

# Landau Phonon-Roton Theory Revisited for Superfluid $^4\text{He}$ and Fermi Gases

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Liquid helium and spin-1/2 cold-atom Fermi gases both exhibit in their superfluid phase two distinct types of excitations, gapless phonons and gapped rotons or fermionic pair-breaking excitations. In the long wavelength limit, revising and extending the theory of Landau and Khalatnikov initially developed for helium [Zh. Exp. Teor. Fiz. **19**, 637 (1949)], we obtain universal expressions for three- and four-body couplings among these two types of excitations. We calculate the corresponding phonon damping rates at low temperature and compare them to those of a pure phononic origin in high-pressure liquid helium and in strongly interacting Fermi gases, paving the way to experimental observations.

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*Introduction.*—Homogeneous superfluids with short-range interactions exhibit, at sufficiently low temperature, phononic excitations  $\phi$  as the only microscopic degrees of freedom. In this universal limit, all superfluids of this type reduce to a weakly interacting phonon gas with a quasi-linear dispersion relation, irrespective of the statistics of the underlying particles and of their interaction strength. Phonon damping then only depends on the dispersion relation close to zero wave number (namely, its slope and third derivative) and on the phonon nonlinear coupling, deduced solely from the system equation of state through Landau-Khalatnikov quantum hydrodynamics [1].

In experiments, however, temperatures are not always low enough to make the dynamics purely phononic. Other elementary excitations can enrich the problem, such as spinless bosonic rotons in liquid helium 4 and spinful fermionic BCS-type pair-breaking excitations in spin-1/2 cold-atom Fermi gases. These excitations, denoted here as  $\gamma$  quasiparticles, exhibit in both cases an energy gap  $\Delta > 0$ . Remarkably, as shown by Landau and Khalatnikov [1], the phonon-roton coupling, and more generally phonon coupling to all gapped excitations as we shall see, depend to leading order in temperature only on a few parameters of the dispersion relation of the  $\gamma$  quasiparticles, namely the value of the minimum  $\Delta$  and its location  $k_0$  in wave number space, their derivatives with respect to density, and the effective mass  $m_*$  close to  $k = k_0$ . We have discovered however that the  $\phi$ - $\gamma$  coupling of Ref. [1] is not exact, a fact apparently unnoticed in the literature. Our goal here is to complete the result of Ref. [1], and to quantitatively obtain phonon damping rates due to the  $\phi$ - $\gamma$  coupling as functions of temperature, a nontrivial task in the considered strongly interacting systems. We restrict to the collisionless regime  $\omega_{\mathbf{q}}\tau_{\gamma} \gg 1$  and  $\omega_{\mathbf{q}}\tau_{\phi} \gg 1$ , where  $\omega_{\mathbf{q}}$  is the angular eigenfrequency of the considered phonon mode of wave vector  $\mathbf{q}$ , and  $\tau_{\gamma}$  ( $\tau_{\phi}$ ) is a typical collision time of thermal  $\gamma$

quasiparticles (thermal phonons). An extension to the hydrodynamic regime  $\omega_{\mathbf{q}}\tau_{\gamma} \lesssim 1$  or  $\omega_{\mathbf{q}}\tau_{\phi} \lesssim 1$  may be obtained from kinetic equations [2]. An experimental test of our results seems nowadays at hand, either in liquid helium 4, extending the recent work of Ref. [3], or in homogeneous cold Fermi gases, which the breakthrough of flat-bottom traps [4] allows one to prepare [5] and to acoustically excite by spatiotemporally modulated laser-induced optical potentials [6,7].

*Landau-Khalatnikov theory revisited.*—We recall the reasoning of Ref. [1] to get the phonon-roton coupling in liquid helium 4, extending it to the phonon-fermionic quasiparticle coupling in unpolarized spin-1/2 Fermi gases. We first treat in first quantization the case of a single roton or fermionic excitation, considered as a  $\gamma$  quasiparticle of position  $\mathbf{r}$ , momentum  $\mathbf{p}$ , and spin  $s = 0$  or  $s = 1/2$ . In a homogeneous superfluid of density  $\rho$ , its Hamiltonian is given by  $\epsilon(\mathbf{p}, \rho)$ , an isotropic function of  $\mathbf{p}$  such that  $p \mapsto \epsilon(\mathbf{p}, \rho)$  is the  $\gamma$ -quasiparticle dispersion relation. In the presence of acoustic waves (phonons), the superfluid acquires position-dependent density  $\rho(\mathbf{r})$  and velocity  $\mathbf{v}(\mathbf{r})$ . For a phonon wavelength large compared to the  $\gamma$ -quasiparticle coherence length [8], here its thermal wavelength  $(2\pi\hbar^2/m_*k_B T)^{1/2}$  [9], and for a phonon angular frequency small compared to the  $\gamma$ -quasiparticle “internal” energy  $\Delta$ , we can write the  $\gamma$ -quasiparticle Hamiltonian in the local density approximation [10,11]:

$$\mathcal{H} = \epsilon(\mathbf{p}, \rho(\mathbf{r})) + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}). \quad (1)$$

The last term is a Doppler effect reflecting the energy difference in the lab frame and in the frame moving with the superfluid. For a weak phononic perturbation of the superfluid, we expand the Hamiltonian to second order in density fluctuations  $\delta\rho(\mathbf{r}) = \rho(\mathbf{r}) - \rho$ :

$$\mathcal{H} \simeq \epsilon(\mathbf{p}, \rho) + \partial_\rho \epsilon(\mathbf{p}, \rho) \delta\rho(\mathbf{r}) + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}) + \frac{1}{2} \partial_\rho^2 \epsilon(\mathbf{p}, \rho) \delta\rho^2(\mathbf{r}), \quad (2)$$

not paying attention yet to the noncommutation of  $\mathbf{r}$  and  $\mathbf{p}$ . Phonons are bosonic quasiparticles connected to the expansion of  $\delta\rho(\mathbf{r})$  and  $\mathbf{v}(\mathbf{r})$  on eigenmodes of the quantum-hydrodynamic equations linearized around the homogeneous solution at rest in the quantization volume  $\mathcal{V}$ :

