,010	-0,011	-0,003	-0,011	-0,011	-0,003	-0,011	-0,011	-0,005	-0,011	-0,010	-0,005	-0,011	-0,010	-0,007	-0,008	-0,008	-0,007	-0,008	-0,007	-0,002	-0,001	-0,245	-0,002	-0,001	-0,002
n=50	-0,002	-0,017	-0,016	-0,002	-0,016	-0,016	-0,004	-0,016	-0,016	-0,004	-0,016	-0,016	-0,008	-0,016	-0,015	-0,008	-0,016	-0,015	-0,011	-0,012	-0,013	-0,011	-0,012	-0,011	-0,003	-0,002	-0,452	-0,004	-0,002	-0,004
n=25	-0,006	-0,036	-0,033	-0,004	-0,032	-0,033	-0,009	-0,033	-0,033	-0,009	-0,033	-0,033	-0,016	-0,031	-0,032	-0,017	-0,032	-0,031	-0,022	-0,024	-0,032	-0,022	-0,025	-0,024	-0,007	-0,004	-0,861	-0,013	-0,007	-0,014
n=10	-0,015	-0,095	-0,076	-0,010	-0,079	-0,078	-0,021	-0,081	-0,081	-0,021	-0,081	-0,081	-0,035	-0,075	-0,086	-0,038	-0,079	-0,080	-0,054	-0,063	-0,115	-0,058	-0,067	-0,073	-0,003	-0,003	-0,999	-0,063	-0,038	-0,071
n=5	-0,037	-0,199	-0,151	-0,025	-0,161	-0,154	-0,047	-0,161	-0,161	-0,047	-0,161	-0,161	-0,069	-0,143	-0,189	-0,079	-0,156	-0,169	-0,104	-0,121	-0,321	-0,122	-0,139	-0,178	0,001	0,000	-1,000	-0,171	-0,108	-0,204
n=4	-0,050	-0,252	-0,175	-0,034	-0,202	-0,183	-0,061	-0,201	-0,201	-0,061	-0,201	-0,201	-0,085	-0,175	-0,247	-0,101	-0,195	-0,216	-0,128	-0,147	-0,440	-0,158	-0,177	-0,239	0,001	0,003	-1,000	-0,229	-0,144	-0,275
n=3	-0,074	-0,340	-0,245	-0,052	-0,270	-0,251	-0,089	-0,267	-0,267	-0,089	-0,267	-0,267	-0,113	-0,224	-0,340	-0,141	-0,260	-0,295	-0,159	-0,187	-0,636	-0,218	-0,246	-0,348	0,001	0,000	-1,000	-0,321	-0,206	-0,390
n=2	-0,128	-0,520	-0,314	-0,095	-0,412	-0,339	-0,155	-0,405	-0,405	-0,155	-0,405	-0,405	-0,170	-0,313	-0,579	-0,233	-0,394	-0,478	-0,201	-0,232	-0,881	-0,355	-0,386	-0,552	0,001	0,000	-1,000	-0,497	-0,314	-0,595
beta(1;1)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	0,000	-0,001	0,000	0,000	-0,001	-0,001	0,000	-0,003	-0,001	0,000	-0,001
n=100	-0,001	-0,007	-0,006	-0,001	-0,006	-0,006	-0,002	-0,006	-0,006	-0,002	-0,006	-0,006	-0,003	-0,006	-0,007	-0,003	-0,006	-0,007	-0,006	-0,006	-0,009	-0,006	-0,006	-0,008	-0,010	-0,005	-0,158	-0,011	-0,006	-0,013
n=75	-0,002	-0,010	-0,008	-0,002	-0,009	-0,008	-0,003	-0,009	-0,009	-0,003	-0,009	-0,009	-0,005	-0,009	-0,010	-0,005	-0,009	-0,010	-0,008	-0,008	-0,012	-0,008	-0,008	-0,011	-0,013	-0,007	-0,255	-0,015	-0,008	-0,018
n=50	-0,003	-0,015	-0,012	-0,002	-0,014	-0,012	-0,004	-0,013	-0,013	-0,004	-0,013	-0,013	-0,007	-0,013	-0,015	-0,007	-0,013	-0,015	-0,012	-0,012	-0,019	-0,012	-0,012	-0,017	-0,018	-0,010	-0,462	-0,025	-0,013	-0,030
