Effects of Spin-Orbit Coupling on the Berezinskii-Kosterlitz-Thouless Transition and the Vortex-Antivortex Structure in Two-Dimensional Fermi Gases

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We investigate the Berezinskii-Kosterlitz-Thouless (BKT) transition in a 2D Fermi gas with spin-orbit coupling (SOC), as a function of the two-body binding energy and a perpendicular Zeeman field. By including a generic form of the SOC, as a function of Rashba and Dresselhaus terms, we study the evolution between the experimentally relevant equal Rashba-Dresselhaus (ERD) case and the Rashba-only (RO) case. We show that in the ERD case, at a fixed nonzero Zeeman field, the BKT transition temperature T_{BKT} is increased by the presence of SOC for all values of the binding energy. We also find a significant increase in the value of the Clogston limit compared to the case without SOC. Furthermore, we demonstrate that the superfluid density tensor becomes anisotropic (except in the RO case), leading to an anisotropic phase-fluctuation action that describes elliptic vortices and antivortices, which become circular in the RO limit. This deformation constitutes an important experimental signature for superfluidity in a 2D Fermi gas with ERD SOC. Finally, we show that the anisotropic sound velocities exhibit anomalies at low temperatures, in the vicinity of quantum phase transitions between topologically distinct uniform superfluid phases.

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In the last few years, ultracold gases have been used as quantum simulators to study many-body systems. Presently, several quantities, such as the interaction strength, population imbalance, and dimensionality are controllable parameters in experiments with ultracold atoms. Recently, the ability to engineer artificial gauge fields has been added to this repertoire [1,2]. One example is the creation of spin-orbit coupling (SOC) in neutral atoms. This capability not only has the potential to shed more light on condensed matter systems such as topological insulators [3], but also to create new states of matter without analogue in other fields of physics, such as synthetic spin-1/2 bosons [4]. In ultracold gases, only equal-Rashba-Dresselhaus (ERD) SOC has been experimentally created so far, first in systems of interacting bosons [5] and subsequently in systems of noninteracting fermions [6,7]. Recently, the interacting spin-orbit coupled Fermi gas near a Feshbach resonance has also been realized [8]. Other proposals to create coupling between momentum and internal states involve rf dressing with an atom chip [9] or using ladderlike optical lattices [10].

On the theoretical side, most investigations of SOC in ultracold fermionic gases focused on 3D systems with either Rashba-only (RO) coupling [11–14] or ERD coupling [15,16]. The RO case has a strong connection to condensed matter literature [17], where the ERD case has no counterpart. Recently, the 2D RO case also received attention [18,19], motivated by the experimental creation of a 2D interacting Fermi gas [20] and by its relation to topological superfluids [21,22]. Additionally, the ERD case was studied

at the mean-field level in the zero temperature limit [23]. However, it is well known that in 2D the finite temperature superfluid transition arises due to the Berezinskii-Kosterlitz-Thouless (BKT) mechanism of vortex-antivortex (VA) unbinding [24,25], where phase fluctuations play a fundamental role. Hence, in 2D, it is essential to go beyond the mean-field approximation and include phase fluctuations [26–28] when dealing with finite temperatures.

In this manuscript, we study the BKT transition in a 2D Fermi gas in the presence of SOC. We use a generic form of SOC, allowing us to investigate the evolution between the experimentally relevant ERD case and the isotropic RO case. We determine the BKT transition temperature T_{BKT} as a function of the two-body binding energy and a perpendicular Zeeman field and we discuss the vortex-antivortex structure of the system. The main results of our work can be summarized as follows. (1) In the ERD case, at fixed nonzero Zeeman field, T_{BKT} is increased by the presence of SOC, for all values of the binding energy. (2) ERD SOC leads to an anisotropic superfluid density tensor. The resulting anisotropic fluctuation action describes elliptic vortices and antivortices, which become circular in the RO limit. (3) The anisotropic sound velocities are sensitive to the presence of quantum phase transitions, where the uniform superfluid (US) state changes its nodal structure from fully gapped (US-0) to gapless with four (US-2) or two nodes (US-1).