$$\begin{pmatrix} \delta\rho(\mathbf{r}) \\ \mathbf{v}(\mathbf{r}) \end{pmatrix} = \frac{1}{\mathcal{V}^{1/2}} \sum_{\mathbf{q} \neq 0} \left[ \begin{pmatrix} \rho_q \\ \mathbf{v}_q \end{pmatrix} \hat{b}_q + \begin{pmatrix} \rho_q \\ -\mathbf{v}_q \end{pmatrix} \hat{b}_q^\dagger \right] e^{i\mathbf{q} \cdot \mathbf{r}}, \quad (3)$$

with modal amplitudes  $\rho_q = [\hbar\rho q/(2mc)]^{1/2}$  and  $\mathbf{v}_q = [\hbar c/(2mpq)]^{1/2} \mathbf{q}$ ,  $m$  being the mass of a superfluid particle and  $c$  the sound velocity. The annihilation and creation operators  $\hat{b}_q$  and  $\hat{b}_q^\dagger$  of a phonon of wave vector  $\mathbf{q}$  and energy  $\hbar\omega_q = \hbar c q$  obey usual commutation relations  $[\hat{b}_q, \hat{b}_{q'}^\dagger] = \delta_{\mathbf{q}, \mathbf{q}'}$ .

For an arbitrary number of  $\gamma$  quasiparticles, we switch to second quantization and rewrite Eq. (2) as

$$\begin{aligned} \hat{H} = & \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} \hat{\gamma}_{\mathbf{k}\sigma}^\dagger \hat{\gamma}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}, \sigma} \frac{\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k}')}{\mathcal{V}^{1/2}} (\hat{\gamma}_{\mathbf{k}'\sigma}^\dagger \hat{\gamma}_{\mathbf{k}\sigma} \hat{b}_q + \text{H.c.}) \\ & \times \delta_{\mathbf{k}+\mathbf{q}, \mathbf{k}'} + \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}, \mathbf{q}', \sigma} \frac{\mathcal{A}_2(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}')}{\mathcal{V}} \hat{\gamma}_{\mathbf{k}'\sigma}^\dagger \hat{\gamma}_{\mathbf{k}\sigma} \delta_{\mathbf{k}+\mathbf{q}, \mathbf{k}'+\mathbf{q}'} \\ & \times \left[ \hat{b}_q^\dagger \hat{b}_q + \frac{1}{2} (\hat{b}_{-q}^\dagger \hat{b}_q + \text{H.c.}) \right], \quad (4) \end{aligned}$$

where  $\hat{\gamma}_{\mathbf{k}\sigma}$  and  $\hat{\gamma}_{\mathbf{k}\sigma}^\dagger$  are bosonic (rotons,  $s=0$ ,  $\sigma=0$ ) or fermionic ( $s=1/2$ ,  $\sigma=\uparrow, \downarrow$ ) annihilation and creation operators of a  $\gamma$  quasiparticle of wave vector  $\mathbf{k} = \mathbf{p}/\hbar$  in spin component  $\sigma$ , obeying usual commutation or anticommutation relations. The first sum in the right-hand side of Eq. (4) gives the  $\gamma$ -quasiparticle energy in the unperturbed superfluid, with  $\epsilon_{\mathbf{k}} \equiv \epsilon(\hbar\mathbf{k}, \rho)$ . The second sum, originating from the Doppler term and the term linear in  $\delta\rho$  in Eq. (2), describes absorption or emission of a phonon by a  $\gamma$  quasiparticle, characterized by the amplitude

$$\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k}') = \rho_q \frac{\partial_\rho \epsilon_{\mathbf{k}} + \partial_\rho \epsilon_{\mathbf{k}'}}{2} + \mathbf{v}_q \cdot \frac{\hbar\mathbf{k} + \hbar\mathbf{k}'}{2}, \quad (5)$$

where  $\mathbf{q}$ ,  $\mathbf{k}$ , and  $\mathbf{k}'$  are the wave vectors of the incoming phonon and the incoming and outgoing  $\gamma$  quasiparticles. Equation (5) is invariant under exchange of  $\mathbf{k}$  and  $\mathbf{k}'$ . This results from symmetrization of the various terms, in the form  $[f(\mathbf{p})e^{i\mathbf{q} \cdot \mathbf{r}} + e^{i\mathbf{q} \cdot \mathbf{r}} f(\mathbf{p})]/2$  with  $\mathbf{r}$  and  $\mathbf{p}$  canonically conjugated operators, ensuring that the correct form of Eq. (2) is Hermitian. The third sum in Eq. (4), originating from the terms quadratic in  $\delta\rho$  in Eq. (2), describes direct scattering of a phonon on a  $\gamma$  quasiparticle, with the symmetrized amplitude

$$\mathcal{A}_2(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') = \rho_q \rho_{q'} \frac{\partial_\rho^2 \epsilon_{\mathbf{k}} + \partial_\rho^2 \epsilon_{\mathbf{k}'}}{2}, \quad (6)$$

where the primed wave vectors are the ones of emerging quasiparticles. It also describes negligible two-phonon absorption and emission. The effective amplitude for  $\phi$ - $\gamma$  scattering is obtained by adding the contributions of the direct process (terms of  $\hat{H}$  quadratic in  $\hat{b}$ ) and of the absorption-emission or emission-absorption process (terms linear in  $\hat{b}$ ) treated to second order in perturbation theory [1]:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') &= \text{diagram 1} + \text{diagram 2} + \text{diagram 3} \\ &= \mathcal{A}_2(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') + \frac{\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k} + \mathbf{q}) \mathcal{A}_1(\mathbf{k}', \mathbf{q}'; \mathbf{k}' + \mathbf{q}')}{\hbar\omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{q}}} \\ &\quad + \frac{\mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}'; \mathbf{k}) \mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}; \mathbf{k}')}{\epsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{q}'} - \epsilon_{\mathbf{k}-\mathbf{q}'}} \quad (7) \end{aligned}$$

