

PRODUCT BUNDLING: THEORY AND APPLICATION

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PRODUCT BUNDLING: THEORY AND APPLICATION¹

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Abstract

For more than three decades, microeconomists have been studying the conditions under which product bundling is a remunerative sales strategy. Only recently, bundling aroused the interest of marketing academics. Until now, no serious attempts have been made to integrate the existing research streams in the bundling literature. A first merit of this paper is the integration of the existing microeconomic and marketing literature on product bundling in a comprehensive framework, which allows us to point out the lacunas.

It turns out that little has been said about the actual implementation of a bundling strategy. One of the hardly addressed issues is the derivation of bundle reservation prices, which are indispensable when determining the optimal bundle composition and price. Therefore, as a second merit, we present a new method to compute bundle reservation prices on the basis of paired comparison choice data and the latent class binomial logit methodology. We then apply our method to the case of multiple textbook bundles and use an existing LP bundling procedure to compute the optimal bundle compositions and prices.

Key words: *Product Bundling, Optimal Bundle Composition, Pricing, Linear Programming, Reservation Prices, Conjoint Analysis, Logit Analysis, Latent Class Estimation*

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1. Introduction

Often products or services are sold in packages: a visit to the zoo is tied to a train journey, subscription TV channels are offered in modulated bundles (cf. Chae, 1992), financial institutions, legally or not, sell tie-ins of mortgages and life policies. And just like many other (fast-food) restaurants, McDonald's entices its customers with complete menus, composed of burger, fries and soft drink. In fact a bicycle could be interpreted as a bundle of assembled parts, with maybe the most important component the assembly itself.

The last 25 years the practice of what is called product *bundling* inspired a lot of microeconomists to develop models which explain the basic principles underlying bundling. Several reasons for the possible remunerativeness of bundling were brought up. The various models often involve very different explicit *and* implicit assumptions, which reduces comparability. In 1987 Guiltinan introduced bundling in the marketing literature as a strategic marketing tool. Although the marketing articles on bundling after Guiltinan (1987) are rather scant, they deal with very different aspects. To our knowledge no attempts have been made to situate the microeconomic and marketing articles on product bundling in an integrating framework. Therefore, as a first vantage point, we integrate the existing research streams in the bundling literature and highlight the major gaps to be bridged. It should be stressed, however, that it is not our intention to give an *exhaustive* list of articles on bundling.

From a methodological point of view, the optimal composition and pricing of product bundles is a demanding job. Only a few authors tackled this issue. In this paper we apply one of the existing bundling procedures, which allows us to bring to life some of the microeconomic principles. At the same time, we come up with some new ideas, a second vantage point of this paper: we develop and apply a new method to infer bundle reservation prices, a quintessential input when working out a bundling strategy. For this

end, we use the implicit bundle utility function, estimated with the latent class binomial logit methodology, on the basis of paired comparison choice data.

In section 2 we define product bundling and its different faces. Sections 3 and 4 guide the reader through the literature on bundling. Section 3 expounds the basic principles underlying bundling by means of the Adams and Yellen model (1976). In section 4 we discern a series of possible motives for bundling, which at the same time enables us to classify the existing literature. Section 5 builds on the ideas presented in the previous sections, putting forward an LP-model for optimal bundle composition and pricing, based on Hanson and Martin (1990). A method for estimating reservation prices, which are used in the LP-problem, is dealt with. We then apply these techniques to the case of textbook packages. Section 6 concludes this paper, pointing out the limitations of our empirical research and the possibilities for future research.

2. Definitions

In the literature there seems to be no generally accepted, clear-cut definition for product bundling. Just in order to point out what will and what will not be the subject of this study, we define product bundling as

the practice of selling two or more different products (or services) in a fixed proportion and at an explicit or implicit price (partly adopted from Carlton and Perloff, 1994, p. 470).

Products do not have to be pre-packed to make a bundle (which would be difficult for services anyway). It suffices that in some way (explicitly or implicitly) a price for the combined products is set and communicated. The price information itself may be bundled (*price bundling*) or unbundled. It is clear then that (product) bundling is a broader concept than price bundling. Offering a package at a single price is both product bundling

and price bundling. A promotion that allows a discount on one product when a second product is bought, is product bundling but *no* price bundling.

Just like Adams and Yellen (1976) we distinguish among two types of bundling, *pure* and *mixed bundling*. The former involves the availability of only the bundle while the latter refers to the availability of the bundle as well as one or more of the separate component products. Although sometimes bundling is used as a synonym for *tie-in sales* (cf. Warhit, 1980, p. 80), we will clearly distinguish among these two notions. Following Carlton and Perloff (1994, p. 470), bundling differs from tie-in sales² in that a tie-in *does not require* products to be “wrapped up” in *fixed* proportions. A classic example of tie-in sales in variable proportions (*requirements tie-in sales*) is the IBM case. In the 30’s IBM obliged purchasers of tabulating machines to draw *all* punch-cards, no matter what quantity, from IBM (Carlton and Perloff, 1994, p. 470; Simon and Fassnacht, 1993, p. 405; Warhit, 1980, pp. 81-82). Besides, a tie-in requires that at least one component cannot be purchased separately. As a consequence, a mixed bundling setting in which *all* component products are separately available, is no tie-in. On the other hand, a bundling strategy in which at least one component is not sold apart, is a so called *package tie-in sale* (Carlton and Perloff, 1994, p. 470). Again, in this paper tie-in sales will be covered only as far as they involve package tie-in sales. The different notions and the links between them are summarized in Exhibit 1.

The difference between bundling and tie-ins is especially relevant from a legal point of view. In the US IBM was prohibited to continue its tie-in arrangement in 1936. And film distributors had to abandon the practice of block booking of films (cf. *infra*) (Simon, 1992, pp. 1229-1230; Simon and Fassnacht, 1993, p. 409). Belgian law imposes some rigid restraints on the “combined offer of products or services”, which seems to correspond to what we defined as tie-ins. The rule is that every such offer is forbidden; it suffices that the offer can be *perceived* or *experienced* as a tie-in by the customer. Exceptions are made, among others, for products or services “that belong together” (De

² For a pioneering article on tie-ins, see Burstein (1960).

Vroede, 1995, pp. 558-562). A brief analysis of case law on this subject learnt that much depends on the actual circumstances (Judit data base, 1999/1).

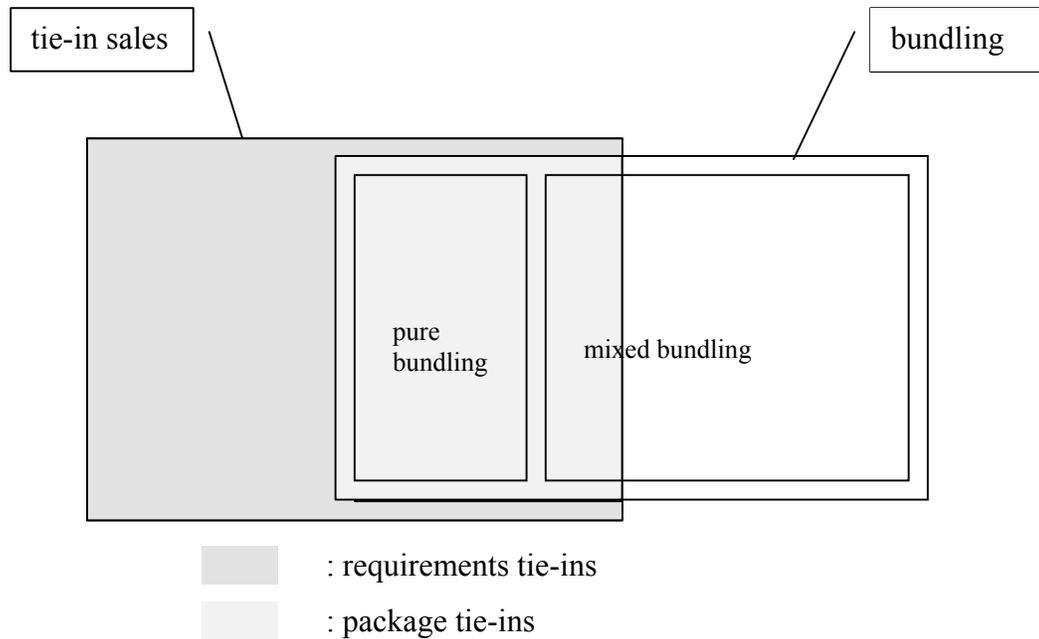


Exhibit 1. Bundling and tie-in sales

3. Basics: the Adams and Yellen model

In a seminal article, Adams and Yellen (1976) introduced the basic bundling principles. Since their model was used as a starting point in many subsequent papers, and since it will be used as a steppingstone in our empirical analysis, we discuss it here at length.

Suppose the following assumptions hold:

- (1) A monopolist offers only two products: product 1 and product 2.
- (2) The marginal costs (c_1 and c_2) are constant with respect to output; the marginal cost of the bundle B is the sum of the marginal costs of its components ($c_B = c_1 + c_2$); there are no fixed costs.

- (3) Reservation prices³ for specific consumers are unknown: perfect, or first-degree, price discrimination is therefore impossible.
- (4) Consumers' reservation prices (r_1 and r_2) for a second unit of the same product are zero. The reservation price for both products bought simultaneously (r_{1+2}), is the sum of the reservation prices for the separate components ($r_{1+2} = r_1 + r_2$).
- (5) *Zero assembly cost*: a consumer can compose the bundle him/herself at no extra cost (Hanson and Martin, 1990, p. 158), i.e. the consumer's reservation price for the bundle (r_B) equals the reservation price for the "self-composed" bundle ($r_B = r_{1+2}$). As a result, in this setting r_B en r_{1+2} can be used interchangeably.
- (6) *Free disposal*: a consumer can dispose of an unwanted component at zero cost (Hanson and Martin, 1990, p. 158).
- (7) Resale among consumers is nonexistent.
- (8) Consumers are surplus maximizing: consumer surplus is the difference between reservation price and product/bundle price; purchasing nothing yields zero consumer surplus.

In the original article only assumptions (1) to (4) are made explicit. Note that by imposing property (2), economies of scale and scope are assumed away⁴. Assumption (4) excludes the possibility of *complementarities* or *substitutabilities* between bundle components. Assumption (5) in turn rules out the possibility that bundling in itself offers extra savings on e.g. search efforts. Given assumption (4), assumption (6) is the same as stating that all (component and bundle) reservation prices should be at least zero⁵. Consequently, every consumer can be situated in the positive quadrant by means of his/her reservation prices, as is done for consumer s in Exhibit 2.

³ Here the reservation price of a consumer for a good is the maximum amount of money that this consumer would be willing to give up to get one unit of that good (Varian, 1992, pp. 152-153, 416).

⁴ As a consequence, if we normalized the reservation prices for each product such that $r_1' = r_1 - c_1$ and $r_2' = r_2 - c_2$, we could carry out the analyses with zero marginal costs and find the same optimal strategy.

⁵ Actually, this is not a stringent assumption since, in this setting, it is always possible to normalize all reservation prices such that they become positive.

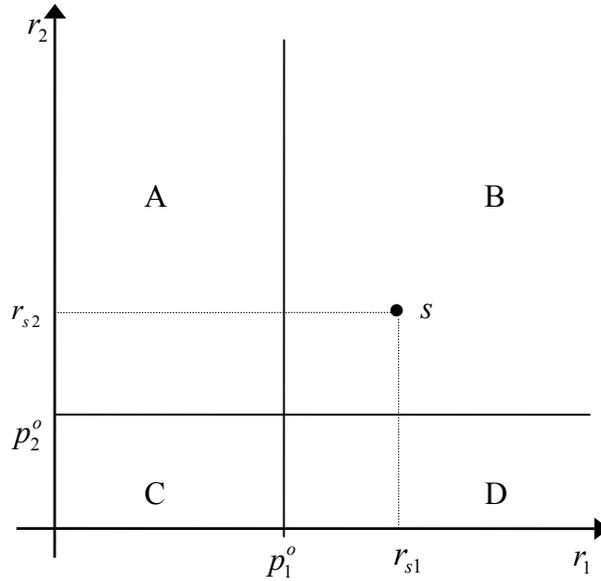


Exhibit 2. Pure components strategy

Given the distributions of both reservation prices, the monopolist could set optimal prices for both products (indicated as p_1^o and p_2^o in Exhibit 2), sorting the population into four segments. Consumers belonging to segment A only buy product 2 as their reservation price for product 2 is higher than p_2^o and their reservation price for product 1 is lower than p_1^o . For similar reasons, consumers in segment B buy both products (like s does), those in segment C buy neither and those in segment D buy only product 1. In the context of bundling, this strategy is often referred to as the *pure components* strategy.