n=25	-0,007	-0,031	-0,024	-0,005	-0,027	-0,024	-0,009	-0,027	-0,027	-0,009	-0,027	-0,027	-0,015	-0,025	-0,031	-0,015	-0,026	-0,030	-0,023	-0,024	-0,042	-0,024	-0,025	-0,036	-0,022	-0,012	-0,860	-0,057	-0,031	-0,067
n=10	-0,018	-0,082	-0,057	-0,014	-0,068	-0,058	-0,023	-0,066	-0,066	-0,023	-0,066	-0,066	-0,035	-0,061	-0,081	-0,037	-0,064	-0,076	-0,055	-0,058	-0,131	-0,061	-0,063	-0,096	0,009	0,005	-0,989	-0,152	-0,085	-0,172
n=5	-0,041	-0,177	-0,118	-0,030	-0,141	-0,121	-0,050	-0,136	-0,136	-0,050	-0,136	-0,136	-0,069	-0,119	-0,177	-0,079	-0,132	-0,161	-0,102	-0,109	-0,325	-0,127	-0,131	-0,207	0,020	0,011	-0,990	-0,289	-0,166	-0,324
n=4	-0,051	-0,219	-0,135	-0,037	-0,172	-0,141	-0,061	-0,167	-0,167	-0,061	-0,167	-0,167	-0,080	-0,143	-0,225	-0,096	-0,163	-0,200	-0,121	-0,130	-0,430	-0,160	-0,166	-0,265	0,020	0,010	-0,990	-0,349	-0,202	-0,392
n=3	-0,073	-0,296	-0,191	-0,053	-0,230	-0,195	-0,086	-0,221	-0,221	-0,086	-0,221	-0,221	-0,105	-0,179	-0,305	-0,132	-0,216	-0,267	-0,144	-0,154	-0,594	-0,218	-0,222	-0,358	0,020	0,010	-0,990	-0,438	-0,254	-0,490
n=2	-0,119	-0,450	-0,248	-0,088	-0,347	-0,268	-0,141	-0,334	-0,334	-0,141	-0,334	-0,334	-0,149	-0,245	-0,501	-0,212	-0,326	-0,418	-0,153	-0,165	-0,794	-0,337	-0,335	-0,522	0,020	0,010	-0,990	-0,591	-0,341	-0,655
beta(3;25)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,002	0,000	-0,004	-0,002	0,000	-0,003
n=100	-0,003	-0,002	-0,001	-0,002	-0,002	-0,001	-0,003	-0,001	-0,001	-0,003	-0,001	-0,001	-0,004	-0,001	-0,002	-0,004	-0,001	-0,002	-0,005	-0,001	-0,003	-0,005	-0,001	-0,003	-0,019	-0,002	-0,072	-0,025	-0,003	-0,032
n=75	-0,003	-0,002	-0,001	-0,003	-0,002	-0,001	-0,004	-0,002	-0,002	-0,004	-0,002	-0,002	-0,005	-0,001	-0,002	-0,005	-0,001	-0,002	-0,006	-0,001	-0,004	-0,007	-0,001	-0,004	-0,021	-0,002	-0,105	-0,033	-0,004	-0,040
n=50	-0,005	-0,004	-0,002	-0,004	-0,003	-0,002	-0,005	-0,002	-0,002	-0,005	-0,002	-0,002	-0,007	-0,002	-0,003	-0,007	-0,002	-0,003	-0,010	-0,002	-0,007	-0,011	-0,002	-0,007	-0,023	-0,003	-0,168	-0,051	-0,006	-0,056
n=25	-0,010	-0,008	-0,004	-0,009	-0,007	-0,004	-0,011	-0,005	-0,005	-0,011	-0,005	-0,005	-0,014	-0,004	-0,007	-0,014	-0,004	-0,007	-0,019	-0,004	-0,014	-0,021	-0,004	-0,012	0,009	0,001	-0,275	-0,097	-0,011	-0,086
n=10	-0,026	-0,020	-0,010	-0,022	-0,016	-0,010	-0,029	-0,013	-0,013	-0,029	-0,013	-0,013	-0,034	-0,010	-0,019	-0,036	-0,011	-0,018	-0,047	-0,009	-0,038	-0,054	-0,010	-0,031	0,118	0,013	-0,306	-0,202	-0,023	-0,134