Hamiltonian.— The starting point of our analysis is the Hamiltonian density: $\mathcal{H}(r) = \mathcal{H}_0(r) + \mathcal{H}_{\text{SOC}}(r) + \mathcal{H}_{\text{int}}(r)$, where $r = (\mathbf{r}, \tau)$ is a three-vector representing position $\mathbf{r}$

and imaginary time τ . The single-particle component is given by $\mathcal{H}_0(r) = \sum_{s,s'} \bar{\psi}_{r,s} (\hat{K}_s \delta_{s,s'} - h_z \sigma_{z,ss'}) \psi_{r,s'}$, where $\hat{K}_s = -\nabla_r^2 - \mu_s$ is the kinetic energy in reference to the chemical potential μ_s of the fermions in spin state $s = (\uparrow, \downarrow)$. Furthermore, h_z is the Zeeman field along the z axis perpendicular to the (x, y) plane, σ_i indicates the i th Pauli matrix and $\bar{\psi}_{r,s}$ and $\psi_{r,s}$ are fermionic fields. The second term

$$\mathcal{H}_{\text{SOC}}(r) = -2 \sum_{s,s'} \bar{\psi}_{r,s} (\alpha \hat{k}_x \sigma_{y,ss'} - \gamma \hat{k}_y \sigma_{x,ss'}) \psi_{r,s'} \quad (1)$$

is the SOC part, where $\hat{k}_i = -i(\partial/\partial r_i)$ and with $\alpha = (v_R + v_D)/2$ and $\gamma = (v_R - v_D)/2$ being the sum and difference of the Rashba and Dresselhaus coupling strengths v_R and v_D , respectively. We have written the SOC part in a generic form, allowing us to study the evolution from the ERD case ($v_R = v_D$) to the RO case ($v_D = 0$). Finally, the third term $\mathcal{H}_{\text{int}}(r) = g \bar{\psi}_{r,\uparrow} \bar{\psi}_{r,\downarrow} \psi_{r,\downarrow} \psi_{r,\uparrow}$, is a local interaction Hamiltonian, where g is the strength of the contact interaction. Henceforth, we will use the units $\hbar = 2m = E_F = 1$, with m and E_F being the atomic mass and the Fermi energy, respectively.

Functional integral derivation.— The main goal of our calculation is to derive the effective action of the system. To this end, we perform a functional integration over the fermionic fields, leading to the partition function $\mathcal{Z} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp[-S(\bar{\psi}, \psi)]$. The action $S(\bar{\psi}, \psi)$ is linked to the Hamiltonian density via the Legendre transformation $S(\bar{\psi}, \psi) = \int dr [\sum_s \bar{\psi}_{r,s} (\partial/\partial \tau) \psi_{r,s} + \mathcal{H}(r)]$. To decouple the fourth order interaction term in $\mathcal{H}_{\text{int}}$ into second order terms, we use the Hubbard-Stratonovich transformation. This introduces a functional integral over the complex pair fields $(\bar{\Delta}_r, \Delta_r)$. Subsequently, we introduce the phase of the order parameter by rewriting $\Delta_r = |\Delta_r| e^{i\theta_r}$, where $|\Delta_r|$ is the amplitude and θ_r is the phase. Furthermore, to make explicit the dependence of the action on the phase θ_r , we perform the gauge transformation: $\psi_{r,s} \rightarrow \psi_{r,s} e^{i\theta_r/2}$.

Integration over the fermionic fields $(\bar{\psi}_{r,s}, \psi_{r,s})$ in the partition function $\mathcal{Z}$ leads to an effective action that is a function of $|\Delta_r|$ and θ_r . However, since phase fluctuations provide the dominant contributions to the physics in two dimensions, we can take the amplitude of the order parameter to be uniform in space and imaginary time ($|\Delta_r| = |\Delta|$). Using this procedure, we arrive at the partition function $\mathcal{Z} = \int \mathcal{D}\theta_r \exp[-S(\theta_r)]$, where the action is given by

$$S = -\frac{1}{2} \text{Tr} \left\{ \ln \left[\beta \begin{pmatrix} \mathbb{A}_+ & \mathbb{D}_+ \\ \mathbb{D}_- & \mathbb{A}_-^* \end{pmatrix} \right] \right\} - \frac{\beta L^2 |\Delta|^2}{g} + \frac{\beta}{2} \sum_{k,s} (-i\omega_n + \mathbf{k}^2 - \mu_s) + \frac{1}{8L^2} \int dr \sum_k [\nabla_r(\theta_r)]^2. \quad (2)$$

