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On the correct use of stepped-sine excitations for the measurement of time-varying bioimpedance

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Abstract. *Objective:* When linear time-varying (LTV) bioimpedance is measured using stepped-sine excitations, a compromise must be made: the temporal distortions affecting the data depend on the experimental time, which in turn sets the data accuracy and limits the temporal bandwidth of the system that needs to be measured. *Approach:* Here, the experimental time required to measure linear time-invariant bioimpedance with a specified accuracy is analyzed for different stepped-sine excitation setups. *Results:* We provide simple equations that allow the reader to know whether LTV bioimpedance can be measured through repeated time-invariant stepped-sine experiments. *Significance:* Bioimpedance technology is on the rise thanks to a plethora of healthcare monitoring applications. The results presented can help to avoid distortions in the data while measuring accurately non-stationary physiological phenomena. The impact of the work presented is broad, including the potential of enhancing bioimpedance studies and healthcare devices using bioimpedance technology.

Article type: Note.

Conflict of interest disclosure: Dr. Sanchez is named as inventor on a patent in the field of electrical impedance. The patent entitled Electrical Impedance Myography is not directly relevant to this publication, as it relates to the development of an automated device for human use. However, since the patent is in the field of electrical impedance, the author has disclosed it. Dr. Sanchez receives consulting income from Maxim Integrated, Inc.

Keywords: linear time-varying (LTV) impedance, impedance spectroscopy (IS), stepped-sine excitations, transients.

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1. Introduction

Nowadays, multi-frequency measurements based on time-*invariant* stepped-sine excitation are the most common approach to measure bioimpedance. This also includes monitoring non-stationary physiological phenomena, i.e. linear time-varying (LTV) impedance $Z_{\text{LTV}}(j\omega, t)$, for example to detect heart failure decomposition (Dovancescu et al. 2015). In this particular case, the impedance measured may be distorted by the inherent non-stationary property of the system under investigation (e.g. figure 1) (Pintelon et al. 2013, Sanchez et al. 2015). Despite its importance, how to identify $Z_{\text{LTV}}(j\omega, t)$ while avoiding time-varying distortions using stepped-sine excitations remains unknown.

Figure 1. Example of a distorted Nyquist plot originating from a time-*invariant* impedance spectroscopy experiment (SFB7, Impedimed Ltd., Queensland, Australia) conducted on a linear time-varying impedance circuit with a 10 Hz temporal bandwidth. Each data point corresponds to a different measured frequency at an *unknown* time instant.

Here, we show that stepped-sine excitations can be used to measure the time-varying dynamics in $Z_{\text{LTV}}(j\omega, t)$. To do so, we present analytical expressions for the calculation of the experimental time required to measure $Z_{\text{LTI}}(j\omega)$ with a pre-specified accuracy. These expressions allow the reader to know whether LTV impedance can be measured accurately through repeated time-*invariant* stepped-sine experiments.

2. Experimental time for stepped-sine excitations

To calculate the experimental time required for measuring $Z_{\text{LTI}}(j\omega)$ and, by extension, for $Z_{\text{LTV}}(j\omega, t)$, we first start by writing down the general expression for the experimental time in a stepped-sine impedance spectroscopy (IS) experiment. We use as notational convention the index variable $m \in (1, 2, \dots, M)$ to denote the experiment number and $k \in (1, 2, \dots, F)$ the frequency bin, where M stands for the total number of performed experiments and F the number of measured frequencies within a single stepped-sine IS experiment.

We assume at least one period of the stepped-sine input is being measured during

the measurement time span, i.e.

$$T_m^{[k]} = \frac{P_m^{[k]}}{f_k} \quad \text{with } P_m^{[k]} \geq 1 \quad (1)$$

where $P_m^{[k]}$ is the number of cycles for each measured (angular) frequency $\omega_k = 2\pi f_k$. In addition, a waiting time $\Delta T_m^{[k]}$ between the end of the k th measurement and the beginning of the $(k+1)$ th measurement record should also be included to allow the bio-system to reach a steady state (Gabrielli et al. 1984). Henceforth, we use the superscript notation $(\bullet)^{[k]}$ to differentiate the frequency index from the mathematical operation of exponentiation.