n=5	-0,053	-0,039	-0,020	-0,042	-0,031	-0,020	-0,058	-0,025	-0,025	-0,058	-0,025	-0,025	-0,064	-0,019	-0,038	-0,074	-0,021	-0,035	-0,081	-0,015	-0,082	-0,111	-0,021	-0,059	0,139	0,016	-0,306	-0,321	-0,037	-0,176
n=4	-0,065	-0,048	-0,023	-0,052	-0,038	-0,024	-0,071	-0,032	-0,032	-0,071	-0,032	-0,032	-0,077	-0,022	-0,048	-0,092	-0,027	-0,043	-0,092	-0,018	-0,104	-0,139	-0,026	-0,072	0,139	0,016	-0,306	-0,369	-0,042	-0,191
n=3	-0,088	-0,065	-0,034	-0,069	-0,051	-0,035	-0,097	-0,043	-0,043	-0,097	-0,043	-0,043	-0,099	-0,029	-0,065	-0,127	-0,036	-0,058	-0,098	-0,019	-0,135	-0,184	-0,035	-0,090	0,139	0,016	-0,306	-0,436	-0,050	-0,211
n=2	-0,129	-0,094	-0,043	-0,099	-0,073	-0,047	-0,144	-0,063	-0,063	-0,144	-0,063	-0,063	-0,130	-0,037	-0,101	-0,193	-0,055	-0,085	-0,079	-0,015	-0,173	-0,287	-0,053	-0,122	0,139	0,016	-0,306	-0,563	-0,064	-0,243
beta(25;3)	$C^R(1,5)$	$F^R(1,5)$	$S^R(-0,25)$	$C^S(1,5)$	$F^S(1,5)$	$S^S(-0,25)$	$C^R(2)$	$F^R(2)$	$S^R(0)$	$C^S(2)$	$F^S(2)$	$S^S(0)$	$C^R(3)$	$F^R(3)$	$S^R(0,5)$	$C^S(3)$	$F^S(3)$	$S^S(0,5)$	$C^R(6)$	$F^R(6)$	$S^R(2)$	$C^S(6)$	$F^S(6)$	$S^S(2)$	$C^R(100)$	$F^R(100)$	$S^R(49)$	$C^S(100)$	$F^S(100)$	$S^S(49)$
population	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
n=1000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,001	-0,001	-0,004	-0,002	-0,002	-0,003
n=100	0,000	-0,002	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	0,000	-0,001	-0,001	-0,001	-0,001	-0,002	-0,001	-0,001	-0,002	-0,001	-0,002	-0,003	-0,001	-0,002	-0,003	0,015	0,014	-0,072	-0,017	-0,017	-0,031
n=75	0,000	-0,003	-0,001	0,000	-0,002	-0,001	0,000	-0,002	-0,002	0,000	-0,002	-0,002	-0,001	-0,002	-0,002	-0,001	-0,002	-0,002	-0,002	-0,002	-0,004	-0,002	-0,003	-0,004	0,034	0,032	-0,105	-0,022	-0,021	-0,040
n=50	-0,001	-0,004	-0,002	0,000	-0,002	-0,002	-0,001	-0,003	-0,003	-0,001	-0,003	-0,003	-0,001	-0,003	-0,004	-0,001	-0,003	-0,004	-0,002	-0,003	-0,006	-0,003	-0,004	-0,006	0,090	0,085	-0,168	-0,032	-0,031	-0,055
n=25	-0,002	-0,009	-0,004	-0,001	-0,005	-0,004	-0,001	-0,005	-0,005	-0,001	-0,005	-0,005	-0,002	-0,005	-0,007	-0,002	-0,006	-0,007	-0,003	-0,005	-0,014	-0,005	-0,008	-0,013	0,324	0,306	-0,275	-0,056	-0,054	-0,087
n=10	-0,005	-0,029	-0,010	-0,002	-0,013	-0,010	-0,003	-0,013	-0,013	-0,003	-0,013	-0,013	-0,004	-0,009	-0,019	-0,006	-0,014	-0,018	-0,001	-0,002	-0,038	-0,013	-0,020	-0,031	0,735	0,694	-0,306	-0,095	-0,091	-0,133