If the monopolist adopts the pure bundling strategy, he sets an optimal price p_B^o for the bundle, given the distribution of the bundle reservation prices. In Exhibit 3 p_B^o is represented by a straight line grouping all reservation price combinations adding up to p_B^o . By assumption (4) all consumers situated above this line (segment A), buy the bundle as their purchase generates a positive consumer surplus. Consumers in segment B do not buy: their bundle reservation prices are lower than p_B^o .

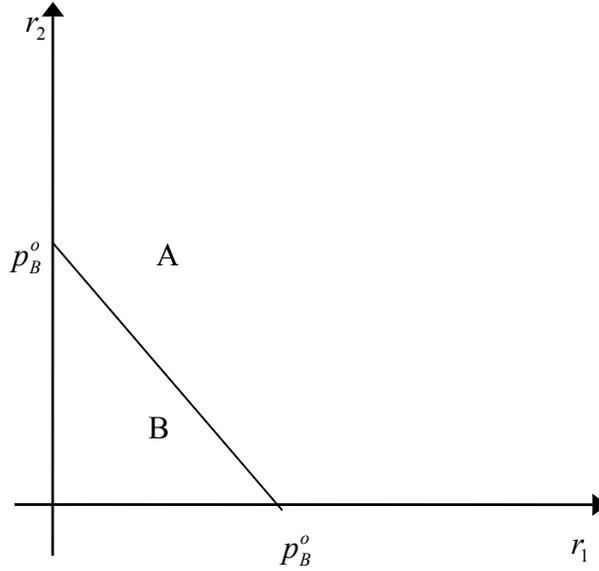


Exhibit 3. Pure price bundling

In the case of mixed bundling, the monopolist sets optimal prices for the separate products and the bundle simultaneously, such that a demand exists for the bundle and at least one component. Zero assembly cost (assumption (5)) forces the optimal bundle price to be lower than the sum of the separate optimal product prices, or, in other words and as shown in Exhibit 4, the line representing p_B^o should lie below the intersection of the lines representing p_1^o , respectively p_2^o . Moreover, because of free disposal (assumption (6)), p_B^o should be higher than $\min\{p_1^o, p_2^o\}$. Constraining the bundle price between $\min\{p_1^o, p_2^o\}$ and $p_1^o + p_2^o$ is called *price subadditivity* (cf. infra). Note that when $p_B \leq \min\{p_1, p_2\}$, the strategy adopted is no longer mixed bundling, but pure bundling. On the other hand, when $p_B \geq p_1 + p_2$, mixed bundling boils down to the pure components strategy. In fact, it is easy to see now that starting from a mixed bundling situation and letting p_B^o , p_1^o and p_2^o behave freely, would allow us to determine the overall optimal strategy.

Adopting mixed bundling, the monopolist divides the market into segments A, B, C and D. Consumers in segment A maximize their consumer surplus by purchasing only product 2. Consider for example consumer s : buying product 1 separately is not an option

as s 's reservation price for product 1 is lower than p_1^o . Buying the bundle would induce a positive consumer surplus b . Purchasing product 2, on the other hand, yields surplus a , which is definitely higher than b . Given that a second unit of the same product is not needed (assumption 4), consumer s only buys product 2. Analogous arguments could be used to show that consumers in segment D only buy product 1 and that those in segment B are bundle buyers. Finally, it is obvious that segment C consists of non-buyers.

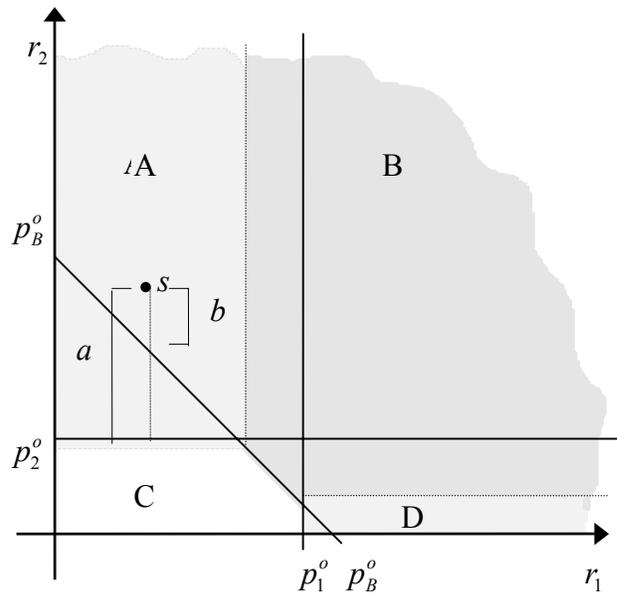


Exhibit 4. Mixed price bundling

4. Motives

Bundling can be motivated by a number of different reasons. It should be noted that these reasons are not mutually exclusive. Rather, it happens that one is a prerequisite for the other. As a matter of fact, one could even argue that many of these motives are derivatives of exactly the same bundling phenomenon. We successively deal with bundling as a vehicle to pursue price discrimination (section 4.1), exploitation of cost synergies (section 4.2), exploitation of demand complementarities (section 4.3), enhancement of customer value (section 4.4), leverage and product differentiation (section 4.5) and manipulation of perceptions (section 4.6).

4.1. Price discrimination

It is beyond questioning that in the microeconomic literature, price discrimination is the motive for bundling that received most attention and support. Indeed, often bundling allows to extract more consumer surplus than were possible when selling the component products (only) separately. Following Pigou (1920), the type of price discrimination described here is called *second-degree* price discrimination, as opposed to *first-degree* price discrimination which allows the producer to extract the *entire* consumer surplus, simply by charging an individualized price which equals the consumer's reservation price (Tirole, 1988, p. 135). Stigler (1963) is generally considered as the first articulating this idea. In his classic article on block booking, he analyzes the movie distributors' practice of offering only a package of movies to exhibitors. Suppose the distributor of movies X and Y, faces the simple reservation price distributions in Exhibit 5.

	Reservation prices for X	Reservation prices for Y	Reservation prices for bundle X+Y
Cinema 1	10	90	100
Cinema 2	40	85	125
Cinema 3	80	50	130
Cinema 4	90	20	110

Exhibit 5. A simple example of bundling with additive bundle reservation prices

Assume costs are zero and first-degree price discrimination is illegal. Charging the optimal price of 80 for film X generates a profit of 160 (cinemas 3 and 4 buy film X). Charging 85 for film Y yields an optimal profit of 170 (cinemas 1 and 2 buy film Y). In sum, pricing the films separately yields a profit of 330. If instead the movie distributor opted for block booking and set the optimal price of 100, he would earn 400 (all cinemas buy the bundle) and thus realize a higher profit. This example shows that even without complementarities (reservation prices for the bundle are the sum of the reservation prices for the component products) and economies of scope (the marginal cost of the bundle is

the sum of the marginal costs of the components) pure bundling allows further erosion of the total consumer surplus.

Stigler's note initiated a whole series of articles, trying to find out under which conditions bundling could function as a remunerative price discriminating device. Adams and Yellen (1976) ("A&Y" from here on) brought up the first elaborate analysis. On the basis of the model presented in section 3, they graphically explain the principles of this type of bundling: which particular strategy - pure components strategy, pure bundling or mixed bundling - prevails in terms of profitability, depends on the extent to which the bundling strategies approach first-degree price discrimination. First-degree price discrimination always satisfies three conditions. The first one, *full extraction*, requires the seller to extract the entire consumer surplus. By the second condition, *exclusion*, the monopolist does not sell to individuals with a reservation price below the marginal cost. Finally, *inclusion* requires the monopolist to sell to all individuals with a reservation price above the marginal cost (A&Y, 1976, p. 481). As a consequence, compliance with these three conditions determines the remunerativeness of the bundling strategy. A&Y (1976) proof that mixed bundling *weakly* dominates pure bundling, and even *strictly* dominates when pure bundling violates exclusion⁶. This turns the whole analysis into a comparison between the pure components strategy and mixed bundling. A&Y (1976) conjecture that mixed bundling is better able to satisfy extraction and inclusion simultaneously when the reservation prices are negatively correlated.

A&Y's study was the basis for numerous sheer theoretical as well as more applied papers. Schmalensee (1982) alters assumption (1) of the original A&Y model, and considers a *single*-product monopolist who bundles his product with another product produced in perfect competition. Schmalensee (1982) first demonstrates that in this setting pure bundling will never do better than pure components pricing. He then illustrates that mixed bundling is typically more profitable than the pure components strategy when the reservation prices for the two goods are negatively correlated. While

⁶ A&Y (1976, p. 483, footnote) show that mixed bundling with bundle price P_B and component prices $P_1 = P_B - c_2$ and $P_2 = P_B - c_2$ will be at least as profitable as pure bundling with bundle price P_B .

A&Y (1976) and Schmalensee (1982) do not presuppose any particular reservation price distribution⁷, Schmalensee (1984) supplements the A&Y model with the assumption that reservation prices follow a bivariate normal distribution, inducing the component and bundle reservation price distributions to be normal too. He rigorously shows that the prevalence of one of the strategies depends on the correlation between the reservation prices, the standardized differences between the average reservation prices and the marginal costs $(E(r_j) - c_j)/\sigma_{r_j}$ ($j = 1, 2$), and the relation between those differences. The major implication is that the negative correlation between the reservation prices is *not* a *conditio sine qua non* for mixed bundling to be more profitable than the pure components strategy. Olderog and Skiera (1998) give further intuitive support to Schmalensee's findings. By altering the means of the marginal (normal) reservation price distributions and the correlation between the two reservation prices, they simulate various scenarios, which demonstrate that, as long as the standardized differences between average reservation prices and marginal costs are high enough, mixed bundling pays. Negatively correlated reservation prices enhance this profitability only to a lesser degree. McAfee et al. (1989) deal with the case where reservation prices follow *any* continuous joint distribution, and derive strict conditions for mixed bundling to be more profitable than the pure components strategy. From their analysis we even learn that mixed bundling strictly dominates pure components when reservation prices are *uncorrelated* (!).

Cairns (1991) applies the A&Y model to the practice of bundling tickets for cultural events. He analyzes two different mixed bundling strategies: the first ties the availability of tickets for a high demand event⁸ to the attendance at a low demand event, still allowing people to attend only the low demand event. In the second strategy, the bundle as well as separate tickets for *both* the high and the low demand event are available. The analysis differs from the one in A&Y (1976) in that Cairns (1991) accounts for the natural capacity constraint, typical of most cultural events. However, Cairns (1991, p. 83) draws more or less the same general conclusions regarding the reservation price distributions. Lan and Kanafani (1993) study park-and-shop programs, which actually

⁷ This is the main reason why most of their conclusions are rather tentative.

bundle parking services and shopping: visitors of a shopping center can make use of the nearby parking facilities at a discount. The authors adopt the A&Y model but in addition assume reservation prices to follow a joint uniform distribution. Under these conditions, it turns out that, in terms of profit, mixed bundling should be preferred to the pure components strategy.

A rather limited number of articles on product bundling motivated by the same price discrimination principle, use models other than the A&Y model. Paroush and Peles (1981) alter assumption (4) of the A&Y model, reckoning with strictly positive reservation prices for a second unit of the same commodity, which invalidates A&Y's graphical analysis. As a result, it can be more lucrative to bundle *several* units of the two components, yet still in a fixed proportion (an essential characteristic of a bundle). Paroush and Peles determine the optimal proportion in rather artificial circumstances⁹ and derive some sufficient conditions for pure bundling to dominate the pure components strategy. On the basis of the A&Y-model, Hanson and Martin (1990) develop an LP-model for optimal bundle composition and pricing when more than two components (and therefore several bundle options) are involved. This model, which will be extensively dealt with in section 5, at the same time relaxes assumptions (2) (absence of scope economies) and (4) (absence of complementarities). Venkatesh and Mahajan (1993) study the impact on profit of a season ticket bundling a particular number of cultural events. Their model is particularly interesting in that it takes into account the expected number of cultural events a consumer will attend. In fact, by reckoning with this expected attendance, the authors indirectly introduce demand interdependencies (substitutabilities) in their model. Assuming reservation prices and expected attendance to be Weibull distributed, Venkatesh and Mahajan illustrate the optimality of mixed bundling on the basis of survey data. Unlike Cairns (1991), however, they do not include capacity constraints. Ansari et al. (1996) extend the model by Venkatesh and Mahajan (1993), determining endogenously the number of cultural events comprised in the season ticket.

⁸ I.e. for which people have high reservation prices.

⁹ The authors consider two segments of consumers. For each segment they presuppose two linear demand curves (one for each product) of a very restrictive nature.