In figure 2 the schematic of the resulting waveforms during the m th experiment are depicted. Summing up the individual contributions in (1), the general expression for the experimental time using stepped-sine excitations becomes

$$\Sigma_m = \sum_{k=1}^F (T_m^{[k]} + \Delta T_m^{[k]}). \quad (2)$$

Figure 2. Schematic illustrating the measurement time for non-uniform impedance spectroscopy experiments using stepped-sine reference excitation $r(t)$, in units of V or A. The measurement time for frequency f_k is $T_m^{[k]} = P_m^{[k]} / f_k$, with $P_m^{[k]}$ the number of periods measured. $\Delta T_m^{[k]}$ is the waiting time between the end and the beginning of the k th and $(k+1)$ th record respectively. Nomenclature: $k \in (1, 2, \dots, F)$ and $m \in (1, 2, \dots, M)$ are, respectively, the frequency and experiment index; F the number of frequencies in an experiment and M the number of experiments performed.

Hereafter, we drop the the experiment dependency $(\bullet)_m$ for notational clarity, i.e. $\Sigma_m = \Sigma$, $T_m^{[k]} = T^{[k]}$, $\Delta T_m^{[k]} = \Delta T^{[k]}$ and $P^{[k]} = P_m^{[k]} \forall m \in \mathbb{N}$. After simplifying the expression in (2) we have for the total experimental time

$$\Sigma = \sum_{k=1}^F (T^{[k]} + \Delta T^{[k]}) \quad \text{with } T^{[k]} = \frac{P^{[k]}}{f_k}. \quad (3)$$

Starting from (3), we now investigate how Σ depends on most common identification setups used. Two types of frequency grids are analyzed:

- (i) Non-uniform: the measurement time, $T^{[k]}$ in (3), corresponds to P periods of the k th frequency, i.e. $P^{[k]} = P$ periods in (3). At the same time, the waiting time

$\Delta T^{[k]}$ depends on the measurement time of the k th frequency. In the end, the experimental time depends on the frequency distribution, which is analyzed for each case separately.

- (ii) Uniform: the measurement time $T^{[k]}$ and the waiting time $\Delta T^{[k]}$ in (3) are constant for all frequencies

$$T^{[k]} = T^{[i]} \text{ and } \Delta T^{[k]} = \Delta T^{[i]} \quad \forall \{k, i\} \in (1, \dots, F), \quad k \neq i$$

In other words, the number of measured periods P increases with frequency, i.e. $P^{[i]} > P^{[k]} \quad \forall i > k$.

2.1. Linear frequency grid

The first case is to distribute the $k \in (1, 2, \dots, F)$ frequencies linearly by the relationship $f_k = f_1 + (k-1)\Delta f$, with $\Delta f = BW/(F-1)$ the frequency resolution and $BW = f_F - f_1$ the measured bandwidth. The experimental time is given in table 1 under the grid category “Linear” and “All”.

2.2. Logarithmic frequency grid

Several decades are usually covered when measuring the impedance in the kHz – MHz frequency range. Hence, it may be wiser not to use a linear grid as in section 2.1 as most frequencies will then be concentrated near the highest measured frequencies. Instead, a logarithmic grid $f_{k+1} = a f_k$ with $\log a = (\log f_F - \log f_1)/(F-1)$ may be used, which distributes the frequencies logarithmically in the measured bandwidth BW . The experimental time is given in table 1 under the grid category “Logarithmic” and “All”.

2.3. Logarithmic frequency grid of F_D frequencies per decade

The last case study is to distribute, consistently, the same number of frequencies per measured frequency decade. Therefore, the relationship between the frequencies is given by $\log a = 1/(F_D - 1)$ with $F_D = F/D \in \mathbb{N}$ the number of frequencies per decade and D the number of decades in BW . Note that, in the end, the number of measured frequencies is still F (see sections 2.1-2.2). The experimental time is shown in table 1 under the grid category “Log-decade” and “All”.

3. Waiting time expressions for LTI impedance

It is obvious from figure 2 that we have to wait $\Delta T^{[k]}$ seconds each time the measured frequency is varied such that the transient power can decrease to a desired level, for example, below the noise level. In general, the parameters upon which the waiting time $\Delta T^{[k]}$ depends are: (i) the impedance under study, (ii) the desired signal-to-noise ratio (SNR) and (iii) the frequency dependent measurement time $T^{[k]}$.