where in the second (third) term the  $\gamma$  quasiparticle first absorbs phonon  $\mathbf{q}$  (emits phonon  $\mathbf{q}'$ ) then emits phonon  $\mathbf{q}'$  (absorbs phonon  $\mathbf{q}$ ). Up to this point this agrees with Ref. [1], except that the first derivative  $\partial_\rho \Delta$  in Eq. (5), thought to be anomalously small in low-pressure helium, was neglected in Ref. [1]. Equation (7), issued from a local density approximation, holds to leading order in a low-energy limit. We then take the  $T \rightarrow 0$  limit with scaling laws

$$q \approx T, \quad k - k_0 \approx T^{1/2} \quad (8)$$

reflecting the fact that the thermal energy of a phonon is  $\hbar c q \approx k_B T$  and the effective kinetic energy of a  $\gamma$  quasiparticle, that admits the expansion

$$\epsilon_{\mathbf{k}} - \Delta_{k \rightarrow k_0} = \frac{\hbar^2 (k - k_0)^2}{2m_*} + O(k - k_0)^3, \quad (9)$$

is also  $\approx k_B T$ . The coupling amplitudes  $\mathcal{A}_1$  and energy denominators in Eq. (7) must be expanded up to relative corrections of order  $T$  [12]. On the contrary, it suffices to expand  $\mathcal{A}_2$  to leading order  $T$  in temperature. We hence get our main result, the effective coupling amplitude of the  $\phi$ - $\gamma$  scattering to leading order in temperature:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') & \\ \xrightarrow{T \rightarrow 0} & \frac{\hbar q}{m c \rho} \left( \frac{1}{2} \rho^2 \Delta'' + \frac{(\hbar \rho k_0')^2}{2m_*} + \frac{\hbar^2 k_0^2}{2m_*} \right. \\ & \times \left\{ \left( \frac{\rho \Delta'}{\hbar c k_0} \right)^2 u u' + \frac{\rho \Delta'}{\hbar c k_0} \left[ (u + u') \left( u u' - \frac{\rho k_0'}{k_0} \right) \right. \right. \\ & \left. \left. + \frac{2m_* c}{\hbar k_0} w \right] + \frac{m_* c}{\hbar k_0} (u + u') w \right. \\ & \left. \left. + u^2 u'^2 - \frac{\rho k_0'}{k_0} (u^2 + u'^2) \right\}. \quad (10) \end{aligned}$$

Here  $\Delta'$ ,  $k'_0$ ,  $\Delta''$  are first and second derivatives of  $\Delta$  and  $k_0$  with respect to  $\rho$ ;  $u = (\mathbf{q} \cdot \mathbf{k}/qk)$ ,  $u' = (\mathbf{q}' \cdot \mathbf{k}/q'k)$ ,  $w = (\mathbf{q} \cdot \mathbf{q}'/qq')$  are cosines of the angles between  $\mathbf{k}$ ,  $\mathbf{q}$ , and  $\mathbf{q}'$ ; our results hold for  $k_0 = 0$  provided the limit  $k_0 \rightarrow 0$  is taken in Eq. (10). In Eq. (3.17) of Ref. [1], the  $\Delta'$  terms were neglected as said, but the last term in Eq. (10), with the factor  $\rho k'_0/k_0$ , was simply forgotten.

*Damping rates.*—A straightforward application of Eq. (10) is a Fermi-golden-rule calculation of the damping rate  $\Gamma_{\mathbf{q}}^{\text{scat}}$  of phonons  $\mathbf{q}$  due to scattering on  $\gamma$  quasiparticles. The  $\gamma$  quasiparticles are in thermal equilibrium with Bose or Fermi mean occupation numbers  $\bar{n}_{\gamma,\mathbf{k}} = [\exp(\epsilon_{\mathbf{k}}/k_B T) - (-1)^{2s}]^{-1}$ . So are phonons in modes  $\mathbf{q}' \neq \mathbf{q}$ , with Bose occupation numbers  $\bar{n}_{b,\mathbf{q}'} = [\exp(\hbar\omega_{\mathbf{q}'}/k_B T) - 1]^{-1}$ ; mode  $\mathbf{q}$  is initially excited (e.g., by a sound wave) with an arbitrary number  $n_{b,\mathbf{q}}$  of phonons. By including both loss  $\mathbf{q} + \mathbf{k} \rightarrow \mathbf{q}' + \mathbf{k}'$  and gain  $\mathbf{q}' + \mathbf{k}' \rightarrow \mathbf{q} + \mathbf{k}$  processes [13] and summing over  $\sigma$ , one finds that  $(d/dt)n_{b,\mathbf{q}} = -\Gamma_{\mathbf{q}}^{\text{scat}}(n_{b,\mathbf{q}} - \bar{n}_{b,\mathbf{q}})$ , with

$$\Gamma_{\mathbf{q}}^{\text{scat}} = \frac{2\pi}{\hbar}(2s+1) \int \frac{d^3k d^3q'}{(2\pi)^6} [\mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}')^2 \times \delta(\epsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}} - \epsilon_{\mathbf{k}'} - \hbar\omega_{\mathbf{q}'}) \frac{\bar{n}_{b,\mathbf{q}'} \bar{n}_{\gamma,\mathbf{k}'} [1 + (-1)^{2s} \bar{n}_{\gamma,\mathbf{k}}]}{\bar{n}_{b,\mathbf{q}}}] \quad (11)$$

and  $\mathbf{k}' = \mathbf{k} + \mathbf{q} - \mathbf{q}'$ . As our low-energy theory only holds for  $k_B T \ll \Delta$ , the gas of  $\gamma$  quasiparticles is nondegenerate, and  $\bar{n}_{\gamma,\mathbf{k}} \approx \exp(-\epsilon_{\mathbf{k}}/k_B T) \ll 1$  in Eq. (11). By taking the  $T \rightarrow 0$  limit at fixed  $\hbar c q/k_B T$  and setting  $\mathcal{A}_2^{\text{eff}} = (\hbar\omega_{\mathbf{q}}/\rho)f$ , where the dimensionless quantity  $f$  only depends on angle cosines, we obtain the equivalent

