Optimizing security patrolling scheduling in chemical industrial parks by using game theory

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ABSTRACT: Protecting chemical clusters from intentional attacks has been a hot topic during the last decade. Besides intrusion security countermeasures such as cameras, entrances control etc., patrolling also fulfils an important role in the security of chemical facilities and industrial parks. Current patrolling strategies in industry are mainly single-plant driven and purely randomized or based on the patroller's preference. Such an approach in a chemical industrial park is on the one hand not able to cover the more hazardous facilities more than the less hazardous plants within the park, and on the other hand is not able to deal with strategic (intelligent) human adversaries w.r.t. terrorism. This paper therefore investigates a game theoretic model for optimizing the schedule of patrolling in chemical clusters. The industrial defender and the intelligent/adaptive attackers are modelled as two players in the game. The defender aims at increasing the probability of detecting the attacker, by randomly but strategically scheduling her patrolling route. The attacker aims at causing maximal consequences with highest success probabilities, by choosing a proper attack time and a proper target. The model is further illustrated by a case study.

1 INTRODUCTION

Since the unfortunate 9/11 attack, the protection of critical infrastructures has been an urgent topic, both in academia and in practice. Chemical industries have an important role in modern society. They provide materials for human being's daily necessities such as clothes, food, energy, etc. However, chemical facilities may also pose huge threat to modern society. The sadly Bhopal disaster (toxic gas leakage), among others, caused more than 3000 deaths and life-long suffering for over 300,000 [1]. In the security aspect, the malicious attack to a chemical plant in France reminded people that there is a possibility of a successful attack to chemical facilities. An investigation carried out by Orum and Rushing [2] concluded that a successful attack to a top 101 dangerous chemical plant in U.S. may result in more than one million casualties.

Furthermore, due to economic and management reasons, chemical plants are nowadays geographically clustered, forming chemical clusters, e.g., the Antwerp port chemical cluster, the Rotterdam port chemical cluster etc. Besides intrusion security countermeasures within each plant, patrolling is also scheduled, for securing these chemical facilities. The patrolling can be either single plant oriented, which is scheduled by the plant itself, or multiple plants oriented, which should be scheduled by a multiple plant council (MPC) [3]. Both types of patrolling have a drawback of not being able to

deal with intelligent attackers. Some patrollers follow a fixed patrolling route, and the attacker thus can predict the patroller's position at a certain time. Other patrollers purely randomize their patrolling, without taking into consideration the hazardous level that each facility/plant holds, and the attacker can attack more dangerous facilities/plants since all the facilities/plants are equally patrolled.

Game theory has been introduced to the security domain to optimally allocate security resources. In a security problem, the attacker (human beings) is able to plan his attack according to the defender's defence, while the defender knows the fact and thus she can also defend accordingly. This procedure is called the 'intelligent interactions' between the defender and the attacker. Game theory was invented to model strategic decision making in multiple actor systems, thus it perfectly fits the necessity of modelling the 'intelligent interactions' in the security domain. Tambe and his co-authors [4] employed game theory for optimizing patrolling of protecting ferries, of protecting wild animals etc. Alpern and his co-authors [5] theoretically studied the optimization problem of patrolling in a graph. Amirali et al. [6] introduced a game theoretic model for optimally scheduling pipeline patrolling. No literature has investigated the use of game theory to optimize patrolling in chemical clusters, neither for the single plant patrolling nor for the multiple plants patrolling.

This paper proposes a Chemical Cluster Patrolling (CCP) game, which answers the question how to optimally randomize the patrolling, to better secure a chemical cluster, by using a game theory model. The reminder of this paper is organized as follows: Section 2 briefly demonstrates the CCP game. A case study is introduced in Section 3 and the results of the case study are given in Section 4. Conclusions are drawn in Section 5.

2 THE CHEMICAL CLUSTER PATROLLING (CCP) GAME

2.1 Graphic modelling

A chemical cluster can be described as a graph $G(V, E)$. The vehicle entrances of each plant and the cross points of the vehicle road form the nodes of the graph. The vehicle roads between different plants (to be more specified, they should be “between different entrances”) are modelled as edges of the graph. Furthermore, all entrance nodes which belong to the same plant are modelled to be full connected, which means edges also exist between every two nodes in these cases.

