VOLUNTARY DISCLOSURE OF SALES BY SMALL AND MEDIUM SIZED ENTERPRISES : AN EXTENDED ANALYTICAL MODEL

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February 2002

Abstract

In Europe, Small and Medium sized Enterprises (SMEs) are allowed to publish their financial statements in an abridged format. One of the main characteristics of the abridged profit and loss account is that disclosure of sales is not mandatory, but is left entirely to the discretion of the SME.

In this paper we study the optimal disclosure decision for an SME that is faced with a large competitor, creditors and customers whose actions towards the firm are influenced by the signal they receive. The setup of the model is a duopoly in which one firm, the SME, holds private information about demand. This information can be truthfully disclosed, signalled by sales, or may be kept private. The rivalling firm is a large firm that uses the information to set its profit maximising output. Further, the interest rate charged by the creditors depends on their assessment of the firm's financial position, which is influenced by the disclosed information. If the SME depends on only one or a few major customers, disclosing sales may imply giving these customer(s) more bargaining power, which may erode the firm's profits.

The paper is closely related to the analytical accounting literature that studies voluntary disclosure of value relevant information to investors and the information sharing literature. An important difference between the existing voluntary disclosure literature in accounting and our model is that we do not take capital market reactions into account. The reason for this is straightforward. Firms that publish financial statements in the abridged format are always unlisted firms. Most shareholders of SMEs tend to be closely related to the firm and have access to all relevant information. Moreover, while other voluntary disclosure models tend to restrict the players of the game to the firm and the capital market, who both wish to maximise firm value, on the one hand and a potential entrant on the other hand, we try to take more interested parties into account. This introduces different types of disclosure costs into the model. The information sharing models exclusively focus on the game of competition and usually consider two firms that both have private information. In our model only the SME has private information. The competitor is modelled as a large firm, for which disclosure is mandatory. The conclusions of the model therefore allow us to gain some new insights.

* I wish to thank, without implications, Carl Reyns, Ann Jorissen, Hylke Vandenbussche, Alfred Wagenhofer, Walter Aerts and Wilfried Pauwels (the referee) for comments on this paper which is based on Chapter III of my doctoral dissertation 'Voluntary Disclosure of Sales by Small and Medium Sized Enterprises'. Two earlier versions of the model were published in Van de Wiele and Vandenbussche (1999) and presented at the 23rd Annual Congress of the European Accounting Association respectively. I am grateful for the financial support provided by the Fund for Scientific Research Flanders (FWO-Vlaanderen).

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I. Introduction

Since the late 1970's - beginning of the 1980's, voluntary disclosure has been one of the emerging topics in accounting research. Undoubtedly, this evolution was inspired by the growing regulation of information disclosure through accounting standards at that time. Researchers were concerned with the desirability of mandating disclosure on the one hand, and the willingness of firms to voluntarily disclose information on the other hand. Analytical models were used to see what the equilibrium disclosure strategy would be for a firm that has private (accounting) information that can be disclosed to the public or kept private. The foundations of analytical disclosure models in the accounting literature go back to industrial economics with papers by e.g. Akerlof (1970), Grossman (1981) and Milgrom (1981) and agency theory with Jensen and Meckling (1976). In industrial economics the voluntary disclosure problem is studied in the context of the seller of a product who has private information about the quality of that product. Disclosure of this information to a potential buyer may influence the price this buyer is willing to pay for the good. Under the assumptions that the disclosed signal is verifiable, that disclosure is free of costs and that the uninformed buyer knows that the seller has private information, full disclosure is the only equilibrium strategy (Grossman, 1981 and Milgrom 1981).