$$\hbar\Gamma_{\mathbf{q}}^{\text{scat}} \underset{T \rightarrow 0}{\sim} (2s+1) \frac{e^{-\Delta/k_B T} k_0^2 q^4 c}{(2\pi)^{9/2} \rho^2} (m_* k_B T)^{1/2} I, \quad (12)$$

with  $I = \int d^2\Omega_{\mathbf{k}} \int d^2\Omega_{\mathbf{q}'} f^2(u, u', w)$  an integral over solid angles of direction  $\mathbf{k}$  and  $\mathbf{q}'$  [14].

One proceeds similarly for the calculation of the damping rate  $\Gamma_{\mathbf{q}}^{\text{aor e}}$  of phonons  $\mathbf{q}$  due to absorption  $\mathbf{q} + \mathbf{k} \rightarrow \mathbf{k}'$  or emission  $\mathbf{k}' \rightarrow \mathbf{q} + \mathbf{k}$  processes by thermal equilibrium  $\gamma$  quasiparticles. We obtain

$$\Gamma_{\mathbf{q}}^{\text{aor e}} = \frac{2\pi}{\hbar}(2s+1) \int \frac{d^3k}{(2\pi)^3} [\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k}')^2 \times \delta(\hbar\omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'}) (\bar{n}_{\gamma,\mathbf{k}} - \bar{n}_{\gamma,\mathbf{k}'})], \quad (13)$$

with  $\mathbf{k}' = \mathbf{k} + \mathbf{q}$ . Low degeneracy of the  $\gamma$  quasiparticles and energy conservation allow us to write  $\bar{n}_{\gamma,\mathbf{k}} - \bar{n}_{\gamma,\mathbf{k}'} \approx \exp(-\epsilon_{\mathbf{k}}/k_B T)/(1 + \bar{n}_{b,\mathbf{q}})$ . Energy conservation leads here to a scaling on  $k$  different from Eq. (8) as it forces  $k$  to be at a nonzero distance from  $k_0$ , even in the low-phonon-energy limit: When  $q \rightarrow 0$  at fixed  $\mathbf{k}$ , the Dirac delta in Eq. (13) becomes

$$\delta(\hbar\omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'}) \underset{q \rightarrow 0}{\sim} (\hbar c q)^{-1} \delta\left(1 - u \frac{de_{\mathbf{k}}}{\hbar c}\right) \quad (14)$$

and imposes that the group velocity  $(1/\hbar)(de_{\mathbf{k}}/dk)$  of the incoming  $\gamma$  quasiparticle is larger in absolute value than that,  $c$ , of the phonons. This condition, reminiscent of Landau's criterion, restricts wave number  $k$  to a domain  $D$  not containing  $k_0$ . In the low- $q$  limit, that is for  $q$  much smaller than the  $k$  significantly contributing to Eq. (13), but with no constraint on the ratio  $\hbar c q/k_B T$ , we write  $\mathcal{A}_1$  in Eq. (5) to leading order  $q^{1/2}$  in  $q$ , and integrate over the direction of  $\mathbf{k}$ , to obtain

$$\Gamma_{\mathbf{q}}^{\text{aor e}} \approx \frac{(2s+1)\rho}{4\pi m c} \int_D \frac{dk k^2 e^{-\epsilon_{\mathbf{k}}/k_B T}}{\left|\frac{de_{\mathbf{k}}}{dk}\right| 1 + \bar{n}_{b,\mathbf{q}}} \left| \partial_{\rho} \epsilon_{\mathbf{k}} + \frac{\hbar^2 c^2 k^2}{\rho \frac{de_{\mathbf{k}}}{dk}} \right|^2 \quad (15)$$

$$\underset{T \rightarrow 0}{\sim} \frac{(2s+1)\rho k_*^2}{4\pi \hbar^2 m c^3} \left| \partial_{\rho} \epsilon_{k_*} + \frac{\hbar c k_*}{\rho \eta_*} \right|^2 \frac{k_B T e^{-\epsilon_{k_*}/k_B T}}{1 + \bar{n}_{b,\mathbf{q}}}. \quad (16)$$

Equation (16) is an equivalent when  $T \rightarrow 0$  at fixed  $\hbar c q/k_B T$ ;  $k_*$  is the element of the border of  $D$  ( $(de_{\mathbf{k}}/dk)|_{k=k_*} = \eta_* \hbar c$ ,  $\eta_* = \pm$ ) with minimal energy  $\epsilon_{k_*}$  (when more than one of such  $k_*$  exists, one has to sum their contributions). As  $\epsilon_{k_*} > \Delta$ , the damping rate due to scattering dominates the one due to absorption-emission in the mathematical limit  $T \rightarrow 0$ ; we shall see, however, that this is not always so for typical temperatures in current experiments.

To be complete, we give a low-temperature equivalent of the damping rate of the  $\gamma$  quasiparticle  $\mathbf{k}$  due to interaction with thermal phonons. With  $k - k_0 = O(T^{1/2})$  as in Eq. (8), we find  $\hbar\Gamma_{\mathbf{k}}^{\gamma\phi} \sim (\pi I/42)(k_B T)^7/(\hbar c \rho^{1/3})^6$ , where the factor  $2s+1$  is gone (no summation over  $\sigma$  is needed) but  $I$  is the same angular integral as in Eq. (12). Here scattering dominates [15]. Using  $\tau_{\gamma} \approx 1/\Gamma_{\mathbf{k}}^{\gamma\phi}$ , we checked that the Figs. 1 and 2 below are in the collisionless regime  $\omega_{\mathbf{q}} \tau_{\gamma} \gg 1$ . Similarly, we checked that  $\omega_{\mathbf{q}} \tau_{\phi} \gg 1$  on the figures.