Based on the graphic model, the chemical cluster patrolling can be described as a graphic patrolling problem: 1) a patroller (team) starts her (In this paper, we denote the patroller/defender as she/her/her, and denote the attacker as he/him/his.) patrolling from a node (the base camp); 2) she moves in the graph; 3) when arriving a node, she may decide whether to stay at the node for a specific period of time t_k^p (i.e., patrol the plant) or not (i.e., move to another plant without patrolling the current plant); 4) after a period T , the patroller terminates the patrolling.

A directed patrolling graph $pG(pV, pE)$ is defined based on the graphic model of the chemical cluster. A node of pG is defined as a tuple of (t, i) , in which $t \in \{1, 2, \dots, |V|\}$ denotes time dimension and $i \in \{1, 2, \dots, |V|\}$ denotes a node in graph $G(V, E)$ (i.e., a plant (entrance) in the chemical cluster). Node (t, i) means that at time t the patroller arrives or leaves node i . A directed edge of pG from node (t_1, i_1) to node (t_2, i_2) therefore means that the patroller moves from node i_1 at time t_1 to node i_2 , and arrives at t_2 . Figure 3 shows the patrolling graph of the case study.

2.2 Game theoretic modelling

A game theoretic model consists of players, strategies, and payoffs.

Players

Players of the chemical cluster patrolling (CCP) game are the patroller team and the potential attackers. The CCP game is a two players game and both players are assumed with perfect rationality.

Strategies

An attacker's strategy consists of three parts: i) which plant to attack; ii) when to attack; and iii) what attack scenario to use, thus can be expressed as:

$$s_a = (t, i, k_i) \quad (1)$$

In which t denotes the attack start time, i represents the target plant, k_i is the attack period (e.g., 7 minutes) which should be determined by both the attack scenario and the target plant.

A mathematic formulation of the defender's strategy is shown in Formula (2).

$$s_d = \prod_{(s,e) \in pE} c_{s-e} \quad (2)$$

In which c_{s-e} denotes the probabilistic number assigned to the edge (of pG) from node s to node e , $\prod$ denotes the Cartesian product of all edges in pG (i.e., all $(s,e) \in pE$).

An important property of these probabilities is that, for each node (of pG), the sum of all the income probabilities must equal the sum of all the outcome probabilities. Formula (3) illustrates the abovementioned property.

$$\begin{aligned} sP_{pv} &= \sum_{in \in \{s \in pV \mid (s,pv) \in pE\}} c_{in-pv} \\ &= \sum_{out \in \{e \in pV \mid (pv,e) \in pE\}} c_{pv-out} \end{aligned} \quad (3)$$

Payoffs

Formulas (4) and (5) define the patroller and the attacker's payoff, in which f is the probability that the attacker would fail, and if the attacker failed, the patroller gets a reward R^d (e.g., obtaining bonus) and the attacker suffers a penalty P^a (e.g., being sent to prison). If the attacker succeeds, the patroller suffers a loss L^d and the attacker obtains a gain G^a .

$$u_d = R^d \cdot f - L^d \cdot (1 - f) \quad (4)$$

$$u_a = G^a \cdot (1 - f) - P^a \cdot f \quad (5)$$

Computing the f

The probability that the attacker would be detected can be calculated by Formula (6), in which f_{cpp} denotes the probability that the intrusion detection systems (IDS) in the target plant would detect the attacker, f_p is the probability that the patroller would detect the attacker

$$f = 1 - (1 - f_{cpp}) \cdot (1 - f_p) \quad (6)$$

Note that f_{cpp} is a plant-specific parameter (a number belongs to $[0, 1]$). While f_p can be calculated by Formula (7), in which r denotes the overlap situ-

ation of that the patroller's staying in the plant and the attacker's intrusion and attack procedure, σ_r is the detection probability of situation r . Furthermore, the probability that the patroller would be in situation r is denoted as τ_r .

$$f_p = \sum_r \sigma_r \cdot \tau_r. \quad (7)$$

Denote the defender's strategy in a vector form as $\vec{c}$. It is worth noting that τ_r would be a linear polynomial of $\vec{c}$, and f_{cp} and σ_r are user provided parameters. Therefore, f is a linear polynomial of $\vec{c}$ as well.