The early models by Grossman-Milgrom (1981) assume that disclosure is free of costs, which results in full disclosure as the only equilibrium strategy. Verrecchia (1983) comments that this is not compatible with the empirical finding that managers do not disclose all available information. The reason should be that in reality disclosure of information is not free of costs. Apart from the cost of preparing and distributing information, for example in the financial statements, there may be costs involved in the sense that disclosure potentially harms the firm. Examples are the reactions of competitors, shareholders or employees to disclosure of strategic information that reduce the firm's future profits or cash flows. Verrecchia (1983) refers to this cost as the 'proprietary cost of disclosure'. In the presence of a proprietary cost the observation of non-disclosure is no longer interpreted as the worst possible information. Non-disclosure may simply indicate that the information is just not good enough to carry the cost that is involved with disclosure. This leads to a discretionary disclosure equilibrium with a threshold level of disclosure. Only information that exceeds this threshold level is disclosed. In Verrecchia (1983) the proprietary cost is exogenous, it is determined outside the model. In Darrough and Stoughton (1990), Wagenhofer (1990) and Feltham and Xie (1992) the proprietary cost is made endogenous. It is modeled as the threat of a new entry in the market. If the opponent decides to enter, the incumbent firm suffers a loss. Depending on the specific assumptions of each of these models and given certain conditions either a full disclosure, a full non-disclosure or a partial disclosure equilibrium with two distinct non-disclosure intervals can be found. Suijs (1999) combines the models in Verrecchia (1983) and Wagenhofer (1990) to focus on the impact of an exogenous versus an endogenous cost of disclosure. He shows that incorporating both types of costs rules out the existence of a full disclosure equilibrium and provides the firm with additional incentives to keep information private.

When it is common knowledge that the manager has value relevant private information, the observation of non-disclosure is unambiguously interpreted as a reluctance to disclose information. However, if the market is uncertain about the manager's endowment of information, non-disclosure may be an indication that the manager has no information at all. Verrecchia (1990) introduces this uncertainty in his earlier model of discretionary disclosure as noise in the private signal that is received by the manager. He interprets the precision of this signal as the 'quality of information'. Again partial disclosure with a threshold level above which information is disclosed is the equilibrium strategy, but the threshold value is higher as the information is less precise. The market in that case discounts the value of the risky asset less sharply than when more accurate information is withheld. The extension in Verrecchia (1990) is clearly inspired by the work in Dye (1985) and Jung and Kwon (1988).

The previously discussed papers all assume that the disclosed signal is verifiable and/or an independent mechanism exists that enforces truthful disclosure. In Farrell and Gibbons (1989) a sender that has private information can disclose this information, with the possibility to lie, to two different audiences who both take actions that affect the sender's payoff after the signal is received. A similar problem with only one audience was studied in Crawford and Sobel (1982). Other papers in this area are Newman and Sansing (1993) and Gigler (1994).

This paper is most closely related to the work in Darrough and Stoughton (1990), Wagenhofer (1990) and Feltham and Xie (1992) and to the information sharing literature that deals with private information about demand in output competition with homogeneous goods. Examples are Ponssard (1979), Novshek and Sonnenschein (1982), Clarke (1983) and Gal-Or (1985). They show that no information sharing is the only equilibrium strategy. In the information sharing literature it is usually assumed that both firms have private information and must commit to a strategy before the private signals are received. Also, the competitive effect of information sharing is the only effect that is taken into account.

The remainder of the paper is organized as follows. First, we introduce the players and assumptions of the model. Then, we discuss the timing of the game and define the profit functions for each of the players under disclosure and non-disclosure. The equilibrium disclosure strategies are derived and discussed in section V. Finally, we sum up the major conclusions and implications of our research.

II. The Model : Players and Assumptions

In this section we briefly describe the players in our model and the way their actions are influenced by the disclosed information or the non-disclosure signal they receive. We also focus on the major assumptions on which the model is built. This allows us to situate the model in the voluntary disclosure and information sharing literature that was briefly discussed in the previous section.