In this expression, β is the inverse temperature, L^2 is the area of the 2D system and $k = (\mathbf{k}, \omega_n)$ is a three-vector

representing the fermionic wave vector $\mathbf{k}$ and the fermionic Matsubara frequency, $\omega_n = (2n + 1)\pi/\beta$. The matrix in Eq. (2) is a 4×4 matrix, written as a function of the 2×2 matrices

$$\mathbb{A}_\pm = \begin{pmatrix} \mp i\omega_n \pm \xi_{\mathbf{k}}^\theta \mp \tilde{\zeta} - \zeta_{\mathbf{k}}^\theta & -h_\perp(\mathbf{k}) \mp h_\perp^\theta \\ -h_\perp^*(\mathbf{k}) \mp h_\perp^{*\theta} & \mp i\omega_n \pm \xi_{\mathbf{k}}^\theta \pm \tilde{\zeta} - \zeta_{\mathbf{k}}^\theta \end{pmatrix} \quad (3)$$

and $\mathbb{D}_\pm = \pm i\sigma_y |\Delta|$. The kinetic terms in Eq. (3) have been divided into phase-independent and phase-dependent contributions, where we defined $\xi_{\mathbf{k}}^\theta = \xi_{\mathbf{k}} + \xi^\theta$. The phase-independent terms are $\xi_{\mathbf{k}} = \mathbf{k}^2 - \mu$ with $\mu = (\mu_\uparrow + \mu_\downarrow)/2$ and $\tilde{\zeta} = \zeta + h_z$ with $\zeta = (\mu_\uparrow - \mu_\downarrow)/2$. The phase-dependent terms are $\xi^\theta = (i/2)(\partial\theta_r/\partial\tau) + \frac{1}{4}[\nabla_r(\theta_r)]^2$ and $\zeta_{\mathbf{k}}^\theta = -\nabla_r(\theta_r) \cdot \mathbf{k}$. The spin-flip terms also contain a phase-independent contribution corresponding to the SOC field $h_\perp(\mathbf{k}) = -2\gamma k_y - 2i\alpha k_x$ and a phase-dependent contribution $h_\perp^\theta = -\gamma(\partial\theta_r/\partial y) - i\alpha(\partial\theta_r/\partial x)$.

Quadratic expansion of the action.— The final step in obtaining the effective action is to perform an expansion of expression (2) to quadratic order in the phase θ_r . Denoting the 4×4 matrix in Eq. (2) by $\mathbb{M}_k(\theta, \partial\theta)$, we can write $\text{Tr}\{\ln[\beta\mathbb{M}_k(\theta, \partial\theta)]\} = \text{Tr}\{\ln[\beta\mathbb{M}_k(0, 0)]\} + \text{Tr}\{\ln[\mathbb{I} + \mathbb{M}_k^{-1}(0, 0)\mathbb{F}_k(\theta, \partial\theta)]\}$, with $\mathbb{F}_k(\theta, \partial\theta) = \mathbb{M}_k(\theta, \partial\theta) - \mathbb{M}_k(0, 0)$ being the phase-fluctuation part. The first term in the expansion leads to the saddle-point part of the action $S_{\text{sp}} = -\frac{1}{2} \text{Tr}\{\ln[\beta\mathbb{M}_k(0, 0)]\} + (\beta/2) \sum_{k,s} (-i\omega_n + \mathbf{k}^2 - \mu_s) - (\beta L^2 |\Delta|^2/g)$, resulting in the saddle-point thermodynamic potential

$$\Omega_{\text{sp}} = \sum_{\mathbf{k}} \left(\frac{-1}{2\beta} \sum_{i=\pm} \ln[2 + 2 \cosh(\beta E_i)] + \xi_{\mathbf{k}} \right) - \frac{L^2 |\Delta|^2}{g}. \quad (4)$$

In Eq. (4) we denoted the eigenvalues of the matrix $\mathbb{M}_k(0, 0)$ by $E_\pm = \sqrt{\epsilon_{\mathbf{k}}^2 + \vartheta^2 \pm 2\sqrt{\epsilon_{\mathbf{k}}^2 \vartheta^2 - |\Delta|^2} |h_\perp(\mathbf{k})|^2}$ where $\epsilon_{\mathbf{k}}^2 = \xi_{\mathbf{k}}^2 + |\Delta|^2$ and $\vartheta^2 = \tilde{\zeta}^2 + |h_\perp(\mathbf{k})|^2$. Finally, we use $\frac{1}{g} = -\int (d\mathbf{k}/(2\pi)^2) [1/(2k^2 + E_b)]$ where E_b is the two-body binding energy.