Stackelberg equilibrium

In the CCP game, the attacker is assumed to be able to collect information of the patroller's patrolling route. A Stackelberg equilibrium $(s_a^*, s_d^*) = (\vec{c}^*, (t^*, i^*, k_i^*))$ for the CCP game is a defender-attacker strategy pair that satisfies the following condition:

$$(t^*, i^*, k_i^*) = \operatorname{argmax} \{u_a(\vec{c}, (t, i, k_i))\} \quad (8)$$

$$\vec{c}^* = \operatorname{argmax} \{u_d(\vec{c}, (t^*, i^*, k_i^*))\} \quad (9)$$

Formula (8) reflects that observing the defender's strategy $\vec{c}$, the attacker would play a strategy which will maximize his own payoff (i.e., a best response). Formula (9) represents that the defender can also work out the attacker's best response to her strategy, thus she plays accordingly.

3 ILLUSTRATIVE CASE STUDY

Figure 1 provides the layout of a chemical cluster from the Antwerp port (data source: Google map). There are 5 plants in this cluster, indexed as plant 'A', plant 'B', and so forth. The yellow dot lines demonstrate the vehicle routes, and the patroller only drives on the vehicle route. Figure 2 shows the graph model of the cluster shown in Figure 1.

Figure 1. Layout of a chemical park in Antwerp port.

Figure 2. Graphic modelling of the chemical park.

As we may see, each plant (i.e., 'A', 'C', 'D', 'E') in Figure 1 is modelled as a node (with the same name) in Figure 2. The cross point of the vehicle road between plant 'D' and 'E' in Figure 1 is also denoted as a node in Figure 2 (i.e., node 'cr'). Moreover, plant 'B' has two vehicle entrances, and two nodes (i.e., nodes 'B1' and 'B2') are used in Figure 2 to denote the two different entrances of plant 'B'. Edges 'e1' to 'e6' reflect the vehicle roads between different plants, while edge 'e7' is added between node 'B1' and 'B2' because these 2 nodes belong to the same plant and hence should be full connected.

We set: $t_1^d = 2, t_2^d = 3, t_3^d = 4, t_4^d = 3, t_5^d = 2, t_6^d = 2$, and further set $t^p(A', B', C', D', E') = [9, 7, 6, 5, 7]$. t_i^d represents the driving time of edge 'e i ' in Figure 2. For instance, t_1^d is the driving time from node 'A' to 'B1'. $t^p(X)$ denotes the time needed to patrol plant 'X'. If the patroller may have multiple patrolling intensity in a plant, then the t^p should not only be a number, but be a set of numbers. In this paper, we only consider one patrolling intensity in each plant and all the temporal data are unified in minutes.

Table 1 shows the time of moving from one node to another node. For instance, from node 'A' to node 'B1' needs $t_1^d = 2$ minutes. It is worth noting that i) numbers in the diagonal denote the time needed to patrol the plant, e.g., patrolling plant 'A' needs $t^p(A) = 9$ minutes; ii) the number from one entrance node to another entrance node of the same plant (e.g., from node 'B1' to 'B2') also represents the time needed to patrol the plant. Case (ii) means that the patroller comes into and leaves the plant from different entrances.

Figure 3 shows the patrolling graph pG for the chemical cluster shown in Figure 1, with the data in Table 1 and further assume a patrolling time $T = 30$. Patroller's base camp is assumed close to the cross road node, thus 'cr' is chosen as the patroller's base camp.

Figure 3. Patrolling graph of the illustrative example.

Table 1. Superior connection matrix for Figure 2 with the illustrative numbers.