*Application to helium.*—Precise measurements of the equation of state (relating  $\rho$  to pressure) and of the roton dispersion relation for various pressures were performed in liquid  $^4\text{He}$  at low temperature ( $k_B T \ll mc^2, \Delta$ ). They give access to the parameters  $k_0, \Delta$ , their derivatives, and  $m_*$ . The measured sound velocities agree with the thermodynamic relation  $mc^2 = \rho(d\mu/d\rho)$ , where  $\mu$  is the zero-temperature chemical potential of the liquid. We plot in Fig. 1 the phonon damping rates  $\Gamma_{\mathbf{q}}$  as functions of temperature, at fixed angular frequency  $\omega_{\mathbf{q}}$ . At the chosen high pressure, the phonon dispersion relation is concave at low  $q$ ; therefore, the Beliaev-Landau [16–21] three-phonon process  $\phi \leftrightarrow \phi\phi$  is energetically forbidden and the Landau-Khalatnikov [1,6,21] process  $\phi\phi \leftrightarrow \phi\phi$  is dominant. Our high yet experimentally accessible [22,23] value of  $\omega_{\mathbf{q}}$  leads to sound attenuation lengths  $2c/\Gamma_{\mathbf{q}}$  short enough to be measured in centimetric cells. As visible on Fig. 1, sound

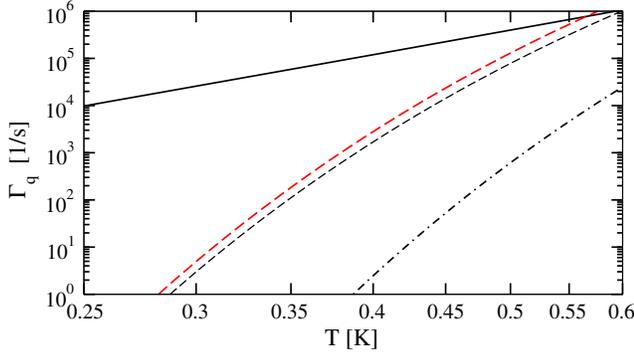


FIG. 1. Phonon damping rates at angular frequency  $\omega_{\mathbf{q}} = 2\pi \times 165$  GHz ( $q = 0.3 \text{ \AA}^{-1}$ ) in liquid  ${}^4\text{He}$  at pressure  $P = 20$  bar as functions of temperature. Solid line: purely phononic damping  $\Gamma_{\phi\phi}$  due to Landau-Khalatnikov four-phonon processes [1,6,21]; it depends on the curvature parameter  $\gamma$  defined as  $\omega_{\mathbf{q}} = cq[1 + (\gamma/8)(\hbar q/mc)^2 + O(q^4)]$ . Interpolating measurements of  $P \mapsto \gamma(P)$  in Refs. [25,26] gives  $\gamma = -6.9$ . Dashed black line [dash-dotted black line]: damping due to scattering [absorption or emission] by rotons, see Eq. (12) [Eq. (15)]. Red dashed line: original formula of Ref. [1] for the damping rate due to phonon-roton scattering. The roton parameters are extracted from their dispersion relation  $k \mapsto \epsilon_{\mathbf{k}}$  measured at various pressures [27]:  $\Delta/k_B = 7.44$  K,  $k_0 = 2.05 \text{ \AA}^{-1}$ ,  $m_*/m = 0.11$ ,  $\rho k'_0/k_0 = 0.39$ ,  $\rho\Delta'/\Delta = -1.64$ ,  $\rho^2\Delta''/\Delta = -8.03$ ,  $\rho m_*/m_* = -4.7$ . In Eq. (15), parabolic approximation Eq. (9) is used (hence,  $\epsilon_{k_*}/\Delta \approx 1.43$ ). The speed of sound  $c = 346.6$  m/s, and the Grüneisen parameter  $d \ln c/d \ln \rho = 2.274$  entering in  $\Gamma_{\phi\phi}$ , are taken from equation of state Eq. (A1) of Ref. [28]. The low values  $\hbar q/mc = 0.13$  and  $k_B T/mc^2 < 10^{-2}$  justify our use of quantum hydrodynamics.

damping is actually dominated by four-phonon Landau-Khalatnikov processes up to temperatures  $T \approx 0.6$  K. In this regime one would directly observe this phonon-phonon damping mechanism, which would be a premiere. The sound attenuation measurements of Ref. [24] in helium at 23 bars and  $\omega_{\mathbf{q}} = 2\pi \times 1.1$  GHz are indeed limited to  $T > 0.8$  K where damping is still dominated by rotons.

*Application to fermions.*—In cold-atom Fermi gases, interactions occur in  $s$  wave between opposite-spin atoms. Of negligible range, they are characterized by the scattering length  $a$  tunable by Feshbach resonance [29–34].

Precise measurements of the fermionic excitation parameters  $k_0$  and  $\Delta$  were performed at unitarity  $a^{-1} = 0$  [35]. Because of the unitary-gas scale invariance [36–38],  $k_0$  is proportional to the Fermi wave number  $k_F = (3\pi^2\rho)^{1/3}$ ,  $k_0 \approx 0.92k_F$  [35], and  $\Delta$  is proportional to the Fermi energy  $\epsilon_F = \hbar^2 k_F^2/2m$ ,  $\Delta \approx 0.44\epsilon_F$  [35]. This also determines their derivatives with respect to  $\rho$ . Similarly, the equation of state measured at  $T = 0$  is simply  $\mu = \xi\epsilon_F$ , where  $\xi \approx 0.376$  [34], and the critical temperature is  $T_c \approx 0.167\epsilon_F/k_B$  [34]. For the effective mass of the fermionic excitations and their dispersion relation at nonvanishing  $k - k_0$ , we must rely on results of a dimensional  $\epsilon = 4 - d$  expansion,  $m_*/m \approx 0.56$  and  $\epsilon_k \approx \Delta + [\hbar^2(k^2 - k_0^2)^2/8m_*k_0^2]$  [39]. We also trust