n=5	-0,011	-0,069	-0,020	-0,004	-0,025	-0,021	-0,007	-0,026	-0,026	-0,007	-0,026	-0,026	-0,002	-0,005	-0,038	-0,012	-0,029	-0,035	0,022	0,034	-0,081	-0,024	-0,038	-0,058	0,790	0,746	-0,306	-0,128	-0,122	-0,175
n=4	-0,015	-0,091	-0,023	-0,005	-0,031	-0,024	-0,009	-0,031	-0,031	-0,009	-0,031	-0,031	0,001	0,002	-0,048	-0,015	-0,035	-0,043	0,043	0,065	-0,103	-0,030	-0,047	-0,071	0,790	0,746	-0,306	-0,139	-0,132	-0,190
n=3	-0,021	-0,130	-0,033	-0,007	-0,041	-0,034	-0,012	-0,042	-0,042	-0,012	-0,042	-0,042	0,008	0,018	-0,064	-0,019	-0,046	-0,057	0,089	0,135	-0,135	-0,039	-0,061	-0,090	0,790	0,747	-0,306	-0,153	-0,146	-0,211
n=2	-0,035	-0,214	-0,044	-0,010	-0,062	-0,048	-0,017	-0,063	-0,063	-0,017	-0,063	-0,063	0,034	0,077	-0,101	-0,029	-0,068	-0,086	0,221	0,338	-0,173	-0,055	-0,085	-0,122	0,790	0,746	-0,306	-0,174	-0,165	-0,242

Note: Population: by definition the small sample bias is zero in the population; $C^R(v)$, $F^R(v)$, $S^R(\alpha)$, $C^S(v)$, $F^S(v)$, $S^S(\alpha)$: denote the small-sample bias of the extended (C), the generalized extended (F) and the symmetric index (S), with superscripts R and S referring to the fractional rank and the small-sample adjusted versions. The Monte Carlo simulations are implemented by taking 20,000 independent samples of size *n*. The population value of the indices are approximated by taking the average of 20,000 samples of size 200,000; and the small-sample bias is calculated as the difference between the latter population values and the average values of the fractional rank versions of the indices (equations (18), (19) and (20)) or the small-sample adjusted versions (equations (29), (30) and (32)).

Table A.2: Comparison of inequality measures (under-five mortality by country)

	Extended CI				Generalized Extended CI				Symmetric Index			
Value ν, α	1.5	2	3	6	1.5	2	3	6	-0.25	0	0.5	2
Bangladesh	-0.036	-0.051	-0.065	-0.084	-0.034	-0.028	-0.024	-0.020	-0.023	-0.028	-0.037	-0.057
Benin	-0.064	-0.096	-0.124	-0.142	-0.079	-0.070	-0.059	-0.045	-0.062	-0.070	-0.083	-0.101
Bolivia	-0.088	-0.135	-0.178	-0.211	-0.079	-0.072	-0.062	-0.048	-0.064	-0.072	-0.082	-0.100
Brazil	-0.082	-0.149	-0.236	-0.352	-0.049	-0.053	-0.054	-0.054	-0.052	-0.053	-0.053	-0.057
Burkina Faso	-0.030	-0.040	-0.042	-0.019	-0.043	-0.034	-0.023	-0.007	-0.030	-0.034	-0.039	-0.046