The second term in the expansion leads to the phase-fluctuation action: $S_{\text{fl}} = -\frac{1}{2} \text{Tr}\{\ln[\mathbb{I} + \mathbb{M}_k^{-1}(0, 0)\mathbb{F}_k(\theta, \partial\theta)]\} + (1/8L^2) \int dr \sum_k [\nabla_r(\theta_r)]^2$, which becomes

$$S_{\text{fl}} = \frac{1}{2} \int dr \left(\mathcal{A} \left(\frac{\partial\theta_r}{\partial\tau} \right)^2 + \sum_{\nu=\{x,y\}} \rho_{\nu\nu} \left(\frac{\partial\theta_r}{\partial\nu} \right)^2 \right), \quad (5)$$

after the explicit expansion in $\nabla_r(\theta_r)$. The expressions for the compressibility $\mathcal{A}$ and the superfluid density tensor components (ρ_{xx}, ρ_{yy}) are given in the Supplemental Material [29]. We note the symmetry relation $\rho_{xx}(v_R, v_D) = \rho_{yy}(v_R, -v_D)$ together with the fact that $\rho_{xx} \neq \rho_{yy}$, provided

that the SOC is anisotropic. The difference between ρ_{xx} and ρ_{yy} is a direct consequence of the anisotropy in the higher-angular-momentum pairing of the SOC-induced triplet component of the order parameter.

Phase diagram and sound velocities.—Henceforth, we particularize to the balanced case $\mu_{\uparrow} = \mu_{\downarrow} = \mu$. To determine the finite temperature phase diagram, we need to find the amplitude $|\Delta|$ of the order parameter, the chemical potential μ and the BKT transition temperature T_{BKT} . This leads to a set of three equations that need to be solved self-consistently: (1) the order parameter equation $\partial\Omega_{\text{sp}}/\partial\Delta = 0$, (2) the number equation $-\partial\Omega_{\text{sp}}/\partial\mu = n$, and (3) the generalized Kosterlitz-Thouless condition $T_{\text{BKT}} = (\pi/2)\rho_s(T_{\text{BKT}})$, where $\rho_s = \sqrt{\rho_{xx}\rho_{yy}}$ is an “effective” superfluid density [29]. The first two equations define the mean-field “transition” temperature T_{MF} , at which the system undergoes a transition between the normal state $|\Delta| = 0$ and the paired state $|\Delta| \neq 0$. However, the transition to a true superfluid state occurs at $T_{\text{BKT}} < T_{\text{MF}}$, for which the underlying mechanism is the unbinding of vortex-antivortex pairs. This mechanism was predicted by Kosterlitz and Thouless [25], after which Nelson and Kosterlitz proved that $T_{\text{BKT}} = (\pi/2)\rho_s$ is universally valid for all Kosterlitz-Thouless-type transitions [32]. Determining this critical temperature requires the simultaneous solution of all three aforementioned equations.

Figures 1(a) and 1(b) show the self-consistency solution for T_{BKT} as a function of the binding energy E_b , for fixed

FIG. 1 (color online). Berezinskii-Kosterlitz-Thouless temperature T_{BKT}/E_F as a function of the two-body binding energy E_b/E_F , at fixed values of the Zeeman field h_z/E_F : (a) $h_z/E_F = 0$ and (b) $h_z/E_F = 0.2$. The Rashba coupling strength is held fixed at $v_R/\tilde{v}_F = 1$ (with $\tilde{v}_F = v_F/2$), while the Dresselhaus coupling strength $v_D/\tilde{v}_F$ is varied. The reference case without SOC ($v = 0$) is also shown. (c) T_{BKT}/E_F as a function of h_z/E_F for ERD SOC ($v_R/\tilde{v}_F = v_D/\tilde{v}_F = 0.8$) and the reference case without SOC ($v = 0$) at fixed E_b . (d) Sound velocities $c_x/\tilde{v}_F$ and $c_y/\tilde{v}_F$ for ERD SOC ($v_R/\tilde{v}_F = v_D/\tilde{v}_F = 0.8$) as a function of h_z , at fixed E_b and varying T . The quantum phase transitions at $T = 0$ between the indirect gapped state (i-US-0) and the gapped state with four (US-2) and two (US-1) nodes are indicated.