Table 1. Experimental times Σ (3) using stepped-sine excitations as a function of the experimental conditions in sections 2.1–2.3.

Grid	Distribution	Experimental time
Linear	Non-uniform ^{*,1}	$\Sigma = \left(\sum_{k=1}^F \frac{P}{f_k} + \Delta T^{[k]} \right) = P \frac{\psi\left(\frac{f_1}{\Delta f} + F\right) - \psi\left(\frac{f_1}{\Delta f}\right)}{\Delta f} + \sum_{k=1}^F \Delta T^{[k]}$
Logarithmic	Non-uniform ²	$\Sigma = \left(\sum_{k=1}^F \frac{P}{f_k} + \Delta T^{[k]} \right) = \frac{P}{f_1} \left(\frac{a - a^{1-F}}{a-1} \right) + \sum_{k=1}^F \Delta T^{[k]}$
Log-decade	Non-uniform ³	$\Sigma = \left(\sum_{k=1}^F \frac{P}{f_k} + \Delta T^{[k]} \right) = \frac{P}{f_1} \left(\frac{a - a^{1-F}}{a-1} \right) + \sum_{k=1}^F \Delta T^{[k]}$
All	Uniform ⁴	$\Sigma = \left(\sum_{k=1}^F T^{[k]} \right) + F \Delta T = F \left(\frac{P^{[1]}}{f_1} + \Delta T \right)$

*The variable $\psi(\bullet)$ stands for the digamma function.

¹ $f_k = f_1 + (k-1)\Delta f$ with Δf the frequency resolution.

² $f_{k+1} = a f_k$ with $f_k = a^{k-1} f_1$.

³ $a = 10^{\frac{1}{(F_D-1)}}$ with F_D the number of frequencies per decade.

⁴ $T^{[k]} = T^{[1]} = \frac{P^{[1]}}{f_1} \quad \forall k \in \mathbb{N}$.

To find the relationship between the waiting time $\Delta T^{[k]}$ and the pre-specified accuracy SNR it is necessary to have prior knowledge of the impedance to be measured (Schoukens et al. 2000). We consider the LTI impedance model proposed by (Fricke & Morse 1925), which is modeled as a continuous-time zero-pole system

$$Z_{\text{LTI}}(s) = \frac{R_1 R_2 C s + R_1}{(R_1 + R_2) C s + 1} \quad (4)$$

where s denotes the Laplace variable. Transfer function (4) models an electrical circuit with a series resistor-capacitor R_2 - C in parallel with a resistor R_1 . The relationship between (4) and its equivalent Randles' series circuit bridge can be found in (Sanchez et al. 2012).

The energy E_z remaining in the impulse response $z_{\text{LTI}}(t)$ from $\Delta T^{[k]}$ until ∞ , after exciting the impedance with a Dirac impulse (unit impulse current), is given by

$$E_z = \int_{\Delta T^{[k]}}^{\infty} |z_{\text{LTI}}(t)|^2 dt = \frac{1}{2} \frac{R_1^4}{(R_1 + R_2)^2 \tau} e^{-2 \frac{\Delta T^{[k]}}{\tau}} \quad (5)$$

$$\text{with } z_{\text{LTI}}(t) = \mathcal{L}^{-1}\{Z_{\text{LTI}}(s)\} = \frac{R_1^2}{\tau (R_1 + R_2)} e^{-\frac{t}{\tau}}, \quad (6)$$

where $\tau = (R_1 + R_2) C$ is the impedance time constant and $\mathcal{L}^{-1}$ is the inverse Laplace transform (see figure 3). Nonetheless, to be strictly correct, the energy expression should be computed considering the input power of a sine excitation instead of that of the unit power of a unit impulse current. This is formally written down in Assumption 2 below.

Before elaborating the waiting time $\Delta T^{[k]}$ as function of the specified SNR and measurement time $T^{[k]}$, two assumptions are needed on the impulse response in order to calculate the voltage transient power.

Figure 3. Transients (modeled by the impedance impulse response $z_{\text{LTI}}(t)$ (6)) are introduced every time an exciting frequency in the stepped-sine reference excitation is changed. The transient energy in the time interval $[\Delta T^{[k]}, T^{[k]} + \Delta T^{[k]}]$ is shown by the shadowed area. The averaged power follows from the energy expression.