Moreover, they consider the possibility of offering several season tickets, each bundling a different number of events¹⁰. Again, mixed bundling is shown to be most profitable. As a matter of fact, both Venkatesh and Mahajan (1993) and Ansari et al. (1996) implicitly illustrate that the distinction between product bundling and multi-unit pricing is only a fuzzy one. Finally, Guiltinan (1987) translates the A&Y model into more operational conditions relating to the bundle and component (reservation) prices, and reckoning with complementarities and (exogenous) competitive prices. Guiltinan in particular stresses the importance of price segmentation, which is a natural consequence of the bundling process: by self-selection, consumers sort themselves into different segments with different reservation price characteristics. As a result, bundling allows a firm to implement three different strategies: *cross-selling* the bundle to those customers that used to buy only one component, *acquisition* of consumers that used to buy neither component or *retention* of those customers that used to buy both components.

4.2. Exploiting cost advantages

In the previous analysis it was assumed that the marginal cost of a bundle was equal to the sum of the marginal costs of the components (assumption (2)). However, often economies of scope (due to savings in production, distribution, transactions and/or information on the part of the seller) offer an extra motive for bundling. It is intuitively clear that in this case bundling becomes an even more attractive strategy since selling the bundle guarantees a higher profit margin than in the absence of economies of scope. It should be noted, though, that part of these cost advantages can also be realized without bundling: if, in a particular time period, one consumer wants only the first component and another consumer only the second component, both components can be produced (distributed) simultaneously, possibly implying cost synergies. For simplicity, we consider here (and in the rest of this paper) those economies of scope which are realized only when the *same* consumer is provided with both components simultaneously.

¹⁰ Ansari et al. (1996, p. 89) call this strategy *amplified mixed bundling*.

Although Eppen et al. (1991, p. 8) illustrate the scope economies motive, and though Hanson and Martin (1990)¹¹ allow for scope economies in their optimal bundling model, to our knowledge, Chae (1992) supplies the only *rigorous* analysis of cost synergies in a bundling context. He considers a monopolist who incurs a constant marginal distribution cost c when a household subscribes to at least one of two TV-channels. This situation is graphically depicted in Exhibit 6. Notice that this is an extreme case of scope economies for the cost of providing one household with *both* TV-channels is equal to the cost of providing only *one* TV-channel.

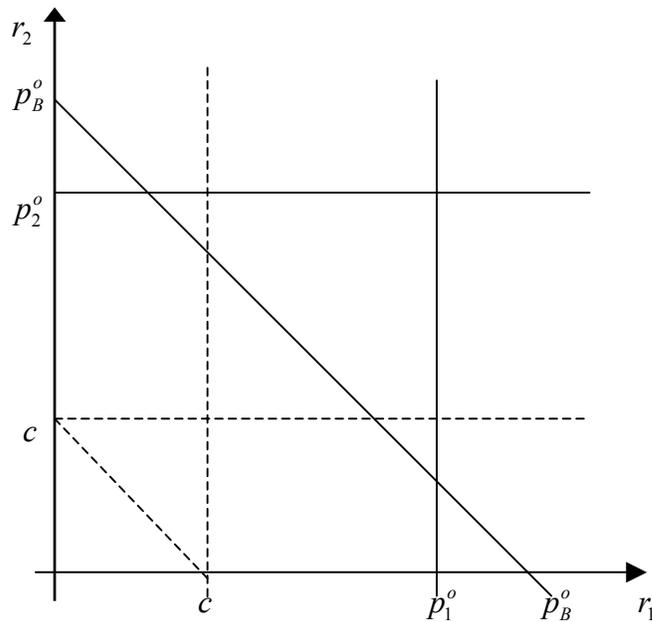


Exhibit 6. Bundling and economies of scope

Assuming that reservation prices follow a symmetric, bivariate uniform distribution, and respecting the other A&Y assumptions, Chae then proves that mixed bundling is the profit maximizing strategy and computes the optimal prices¹². In fact, in these circumstances bundling by definition still functions as a price discriminating tool since it

¹¹ Hanson and Martin (1990, p. 165) provide an example showing that, when marginal costs are subadditive, bundling may be even necessary in order to make positive profits.

¹² Apart from the profit maximizing objective, Chae also considers a welfare maximizer and a profit-constrained (i.e. subject to the constraint that the profit is nonnegative) welfare maximizer.

more completely complies with the price discrimination conditions than the pure components strategy¹³. The difference between the present situation and the one described in section 4.1 is that here the bundle's remunerativeness is no longer exclusively attributable to the preference structure.

4.3. Exploiting demand complementarities

Complementarities induce a consumer to be willing to pay a higher price for both products than just the sum of the separate reservation prices, i.e. $r_{1+2} (=r_B) > r_1 + r_2$ (cf. assumption (4)). The presence of complementarities invalidates the graphical A&Y model since the preference structure of a single consumer can no longer be described by two coordinates in the reservation price space, unless r_{1+2} can be written as a function of (r_1, r_2) . From the previous sections, we know that these complementarities are no prerequisite for bundling to be remunerative. Yet, complementary components make profitable bundling a much simpler task (Paun, 1993, p. 31; Lawless, 1991, p. 271). Although complementarities (and inter-product relationships in general) can also be exploited - to a certain extent - by optimal product line pricing (cf. Mulhern and Leone, 1991)¹⁴, often bundling is better able to do so thanks to its price discrimination characteristic.

Balderston (1956) distinguishes two types of complementarities (Mulhern and Leone, 1991, p. 65). *Use complementarity* is the enhancement of consumers' level of satisfaction when products are consumed together. Use complementarity can refer to the functional relation between the products, but also to the satisfaction derived from buying several

¹³ Chae also introduces production costs: prior to the pricing decision, the monopolist decides whether to "produce" only the first channel at a single setup cost C_1 , only the second channel at a setup cost C_2 , or both, at a setup cost C_B (which can be lower than, equal to, or higher than the sum of C_1 and C_2). As a consequence, although mixed bundling is optimal *once both channels are "produced"* (i.e. when C_B is a sunk cost), it could be that offering only one channel is overall a more profitable strategy.

¹⁴ Mulhern and Leone (1991) use the term "implicit bundling" to denote product line pricing: a rather unfortunate choice, since by no means product line pricing implies price discrimination, which is typical of product bundling.

products under one known brand name, or to the ease of obtaining after-sales support from a single source (Lawless, 1991, p. 274). *Purchase complementarity* involves savings in time, effort and transportation costs from purchasing products together (cf. Gultinan, 1987, p. 79). As a result, one could regard all products available in one store as complements. Anyway, note that buying the bundle is no prerequisite for consumers to realize whatever complementarities: consumers can just as well compose the bundle themselves¹⁵.

Although some attention has been given to complementarities in the tie-in literature (cf. Burstein, 1960; Warhit, 1980), which is partly relevant for product bundling, the issue remains to be fully explored in the bundling literature. Gultinan (1987) and Hanson and Martin (1990) account for complementarities (superadditive bundle reservation prices) in their respective models, but the most compelling (and complex) analysis motivated by the presence of complementarities is due to Telser (1979). Telser assumes that n consumers buy m components only in fixed proportions¹⁶, that the n consumer-specific proportion vectors are no linear combinations of each other, that cost synergies are absent and that the seller has a monopoly over at least one component. He then proves that it is remunerative to compose and price n customized bundles (possibly with several units of the same component) in lieu of pricing the m separate components.

4.4. Increasing customer value

Until now, we assumed that a customer can compose the bundle himself at no extra cost, or, in other words, that the bundle offers no additional value above the value of the “self-composed” bundle (assumption (5)). This assumption may not always be a realistic one: it could be that $r_B > r_{1+2}$. Different sources of this *intrinsic* bundle value can be pointed out. By buying the bundle, a customer can save on *search* costs. Especially in the beginning of the product life cycle, obtaining bundled products can be a cost saving

¹⁵ Albeit that bundling in itself creates *de facto* complementarities between the bundled products, since the products are *bought together*.

¹⁶ This means that the components are *perfect* compliments, at consumer level.

alternative for the search for further information on the separate products (Lawless, 1991, p. 271; Paun, 1993, pp. 30, 32). Often the seller has better expertise and information on what products work properly together (Eppen et al., 1991, p. 11) (cf. computers and peripherals). Eppen et al. (1991, pp. 11-12) state that, for new products, bundling “helps consumers understand the full range of product and service benefits”, which they call *product definition bundling*. In addition to savings on search efforts, bundles also allow consumers to realize extra savings on *transaction* costs (Lawless, 1991, p. 274). This is especially true for pre-packed bundles. Dansby and Conrad (1984) seem to be the only ones who allow for this additional bundle value in a formal analysis. By introducing a bundle reservation price function $r_B = b(r_1, r_2)$, they construct a graphical, two-dimensional model, comparable to the A&Y model. However, their conclusions with respect to profit implications are trivial and/or intuitive. Among other things they conclude that bundling is the preferred strategy when the bundle yields additional value (i.e. $r_B = b(r_1, r_2) \geq r_1 + r_2 = r_{1+2}$).

4.5. Strategic motives

In the strategic bundling literature, authors seem to discern two related bundle effects: extension of market power (*leverage*) and product differentiation. The first involves the penetration of a competitive market by bundling a monopolistically produced component with the component produced in the competitive market. Whereas Schmalensee (1982) argues that bundling a monopolistically produced good with a good produced in *perfect competition* is no leverage¹⁷, Carbajo et al. (1990) show how a monopolist can extend his market power (and enhance profit) by bundling his product with some other good produced in a *duopolistic* market. The authors presume that reservation prices are perfectly positively correlated and uniformly distributed, and respect the other relevant A&Y assumptions. Under both Bertrand price and Cournot quantity competition, they

¹⁷ As was mentioned in section 4.1, Schmalensee first proves that a monopolist will never choose to tie in his product with a product of a perfectly competitive industry in a *pure* bundle. Schmalensee then argues that mixed bundling (the optimal strategy) should not be considered as leverage since the monopolist is indifferent between producing the competitive product himself and buying it at cost from the competitive industry.

derive strict conditions¹⁸ for pure bundling (which dominates mixed bundling) to be remunerative. It turns out that, under Bertrand competition, both firms benefit from pure bundling. However, under Cournot competition, the profits of the non-bundling firm fall. Martin (1999) presupposes simple linear demand curves and Cournot-quantity setting duopolists, and retains the other A&Y assumptions. He, too, finds that the monopolist benefits from pure bundling, unlike the non-bundling firm.

The second strategic bundling implication is the differentiation of a good produced in a competitive market, by bundling it with another product. Chen (1997) analyzes a two-stage game. In the first stage, two firms decide simultaneously what products to offer and which strategy (pure/mixed bundling, pure components) to implement. One component is produced in duopolistic competition, the other in perfect competition. In the second stage, the firms set Bertrand-prices for their products/bundle. Consumers' reservation prices can follow any continuous distribution on a finite support. Maintaining the other A&Y assumptions, Chen then proves that pure bundling weakly dominates mixed bundling and that the Nash-equilibrium dictates one firm to offer the (pure) bundle and prescribes the other to sell only the duopolistic product.

4.6. Molding perceptions

Recent marketing literature on bundling particularly focuses on the psychological rather than on the economic aspects of bundling. While all previous motives consider each customer as a rational *homo economicus*, this new research stream analyzes the way price information is *processed* in a bundle context and the way the bundle value and quality is *perceived*.

A first focal point in this research tradition concerns *framing effects*, or the impact of the presentation of the bundle price information on price perceptions. Under perfect rationality, a customer would be indifferent between e.g. 1) buying products A and B together at \$10 (price bundling), 2) buying A at \$6 and B at \$4 when A and B are bought

¹⁸ These conditions relate to the (constant) marginal costs.

together, and 3) buying A at \$10 and getting B for free (or vice versa). Also, a customer would be indifferent between e.g. 1) \$2 off the bundle price, 2) \$2 off the price of A when B is bought at the regular price (or vice versa), and 3) \$1 off the price of A and \$1 off the price of B when A and B are bought simultaneously. In other words, under perfect rationality, consumer surplus, and therefore perceived prices and price discounts, are supposed to be *additive* and *completely transferable* from one product to another (Yadav, 1995, p. 207). Several authors consider departures from these (implicit) assumptions.