Zeeman field h_z . The Rashba coupling strength v_R is held fixed in a given figure at $v_R/\tilde{v}_F = 1$ (with $\tilde{v}_F = v_F/2$ and v_F the Fermi velocity), while the Dresselhaus coupling strength $v_D/\tilde{v}_F$ is varied. This allows us to study the evolution between the ERD case and the RO case. In Fig. 1(a), where $h_z = 0$, the highest T_{BKT} is always achieved in the case without SOC ($v = 0$, black solid line). This is because SOC makes zero momentum pairing difficult by introducing orbital frustration. In the ERD case, however, this orbital frustration can be removed by a gauge transformation; hence, T_{BKT} (green diamonds) does not decrease. In Fig. 1(b), where $h_z \neq 0$, the presence of SOC offsets the effect of the Zeeman field, by introducing a triplet pairing component. In the ERD case (green diamonds), T_{BKT} increases for all values of E_b , compared to the case without SOC (black solid line). In the “hybrid” case (red squares) and the RO case (blue circles), an increase of T_{BKT} occurs for small values of E_b . For large values of E_b , however, the orbital frustration effect becomes dominant and T_{BKT} decreases compared to the case without SOC.

In Fig. 1(c), T_{BKT} is plotted as a function of the Zeeman field h_z for the ERD case $v_R/\tilde{v}_F = v_D/\tilde{v}_F = 0.8$, together with the reference case without SOC ($v = 0$), for two values of the binding energy: $E_b/E_F = 0.05$ (blue empty and solid circles) and $E_b/E_F = 0.5$ (red empty and solid squares). This figure shows that ERD SOC stabilizes the superfluid state against the effect of a Zeeman field. In general, a Zeeman field tends to break singlet superfluid pairs, leading to a superfluid-to-normal transition at a critical field, known as the Clogston limit [33]. Thus, as a result, the Clogston limit, at which T_{BKT} jumps discontinuously to zero, lies at a significantly higher value in the case with ERD SOC [$h_z^{(c)}/E_F \approx 1.1$ for $E_b/E_F = 0.05$] compared to when no SOC is present [$h_z^{(c)}/E_F \approx 0.21$ for $E_b/E_F = 0.05$]. This larger generalized Clogston limit occurs in the phase locked case ($\theta_{r,\uparrow} = \theta_{r,\downarrow}$), which becomes orbitally frustrated, even though there is an induced triplet component of the order parameter.

In Fig. 1(d), we show the speed of sound along the x and y directions as a function of h_z , for the ERD case at low temperatures. Notice that for $E_b/E_F \ll 1$ and $h_z = 0$, the sound velocities $c_x/\tilde{v}_F = c_y/\tilde{v}_F = \sqrt{2}$ reduce to the standard value in the BCS regime given by $c_x = c_y = v_F/\sqrt{2}$. In Fig. 1(d), we have indicated the quantum phase transitions (QPTs) at $T = 0$. In the ERD case, the US phases can be classified according to the nodal structure of the quasiparticle energies $E_{\pm}$ [see Eq. (4)], leading to four cases [23]. The first two cases correspond to a uniform superfluid phase without nodes, which can have an indirect gap (i-US-0) at nonzero momentum, or a direct gap (d-US-0) at zero momentum. The last two cases correspond to uniform superfluid phases with two nodes (US-1) or four nodes (US-2). Figure 1(d) shows that the sound velocity at

low temperature is sensitive to the QPT between US-2 and US-1. The reason is that at this transition two nodal Dirac quasiparticles with opposite topological charges annihilate at zero momentum (long-wavelength limit). However, the sound velocities are much less sensitive to the QPT between i-US-0 and US-2, because there the transition occurs at finite quasiparticle momenta. When approached from the i-US-0 side, this transition can be viewed as the softening of the quasiparticle excitation spectrum of the gapped i-US-0 phase at two finite momenta near $\pm k_F$. When approached from the US-2 side, the transition can be understood as the annihilation of two Dirac quasiparticles with opposite topological charges at nonzero momentum. We suggest the sensitivity of the speed of sound to the presence of QPTs to be an experimental hallmark of ERD SOC. However, note that measurements on the time scale needed to determine the speed of sound are challenging to attain with current state of the art techniques. The main reason being that the spontaneous emission is large when SOC is created using the optical dressing of internal states. A recent proposal that eliminates this spontaneous emission involves the use of radio-frequency driven Raman transitions in the near field of an atom chip [9].