Assumption 1: We assume that the transient energy, which is still remaining at $t > T^{[k]} + \Delta T^{[k]}$, is negligible small (with $T^{[k]}$ the measurement time at one particular frequency, see figure 3). Therefore, the averaged power of the transient signal is

$$P_z = \frac{E_z}{T^{[k]}}. \quad (7)$$

Assumption 2: The amplitude spectrum of the voltage transient response is proportional to the LTI impedance $Z_{\text{LTI}}(j\omega)$. In that case, after scaling with the actual input power instead of that of a Dirac impulse, the power spectrum of the voltage transient response in V^2s^2 becomes

$$P_V(\omega) = \tilde{P}_z |Z_{\text{LTI}}(j\omega) I(j\omega)|^2 = \left(\frac{1}{2T^{[k]}} \frac{R_1^4}{(R_1 + R_2)^2 \tau} e^{-2\frac{\Delta T^{[k]}}{\tau}} \right) |Z_{\text{LTI}}(j\omega) I(j\omega)|^2, \quad (8)$$

where $\tilde{P}_z$ is the normalized power (dimensionless) and $I(j\omega)$ is the current spectrum.

Below, we provide simple expressions for calculating the waiting time $\Delta T^{[k]}$. The first case (section 3.1) balances the transient error and measurement noise when an intermediate *SNR* is considered. The second case (section 3.2) analyzes a very good *SNR* where the transient error dominates over the measurement noise.

3.1. Intermediate signal-to-noise ratio (*SNR*)

In general, the impedance accuracy will take a moderate value, for instance, an *SNR* around 30–40 dB. Under these circumstances, (9)

$$\Delta T^{[k]} = \frac{\tau}{2} \ln \left(\frac{\text{SNR}}{2T^{[k]}} \frac{R_1^4}{(R_1 + R_2)^2 \tau} \right) \quad (9)$$

gives us a rough approximation for the waiting time between frequencies such that the noise floor balances the transient errors. Expression (9) has been obtained using (8) where the voltage transient power equals the noise power, i.e. $P_V(\omega) = \sigma_{\text{noise}}^2(\omega)$, and where *SNR* is given as the ratio of the voltage spectrum and the voltage noise power, i.e. $\text{SNR} = |Z_{\text{LTI}}(j\omega) I(j\omega)|^2 / \sigma_{\text{noise}}^2(\omega)$.

3.2. Very good signal-to-noise ratio (SNR)

For accurate measurements where the transient error dominates over the noise level, the waiting time is determined by the accepted relative quality ε ($SNR \rightarrow \varepsilon^2$ in (9))

$$\Delta T^{[k]} = \frac{\tau}{2} \ln \left(\frac{\varepsilon^2}{2T^{[k]}} \frac{R_1^4}{(R_1 + R_2)^2 \tau} \right). \quad (10)$$

Hence, the SNR term in (9) can be replaced by the inverse of the desired relative error.

It is worth nothing that (9)-(10) allow us to calculate the non-uniform waiting time associated with the k th frequency, which is explicitly indicated by the superscript $(\bullet)^{[k]}$. The uniform waiting time follows then straightforwardly after dropping the frequency dependence.

4. Numerical example

The minimum experimental time in (3) for the measurement of $Z_{LTI}(j\omega)$ can now be predicted given the setup, the specified SNR and the impedance model. For example, consider the non-uniform measurement (3) of $F = 50$ sine-waves linearly (section 2.1) as well as logarithmically (section 2.2) distributed in the bandwidth $BW = [f_1, f_F]$ with $f_1 = 1$ kHz and $f_F = 1$ MHz. The pre-specified accuracy is $SNR = 50$ dB and the impedance model parameters in (4) are set to $R_1 = 330 \Omega$, $R_2 = 100 \Omega$ and $C = 10$ nF. For both grids the number of periods is chosen to be $P = 10$. The corresponding non-uniform waiting times can be calculated using (9) for all measured frequencies. In the end, the experimental times computed with the expressions in table 1 are $\Sigma_{lin} = 16.96$ ms and $\Sigma_{log} = 80.53$ ms.