Some authors refute the assumption that (perceived) prices and price discounts of the separate components can simply be added up to a single perceived price, respectively price discount. Johnson et al. (1999) apply Thaler's mental accounting theory (1985) to product bundling. This theory posits that segregated (monetary) gains x and y induce a higher value than the corresponding integrated gain $x + y$ (i.e., if $v()$ is the value function, $v(x) + v(y) > v(x + y)$), and that segregated (monetary) losses $-x$ and $-y$ are perceived as less harmful when they are integrated into a single loss $-x - y$ (i.e. $v(-x) + v(-y) < v(-x - y)$). As a result, consumers prefer a single bundle price (considered as a loss) to separate product prices and consider separate discounts (considered as gains) as more favorable than a single bundle discount (Johnson et al., 1999, pp. 130-132). Johnson et al. test and confirm their hypotheses in an experimental pure bundling design, where the bundle consists of a basic model of a particular automobile and optional extras. Consumer evaluations of the bundle offer include satisfaction with the offer, likelihood of recommending the offer to other people, and likelihood of (re)purchasing the bundle. Yadav and Monroe (1993) draw on the same mental accounting principles, though in a slightly different experimental setting. They consider a mixed bundling design with a garment bag and a suitcase as component products. Discounts on the products, when bought separately, as well as the *extra* discount on the bundle differ across the bundle/price-profiles. The results seem to indicate that consumers rate the transaction value of an offer with segregated discounts (e.g. a \$10 discount on each separate product and an extra \$20 discount on the bundle) higher than the transaction value of the offer with corresponding integrated bundle discount (no discount on the separate products, a \$40 discount on the bundle price). Harlam et al. (1995), too, focus on framing effects. In

an interactive computer experiment respondents were first asked to enter their reservation prices for 10 individual branded products (VCR, TV, watch, videotapes, backpack, shampoo, conditioner, soap, chocolate and cookies). The computer program then presented two-component bundles at a segregated or integrated price. Prices equaled the (sum of the) corresponding reservation prices plus a premium or minus a discount. The results support mental accounting insofar that integrated price information gives rise to a higher purchase intent than segregated price information. However, the results regarding the price premiums and discounts go exactly in the opposite direction. One of the theories explaining these findings, is Weber's law, which implies that a large price change is more likely to be noticed than a small price change. As a consequence, on the basis of this law, one would opt for segregated price premiums and integrated price discounts. One reason for the "failure" of the mental accounting principles is that, in the experiment, neither premiums nor discounts were highlighted as such (Harlam et al., 1995, p. 59).

Yadav (1995) questions the perfect transferability of consumer surplus from one bundle component to another. In particular, he examines whether consumers are indifferent between a discount on one of two magazines, purchased together, (a sports magazine and an entertainment magazine) and a monetarily equivalent discount on the other magazine. It turns out that discounts are more effective (in terms of bundle utility and choice probability) when offered on a consumer's preferred magazine. Yadav's results show that the preference heterogeneity, which underlies the success of price discrimination, at the same time influences the way price information should be framed.

Apart from the semantic effects of price information, the recent behavioristic marketing literature also investigates how bundle composition affects the perceived bundle value. In a pure bundling experiment, where respondents evaluated bundles made up of a primary product and a free product (VCR + free videotapes, respectively typewriter + free calculator), Gaeth et al. (1990) find that bundle quality and usefulness perceptions are equal-weight-averages of the separate component perceptions. They also demonstrate that the freebie-equivalent-discount (i.e. the discount that would make consumers indifferent between the bundle and the primary product with that discount) is higher than

the retail value of the free product, and increases with the quality of the free product and, to a lesser extent, with the quality of the *primary* product. Further, Gaeth et al. show that bundle reservation prices are influenced by the quality of the respective bundle components: for functionally related bundles (VCR + tapes), for which overall superadditivity could have been expected, a free low quality product triggers reservation price *subadditivity*. For unrelated products (typewriter + calculator), a primary low quality product bundled with a free product of any quality induces superadditivity. In the computer experiment mentioned above, Harlam et al. (1995) find that, overall, functionally related products (e.g. shampoo + conditioner) imply a higher bundle purchase intent than monetarily equivalent unrelated products (e.g. shampoo + cookies).

Summing up, one needs little imagination to realize that ignoring perceptual effects could highly bias the alleged positive impact of a bundling strategy. The other side of the coin is that bundling can be an opportunity to mold favorable perceptions at a limited cost.

From the above discussions, it may seem that bundling only pays in terms of profit. However, most of the aforementioned articles (except for those in section 4.6) at least succinctly dwell on the possibly (!) favorable *welfare* implications. Yet, it is evident that the welfare maximizing strategy and prices do not always coincide with the profit optima. As a detailed analysis of the welfare implications is outside the scope of this paper, the interested reader is in particular referred to Ansari et al. (1996) who put forward compelling objectives other than profit (or revenue) maximization.

Exhibit 7 systematically arranges the motives and the literature mentioned above. It should be stressed that this classification does *not* contain an *exhaustive* list of articles on bundling. Our only intention was to integrate the *main research streams available in the bundling literature*. Moreover, due to its inductive character, our framework is possibly conceptually incomplete. Yet, we are convinced that this framework, together with our empirical research presented in the next section, will enable us to touch the sore spots in the bundling literature.

Motive	Authors	Demand side characteristics			Supply side characteristics		Remarks/applications	Major findings
		Component preferences	Demand interdependencies	Bundle preference	Cost structure	Competitive situation		
Price discrimination	Stigler (1963)	- any discrete r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	NA	monopoly for both components	theoretical/empirical (pure bundling - two components - block booking of movies)	intuitive support for bundling to be remunerative when relative reservation prices vary across customers
	Adams and Yellen (1976)	- any discrete r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	monopoly for both components	theoretical (pure/mixed bundling - two components)	- Mixed bundling at least weakly dominates pure bundling - Mixed bundling <i>seems</i> to dominate pure components when reservation prices are negatively correlated
	Paroush and Peles (1981)	simple linear demand curves	none	no additional bundle value	no scale or scope economies	monopoly for both components	theoretical (pure bundling - two components – possibly several units of the same component)	- determination of the optimal bundle proportion - sufficient conditions for pure bundling to dominate pure components
	Schmalensee (1982)	- any discrete r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	monopoly for one component, other component produced in perfect competition	theoretical (pure/mixed bundling - two components)	- Pure components weakly dominates pure bundling. - Mixed bundling <i>seems</i> to dominate pure components when reservation prices are negatively correlated.
	Schmalensee (1984)	- normal r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	monopoly for both components	theoretical (pure/mixed bundling - two components)	Correlation between reservation prices, and $E(r_j) - c_j$ ($j = 1, 2$) determine σ_{r_j} optimal strategy.
	Guiltinan (1987)	- any discrete r -distribution - r for second unit of same component = 0	complements	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	duopoly for both components (without strategic interaction)	theoretical (pure/mixed bundling - two components)	- derivation of conditions (relating to bundle and component prices and preferences) for bundle to be effective - profitability considerations
	McAfee et al. (1989)	- any continuous r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	monopoly for both components	theoretical (mixed bundling - two components)	derivation of strict conditions for mixed bundling to be more profitable than pure components
	Hanson and Martin (1990)	- any discrete r -distribution - r for second unit of same component = 0	substitutes/complements	no additional bundle value	no scale economies, scope economies possible	monopoly for all components	theoretical/ empirical (pure/mixed bundling - unlimited number of components - bundles of home services: ironing, shopping, etc.)	mixed integer LP model for optimal bundle composition and pricing

Motive	Authors	Demand side characteristics			Supply side characteristics		Remarks/applications	Major findings
		Component preferences	Demand interdependencies	Bundle preference	Cost structure	Competitive situation		
Price discrimination (cont.)	Cairns (1991)	- any discrete r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B^* = r_{1+2}$)	NA	monopoly for both components	theoretical (mixed bundling - two components: high demand event (with capacity constraint) and low demand event)	W.r.t. mixed bundling, tying high demand event to low demand event: The more popular is the high demand event, the more likely revenue will fall. The less popular is the low demand event, the more likely revenue will rise.
	Lan and Kanafani (1993)	- uniform r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B^* = r_{1+2}$)	no scale or scope economies	monopoly for both components	theoretical/empirical (mixed bundling - two components – park-and-shop bundles)	Mixed bundling dominates pure components.
	Venkatesk and Mahajan (1993)	- Weibull r -distribution - r for second unit of same component = 0	substitutes	no additional bundle value ($r_B^* = r_{1+2}$)	only fixed costs	monopoly for all components	empirical (pure/mixed bundling – season ticket for 10 events)	Mixed bundling is optimal strategy.
	Ansari et al. (1996)	- Weibull r -distribution - r for second unit of same component = 0	substitutes	no additional bundle value ($r_B^* = r_{1+2}$)	only fixed costs	monopoly for all components	empirical (pure/mixed bundling – season ticket for variable number of performances)	- Derivation of optimal number of bundled events - Mixed bundling is optimal strategy.
	Olderog and Skiera (1998)	- normal r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B^* = r_{1+2}$)	no scale or scope economies	monopoly for both components	theoretical (pure/mixed bundling – two components)	When $\frac{E(r_j) - c_j}{\sigma_{r_j}}$ ($j = 1, 2$) are high enough, mixed bundling pays.
Exploiting cost advantages	Hanson and Martin (1990)	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra
	Chae (1992)	- uniform r -distribution - r for second unit of same component = 0	none	no additional bundle value ($r_B^* = r_{1+2}$)	scope economies	monopoly for both components	theoretical (pure/mixed bundling – two components – bundle of subscription TV channels)	Mixed bundling is most profitable strategy.
Exploiting complementarities	Telser (1979)	linear demand curves	complements (consumer-specific)	- (consumer-specific) linear demand curves - no additional bundle value	no scale or scope economies	monopoly for at least one component	theoretical (pure bundling – possibly with several units of same component – unlimited number of components)	Pure bundling (customized bundles) dominates pure components strategy.
	Guiltinan (1987)	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra
	Hanson and Martin (1990)	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra	cf. supra

Motive	Authors	Demand side characteristics			Supply side characteristics		Remarks/applications	Major findings
		Component preferences	Demand interdependencies	Bundle preference	Cost structure	Competitive situation		
Increasing customer value	Dansby and Conrad (1984)	- any continuous r -distribution - r for second unit of same component = 0	none	additional bundle value possible, $r_B = b(r_1, r_2)$	no scale or scope economies	monopoly for both components	theoretical (pure/mixed bundling – two components)	Bundling pays if $b(r_1, r_2) \geq r_1 + r_2 = r_{1+2}$.
Strategic motives	Carbajo et al. (1990)	- uniform r -distribution - r for second unit of same component = 0 - r_1 en r_2 perfectly positively correlated	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	monopoly for one component, duopoly for other component	theoretical (pure/mixed bundling – two components)	- Pure bundling (weakly) dominates mixed bundling for both firms. - derivation of strict conditions (w.r.t. marginal costs) for (pure) bundling to be remunerative
	Chen (1997)	- any continuous r -distribution on finite support - r for second unit of same component = 0	none	no additional bundle value ($r_B = r_{1+2}$)	no scale or scope economies	duopoly for one component, perfect competition for other component	theoretical (pure/mixed bundling – two components)	- Pure bundling (weakly) dominates mixed bundling for both firms. - Equilibrium: one firm bundles, other firm sells only duopolistic product.
	Martin (1999)	linear aggregated demand curves	none	no additional bundle value	no scale or scope economies	monopoly for one component, duopoly for other component	theoretical (pure bundling – two components)	Bundling firm enhances its profit at the competitor's expense.
Molding perceptions	Gaeth et al. (1990)	Any	complements	tested (see Major findings)	NA	NA	empirical (pure bundling - bundle composed of VCR and tapes, resp. typewriter and calculator)	- Consumer evaluations of bundle quality and usefulness are equal-weight-averages of the component evaluations. - Free low quality product, bundled with functionally related product, induces r -subadditivity. Free product, bundled with unrelated low quality product, induces r -superadditivity. - Freebie-equivalent-discount is higher than retail value of freebie.
	Yadav and Monroe (1993)	NA	NA	NA	NA	NA	empirical (mixed bundling – bundle of suitcase and garment bag)	Consumer evaluations of bundle offer increase when discount information is split up into discounts on separate products + <i>extra</i> discount on bundle.

Motive	Authors	Demand side characteristics			Supply side characteristics		Remarks/applications	Major findings
		Component preferences	Demand interdependencies	Bundle preference	Cost structure	Competitive situation		
Molding perceptions (cont.)	Harlam et al. (1995)	Any	Complements	NA	NA	NA	empirical (pure bundling - two-component bundles composed of VCR, video tapes, watch, backpack, shampoo, conditioner, soap, cookies, chocolate)	Consumer evaluations of bundle offer increase as price information is integrated, as (non-highlighted) price premiums are segregated, as (non-highlighted) discounts are integrated, or when components are complements.
	Yadav (1995)	heterogeneous, negatively correlated preferences	NA	NA	NA	NA	empirical (pure bundling - bundle of entertainment and sports magazine)	Consumer evaluations of bundle offer increase when discount is offered on preferred component.
	Johnson et al. (1999)	NA	NA	NA	NA	NA	empirical (pure bundling - bundle of a basic car model and 12 optional extras)	Consumer evaluations of bundle offer increase as price information is integrated or as discount information is segregated.