Cameroon	-0.094	-0.150	-0.219	-0.319	-0.101	-0.096	-0.091	-0.088	-0.084	-0.096	-0.114	-0.146
CAR	-0.056	-0.078	-0.097	-0.133	-0.060	-0.049	-0.040	-0.036	-0.035	-0.049	-0.069	-0.102
Chad	-0.003	0.002	0.014	0.056	-0.004	0.002	0.008	0.020	-0.001	0.002	0.005	0.009
Colombia	-0.083	-0.132	-0.189	-0.264	-0.022	-0.020	-0.019	-0.018	-0.017	-0.020	-0.024	-0.029
Comoros	-0.042	-0.068	-0.094	-0.109	-0.036	-0.035	-0.031	-0.024	-0.032	-0.035	-0.036	-0.037
Cote D'Ivoire	-0.059	-0.093	-0.136	-0.202	-0.061	-0.057	-0.054	-0.053	-0.048	-0.057	-0.070	-0.090
Dominican	-0.105	-0.174	-0.261	-0.421	-0.043	-0.042	-0.041	-0.043	-0.036	-0.042	-0.052	-0.070
Egypt	-0.072	-0.125	-0.191	-0.284	-0.049	-0.050	-0.050	-0.049	-0.045	-0.050	-0.057	-0.065
Ghana	-0.028	-0.035	-0.040	-0.060	-0.021	-0.016	-0.012	-0.012	-0.011	-0.016	-0.025	-0.047
Guatemala	-0.066	-0.094	-0.116	-0.143	-0.036	-0.030	-0.024	-0.020	-0.024	-0.030	-0.040	-0.057
Haiti	-0.039	-0.060	-0.077	-0.079	-0.045	-0.041	-0.034	-0.023	-0.037	-0.041	-0.045	-0.045
Indonesia	-0.108	-0.171	-0.248	-0.376	-0.062	-0.058	-0.055	-0.055	-0.047	-0.058	-0.074	-0.096
Kazakhstan	-0.088	-0.130	-0.170	-0.257	-0.062	-0.055	-0.047	-0.046	-0.047	-0.055	-0.072	-0.116
Kenya	-0.067	-0.110	-0.165	-0.214	-0.055	-0.054	-0.052	-0.045	-0.049	-0.054	-0.058	-0.061
Kyrgyzstan	-0.091	-0.141	-0.187	-0.177	-0.070	-0.065	-0.056	-0.035	-0.063	-0.065	-0.065	-0.062
Madagascar	-0.065	-0.102	-0.143	-0.194	-0.081	-0.075	-0.069	-0.062	-0.069	-0.075	-0.087	-0.110
Malawi	-0.032	-0.042	-0.041	-0.027	-0.021	-0.016	-0.010	-0.004	-0.013	-0.016	-0.021	-0.030
Mali	-0.044	-0.062	-0.077	-0.098	-0.075	-0.062	-0.050	-0.043	-0.050	-0.062	-0.084	-0.121
Morocco	-0.096	-0.155	-0.220	-0.279	-0.050	-0.048	-0.044	-0.037	-0.044	-0.048	-0.053	-0.058

Mozambique	-0.018	-0.016	0.002	0.070	-0.029	-0.015	0.001	0.028	-0.021	-0.015	-0.011	-0.019
Namibia	-0.054	-0.082	-0.094	-0.077	-0.021	-0.019	-0.014	-0.007	-0.017	-0.019	-0.020	-0.020
Nepal	-0.044	-0.068	-0.099	-0.130	-0.047	-0.044	-0.041	-0.036	-0.041	-0.044	-0.049	-0.061
Nicaragua	-0.073	-0.122	-0.178	-0.255	-0.032	-0.032	-0.030	-0.028	-0.027	-0.032	-0.036	-0.041
Niger	-0.043	-0.055	-0.054	-0.024	-0.098	-0.075	-0.049	-0.014	-0.069	-0.075	-0.090	-0.124
Nigeria	-0.093	-0.142	-0.190	-0.218	-0.151	-0.137	-0.119	-0.090	-0.123	-0.137	-0.155	-0.180
Pakistan	-0.061	-0.085	-0.101	-0.111	-0.056	-0.047	-0.036	-0.026	-0.043	-0.047	-0.058	-0.084
Peru	-0.100	-0.159	-0.221	-0.275	-0.065	-0.061	-0.055	-0.045	-0.054	-0.061	-0.068	-0.076