Vortex-antivortex structure.—An important effect of SOC is that the superfluid density tensor can become anisotropic, as shown in expression (5). In Fig. 2, we show the components ρ_{xx} and ρ_{yy} of the superfluid density, as a function of E_b , for fixed Rashba coupling strength ($v_R/\tilde{v}_F = 1$) and varying Dresselhaus coupling strength $v_D/\tilde{v}_F$. In Fig. 2(a), $\rho_{xx} = \rho_{yy}$ in two situations: (1) in the ERD case (green diamonds and green solid line), where the effect of SOC can be gauged away because $h_z = 0$, and (2) in the RO case (black circles and black solid line), which is isotropic. In Fig. 2(b), we show that $\rho_{yy} > \rho_{xx}$ when $h_z \neq 0$, provided that the SOC is anisotropic. The emergence of an anisotropic superfluid density tensor can be caused both by extrinsic constraints, such as hard-wall potentials [34], or by intrinsic properties of the system such as anisotropic p -wave pairing, the presence of anisotropic optical lattices [35], or the inhomogeneity of the superfluid state [36].

FIG. 2 (color online). The components of the anisotropic superfluid density tensor ρ_{xx}/E_F and ρ_{yy}/E_F as a function of the binding energy E_b/E_F , for fixed Rashba coupling strength $v_R/\tilde{v}_F = 1$ and varying Dresselhaus coupling strength $v_D/\tilde{v}_F$: (a) $h_z/E_F = 0$ and (b) $h_z/E_F = 0.2$.

The SOC-induced anisotropy of the superfluid density has important effects on the vortex structure of the superfluid state. In the case without SOC or in the RO case, $\rho_{xx} = \rho_{yy}$ and thus the vortex (and antivortex) solutions exhibit circular symmetry, as shown in Fig. 3(a). However, the presence of ERD SOC leads to elliptic rather than circular vortices, as demonstrated in Fig. 3(b). The general solutions used in Figs. 3(a) and 3(b) are of the form $\theta_V(x, y) = \pm \arctan(\tilde{\rho}^2 y/x)$ with $\tilde{\rho} = (\rho_{xx}/\rho_{yy})^{1/4}$ for a vortex located at coordinates $(x = 0, y = 0)$. The general solution for the VA pair is $\theta_{VA}(x, y) = \arctan[2\tilde{a}\tilde{y}/(\tilde{a}^2 - \tilde{x}^2 - \tilde{y}^2)]$, where $\tilde{x} = x/\tilde{\rho}$, $\tilde{y} = y\tilde{\rho}$ and $\tilde{a} = a/\tilde{\rho}$. The parameter a indicates half the VA pair size: the location of the vortex-antivortex is at $(x = \pm a, y = 0)$. Plots of these solutions are shown in Figs. 3(c) and 3(d) for the RO case and for the ERD case, respectively. The parameters used in Fig. 3 are $E_b/E_F = 0.01$ and $h_z/E_F = 0.2$. The emergence of elliptic vortices and the structure of the VA pairs in a 2D Fermi superfluid constitute important signatures for the experimentally relevant ERD case. These signatures could be detected during a time-of-flight expansion of the trapped system, or via Bragg spectroscopy, which is also sensitive to the direction of rotation of the supercurrents.

FIG. 3 (color online). Single antivortex and vortex-antivortex structure for RO SOC [(a) and (c)], and ERD SOC [(b) and (d)]. In the former case, the antivortex exhibits circular symmetry around its core, whereas in the latter case the antivortex is elongated along one axis and contracted along the other. The parameters used are $h_z/E_F = 0.2$, $E_b/E_F = 0.01$, $T \approx T_{\text{BKT}}$, with $v_R/\tilde{v}_F = 1$ and $v_D/\tilde{v}_F = 0$ in (a) and (c), and $v_R/\tilde{v}_F = v_D/\tilde{v}_F = 1$ in (b) and (d).

Conclusion.— We have studied the effects of SOC with an arbitrary mixture of Rashba and Dresselhaus terms on the BKT transition in a 2D Fermi gas. We have found that in the ERD case, at fixed nonzero Zeeman field, the BKT transition temperature T_{BKT} increases for all values of the binding energy, due to the emergence of a SOC-induced triplet component of the order parameter. However, T_{BKT} never becomes larger than the case of vanishing Zeeman and SOC fields, because of residual orbital effects. In addition, we found a significant increase in the value of the Clogston limit, compared to the case without SOC. Furthermore, we have shown that at a nonzero Zeeman field, the superfluid density tensor becomes anisotropic due to the presence of SOC (except in the Rashba-only case). This anisotropy leads to vortices which exhibit elliptical symmetry in the experimentally relevant ERD case. This deformation constitutes an important experimental signature for superfluidity in a 2D Fermi gas with anisotropic SOC. Finally, we have shown that the anisotropic sound velocities exhibit anomalies near the quantum phase transition between the US phase with four nodes (US-2) and two nodes (US-1).

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