Exhibit 7. Summary of the mainline research streams available in the bundling literature (r = reservation price, r_B = bundle reservation price, r_{1+2} = reservation price of the self-composed bundle with components 1 and 2)

5. Bundling applied

Walking through the above literature review, one will notice that much research investigates *why* bundling may or may not work. Less research addresses the question *how* a bundling strategy can be implemented. When only two bundle components and a small number of consumers are involved, computing an optimal strategy by hand is feasible (Simon and Fassnacht, 1993, p. 409). Once one has to deal with more than two component products and/or a huge number of heterogeneous consumers, the problem becomes much more complicated. Most of the few articles that do tackle this issue, are too product-specific (cf. Ansari et al., 1996; Goldberg et al., 1984; Venkatesh and Mahajan, 1993), or are only applicable to two-component situations (cf. Chae, 1992; Lan and Kanafani, 1993). The model we present is a mixed integer linear program, adapted from Hanson and Martin (1990), and computes optimal bundles and their corresponding optimal prices for, theoretically, an unlimited number of consumers (or classes of consumers with similar reservation prices) and bundle components. This model is capable of handling at least three of the aforementioned motives: price discrimination, exploiting cost advantages and exploiting complementarities. Section 5.1 sheds light on the assumptions underlying the model, which is then systematically elaborated. As will turn out in this section, (bundle) reservation prices are essential input parameters to the considered model (as well as to the other models brought up supra). Yet, in all applied bundling articles, the derivation of these reservation prices is considered only in a stepmotherly way. In section 5.2, we therefore present a new method to infer bundle reservation prices, based on the implicit bundle utility function estimated by means of a latent class binomial logit model. Both our reservation price derivation procedure and the LP model are then applied to the case of syllabus bundles: in section 5.3 we discuss the survey data and section 5.4 reports the results.

5.1. Optimal bundle composition and pricing model

The model presented here is based on Hanson and Martin (1990). The authors omit or relax some of the assumptions of the A&Y model, and preserve others. In what follows, *bundle* can refer to a “real” bundle, composed of several components, as well as to a single component (a “bundle” with only one element).

- (1) The seller has the monopoly over the components.
- (2) The marginal costs c_i are constant with respect to output (production and/or distribution) and there are no fixed costs; For every possible division of bundle i into two disjoint¹⁹ component bundles k and l , $0 \leq \max\{c_k, c_l\} \leq c_i$ holds.
- (3) Reservation prices for specific consumers are unknown. Therefore first-degree price discrimination is impossible.
- (4) The benefit of a second unit of the same component is zero.
- (5) Zero assembly cost: a consumer can compose a bundle him/herself at no extra (search or transaction) cost.
- (6) Free disposal: a consumer can dispose of an unwanted component at zero cost.
- (7) Resale among consumers is nonexistent.
- (8) Consumers are surplus maximizing; purchasing nothing yields zero consumer surplus.

First it should be stressed that the analysis is no longer limited to two components and therefore allows for multiple bundle options. More specifically, J components give rise to $I = 2^J - 1$ bundle options (including bundles comprising only one component). Assumption (2) rules out economies of scale but includes economies of scope. Also notice that the only assumptions restraining the behavior of reservation prices are zero assembly cost (assumption (5)) and free disposal (assumption (6)). This is equivalent to requiring that for every possible division of bundle i into two disjoint component bundles k and l , $0 \leq \max\{r_k, r_l\} \leq r_i = r_{k+l}$ holds. As a consequence, complementarities and even substitution effects are possible.

¹⁹ I.e. they do not have any component in common.

The idea behind the model is that initially all possible bundles I are considered and priced. Those bundles that are not bought by the surplus maximizing consumers, need not be offered and will not be part of the optimal solution. After all, if there are S *representative consumers* (i.e. consumers, each representing a segment or class of consumers with similar reservation prices), at most S different bundles will be part of the optimal solution (cf. infra).

The model will be presented as non-linear, which is the most understandable way. Yet, the problem can be linearized with the help of auxiliary variables, as will be demonstrated for the case of syllabus bundles in section 5.4. The objective function is formulated from the monopolist's perspective and represents his profit:

$$\max_{p_i, \theta_{si}} \sum_{s=1}^S \sum_{i=1}^I N_s \theta_{si} (p_i - c_i) \quad (5.1)$$

where p_i and θ_{si} are decision variables; p_i representing the price of bundle i ($=1, \dots, I$) and θ_{si} being the number of units of bundle i , bought by representative consumer s ($= 1, \dots, S$).

N_s is a parameter, being the number of consumers in class s ($\sum_{s=1}^S N_s = N$).

c_i is a parameter, representing the constant marginal cost of bundle i , and satisfying assumption (2).

The first group of constraints relates to *price subadditivity*:

$$\text{For every possible split of bundle } i \text{ into two disjoint} \\ \text{component bundles } k \text{ and } l: \max\{p_k, p_l\} \leq p_i \leq p_k + p_l \quad (5.2)$$

As a matter of fact, the first inequality in (5.2) is redundant since the monopolist will never be able to increase his profit by setting a bundle price below the price of a

component bundle. If he did set a lower bundle price, every surplus maximizing consumer interested in the component bundle, would, because of free disposal, purchase the bundle, which would generate a lower profit margin than the component bundle (cf. assumption (2)). The second inequality in (5.2) imposes the zero assembly cost assumption. Pricing a bundle above the summed prices of two collectively exhaustive component bundles would induce the consumer to buy the separate component bundles and compose the bundle him/herself.

The second group of constraints concerns consumer surplus maximization (assumption (8)):

$$\begin{aligned}
\forall s : \max_{\theta_{si}} \sum_{i=1}^I (r_{si} - p_i) \theta_{si} \\
\text{s.t. } \forall i : \theta_{si} \text{ integer} \geq 0 \\
\sum_{i=1}^I \theta_{si} \leq 1
\end{aligned} \tag{5.3}$$

where r_{si} is a parameter, being representative consumer s 's reservation price for bundle i , and satisfying assumption (6).

The sub-constraints in (5.3) together state that representative consumer s purchases at most one bundle, and only one unit of it (cf. assumption (4)). When a consumer s is indifferent between several bundles, he/she is supposed to choose the bundle with the highest price, since the objective function represents the monopolist's profit. We can see now that, as there are S representative consumers, at most S distinct bundles will be offered in the end. Even when all representative consumers buy, it is still possible that some of them buy the same bundle.

5.2. Derivation of reservation prices

One of the major issues when putting the above analysis into practice, is the determination of consumers' reservation prices. Following Hanson and Martin (1990),

Simon (1992), Simon and Fassnacht (1993) and Venkatesh and Mahajan (1993), one could simply ask consumers for their maximum willingness to pay. When applying this method, it may be expected, though, that the stated prices will reflect what consumers rationally consider as reasonable, rather than what they are really willing to pay, given their needs. Another type of techniques uses conjoint analysis to elicit reservation prices in an implicit way, as suggested in Hanson and Martin (1990, pp. 161-162). For the case of textbook packages, a conjoint analysis based on paired comparison choice data was opted for and reservation prices were inferred by estimating a binomial logit model. The most important advantage of choice data over ratings data is that “choosing” is a more natural task than “rating”. The main *disadvantage* is that (non-aggregated) choice data are most amendable to the complicated maximum likelihood logit analysis (Elrod et al., 1992, p. 368).

In principle, the model in section 5.1 could be solved for a large number of *individual* reservation prices, such that each class s consists of one consumer (or in (5.1): $\forall s: N_s = 1$). However, this would lead to unacceptable solution times (Hanson and Martin, 1990, p. 162) and possibly an unmanageable number of customized bundles. Moreover, estimating reservation prices by means of conjoint analysis at an individual level would burden respondents’ task, if one wants to guarantee a minimum number of degrees of freedom per respondent. Since it was not possible here to aggregate respondents into a priori classes of consumers with similar reservation prices, a latent class estimation was carried out, which simultaneously estimates parameters and sorts respondents into *fuzzy* clusters.

The latent class binomial logit methodology, presented here, is similar to (among others) Wedel and DeSarbo (1992). We will show how it can be used in a conjoint analysis to derive reservation prices. Suppose that N respondents are instructed to consider T different fictitious bundles at a given price, and to indicate for each bundle whether they would buy it or not (1/0). The T bundles involve a draw from the set of I possible bundles and can differ across respondents. (5.4) describes the implicit utility function for

consumer/respondent z , given that z is a member of class s which consists of consumers with the same bundle utility function.

$$U_{i|z \in s} = \sum_{j=1}^J u_{js} X_{ji} + \sum_{\substack{j=1 \\ j \neq n}}^J v_{(j,n)s} X_{ji} X_{ni} + up_s (W_z - P_i) + \varepsilon_{i|z \in s} \quad (5.4)^{20}$$

where $U_{i|z \in s}$ is the implicit utility for respondent z , derived from buying bundle i , given that z is a member of class s .

X_{ji} is a binary variable, indicating whether component j is present ($X_{ji} = 1$) or not ($X_{ji} = 0$) in bundle i .

W_z is a variable, being consumer z 's level of pre-purchase cash.

P_i is a variable, being the price of bundle i .

u_{js} is a parameter, representing the marginal utility of component j for consumers of class s .

$v_{(j,n)s}$ is a parameter, representing the utility interaction effect between components j and n for members of class s .

up_s is a parameter, being the marginal utility of money for class s .

$\varepsilon_{i|z \in s}$ are random error terms, following independent and identical extreme value distributions with mean $\Gamma(1)$ and variance $\pi^2/6$,

i.e.: $\Pr(\varepsilon_{i|z \in s} \leq b) = 1 - \exp(-\exp(b))$ (Lilien et al., 1992, p. 604). (5.5)

As a consequence, consumer z 's utility derived from buying nothing, can be written as:

$$U_{0|z \in s} = up_s W_z + \varepsilon_{0|z \in s} \quad (5.6)$$

Then, the probability that consumer z purchases bundle i , is:

²⁰ In (5.4) only two-way interactions are taken into account, but, of course higher-degree interactions could be tested for.

$$\Pr_{z \in s}(Y_{iz} = 1) = \Pr(U_{i|z \in s} > U_{0|z \in s}) \quad (5.7)$$

where Y_{iz} is a Bernoulli random variable (Aldrich and Nelson, 1984, p. 20), indicating whether consumer z purchases bundle i ($Y_{iz} = 1$), or not ($Y_{iz} = 0$),

and hence:

$$\Pr_{z \in s}(Y_{iz} = 1) = \Pr(\varepsilon_{0|z \in s} - \varepsilon_{i|z \in s} < \sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s P_i) \quad (5.8)$$

Since (5.5) holds, it follows that $\varepsilon_{0|z \in s} - \varepsilon_{i|z \in s}$ is distributed logistically with mean zero²¹ and variance $\pi^2 / 3$ (Aldrich and Nelson, 1984, p. 41; Lilien et al., 1992, p. 607), which allows us to write (5.8) as:

$$\begin{aligned} \Pr_{z \in s}(Y_{iz} = 1) &= \frac{\exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s P_i\right)}{1 + \exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s P_i\right)} \\ &= \Pr_{i|z \in s}(\cdot) \end{aligned} \quad (5.9)$$

The likelihood that purchase decisions $y_{1z}, y_{2z}, \dots, y_{Tz}$ (equaling 1 (purchase) or 0 (non-purchase)) are observed, given that z belongs to class s , is then (assuming independence over all purchase decisions):

$$L_{z \in s} = \prod_{i=1}^T \Pr_{i|z \in s}(\cdot)^{y_{iz}} (1 - \Pr_{i|z \in s}(\cdot))^{1-y_{iz}} \quad (5.10)$$

²¹ Setting the mean equal to zero is quite a stringent assumption when one does *not* include a constant term (cf. (5.4)) (Greene, 1993, p. 642). However, given the rationale underlying the choice process and the estimation results in section 5.4, a model without intercept is opted for.