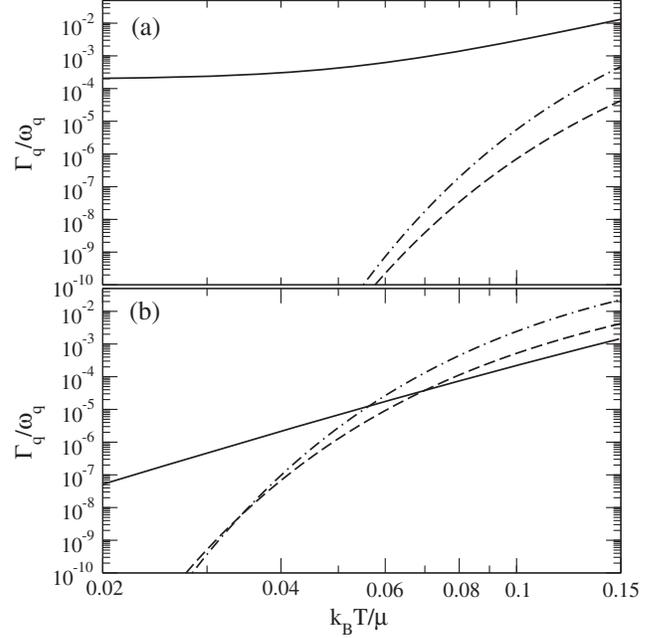


FIG. 2. Phonon damping rates at wave number  $q = mc/2\hbar$  in unpolarized homogeneous cold-atom Fermi gases in thermodynamic limit as functions of temperature. (a) At unitarity  $a^{-1} = 0$ , where most parameters of the phonons and fermionic quasiparticles are measured (see text). (b) On the BCS side  $1/k_F a = -0.389$ , these parameters are estimated in BCS theory ( $\mu/\epsilon_F \approx 0.809$ ,  $\Delta/\mu \approx 0.566$ ,  $m_*/m = \Delta/2\mu$ ,  $\rho\mu'/\mu \approx 0.602$ ,  $\rho\Delta'/\Delta \approx 0.815$ ,  $\rho^2\Delta''/\Delta \approx -0.209$ ,  $d \ln c/d \ln \rho \approx 0.303$ ). In both cases the curvature parameter  $\gamma$  defined in the caption of Fig. 1 is estimated in the RPA [42]. Solid line: phonon-phonon (a) Beliaev-Landau damping  $\phi \leftrightarrow \phi\phi$  (for  $\gamma > 0$ ) as in Eqs. (121) and (122) of Ref. [21] (independent of  $|\gamma|$ ) and (b) Landau-Khalatnikov damping  $\phi\phi \leftrightarrow \phi\phi$  (for  $\gamma \approx -0.30 < 0$ ) [6,21]. Dashed line [dash-dotted line]: scattering [absorption or emission] phonon-fermionic quasiparticle processes, as in Eq. (12) [Eq. (15)]. In Eq. (15), we took for  $\epsilon_k$  (a) the form proposed in Ref. [39] (hence,  $\epsilon_{k_*}/\Delta \approx 1.12$ ) and (b) the BCS form (hence,  $\epsilon_{k_*}/\Delta \approx 1.14$ ).  $\mu$  is the  $T = 0$  gas chemical potential, and the plotted quantities are in fact inverse quality factors. Here  $k_B T/mc^2 > 0.03$  in contrast to Fig. 1 where  $k_B T/mc^2 < 0.01$ : cold atoms are effectively farther from the  $T \rightarrow 0$  limit than liquid helium, hence the inversion of the  $\Gamma_{\mathbf{q}}^{\text{scat}} - \Gamma_{\mathbf{q}}^{\text{a or e}}$  hierarchy.

Anderson’s RPA prediction [40,41] that the  $q = 0$  third derivative of the phononic dispersion relation is positive [42]. The damping rates of phonons with wave number  $q = mc/2\hbar$  are plotted in Fig. 2(a). The contribution of the three-phonon Landau-Beliaev processes  $\phi \leftrightarrow \phi\phi$ , here energetically allowed, is dominant; it is computed in the quantum-hydrodynamic approximation where it is independent of the aforementioned third derivative.

The phononic excitation branch becomes concave in the BCS limit  $k_F a \rightarrow 0^-$  [43]. As visible on Fig. 2(b), the phonon-phonon damping (now governed by the Landau-Khalatnikov processes mentioned earlier) is much weaker, and dominates the  $\phi$ - $\gamma$  damping only at very low temperatures. At commonly reached temperatures  $T > 0.05\epsilon_F/k_B$  [44], the damping is in

fact dominated by absorption-emission  $\phi$ - $\gamma$  processes which, unlike in liquid helium, prevail over scattering ones because of the smaller value of  $\epsilon_{k^*}/\Delta$ . Although the associated quality factors  $\omega_{\mathbf{q}}/\Gamma_{\mathbf{q}}$  may seem impressive, the mode lifetimes  $\Gamma_{\mathbf{q}}^{-1}$  do not exceed 1 s in a gas of  ${}^6\text{Li}$  with a typical Fermi temperature  $T_F = 1 \mu\text{K}$ , which is shorter than what was observed in a Bose-Einstein condensate [45]. Our predictions, less quantitative than on Fig. 2(a), are based on the BCS approximation for the equation of state and the fermionic excitation dispersion relation  $\epsilon_k \simeq \epsilon_k^{\text{BCS}} = [(\hbar^2 k^2/2m - \mu)^2 + \Delta_{\text{BCS}}^2]^{1/2}$  and on the RPA for the  $q = 0$  third derivative of  $\omega_q$  (whose precise value matters here). A cutting remark on Ref. [46]: even in the BCS approximation to which it is restricted, we disagree with its expression of  $\Gamma_{\mathbf{q}}^{\text{a or e}}$ .