5. Discussion

The measurement of the time-*invariant* part of $Z_{LTV}(j\omega, t)$ using stepped-sine inputs was originally subject to investigation by Stoynov in (Stoynov & Savova-Stoynov 1985) and further studied in (Stoynov 1992, Savova-Stoynov & Stoynov 1992, Stoynov 1993). In one of their first works, the authors aimed at correcting the time-varying distortions in the measured impedance at frequencies < 1 Hz. The authors developed a correction method based on a cubic spline interpolation scheme to account for the measurement delay between the excited sine-waves. Further, (Gabrielli et al. 1984) showed that the experimental time depends on the waiting times between the excited sine-waves to avoid transient regimes. If the measurement is carried out during the transient regime, an error will arise due to a too fast sweeping of the sine-waves. The authors used Randles' impedance model (Randles 1947) to compute the impedance transient response and waiting times between the sine-waves. In both approaches, however, the authors did not intend to measure $Z_{LTV}(j\omega, t)$ *per se*.

The ultimate goal of this work is to help the reader evaluate whether it is possible to measure accurately non-stationary bioimpedance using stepped-sine excitations. To do

so, we have provided expressions that allow one to calculate the experimental time for a specified *SNR*. These equations depend explicitly on the Fricke-Morse model parameters, a commonly used model used in bioimpedance. Importantly, in order to track the dynamics of $Z_{\text{LTV}}(j\omega, t)$ over time, it is necessary to repeat time-invariant stepped-sine experiments over time. As can be predicted from the equations presented, the measurement time will increase for a higher desired accuracy. This will be even more restrictive when a *time-varying* impedance is measured by virtue of repeated time-invariant experiments. Specifically, to avoid under-sampling problems, the Nyquist-Shannon sampling condition in (11) should be fulfilled, namely

$$W_{\max} < \frac{\pi}{\Sigma_m}, \quad (11)$$

where $W_{\max}$ denotes the maximum angular frequency of the time-variation in $Z_{\text{LTV}}(j\omega, t)$ $\forall \omega$ and Σ_m stands for the duration of the m th stepped-sine experiment.

Although in case the sampling criterion (11) is obeyed, one can realize that, through the course of the m th experiment, the impedance at angular frequencies ω_k and ω_l are measured at different time instants, $t_m^{[k]}$ and $t_m^{[l]}$, respectively. Therefore, strictly speaking, the measured impedance (F frequencies in number) *is not* a meaningful instantaneous description of the time-varying impedance. Future research is thus warranted to retrieve an approximation of the instantaneous impedance at non-measured time instants provided that the condition in (11) is fulfilled.

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6. References

- Dovancescu S, Saczynski J S, Darling C E, Riistama J, Sert Kuniyoshi F, Meyer T, Goldberg R & McManus D D 2015 *JMIR Res. Protoc.* **4**(4), e121.
- Fricke H & Morse S 1925 *J. Gen. Physiol.* **9**(2), 153–67.
- Gabrielli C, Keddam M & Lizzee J F 1984 *J. Electroanal. Chem. Interfacial Electrochem.* **163**(1-2), 419–427.
- Pintelon R, Louarroudi E & Lataire J 2013 *IEEE Trans. Instrum. Meas.* **62**(4), 853–863.
- Randles J E B 1947 *Discuss. Faraday Soc.* **1**, 11–19.
- Sanchez B, Louarroudi E & Pintelon R 2015 *Physiol. Meas.* **36**(3), 595–620.
- Sanchez B, Vandersteen G, Bragos R, Schoukens J & Rojas C R 2012 *Meas. Sci. Technol.* **23**(23), 085702.
- Savova-Stoynov B & Stoynov Z 1992 *Electrochim. Acta* **37**(12), 2353–2355.
- Schoukens J, Pintelon R & Rolain Y 2000 *IEEE Trans. Instrum. Meas.* **49**(2), 275–278.
- Stoynov Z 1992 *Electrochim. Acta* **37**(12), 2357–2359.
- Stoynov Z 1993 *Electrochim. Acta* **38**(14), 1919–1922.
- Stoynov Z & Savova-Stoynov B 1985 *J. Electroanal. Chem. Interfacial Electrochem.* **183**, 133–144.