If it were possible to divide consumers a priori into classes, we would estimate the parameters in (5.9) for each class s , simply by maximizing the likelihood over all members of class s :

$$L_s = \prod_{z=1}^{N_s} \prod_{i=1}^T \Pr_{i|z \in s}(\cdot)^{y_{iz}} (1 - \Pr_{i|z \in s}(\cdot))^{1-y_{iz}}$$

As a priori membership is unknown, (5.9) can only be considered as the probability that respondent z purchases bundle i , *conditional upon belonging to class s* . In order to derive *unconditional* purchase probabilities, we have to account for the unconditional probabilities α_s of belonging to class s (such that $\sum_{s=1}^S \alpha_s = 1$), and thus:

$$L_z = \sum_{s=1}^S \left[\alpha_s \prod_{i=1}^T \Pr_{i|z \in s}(\cdot)^{y_{iz}} (1 - \Pr_{i|z \in s}(\cdot))^{1-y_{iz}} \right] \quad (5.11)$$

is the unconditional likelihood for respondent z that observations $y_{1z}, y_{2z}, \dots, y_{Tz}$ are recorded. The complete likelihood over the entire sample of N consumers is then:

$$L = \prod_{z=1}^N \sum_{s=1}^S \left[\alpha_s \prod_{i=1}^T \Pr_{i|z \in s}(\cdot)^{y_{iz}} (1 - \Pr_{i|z \in s}(\cdot))^{1-y_{iz}} \right] \quad (5.12)$$

Implicitly imposing $\sum_{s=1}^S \alpha_s = 1$ can be done by considering α_s as a logit transformation of a population characteristic a :

$$\alpha_s = \frac{\exp(a_s)}{\sum_{s'=1}^S \exp(a_{s'})}, \quad (5.13)$$

which guarantees that all α_s are positive and add up to one. By maximizing L , estimates of u_{js} , $v_{(j,n)s}$, up_s and a_s can be obtained simultaneously. Using Bayes' rule, one can now compute the estimated a posteriori probability for each consumer c to belong to class s :

$$\hat{\delta}_{zs} = \frac{\hat{\alpha}_s \hat{L}_{z \in s}}{\sum_{s'=1}^S \hat{\alpha}_{s'} \hat{L}_{z \in s'}} \quad (5.14)$$

A consumer is assigned to the class for which he/she has the highest a posteriori probability.

Finally, reservation prices r_{si} for each bundle i and each class (or representative consumer) s can be inferred by requiring:

$$\frac{\exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n} v_{(j,n)s} X_{ji} X_{ni} - up_s r_{si}\right)}{1 + \exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n} v_{(j,n)s} X_{ji} X_{ni} - up_s r_{si}\right)} = 0.5 \quad (5.15)$$

I.e. r_{si} is equal to the price which implies a purchase probability of 0.5, or:

$$r_{si} = \frac{\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n} v_{(j,n)s} X_{ji} X_{ni}}{up_s} \quad (5.16)$$

In fact, r_{si} is the mean of the logistic reservation price density

$$f_{si}(r) = up_s \frac{\exp((r_{si} - r)up_s)}{[1 + \exp((r_{si} - r)up_s)]^2} \quad (5.17)$$

with class-specific variance $\sigma_s^2 = \frac{\pi^2(1/np_s)^2}{3}$ (see appendix 1 for the derivation of this distribution).

5.3. Data

As an example of product bundling, bundle opportunities for 6 textbooks, used by undergraduates in business economics, were explored. It was thought that syllabi comply quite well with most of the assumptions of the optimal bundling model (see section 5.1). However, the monopoly assumption should not be accepted without reservation: even though each of the considered textbooks is published and sold by a single store, it may be expected that price setting is influenced by e.g. photocopy rates. On the other hand, one could argue that the estimated reservation prices will be biased, to a certain extent, in the same direction and because of the same reasons.

Brief descriptions of the textbooks, together with the corresponding store prices, are presented in Exhibit 8. In order to make the case didactically interesting, the 6 selected textbooks relate to generally mandatory, as well as to orientation-specific and optional courses, such that reservation prices could be expected to vary considerably across students. Moreover, syllabi 1 and 2, respectively 3 and 4 could be considered as complementary, for they were used in the same courses.

10 different questionnaires were designed, each containing a different list of 15 - out of the 63 ($= 2^6 - 1 = I$) possible - syllabus packages with a corresponding, mostly fictitious price. A price could attain one of three possible price levels: store price, 20 percent below store price and 20 percent above store price²² (see appendix 2 for an extract from one of

²² We chose 20% since it is a common discount level (Harlam et al., 1995, p. 61). Moreover, as will turn out later, it allowed us to build in enough variation in the price levels in order to avoid multicollinearity as much as possible.

the questionnaires). All 10 designs were framed quasi orthogonally²³. The questionnaires were randomly distributed over nearly 150 senior students in business economics. It was the respondents' task to indicate which bundles they would buy, and which they would not buy, given their particular situation (orientation chosen for etc.). Students were urged to consider the 15 purchase decisions as being independent from one another: students were told that for each fictitious purchase occasion the bundle was the only opportunity to acquire the considered books and that the other books, not comprised in the bundle, were unavailable. Exactly 120 usable questionnaires were returned, yielding a data set of 1800 (120 x 15) observations.

Number	Description	Store price
1	Textbook for the course "Strategic Management"	290 BEF
2	Students' manual for a strategic game, part of the course "Strategic Management"	50 BEF
3	Textbook for the course "Social Law", part I	185 BEF
4	Textbook for the course "Social Law", part II	250 BEF
5	Textbook for the course "Religious Studies"	250 BEF
6	Textbook for the course "Marketing Decision Support Systems"	315 BEF

Exhibit 8. Descriptions and store prices of selected textbooks

5.4. Results

Likelihood function (5.12), for $N = 120$ (number of respondents), $S = 5$ ²⁴ (number of classes), $T = 15$ (number of purchase decisions per respondent) and $J = 6$ (number of components/books) was programmed in GAUSS 3.4.1 and optimized with GAUSS' *maxlik*-subroutine version 3.1.3 (see appendix 3):

²³ The zero-stimulus, being a bundle without any components, was removed from each of the originally perfectly orthogonal designs. However, this quasi orthogonality is only a relative notion because the *absolute* prices P_i can still be collinear with the bundle component variables X_{ji} .

²⁴ We chose $S=5$ mainly because of didactic reasons. The *CAIC* (see section 6) for $S=5$ was 1973.88, but a few trials with different S 's revealed that it is possible to achieve lower values.

$$L = \prod_{z=1}^{120} \sum_{s=1}^5 \left[\alpha_s \prod_{i=1}^{15} \Pr_{i|z \in s}(\cdot)^{y_{iz}} (1 - \Pr_{i|z \in s}(\cdot))^{1-y_{iz}} \right] \exp \left(\sum_{j=1}^6 u_{js} X_{ji} + v_{(1,2)s} X_{1i} X_{2i} + v_{(3,4)s} X_{3i} X_{4i} - u p_s P_i \right) \quad (5.18)$$

where $\Pr_{i|z \in s}(\cdot) = \frac{\exp \left(\sum_{j=1}^6 u_{js} X_{ji} + v_{(1,2)s} X_{1i} X_{2i} + v_{(3,4)s} X_{3i} X_{4i} - u p_s P_i \right)}{1 + \exp \left(\sum_{j=1}^6 u_{js} X_{ji} + v_{(1,2)s} X_{1i} X_{2i} + v_{(3,4)s} X_{3i} X_{4i} - u p_s P_i \right)}$

Prices P_i were divided by 100 in order to make the problem computationally feasible. Note that (5.18) includes interaction effects between components 1 and 2, respectively 3 and 4.

A typical problem in conjoint analysis is the trade-off between the experimental design's orthogonality and its consistency (Goldberg et al., 1984, pp. S112-S113). Often one can create a (perfectly) orthogonal design only at the expense of the stimuli's realism. In the present case, "realistic stimuli" means that one could expect at least some collinearity between the bundle's price P_i and its composition variables X_{ji} . This collinearity of course reduces the model's capability to separate out the effects of the bundle components and price on total utility. Fortunately, in the present case, there was no reason to panic. (see appendix 4 for the results of the Belsley test).

Exhibit 9 reports the results of the optimization of (5.18). First, most striking is the significant negativity of the marginal utility of the 6th textbook, in class 3. Assuming rational economic behavior, one would expect marginal utilities of at least zero. This is a rather undesirable result, since a negative marginal utility gives rise to a negative reservation price and, consequently, the free disposal assumption fails. It seems that the presence of unwanted components, in casu syllabi, can have a strongly negative impact on the bundle reservation price. Second, syllabi 1 and 2, and syllabi 3 and 4 - both pairs were expected to be complementary - seem to generate sometimes negative, though never highly significant, interaction effects. Possibly this reflects the respondents' expectation of price reductions for bundled products.

	class 1		class 2		class 3		class 4		class 5	
	estimate	t-value	estimate	t-value	estimate	t-value	estimate	t-value	estimate	t-value
u_{1s}	-0.41	-0.31	4.05	7.02	6.09	4.40	2.19	4.78	7.20	2.15
u_{2s}	-1.73	-1.62	1.80	4.79	0.93	1.33	0.71	2.23	3.20	1.46
u_{3s}	-1.88	-1.34	1.93	5.11	5.01	4.91	1.11	3.00	-3.29	-1.88
u_{4s}	-1.65	-1.15	2.29	4.68	5.80	4.81	0.58	1.38	-2.06	-1.10
u_{5s}	0.78	0.89	2.84	6.69	5.62	4.60	1.43	3.99	4.26	2.22
u_{6s}	-1.82	-1.17	1.74	4.59	-4.43	-4.99	1.79	4.61	-0.54	-0.23
$v_{(1,2)s}$	2.92	1.60	-0.68	-1.24	0.48	0.50	-0.27	-0.59	-2.15	-0.91
$v_{(3,4)s}$	5.92	3.33	1.62	2.77	-0.50	-0.55	0.88	1.81	-0.25	-0.01
$-up_s$	-0.31	-1.01	-1.09	-8.21	-1.88	-4.97	-0.74	-6.02	-1.73	-1.88
N_s	12		31		29		30		18	
% correct hits	88.89		78.71		93.56		70.22		97.04	
total % correct hits = 83.94										
overall chi-square $-2\ln(\hat{L}_0/\hat{L}) = 937.73$ (significance = 0.0000)										
$(\hat{L}_0 = L(u_{1s} = 0, \dots, u_{6s} = 0, v_{(1,2)s} = 0, v_{(3,4)s} = 0, up_s = 0))$										

Exhibit 9. Estimation results for $S = 5$

Two different measures of fit were computed. The *(total) percentage of correct hits* counts the number of times the estimated model correctly predicts the purchase decisions²⁵ of the members of a given class (or of the whole sample). The second measure, the *overall chi-square*, allows for testing the hypothesis that all coefficients, except the a_s 's, are zero (Aldrich and Nelson, 1984, p. 55). Note that both measures indicate a more than acceptable fit.

Significant as well as non-significant parameters were used in (5.16) in order to derive reservation prices, which are presented in Exhibit 10, rounded off to the nearest integer.

²⁵ A consumer is considered to buy the bundle when his purchase probability is 0.5 or higher.

Bundle number i	Bundle composition ²⁶	Reservation prices				
		class 1: r_{1i}	class 2: r_{2i}	class 3: r_{3i}	class 4: r_{4i}	class 5: r_{5i}
1	000001	-581	159	-236	243	-31
2	000010	250	260	299	194	247
3	000011	-331	419	63	437	216
4	000100	-526	210	309	79	-119
5	000101	-1108	369	73	322	-150
6	000110	-277	470	608	273	127
7	000111	-858	629	372	516	96
8	001000	-600	177	267	150	-190
9	001001	-1182	336	31	393	-221
10	001010	-350	437	566	345	56
11	001011	-932	596	330	587	25
12	001100	758	535	548	348	-324
13	001101	177	694	313	591	-355
14	001110	1008	795	848	543	-77
15	001111	427	954	612	785	-108
16	010000	-550	164	50	96	185
17	010001	-1132	324	-186	338	154
18	010010	-300	425	349	290	432
19	010011	-882	584	113	533	401
20	010100	-1077	374	358	174	66
21	010101	-1658	533	123	417	35
22	010110	-827	634	657	369	312
23	010111	-1408	794	422	611	281
24	011000	-1150	341	316	246	-5
25	011001	-1732	501	81	489	-36
26	011010	-901	601	615	440	241
27	011011	-1482	761	380	683	210
28	011100	208	699	598	444	-139
29	011101	-373	859	362	687	-170
30	011110	458	959	897	638	108
31	011111	-123	1119	661	881	77
32	100000	-131	370	324	297	416
33	100001	-712	530	88	540	385
34	100010	119	630	623	492	663
35	100011	-462	790	387	734	632
36	100100	-657	580	632	376	297
37	100101	-1239	739	397	619	266
38	100110	-407	840	932	570	544
39	100111	-989	999	696	813	513
40	101000	-731	547	590	447	226
41	101001	-1313	706	355	690	195
42	101010	-481	807	889	642	473

²⁶ The second column contains binary codes for the numbers in the first column and indicates with 1's which components the bundle is composed of. For example, bundle 5 (000101) is composed of components 4 and 6.