*Conclusion.*—By complementing the local density approximation in Ref. [1] with a systematic low-temperature expansion, we derived the definitive leading order expression of the phonon-roton coupling in liquid helium and we generalized it to the phonon-pair-breaking excitation coupling in Fermi gases. The ever-improving experimental technics in these systems give access to the microscopic parameters determining the coupling and allow for a verification in the near future. Our result also clarifies the regime of temperature and interaction strength in which the purely phononic  $\phi\phi \leftrightarrow \phi\phi$  Landau-Khalatnikov sound damping in a superfluid, unobserved to this day, is dominant.

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- [1] L. Landau and I. Khalatnikov, *Teoriya vyazkosti Geliya-II*, *Zh. Eksp. Teor. Fiz.* **19**, 637 (1949) and in *Collected Papers of L. D. Landau*, edited by D. ter Haar (Pergamon, New York, 1965), Chap. 69, pp. 494–510.
- [2] I. M. Khalatnikov and D. M. Chernikova, Relaxation phenomena in superfluid Helium, *Zh. Eksp. Teor. Fiz.* **49**, 1957 (1965) [*JETP* **22**, 1336 (1966)].
- [3] B. Fåk, T. Keller, M. E. Zhitomirsky, and A. L. Chernyshev, Roton-Phonon Interaction in Superfluid  ${}^4\text{He}$ , *Phys. Rev. Lett.* **109**, 155305 (2012).
- [4] A. L. Gaunt, T. F. Schmidutz, I. Gotlibovych, R. P. Smith, and Z. Hadzibabic, Bose-Einstein Condensation of Atoms in a Uniform Potential, *Phys. Rev. Lett.* **110**, 200406 (2013).
- [5] B. Mukherjee, Zhenjie Yan, P. B. Patel, Z. Hadzibabic, T. Yefsah, J. Struck, and M. W. Zwierlein, Homogeneous Atomic Fermi Gases, *Phys. Rev. Lett.* **118**, 123401 (2017).
- [6] H. Kurkjian, Y. Castin, and A. Sinatra, Landau-Khalatnikov phonon damping in strongly interacting Fermi gases, *Europhys. Lett.* **116**, 40002 (2016).
- [7] M. Zwierlein (private communication).
- [8] C. Cohen-Tannoudji, in *Atomic Motion in Laser Light, Proceedings of the Les Houches Summer School*, Session LIII, edited by J. Dalibard, J.-M. Raimond, and J. Zinn-Justin (North-Holland, Amsterdam, 1992), Sec. 2.3.
- [9] For a thermal phonon wave number, this requires  $k_B T \ll m_* c^2$ , a meaningful condition even in the BEC limit of Fermi gases where  $k_0 = 0$ .
- [10] L. H. Thomas, The calculation of atomic fields, *Proc. Cambridge Philos. Soc.* **23**, 542 (1927).
- [11] E. Fermi, Un metodo statistico per la determinazione di alcune proprieta dell'atomo, *Rend. Accad. Naz. Lincei* **6**, 602 (1927) A statistical method to evaluate some properties of the atom, and in *Collected papers, Note e memorie, of Enrico Fermi, volume I*, edited by E. Amaldi *et al.* (The University of Chicago Press, Chicago, 1962).
- [12] One expands to order  $T^{3/2}$  for  $A_1$  and  $T^2$  for energy denominators,  $q - q' = [\hbar(k - k_0)q(u - u')/m_*c] \{1 + [\hbar(k - k_0)u'/m_*c]\} + [\hbar q^2(u - u')^2/2m_*c] + O(T^{5/2})$  with  $u$  and  $u'$  defined below Eq. (10).
- [13] We use energy conservation and relations  $1 + (-1)^{2s} \bar{n} = e^{\epsilon/k_B T} \bar{n}$  to transform the difference of the gain and loss quantum statistical factors.
- [14]  $I/(4\pi)^2 = (\hbar^2 k_0^2/2m_*mc^2)^2 \{ \frac{1}{25} - (4\alpha/15) + \frac{28}{45}\alpha^2 + (2\beta^2/9) + A[\frac{2}{9} - (4\alpha/3)] + A^2 + 4\beta B[\frac{1}{15} - (\alpha/9)] + B^2[\frac{2}{15} - (4\alpha/9) + (2\alpha^2/3) + (4\beta^2/3)] + (4\beta/9)B^3 + (B^4/9) \}$ , with  $\alpha = (\rho k'_0/k_0)$ ,  $\beta = (m_*c/\hbar k_0)$ ,  $A = [m_*\rho^2\Delta''/(\hbar k_0)^2] + \alpha^2$ ,  $B = (\rho\Delta'/\hbar ck_0)$ .
- [15] At low  $T$ ,  $k$  is close to  $k_0$ , emission  $\mathbf{k} \leftrightarrow \mathbf{q} + \mathbf{k}'$  is forbidden by energy conservation, and absorption  $\mathbf{k} + \mathbf{q} \leftrightarrow \mathbf{k}'$ , conserving energy for  $q \geq q_* \simeq 2m_*c/\hbar$ , is  $O(e^{-\hbar\omega_{q_*}/k_B T})$ .
- [16] S. T. Beliaev, Energy-Spectrum of a non-ideal Bose gas, *Zh. Eksp. Teor. Fiz.* **34**, 433 (1958) [*JETP* **7**, 299 (1958)].
- [17] L. P. Pitaevskii and S. Stringari, Landau damping in dilute Bose gases, *Phys. Lett. A* **235**, 398 (1997).
- [18] S. Giorgini, Damping in dilute Bose gases: A mean-field approach, *Phys. Rev. A* **57**, 2949 (1998).
- [19] B. M. Abraham, Y. Eckstein, J. B. Ketterson, M. Kuchnir, and J. Vignos, Sound propagation in liquid  ${}^4\text{He}$ , *Phys. Rev.* **181**, 347 (1969).
- [20] N. Katz, J. Steinhauer, R. Ozeri, and N. Davidson, Beliaev Damping of Quasiparticles in a Bose-Einstein Condensate, *Phys. Rev. Lett.* **89**, 220401 (2002).
- [21] H. Kurkjian, Y. Castin, and A. Sinatra, Three-phonon and four-phonon interaction processes in a pair-condensed Fermi gas, *Ann. Phys. (Berlin)* **529**, 1600352 (2017).
- [22] N. A. Lockerbie, A. F. G. Wyatt, and R. A. Sherlock, Measurement of the group velocity of 93 GHz phonons in liquid  ${}^4\text{He}$ , *Solid State Commun.* **15**, 567 (1974).
- [23] W. Dietsche, Superconducting Al-PbBi Tunnel Junction as a Phonon Spectrometer, *Phys. Rev. Lett.* **40**, 786 (1978).
- [24] P. Berberich, P. Leiderer, and S. Hunklinger, Investigation of the lifetime of longitudinal phonons at GHz frequencies in liquid and solid  ${}^4\text{He}$ , *J. Low Temp. Phys.* **22**, 61 (1976).
- [25] D. Rugar and J. S. Foster, Accurate measurement of low-energy phonon dispersion in liquid  ${}^4\text{He}$ , *Phys. Rev. B* **30**, 2595 (1984).
- [26] E. C. Swenson, A. D. B. Woods, and P. Martel, Phonon Dispersion in Liquid Helium under Pressure, *Phys. Rev. Lett.* **29**, 1148 (1972).
- [27] M. R. Gibbs, K. H. Andersen, W. G. Stirling, and H. Schober, The collective excitations of normal and superfluid  ${}^4\text{He}$ : The dependence on pressure and temperature, *J. Phys. Condens. Matter* **11**, 603 (1999).