A. The players

Our major player is firm S, an SME that is not obliged to disclose sales but can choose to voluntarily disclose sales to its competitor L, a large firm for which disclosure of sales is mandatory.[1] We assume that total industry sales contain information about the intercept of the demand function. If sales are not disclosed, then S is the only player that knows the exact demand function so S holds private information about demand. If S decides to disclose sales both firms act under symmetric information. The disclosed sales figure is also observed by the SME's creditors and customers. These creditors should not be seen as a bank, but rather as suppliers or any creditors for whom the financial statements are the only source of information. Banks only grant credit to a firm after an assessment of that firm's creditworthiness for which they usually require the firm to privately disclose all necessary information. If the SME depends on only one or a few major customer(s), and sales are disclosed, then these customers can perfectly infer their share in the sales figure and use this information, for example, to bargain lower prices. The reaction of both the creditors and the customers is modelled as a shift in the SME's profit function that does not fluctuate with output but is different under disclosure and non-disclosure.

If the competitor L does not know the exact demand function, it will set an output that can be either higher or lower than the output that L would have chosen under complete information. Not considering other effects of disclosure, this implies that when expected demand is higher than true demand, firm S has an incentive to disclose sales while it has an incentive not to disclose sales when expected demand is lower. The interest cost that is charged by the creditors depends on the SME's financial position. The sales figure, if it is disclosed, is taken into account in the creditors' assessment of the SME's financial position. If, for example, a high sales figure is interpreted as 'good news' and leads to a lower interest cost, but also has an unfavourable effect on the game of output competition, the SME faces a trade-off in the disclosure decision. If the SME has only one customer, disclosure of sales will give this customer important bargaining power. This gives the SME an additional incentive to keep information private. The creditors' and customer's effect of disclosure is modelled as a shift in the profit function.

B. The assumptions

The SME is endowed with private information about demand. All players know this so non-disclosure is always interpreted as withholding information. The SME cannot signal that it has no information.

We assume truthful disclosure of sales. This means that the sales figure that is disclosed in the published financial statements is the sales figure that can be found in the firm's accounts. An external auditor does not certify the financial statements before they are published, but some verification is done afterwards by a tax auditor. Although this does not allow to completely rule out cheating on the level of accounting for sales, it guarantees that the published financial statements are in accordance with the firm's books. This is the way our assumption of truthful disclosure should be interpreted.

Both firms compete in output (Cournot) and produce a homogeneous product. We assume that there is an exact relation between total industry sales at time t_0 and the intercept of the demand function at time t_2. In between, at time t_1, firm S decides whether or not to disclose sales. The specifications of the demand-, cost- and profit functions can be found in section IV. To avoid unnecessary complicated calculations, we work with linear demand- and cost functions. The intercept of the demand function is a random variable a that is normally distributed with means $\hat{a}$ and variance $V(a)$ [2]. The prior distribution of a is public information. This implies first, that the competitor, the creditors and the customers all act on the same information set and thus hold the same beliefs about a and second, that these beliefs are public information.

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[1] The large firm can be a diversified firm that produces and sells different products and is only competing with the SME in one specific product market. According to he 4th European directive segmental disclosure of sales for each type of product is mandatory for large firms.

[2] The assumption of a normal distribution for the intercept of the demand function is also found in the information sharing literature, for example in Vives (1984) and Gal-Or (1985). The shape of this type of distribution function is perfectly compatible with the other assumptions in our model. In the case of non-disclosure the competitor L, which is a large firm, may try to gather information about demand for example through market research. This leads to the prior means $\hat{a}$ which has a relatively high probability, while the probability of other values of a decreases as the distance from the means is higher.

The competitor is a large firm. We therefore assume that S and L have different marginal costs of production, c_S and c_L respectively. We say that the cost for the large firm is lower, given that it may have advantages of scale in its production process, or use a more advanced technology, so $c_L \leq c_S$. Apart from the intercept of the demand function a all other parameters, including the marginal cost of production, are public information.