Bundle number i	Bundle composition ²⁷	Reservation prices				
		class 1: r_{1i}	class 2: r_{2i}	class 3: r_{3i}	class 4: r_{4i}	class 5: r_{5i}
43	101011	-1063	967	654	885	442
44	101100	628	905	872	646	92
45	101101	46	1064	637	888	61
46	101110	877	1165	1171	840	339
47	101111	296	1325	936	1083	308
48	110000	250	473	399	357	477
49	110001	-331	632	163	599	446
50	110010	500	733	698	551	724
51	110011	-81	892	463	794	693
52	110100	-276	682	708	435	358
53	110101	-857	842	472	678	327
54	110110	-26	942	1007	630	604
55	110111	-607	1102	771	873	573
56	111000	-350	649	666	507	287
57	111001	-931	809	430	750	256
58	111010	-100	910	965	701	534
59	111011	-681	1069	729	944	503
60	111100	1009	1007	948	705	153
61	111101	428	1167	712	948	122
62	111110	1259	1268	1247	899	400
63	111111	677	1427	1011	1142	369

Exhibit 10. Estimated reservation prices

Combining information from Exhibit 9 and Exhibit 10, we see, for example, that the first class is characterized by highly negative reservation prices and that the only positive reservation prices are mainly due to interaction effects; i.e. book 1 is only wanted when it is bought in combination with book 2 and vice versa. The same holds, even more significantly, for books 3 and 4. Analogous conclusions can be drawn with respect to the behavior of other classes.

The estimated reservation prices and the class sizes were inputted to a C-module which formulated a mixed integer LP-model in Industrial LINDO 6.01 for the optimal product bundling problem of section 5.1. Linearization of model (5.1)-(5.3) was carried out by introducing the auxiliary decision variable p_{si} , the price representative consumer s pays

²⁷ The second column contains binary codes for the numbers in the first column and indicates with 1's which components the bundle is composed of. For example, bundle 5 (000101) is composed of components 4 and 6.

for bundle i . In other words, p_{si} equals zero when s does not buy bundle i , and equals p_i when s does purchase bundle i . Since cost data were unavailable, the model considered a *revenue*-maximizing (instead of *profit*-maximizing) monopolist. For the case of syllabus bundles, the linearized objective function (being the monopolist's revenue) is:

$$\max_{p_{si}} \sum_{s=1}^5 \sum_{i=1}^{63} N_s p_{si} \quad (5.19)$$

p_{si} should satisfy the following constraints (Hanson and Martin, 1990, p. 159):

$$\begin{aligned} \forall s, \forall i: p_{si} &\geq p_i - \left(\max_{s,i} \{r_{si}\} \right) (1 - \theta_{si}) \\ p_{si} &\leq p_i \\ p_{si} &\geq 0 \end{aligned} \quad (5.20)$$

Except for a minor adjustment (printed in italics) due to failure of the free disposal assumption, the constraints in (5.2) remain the same, since they were linear already:

$$\begin{aligned} \text{For every possible split of bundle } i \text{ into two, } &\textit{not necessarily disjoint}, \\ \text{component bundles } k \text{ and } l: & p_i \leq p_k + p_l \end{aligned} \quad (5.21)$$

Finally, linearizing (5.3) yields:

$$\begin{aligned} \forall s, \forall i: \sum_{i'=1}^{63} (r_{si'} \theta_{si'} - p_{si'}) &\geq r_{si} - p_i \\ \sum_{i'=1}^{63} (r_{si'} \theta_{si'} - p_{si'}) &\geq 0 \\ \sum_{i'=1}^{63} \theta_{si'} &\leq 1 \\ \theta_{si} &\text{ integer } \geq 0 \end{aligned} \quad (5.22)$$

Exhibit 11 reports the results.

Bundle number i	Bundle composition	Purchase decisions					Price (BEF) P_i
		class 1: θ_{1i}	class 2: θ_{2i}	class 3: θ_{3i}	class 4: θ_{4i}	class 5: θ_{5i}	
1	000001	0	0	0	0	0	1427
2	000010	0	0	0	0	0	614
3	000011	0	0	0	0	0	1427
4	000100	0	0	0	0	0	1427
5	000101	0	0	0	0	0	1427
6	000110	0	0	0	0	0	813
7	000111	0	0	0	0	0	714
8	001000	0	0	0	0	0	1427
9	001001	0	0	0	0	0	1427
10	001010	0	0	0	0	0	714
11	001011	0	0	0	0	0	1427
12	001100	0	0	0	0	0	1427
13	001101	0	0	0	0	0	813
14	001110	0	0	0	0	0	996
15	001111	0	0	0	0	0	1427
16	010000	0	0	0	0	0	614
17	010001	0	0	0	0	0	1427
18	010010	0	0	0	0	0	614
19	010011	0	0	0	0	0	614
20	010100	0	0	0	0	0	1427
21	010101	0	0	0	0	0	714
22	010110	0	0	0	0	0	1427
23	010111	0	0	0	0	0	714
24	011000	0	0	0	0	0	1427
25	011001	0	0	0	0	0	714
26	011010	0	0	0	0	0	714
27	011011	0	0	0	0	0	714
28	011100	0	0	0	0	0	1427
29	011101	0	0	0	0	0	1427
30	011110	0	0	0	0	0	897
31	011111	0	0	0	0	0	1427
32	100000	0	0	0	0	0	1427
33	100001	0	0	0	0	0	1427
34	100010	0	0	0	0	0	714
35	100011	0	0	0	0	0	1427
36	100100	0	0	0	0	0	1427
37	100101	0	0	0	0	0	813
38	100110	0	0	0	0	0	1427
39	100111	0	0	0	0	0	813
40	101000	0	0	0	0	0	1427
41	101001	0	0	0	0	0	813
42	101010	0	0	0	0	0	1427
43	101011	0	0	0	0	0	1427
44	101100	0	0	0	0	0	1427
45	101101	0	0	0	0	0	1427

Bundle number i	Bundle composition	Purchase decisions					Price (BEF) P_i
		class 1: θ_{1i}	P_i	class 3: θ_{3i}	class 4: θ_{4i}	class 5: θ_{5i}	
46	101110	0	0	0	0	0	1171
47	101111	0	0	0	0	0	1083
48	110000	0	0	0	0	0	714
49	110001	0	0	0	0	0	1427
50	110010	0	0	0	0	1	724
51	110011	0	0	0	0	0	794
52	110100	0	0	0	0	0	1427
53	110101	0	0	0	0	0	1427
54	110110	0	0	0	0	0	1007
55	110111	0	0	0	0	0	1427
56	111000	0	0	0	0	0	1427
57	111001	0	0	0	0	0	1427
58	111010	0	0	0	0	0	1427
59	111011	0	0	0	0	0	944
60	111100	0	0	0	0	0	1427
61	111101	0	0	0	0	0	948
62	111110	1	0	1	0	0	1247
63	111111	0	1	0	1	0	1142
Consumer surplus		12	285	0	0	0	

Exhibit 11. Optimal bundle solution

It turns out that in the optimal bundle solution only bundles 50 (containing books 1, 2 and 5), 62 (composed of books 1, 2, 3, 4 and 5) and 63 (containing all syllabi) will be offered and sold. Classes 1 and 3 purchase bundle 62 at a price of 1247 (BEF) and their members realize a surplus of 12²⁸, respectively 0. Bundle 63 is sold to classes 2 and 4 at a price of 1142, implying a consumer surplus of 285, respectively 0. Class 5, finally, buys bundle 50 at a price of 724, which yields zero consumer surplus. Notice that due to failure of the free disposal assumption, the price of bundle 63 is lower than the price of bundle 62, containing one component less than bundle 63.

6. Limitations and future research

²⁸ This consumer surplus can be derived as the difference between class 1's reservation price for bundle 62 (see Exhibit 10) and the optimal price for bundle 62.

Considering our approach for the estimation of reservation prices as a stand-alone procedure, there is still room for methodological fine-tuning. First, estimates of the parameters in (5.4) could be refined by iteratively updating the unconditional membership probabilities δ_{zs} in:

$$L = \prod_{z=1}^{120} \sum_{s=1}^S \left[\delta_{zs} \prod_{i=1}^{15} \Pr_{i|z \in S}(\cdot)^{Y_{iz}} (1 - \Pr_{i|z \in S}(\cdot))^{1-Y_{iz}} \right] \quad (6.1)$$

In the first iteration, the δ_{zs} 's are the same for all consumers z and are estimated like the α_s 's in (5.12). From iteration $t = 2$ on, the δ_{zs} 's are updated in (6.1) before re-estimating the parameters of (5.4):

$$\hat{\delta}_{zs}^t = \frac{\hat{\delta}_{zs}^{t-1} \hat{L}_{z \in S}^{t-1}}{\sum_{s'=1}^S \hat{\delta}_{zs'}^{t-1} \hat{L}_{z \in S'}^{t-1}} \quad (6.2)$$

where superscripts t and $t-1$ refer to the corresponding iterations.

This process continues as long as the marginal improvement in \hat{L} exceeds a predetermined treshold.

The previous iterative process in turn could be included in an iteration that scans for the optimal number of classes. For this end Wedel and DeSarbo (1992, p. 16) use the *consistent Akaike's information criterion (CAIC)* as a heuristic:

$$CAIC = -2 \ln \hat{L} + M_s (\ln N + 1) \quad (6.3)$$

where M_s is the number of parameters estimated in an S -class solution.

N is the total number of observations.

One should select the S -value that generates the minimum *CAIC*.

A final methodological concern with respect to our reservation price estimation model involves the presence of heteroscedasticity. Although we did not test for heteroscedasticity, recent research by Dellaert et al. (1998) shows that choices between modularized packages that share identical modules have lower variances than choices between packages that differ in several modules. Applying this to our model, where respondents choose between purchase of the bundle or non-purchase, this could mean that “simple” bundles (composed of only one or two components) induce a lower variance than “complex” bundles (composed of several components). Therefore a model should be built that allows for heteroscedasticity, similar to Dellaert et al (1998).

The optimal bundle pricing model by Hanson and Martin (1990), too, is capable of improvement. In general, relaxation of the assumptions would be beneficial for the analysis of other, less artificial bundle opportunities. Competitive effects should be accounted for. Moreover, though not really important for the case of textbooks, the model should be extended such that it can manage situations where several units of the same component are needed. Further, the LP-model entails a drawback, which only applies to our global approach: the switch from a probabilistic estimation model to a deterministic optimization model (using *expected* reservation prices) not only suffers from a lack of elegance but also, and more important, implies a loss of information. Consequently, it would be worthwhile to develop an optimal bundle pricing model which preserves the probabilistic information. Computation of reservation prices would become redundant. Probably, this would also mean that the paired comparison choice task, preceding the binomial logit estimation procedure, has to be redesigned into a given-N-pick-one choice task, followed by a multinomial logit analysis.

It remains to be seen whether all these adjustments lead to more plausible bundle solutions. Although we know from section 4.6 (Molding perceptions) and from the results of the reservation price estimation that consumers do not necessarily behave like rational decision makers, it is doubtful whether the results found in section 5.4 offer a workable bundle solution. A serious drawback of our global approach is that the experimental bundle design, used to infer reservation prices, is completely different from

the final bundle solution. In the experimental design, it is acceptable that the members of a particular segment value a first bundle, which meets their needs, higher than a second bundle which comprises the first bundle and an additional unneeded component, even when the price of the first bundle exceeds the price of the second bundle. It may be irrational, but it *is* explicable. After all, a huge number of bundle profiles was randomized over a huge number of respondents and comparison of the bundle offers was therefore difficult or even impossible (and was not encouraged either). However, in the final bundle solution, consisting of only three bundles, it is much easier to compare bundle offers. Since the optimal bundles share many components, there is less room for variance in choice, as was pointed out above. It is not very likely that in these circumstances perceptual biases can be exploited. Ideally, an optimal bundling model that explicitly accounts for perceptual biases should be developed.

For that reason, more research is needed that traces the bundle effects on perceptions. The existing behavioristic literature on bundling provides divergent and thus hardly generalizable results that are based on experiments of a very restrictive nature. As bundling is gaining increasing acceptance with marketing practitioners, data are available and artificial experiments can be dropped. The effect of the number of bundle components on preferences should be fully explored. In particular, are preferences for complex bundles just more susceptible to random error than preferences for simple bundles, or do consumers display any systematic aversion to/preference for complex bundles? Do consumers use other cues for the evaluation of complex bundle offers than for the evaluation of simple bundles? Further, the relation between consumer characteristics and bundle preferences should be studied in depth. Among other things, this research might shed light on the impact of variety seeking and brand loyalty on bundle evaluations. Until now, only familiarity/experience with the considered product category has been considered as a mediating consumer characteristic (cf. Gaeth et al., 1990; Johnson et al., 1999) which, furthermore, often turned out to be insignificant. Finally, as Harlam et al. (1995, p. 64) point out, future research should address the various definitions of compliments and the relative success of bundles based on those different definitions. In this context, it might be interesting to study the interaction

between attribute levels of bundled components. As Dhar and Simonson (1999) suggest, in particular circumstances consumers tend to *highlight* attribute levels by selecting similar attribute levels for items consumed in the same episode, while in other circumstances, consumers may *balance* attribute levels for co-consumed items. This would mean that businesses can take advantage of the observed consumption episode effects by offering bundles that highlight, respectively balance attribute levels.