	A	B1	B2	cr	C	D	E
A	9	2	∞	∞	∞	∞	∞
B1	2	7	7	∞	3	∞	∞
B2	∞	7	7	3	∞	∞	∞
cr	∞	∞	3	∞	∞	2	2
C	∞	3	∞	∞	6	4	∞
D	∞	∞	∞	2	4	5	∞
E	∞	∞	∞	2	∞	∞	7

In Figure 3, the x axis denotes the time dimension, while the y axis represents the different nodes in Figure 2. Therefore, any coordinates in Figure 3 can be a possible node for pG . As we may see, node 1 (at the left hand side of the figure) in Figure 3 is (0, cr), and it means that at time 0, the patroller starts from her base camp (i.e., ' cr '). Thereafter she has 3 choices: i) to come to plant ' B ' (more accurately, entrance ' $B2$ ') with a driving time t_4^d , and reaches node 2; ii) to come to plant ' D ' with a driving time t_5^d , and reaches node 3; and iii) to come to plant ' E ' with a driving time t_6^d , and reaches node 4. Subsequently, at new nodes (e.g., 2, 3, or 4), the patroller has the same choice problem, that is, to patrol the current plant or to come to another plant. In Figure 3, the indexes of some nodes and the weight of some edges are not shown, for the clarity of the figure.

Table 2. Model inputs.

	R^d	L^d	G^a	P^a	f_{cpp}
'A'	1	16	10	3	0.45
'B'	1	11.2	6	3	0.3
'C'	1	14	8.3	3	0.42
'D'	1	12	7.1	3	0.45
'E'	1	15	10	3	0.5

For example, the bold (and black) line in the figure, denotes a patrolling route as: ' cr ' $\rightarrow$ ' C ' $\rightarrow$ ' C ' $\rightarrow$ patrol plant ' C ' $\rightarrow$ ' $B1$ ' $\rightarrow$ patrol plant ' B ' $\rightarrow$ leave plant ' B ' from ' $B2$ ' $\rightarrow$ ' cr ' $\rightarrow$ ' E ' $\rightarrow$ ' cr '.

Finally, when time comes to the end of the patrol, the patroller terminates the patrolling and comes back to her base camp. In this research, to keep the continuity of coverage of each plant, the patroller is required to prolong their patrolling in the plant until that the next patroller team might be able to arrive the plant. For instance, in Figure 3, though the patrolling time is set as $T = 30$, however, the patrolling in plant ' A ' is not stopped until $t = 41$. The reason is that, the shortest time that the next patrolling team can arrive plant ' A ' (from ' cr ') is 11 (By following a path ' cr ' $\rightarrow$ ' $B2$ ' $\rightarrow$ ' $B1$ ' $\rightarrow$ ' A '). If the current patroller team does not prolong their patrolling, and the next patroller team starts at time 30 and starts from their base camp (i.e., ' cr '), then plant ' A ' will definitely not be covered during time (30, 41). This approach may increase the patroller's workload. However, if we set T slightly smaller than the patroller's real workload, the problem will be solved. For example, if a patroller team's workload is 240 minutes per day, and we may set $T = 220$.

For the sake of clarity, only one type of attacker and only one attack scenario is considered. Further assume that the intrusion and attack procedure of the employed scenario would last for 10 minutes. For instance, the two horizontal bold dot red lines in Figure 3 represent attack strategies that attack plant ‘A’ start at time 9 (the line at below) and attack plant ‘E’ start at time 4 (the line at above), with an intrusion and attack period of ten time units, respectively.

Table 2 gives the model inputs, i.e., the defender’s reward (loss) of (not) detecting an attacker; the attacker’s gain (penalty) from a (not) successful attack; the probability that the intrusion detection system (IDS) can detect the attacker. The probability that the patroller can detect the attacker (i.e., σ_i) should also be provided by security experts. However, in this paper, we simply assume that in each time unit, if the attacker and the patroller stay in the same plant (i.e., overlap), there is a probability of 0.05 that the attacker would be detected by the patroller.

4 RESULTS

4.1 Stackelberg equilibrium

Figure 4 shows the Stackelberg equilibrium (SE) of the case study. The black (and narrow) lines demonstrate the patroller’s optimal patrolling strategy. The associated numbers on the line denotes the probability that the defender will take this action. For instance, $c_1 = 0.22747$ means that at time 0, the patroller should drive to node ‘B2’ at probability 0.22747. Furthermore, in patrolling practice, if the patroller arrives at a node in the

figure, the conditional probabilities of following actions can be calculated as $cP = c/sP$, in which c denotes the probability assigned to the edge, sP_v denotes the probability that the patroller would be at the node. For instance, the probability that the patroller would arrive at the red node (6, ‘C’) in Figure 4 is $sP_v = 0.41734$, and the conditional probabilities that she should take the 2 actions are $cP_1 = 0.4979$, $cP_2 = 0.5021$.