III. The Timing of the Game

The game is played in three stages, as shown in figure 1. At time t_0, when both firms are new in the market, the intercept of the demand function is unknown to both firms. Each firm privately collects information about demand, for example through market research, and produces a certain output based on its own expectations. The output decision is simultaneous. Without disclosure in the next stage of the game, no firm knows the output produced by the other firm. Total industry output is sold at the market-clearing price. At time t_1 S decides whether or not to disclose sales. For L disclosure of sales is mandatory[3]. This way S always learns the exact demand function from t_0., given the market price and total output. Under non-disclosure L only knows the market price and his own output which it uses to make inferences about demand. We assume that the intercept of the demand function at time t_0 is an unbiased signal for the intercept of the demand function at time t_2.[4] If sales are disclosed, at time t_2 both firms simultaneously choose the Cournot profit maximising output. Under non-disclosure, L sets its profit maximising output depending on his expectations about demand. These expectations are based on the prior distribution of a and the interpretation of the non-disclosure signal. The output produced by L is exactly known to S who sets its own profit maximising output given the output

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[3] We assume that the disclosure decision for S is made *after* mandatory disclosure of sales by L is observed. This assumption of a sequential move can be interpreted as a simplification of a similar game played in 4 stages but with simultaneous disclosure by S and L :

t_0	outputs based on market expectations sold at market clearing price
t_1	simultaneous disclosure (or non-disclosure) of sales t_0 by S and L
t_2	game of output competition
t_3	disclosure or non-disclosure of sales t_2 given the information disclosed at t_1
t_4	game of output competition

The solution of this type of game is obviously more complex, while the probability that the results would provide additional insights, is low. The simultaneous disclosure decision at t_1 is equivalent to a precommitment to (non) disclosure. From the information sharing literature we learn that for private information about demand and output competition for substitutes, non-disclosure is the optimal decision. This implies that the game at t_2 is played under non-disclosure. The disclosure decision at t_3 is then equivalent to the disclosure decision at t_1 as it is formulated in figure 1. The game of output competition at t_4 is equivalent to the game at t_2 in figure 1. Another motivation for the assumption of a sequential move at t_1 is our option to stress the difference between large firms facing mandatory disclosure and SMEs with voluntary disclosure of sales, by allowing S to observe sales disclosed by L before making its own disclosure decision.

[4] An interesting extension would be to introduce noise in the signal about demand, so that S still holds better information about demand than L under non-disclosure but also acts under imperfect information.

chosen by L and the true demand function.[5] Total industry output is again sold at the market-clearing price. The exact profit for S at time t_2 is also dependent on the shift in the profit function caused by the creditors' and the customers' reaction to the signal they received at time t_1. In this model the game ends after the second stage. An extension would be to (infinitely) repeat the game, which would obviously have an impact on the results found here.

Figure 1 : Timing of the game

t_0	Outputs based on market expectations are sold at market clearing price.
t_1	1. Mandatory disclosure of sales t_0 by firm L. 2. Voluntary disclosure of sales t_0 by firm S.
t_2	Outputs set by firms L and S based on available information. Other disclosure effects modelled as a shift in the profit function for S : K^D under disclosure or K^{ND} under non-disclosure. Outputs sold at market clearing price.

IV. Profit Functions

The linear demand function at time t_2 takes the form :

$$p(Q) = a - bQ \tag{1}$$

with $a=a_0 = f(sales_0)$ the intercept of the demand function at time t_0 which is a function of sales

a is normally distributed with means $\hat{a}$ and variance $V(a)$

$Q = q_S + q_L$ total industry output

with q_S output firm S

q_L output firm L

$b > 0$[6]

[5] The output choice at t_2 under non-disclosure is really a simultaneous choice. However, because S and L act on the same information set (except for the intercept of the demand function), S can perfectly infer the output chosen by L. To set its own profit maximizing output, S must take the output chosen by L into account. For the calculations, this is equivalent to a sequential game of output competition in which L moves first.

[6] We rule out the special case of Giffen goods and focus the analysis on 'normal' goods for which there is a negative relation between price and demand.