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Appendix 1: Derivation of the logistic reservation price density (5.17)

r_{si} is the reservation price r for which

$$\frac{\exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s r\right)}{1 + \exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s r\right)} = 0.5,$$

which is equivalent to:

$$\frac{1}{1 + \exp\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni} - up_s r\right)} = 0.5.$$

This can also be written as:

$$\frac{1}{1 + \exp\left(\left(\frac{\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni}}{up_s} - r\right) up_s\right)} = 0.5$$

It is clear now that the r we are looking for, is the mode of the logistic distribution with

mean $\left(\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni}\right) / up_s$ and variance $\sigma_s^2 = \frac{\pi^2 (1/up_s)^2}{3}$. Since the

logistic distribution is a symmetric one, the mode equals the mean and thus:

$$r_{si} = \frac{\sum_{j=1}^J u_{js} X_{ji} + \sum_{j \neq n}^J v_{(j,n)s} X_{ji} X_{ni}}{u p_s}.$$

The corresponding logistic density function is then:

$$f_{si}(r) = u p_s \frac{\exp((r_{si} - r) u p_s)}{[1 + \exp((r_{si} - r) u p_s)]^2}.$$

Appendix 2: Extract from questionnaire 1 (translated from Dutch)

Below you will find a series of 15 textbook packages with a corresponding price. These packages comprise syllabi used by undergraduates in economics. Please indicate which packages you would buy and which packages you would not buy.

Syllabus package + price			Yes	No
1	- Strategic Management - Manual Strategic Business Game - Social Law Part II - Religious Studies	672 BEF	<input type="radio"/>	<input type="radio"/>
2	- Strategic Management - Religious Studies	648 BEF	<input type="radio"/>	<input type="radio"/>
3	- Strategic Management - Social Law Part I - Social Law Part II	580 BEF	<input type="radio"/>	<input type="radio"/>
4	- Manual Strategic Business Game - Social Law Part II	360 BEF	<input type="radio"/>	<input type="radio"/>
5	Strategic Management - Manual Strategic Business Game - Social Law Part I - Social Law Part II - Religious Studies - Marketing Decision Support Systems	1608 BEF	<input type="radio"/>	<input type="radio"/>
6	- Social Law Part I - Social Law Part II - Religious Studies	685 BEF	<input type="radio"/>	<input type="radio"/>

Appendix 3: GAUSS-code

```
/* ESTIMATION OF A LATENT CLASS BINOMIAL LOGIT MODEL WITH LINEAR IMPLICIT  
UTILITY FUNCTION - BRAM FOUBERT 1999 */
```

```
NEW;  
LIBRARY maxlik;  
#include maxlik.ext;  
CLEAR  
data,colnr,rownr,segr,obs_resp,nom,b0,prob_seg,lh_r_s,estimate,mllh,gradient,varcov,retcode,aprior_p,m  
ember;
```

```
PROC (0)=readin;  
  /*Reads an ASCII-matrix, number of latent classes and initial parameter estimates.*/  
  LOCAL filename,answer,i;  
  DOS CLS;  
  PRINT "Enter disk\path\asciidatafile.\nData should be ordered per respondent. First column should  
represent dependent\variable.";  
  filename=CONS;  
  PRINT;  
  PRINT "Enter number of columns:";  
  colnr=CON(1,1);  
  PRINT "Enter number of rows:";  
  rownr=CON(1,1);  
  LOAD data[rownr,colnr]=^filename;  
  PRINT "Enter number of observations per respondent:";  
  obs_resp=CON(1,1);  
  data=RESHAPE(data,rownr/obs_resp,obs_resp*colnr);  
  PRINT "Enter required number of latent classes:";  
  segr=CON(1,1);  
  answer="w";  
  DO WHILE ((answer $/= "y") AND (answer $/= "n"));  
    PRINT "Do you want to enter initial estimates of parameters? (y/n)\nIf not, this program will do for  
you.";  
    answer=CONS;  
    PRINT;  
  ENDO;  
  IF (answer $== "y");  
    i=1;  
    b0={};  
    FORMAT /RD 3,0;  
    DO WHILE (i<=segr);  
      PRINT "\nEnter " colnr-1 "values for segment " i;  
      b0=b0 | CON(colnr-1,1);  
      i=i+1;  
    ENDO;  
    FORMAT /ROS 16,8;  
  ELSE;  
    b0=ONES(segr*(colnr-1),1);  
  ENDIF;  
  IF (segr>1);  
    b0=b0 | ZEROS(segr-1,1);
```

```

ENDIF;
ENDP;

PROC (1)=function(b,x);
/*Specifies the maximum likelihood function.*/
LOCAL i,denom,lh_o_s;
x=RESHAPE(x,rownr,colnr);
b=b|0;
nom=EXP(x[:,2:colnr]*(RESHAPE(b[1:(colnr-1)*segnr],segnr,colnr-1)));
/*nom is a (rownr X segnr)-matrix where one cell represents the nominator of the binomial logit model
for each observation and each segment.*/
denom=1+nom;
/*denom is a (rownr X segnr)-matrix where one cell represents the denominator of the model, for each
observation and each segment.*/
lh_o_s=((nom ./ denom).^x[:,1]).*((1./ denom).^(1.-x[:,1]));
/*lh_o_s is a (rownr X segnr)-matrix where one cell represents the likelihood, per
observation, per segment.*/
i=1;
lh_r_s={};
DO WHILE (i<=rownr-obs_resp+1);
  lh_r_s=lh_r_s | (PRODC(lh_o_s[i:(i+obs_resp-1),.]));
  i=i+obs_resp;
ENDDO;
/*lh_r_s is a (rownr/obs_resp) X segnr matrix where one cell represents the likelihood per respondent
and per segment.*/
prob_seg=EXP(b[(segnr*(colnr-1)+1):ROWS(b)]);
prob_seg=prob_seg./ SUMC(prob_seg);
aprior_p=RESHAPE(prob_seg,rownr/obs_resp,segnr);
/*prob_seg is a (segnr X 1)-vector where one cell represents the unconditional membership
probability.*/
RETP(LN(lh_r_s * prob_seg));
ENDP;

```

```

PROC (1)=gradfie(b,x);
LOCAL grad1,grad2,grad,i;
CALL function(b,x);
x=RESHAPE(x,rownr,colnr);
grad1=x[:,1]-(nom./(1+nom));
grad2={};
i=1;
DO WHILE (i<=segnr);
  grad2=grad2 ~ (x[:,2:colnr] .* grad1[:,i]);
  i=i+1;
ENDDO;
i=1;
grad1={};
DO WHILE (i<=rownr-obs_resp+1);
  grad1=grad1 | (SUMC(grad2[i:(i+obs_resp-1),.]));
  i=i+obs_resp;
ENDDO;
grad2=(prob_seg' .* lh_r_s) ./ (lh_r_s * prob_seg);
i=0;
grad={};
DO WHILE (i<segnr);
  grad=grad ~ (grad2[:,i+1] .* grad1[:,i*(colnr-1)+1:(i+1)*(colnr-1)]);

```

```

        i=i+1;
    ENDO;
    IF (segr>1);
        grad=grad~(grad2[.,1:segr-1]-(prob_seg[1:segr-1]'));
    ENDIF;
    RETP(grad);
ENDP;

PROC (1)=postprob;
    /*finds the posterior membership probabilities per respondent.
    input: aprior_p=((rownr/obs_resp)Xsegr)-matrix where one cell represents the prior membership
    probability for each respondent and each segment.
    output: ((rownr/obs_resp)Xsegr)-matrix where one cell represents the posterior membership
    probability.*/
    LOCAL apost_p,i;
    apost_p=lh_r_s.* aprior_p;
    member=(apost_p==MAXC(apost_p));
    PRINT "\nMembership on the basis of a posteriori probabilities:\n";
    FORMAT /RD 5,0;
    i="r/s"~SEQA(1,1,segr);
    PRINT $i[1];;
    PRINT i[2:segr+1];
    PRINT SEQA(1,1,rownr/obs_resp)~member;
    PRINT "tot" SUMC(member);
    FORMAT /ROS 16,8;
    RETP(apost_p ./ SUMC(apost_p));
ENDP;

PROC (0)=fit;
    LOCAL member2,i,hits,x,caic,chi,tot;
    caic=-2*mllh*rownr/obs_resp+(segr*(colnr-1)+segr-1)*(LN(rownr)+1);
    x=RESHAPE(data,rownr,colnr);
    member2={};
    i=1;
    DO WHILE (i<=rownr/obs_resp);
        member2=member2|RESHAPE(member[i,],obs_resp,segr);
        i=i+1;
    ENDO;
    hits=SUMC((((nom./(1+nom)).>=0.5).==x[.,1]).*member2);
    tot=SUMC(member);
    tot=tot+(tot==0);
    i=ZEROS((colnr-1)*segr,1);
    IF (segr>1);
        i=i|estimate[(colnr-1)*segr+1:colnr*segr-1];
    ENDIF;
    chi=-2*(SUMC(function(x,data))-(rownr/obs_resp)*mllh);
    PRINT "\n\nCAIC=" caic "\n";
    PRINT "\n\ncomputed chi-square=" chi " DF=" segr*(colnr-1) "\n";
    PRINT "\n\n% correct hits (cut-off 0.5) per segment:\n" SEQA(1,1,segr)~((100/obs_resp)*hits./tot);
    PRINT "\n\nTotal % correct hits (cut-off 0.5):" 100*SUMC(hits)/rownr;
ENDP;

PROC (0)=colldiag;
    /*belsley test*/
    LOCAL x,s,va,ve,varprop,condit,outp;

```

```

x=RESHAPE(data,rownr,colnr);
x=x[:,2:colnr];
x=x.*(DIAG((x*x).^(-0.5)));/*Belsley,1991,p.66*/
{va,ve}=EIGRS2(x*x);
varprop=(((ve.^2)./va')./SUMC(((ve.^2)./va')));/*Belsley, 1991, p.58*/
condit=MAXC(va.^0.5)*(va.^-0.5);/*Belsley, 1991, p.55*/
outp=REV(va~condit~varprop);
FORMAT /RD 6,3;
PRINT;
PRINT "\nScaled variance-decomposition proportions:";
PRINT $"eigval"~"CI";;
PRINT SEQA(1,1,colnr-1);
PRINT outp;
PRINT;
FORMAT /ROS 16,8;
ENDP;

maxset;
_mlgdprc=&gradfie;
readin;
OUTPUT FILE=\gauss\output.out RESET;
/* {estimate,mllh,gradient,varcov,retcode}=maxgrd(data,0,&function,b0);*/
{estimate,mllh,gradient,varcov,retcode}=maxprt(maxlik(data,0,&function,b0));
CALL function(estimate,data);
aprior_p=postprob;
fit;
colldiag;
OUTPUT OFF;
PRINT "In order to scroll through the output, type 'edit \\gauss\ouput.out'."
END;

```

Appendix 4: Output of Belsley test for data of utility function (5.4)

Eigenvalue	Condition index	Variance proportions of								
		X_1	X_2	X_3	X_4	X_5	X_6	X_1X_2	X_3X_4	P
5.239	1.000	0.004	0.006	0.004	0.003	0.005	0.004	0.003	0.004	0.001
1.183	2.104	0.015	0.021	0.015	0.012	0.000	0.000	0.063	0.055	0.000
0.776	2.599	0.000	0.011	0.004	0.003	0.093	0.063	0.048	0.065	0.002
0.510	3.206	0.124	0.212	0.018	0.017	0.047	0.005	0.000	0.000	0.001
0.500	3.237	0.029	0.040	0.151	0.108	0.020	0.027	0.000	0.000	0.000
0.500	3.237	0.003	0.005	0.025	0.032	0.198	0.185	0.000	0.000	0.000
0.198	5.148	0.054	0.153	0.097	0.081	0.128	0.073	0.223	0.295	0.001
0.073	8.466	0.476	0.552	0.341	0.250	0.004	0.001	0.647	0.532	0.006
0.022	15.424	0.295	0.000	0.344	0.495	0.505	0.643	0.015	0.050	0.989