Senegal	-0.104	-0.172	-0.255	-0.341	-0.106	-0.104	-0.100	-0.088	-0.096	-0.104	-0.113	-0.122
Tanzania	-0.050	-0.070	-0.084	-0.079	-0.051	-0.042	-0.033	-0.021	-0.037	-0.042	-0.050	-0.064
The Philippines	-0.124	-0.211	-0.327	-0.482	-0.046	-0.046	-0.047	-0.045	-0.042	-0.046	-0.051	-0.058
Togo	-0.052	-0.080	-0.108	-0.147	-0.059	-0.053	-0.047	-0.042	-0.044	-0.053	-0.066	-0.081
Turkey	-0.067	-0.107	-0.154	-0.238	-0.043	-0.040	-0.037	-0.038	-0.032	-0.040	-0.051	-0.070
Uganda	-0.051	-0.080	-0.114	-0.171	-0.058	-0.053	-0.050	-0.049	-0.046	-0.053	-0.066	-0.094
Uzbekistan	-0.001	0.010	0.042	0.122	-0.001	0.002	0.007	0.014	0.000	0.002	0.005	0.002
Vietnam	-0.113	-0.168	-0.208	-0.173	-0.043	-0.038	-0.030	-0.017	-0.034	-0.038	-0.041	-0.040
Zambia	-0.070	-0.108	-0.152	-0.206	-0.079	-0.073	-0.066	-0.059	-0.063	-0.073	-0.087	-0.110
Zimbabwe	-0.069	-0.100	-0.124	-0.126	-0.038	-0.033	-0.026	-0.018	-0.031	-0.033	-0.037	-0.049

Table A.3: Country rankings of the generalized extended concentration index and the symmetric index

	Generalized Extended CI				Symmetric Index			
Value ν, α	1.5	2	3	6	-0.25	0	0.5	2
Bangladesh	9	8	9	13	8	8	10	16
Benin	36	35	36	28	34	35	35	33
Bolivia	37	36	37	32	37	36	34	32
Brazil	20	25	32	36	32	25	22	15
Burkina Faso	15	12	8	5	11	12	12	11
Cameroon	40	40	40	40	40	40	41	41
CAR	28	23	19	21	16	23	30	34
Chad	2	2	1	2	2	2	1	1
Colombia	6	7	7	10	6	7	6	5
Comoros	11	13	14	16	13	13	9	7
Cote D'Ivoire	29	30	31	35	29	30	31	29
Dominican	14	17	20	26	17	17	20	24
Egypt	21	24	28	34	25	24	23	23
Ghana	5	4	5	7	3	4	7	12
Guatemala	10	9	10	12	9	9	13	14
Haiti	17	16	16	15	19	16	15	10
Indonesia	30	31	34	37	28	31	33	31
Kazakhstan	31	29	24	31	27	29	32	37
Kenya	24	28	30	27	30	28	24	19
Kyrgyzstan	33	34	35	19	35	34	26	21
Madagascar	38	38	39	39	39	38	37	36
Malawi	4	5	4	4	4	5	5	6
Mali	34	33	29	25	31	33	36	38
Morocco	22	22	22	22	23	22	21	17
Mozambique	7	3	3	1	7	3	3	3
Namibia	3	6	6	6	5	6	4	4
Nepal	19	19	21	20	20	19	16	20
Nicaragua	8	10	12	18	10	10	8	9
Niger	39	39	26	8	38	39	39	40
Nigeria	42	42	42	42	42	42	42	42
Pakistan	25	21	17	17	22	21	25	28
Peru	32	32	33	29	33	32	29	26
Senegal	41	41	41	41	41	41	40	39
Tanzania	23	18	15	14	18	18	17	22
Philippines	18	20	23	30	21	20	19	18
Togo	27	26	25	24	24	26	28	27
Turkey	13	15	18	23	14	15	18	25
Uganda	26	27	27	33	26	27	27	30
Uzbekistan	1	1	2	3	1	1	2	2
Vietnam	16	14	13	9	15	14	14	8
Zambia	35	37	38	38	36	37	38	35
Zimbabwe	12	11	11	11	12	11	11	13