If S decides not to disclose sales at t_1 then the true value of the demand function intercept *a* is unknown to L.[7] The competitor then makes all its decisions based on an expected demand function :

$$p(Q) = E(a|s_{ND}) - b(Q) \tag{2}$$

with $E(a|s_{ND})$ = the expected value of *a* given the observation of non-disclosure at t_1

$= (1-t)\hat{a} + ts_{ND}$

with $s_{ND} = a + \varepsilon$ = non-disclosure signal

with ε normally distributed, independent of *a*, with means zero and variance $V(\varepsilon)$

$t = V(a) / [V(a) + V(\varepsilon)]$

In the specification of the expected demand function we follow Vives (1984)[8]. The variance of the disturbance term in the non-disclosure signal, $V(\varepsilon)$, and the variance of the prior distribution of *a*, $V(a)$, determine the informative value of the non-disclosure signal. Further in the analysis it will be clear that the non-disclosure signal under certain conditions contains information about the true value of *a*. In fact, non-disclosure can be interpreted as disclosure of *a* plus noise. If this noise is infinitely high, then $V(\varepsilon)$ is infinitely high and *t* reduces to zero. The non-disclosure signal in this case contains no information and the expected intercept of the demand function is the prior means $\hat{a}$. However, if the noise in the non-disclosure signal approaches zero, there is a lot of information in the signal and *t* is close to one. For example, if the optimal disclosure strategy for the SME is not to disclose sales only for a small range of values for *a*, then the observation of non-disclosure implies that *a* lies within this range so the non-disclosure signal is very informative. The intercept of the expected demand function is then the prior means $\hat{a}$ updated with the information in the non-disclosure signal.

To identify the size difference between the two competing firms, we assume that firm L has a lower marginal cost of production than S : $c_L \leq c_S$. Both firms have constant marginal costs. This gives the following cost functions for S and L :

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[7] In equation (1) $a = p + bq_S + bq_L$ so L needs to know q_S to be able to calculate the exact value of *a*. If q_S and q_L are disclosed, *a* is revealed by the market clearing price.

[8] We apply the standard updating procedure for random variables that are normally distributed. To guarantee positive outputs for both players under disclosure we must assume however that *a* is truncated by $a>2c_S-c_L$. Although this may have a marginal effect on the technical outcomes of the analysis, it does not affect the intuition behind it and the types of equilibrium strategies that are found. Therefore we apply the standard updating procedure in the calculations.

$C(q_L) = c_L q_L$ cost function firm L (3)

$C(q_S)^D = c_S q_S^D$ cost function firm S under disclosure (4)

$C(q_S)^{ND} = c_S q_S^{ND}$ cost function firm S under non-disclosure (5)

with c_L = marginal cost of production firm L

c_S = marginal cost of production firm S

Given the demand and cost functions, we can now derive the profit maximising outputs for both firms under disclosure and non-disclosure. The case of disclosure coincides with the textbook case of Cournot competition with homogeneous goods and different marginal costs. The profit maximising outputs are :

$$q_S^D = (a-2c_S+c_L)/3b \quad (6)$$

$$q_L^D = (a-2c_L+c_S)/3b \quad (7)$$

To guarantee positive outputs for both firms under disclosure we assume that $a > 2c_S-c_L$ throughout the analysis. Although in theory there is no need to impose a maximum value on a, we assume that a is not expected to be higher than a maximum value A with $2c_S-c_L<A\leq\infty^+$. This simplifies the notations and interpretations further in the analysis.

Firm S additionally faces a shift in the profit function, which represents the creditors' and customers' effect of disclosure. Examples are interests charged on outstanding debt or price bargaining. The latter clearly depends on the number of customers for S. If there is only one customer and this customer is aware of the fact that it is the single customer, it has considerable bargaining power so the shift in the profit function may be very strong. The customer's information about his bargaining position, depends on the disclosure decision at t_1. Therefore the shift is different under disclosure and non-disclosure. The creditors' perception of this financial position is different under disclosure and non-disclosure because it depends, among other things, on the sales figure. So the creditors' reaction is also different under disclosure and non-disclosure. To simplify the calculations, we model both effects as a shift in the profit function, which has size K^D under disclosure and K^{ND} under non-disclosure[9]. Depending on the creditors' beliefs about sales and depending on the number of customers for S, both $K^D \geq K^{ND}$ and $K^D \leq K^{ND}$ are possible. We assume that there is no shift in the profit function for L.