- [28] H. J. Maris and D. O. Edwards, Thermodynamic properties of superfluid  $^4\text{He}$  at negative pressure, *J. Low Temp. Phys.* **129**, 1 (2002).
- [29] K. M. O'Hara, S. L. Hemmer, M. E. Gehm, S. R. Granade, and J. E. Thomas, Observation of a strongly interacting degenerate Fermi gas of atoms, *Science* **298**, 2179 (2002).
- [30] T. Bourdel, J. Cubizolles, L. Khaykovich, K. M. Magalhães, S. J. J. M. F. Kokkelmans, G. V. Shlyapnikov, and C. Salomon, Measurement of the Interaction Energy Near a Feshbach Resonance in a  $^6\text{Li}$  Fermi Gas, *Phys. Rev. Lett.* **91**, 020402 (2003).
- [31] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. H. Denschlag, and R. Grimm, Collective Excitations of a Degenerate Gas at the BEC-BCS Crossover, *Phys. Rev. Lett.* **92**, 203201 (2004).
- [32] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Condensation of Pairs of Fermionic Atoms Near a Feshbach Resonance, *Phys. Rev. Lett.* **92**, 120403 (2004).
- [33] S. Nascimbène, N. Navon, K. J. Jiang, F. Chevy, and C. Salomon, Exploring the thermodynamics of a universal Fermi gas, *Nature (London)* **463**, 1057 (2010).
- [34] M. J. H. Ku, A. T. Sommer, L. W. Cheuk, and M. W. Zwierlein, Revealing the superfluid lambda transition in the universal thermodynamics of a unitary Fermi gas, *Science* **335**, 563 (2012).
- [35] A. Schirotzek, Y. Shin, C. H. Schunck, and W. Ketterle, Determination of the Superfluid Gap in Atomic Fermi Gases by Quasiparticle Spectroscopy, *Phys. Rev. Lett.* **101**, 140403 (2008).
- [36] T.-L. Ho, Universal Thermodynamics of Degenerate Quantum Gases in the Unitarity Limit, *Phys. Rev. Lett.* **92**, 090402 (2004).
- [37] T. Enss, R. Haussmann, and W. Zwerger, Viscosity and scale invariance in the unitary Fermi gas, *Ann. Phys. (Amsterdam)* **326**, 770 (2011).
- [38] Y. Castin and F. Werner, in *BCS-BEC Crossover and the Unitary Fermi Gas*, Springer Lecture Notes in Physics, edited by W. Zwerger (Springer, Berlin, 2011).
- [39] Y. Nishida and D. T. Son,  $\epsilon$  Expansion for a Fermi Gas at Infinite Scattering Length, *Phys. Rev. Lett.* **97**, 050403 (2006).
- [40] P. W. Anderson, Random-phase approximation in the theory of superconductivity, *Phys. Rev.* **112**, 1900 (1958).
- [41] R. Combescot, M. Y. Kagan, and S. Stringari, Collective mode of homogeneous superfluid Fermi gases in the BEC-BCS crossover, *Phys. Rev. A* **74**, 042717 (2006).
- [42] H. Kurkjian, Y. Castin, and A. Sinatra, Concavity of the collective excitation branch of a Fermi gas in the BEC-BCS crossover, *Phys. Rev. A* **93**, 013623 (2016).
- [43] M. Marini, F. Pistolesi, and G. C. Strinati, Evolution from BCS superconductivity to Bose condensation: Analytic results for the crossover in three dimensions, *Eur. Phys. J. B* **1**, 151 (1998).
- [44] Z. Hadzibabic, S. Gupta, C. A. Stan, C. H. Schunck, M. W. Zwierlein, K. Dieckmann, and W. Ketterle, Fiftyfold Improvement in the Number of Quantum Degenerate Fermionic Atoms, *Phys. Rev. Lett.* **91**, 160401 (2003).
- [45] F. Chevy, V. Bretin, P. Rosenbusch, K. W. Madison, and J. Dalibard, Transverse Breathing Mode of an Elongated Bose-Einstein Condensate, *Phys. Rev. Lett.* **88**, 250402 (2002).
- [46] Z. Zhang and W. V. Liu, Finite-temperature damping of collective modes of a BCS-BEC crossover superfluid, *Phys. Rev. A* **83**, 023617 (2011).