Table 3. The patroller’s actions that may detect the attacker.

Edge	τ	Overlap	σ
25	0.00220	[9,13]	0.20
41	0.09935	[9,16]	0.35
85	0.11143	[11,18]	0.35
159	0.09935	[16,19]	0.15
186	0.00220	[17,19]	0.10
206	0.11143	[18,19]	0.05

Table 4. Comparison of the CCP strategy and the purely randomized strategy.

Edge	Overlap	τ_c	τ_{rc}	σ
82	[11,19]	0.1926	0.0046	0.4
98	[12,19]	0.1942	0.0139	0.35
156	[15,19]	0	0.0019	0.2
176	[16,19]	0	0.0071	0.15
196	[17,19]	0	0.0024	0.1
216	[18,19]	0	0.0039	0.05
425	[9,10]	0	0.0100	0.05
430	[9,11]	0.3358	0.0274	0.1

Figure 4. The optimal patrolling strategy and the attacker’s best response (Stackelberg equilibrium).

The attacker's best response in the SE is to attack plant 'E' at time 9, as shown in the figure as a red bold line. The short lines above the attacker's best response line represent the defender's patrolling actions which would have overlap with the attacker's strategy. Table 3 shows detail information of these patrolling actions.

Based on the result in Table 3, we have that: $f_p = \sum_r \tau_r \cdot \sigma_r = 0.089101$, $f = 1 - (1 - 0.5) \cdot (1 - f_p) = 0.09491$, $u_a = 2.88311$, $u_d = -6.24074$.

4.2 Comparing to random patrolling

In the current patrolling practise, patrollers may randomly schedule their patrol route. This situation, one looks at Figure 3, is simply assigning the same probabilities to edges that start from the same node. For instance, at the starting node (i.e., (0, 'cr')), the patroller would come to plant (entrance) 'B2', 'D', and 'E' at the same probability, and the probability is 1/3.

In the case study, if the defender would purely randomize her patrolling, then the attacker's best response would be attacking plant 'A' at time 9. The attacker and the defender would obtain a pay-off of 4.0653 and -8.2393, respectively. Comparing to the result of the CCP game, the defender suffers a higher lose.

Table 4 illustrates the differences between the CCP strategy and the purely randomized strategy. The 'Edge' column in Table 4 shows the edges in the patrolling graph that have an overlap with the attacker's strategy (i.e., attack plant 'A' at time 9). The overlap column illustrates which period of the attack procedure is overlapped by the edge. The 'c' and 'rc' column show the probability that the patroller will go the edge, resulting from the CCP game and from the randomized strategy respectively. The sigma column shows the probability that the attacker will be detected by the patroller by this edge and it is simply calculated as 0.05 multiplied by the overlapped time units. According to the result in Table 4, we can calculate the probability that the attacker would be detected by the patroller (see Formula 7), and the results are: $f_p^c = 0.17860$, $f_p^{rc} = 0.01183$. These results reveal that the CCP strategy has a higher probability of detecting the attacker at plant 'A', and thus transfers the attacker's best response target from plant 'A' to plant 'E'.

5 CONCLUSION

Terrorism has been a global problem. The chemical industry can be an attractive target for terrorists, due to the existence of hazardous materials. A chemical cluster is formed by multiple chemical plants, and can be of extra interest for attackers.

Besides intrusion security countermeasures of each plant, security countermeasures at the cluster level are also recommended. The current patrolling in chemical clusters are either single plant based or purely randomized, being economically not efficient and theoretically not optimized. The security adversaries are human beings, and they may learn the patroller's daily patrolling routes and plan their attack accordingly.

This paper therefore proposes a chemical cluster patrolling (CCP) game. The CCP game generates randomized but strategic patrolling routes for the cluster patrolling team. The intelligent interactions between the patroller and the potential attackers are modelled in the CCP game. An illustrative case study shows that the patrolling routes generated by our CCP game outperforms the purely randomized patrolling strategy.

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