[9] The combination of all other effects of disclosure in one shift K^D under disclosure and K^{ND} under non-disclosure is a strong simplification of reality. In an earlier version of the paper; presented at the 23rd annual congress of the European Accounting Association, the customer's and creditors' effect of disclosure were considered separately. The interest rate charged by the creditors on the SME's outstanding debt was dependent on the sales figure or the non-disclosure signal. The customer's effect was modeled as the risk that the single customer would bargain a price equal to marginal costs. The equilibrium strategies

The profit functions under disclosure are :

$$\pi_S^D = [(a-2c_S+c_L)^2/9b] - K^D \tag{8}$$

$$\pi_L^D = (a-2c_L+c_S)^2/9b \tag{9}$$

Theoretically, the shift in the profit function for S may be either positive or negative. This does not influence the results of the model. Intuitively however, we might argue that an unfavourable reaction to disclosure revises the firm's profits downwards, while a favourable reaction implies that there is no price bargaining and no additional interest costs are charged so that K^D is zero. It would be difficult to accept that customers would bargain prices that are higher than the market price or that creditors would charge negative interest rates.

The shift in the profit function does not influence the profit maximising outputs. K^D or K^{ND} are treated as 'sunk costs' in the game of output competition. Even if S decides not to produce it may suffer a loss $-K^D$ or $-K^{ND}$ due to an unfavourable reaction by the other players. So the size of K^D and K^{ND} is not relevant in the decision at time t_2 but does influence the disclosure decision at time t_1.

If S does not disclose sales at time t_1 then the game at time t_2 is a sequential game of output competition with asymmetric information in which L moves first and makes its output decision based on the expected demand function (2), the cost function (3) and the reaction curve of firm S. This results in the profit maximising output :

$$q_L^{ND} = [\hat{a}+t(s_{ND}-\hat{a})-2c_L+c_S]/3b \tag{10}$$

The calculations are shown in appendix A. By assumption q_L^{ND} is public information. Moreover, S knows the true demand function (1). Therefore, S can choose its profit maximising output based on the demand function (1) and the output chosen by L (10). This gives the following profit maximising output :

$$q_S^{ND} = [3a-\hat{a}-t(s_{ND}-\hat{a})-4c_S+2c_L]/6b \tag{11}$$

Note that S only produces a positive output if $a > [\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$. Under non-disclosure it may be optimal for S not to produce. The assumption that $\hat{a} > 2c_S-c_L$ guarantees a positive output for L both under disclosure and non-disclosure. However, the market clearing price

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we derived in this more complex model are not significantly different from the results we find in this chapter. This justifies the simplifications that are made here.

under non-disclosure may be zero or smaller than L's marginal cost c_L if L is the only firm to produce a positive output so L risks suffering a loss if $E(a/s_{ND})$ is much larger than the true value of a.

With total industry output sold at the market clearing price $p^{ND}(Q^{ND}) = a - b(q_S^{ND} + q_L^{ND})$ the profit functions under non-disclosure are :

$$\pi_S^{ND} = [3a-\hat{a}-t(s_{ND}-\hat{a})+2c_L-4c_S]^2/36b - K^{ND} \qquad \text{for } q_S^{ND} > 0 \qquad (12)$$

$$\pi_S^{ND} = -K^{ND} \qquad \text{for } q_S^{ND} = 0 \qquad (12')$$

$$\pi_L^{ND} = [\hat{a}+t(s_{ND}-\hat{a})-2c_L+c_S][3a-\hat{a}-t(s_{ND}-\hat{a})-4c_L+2c_S]/18b \qquad \text{for } p^{ND}(Q^{ND}) > c_L \qquad (13)$$

$$\pi_L^{ND} = [p^{ND}(Q^{ND})-c_L]q_L^{ND} \qquad \text{for } p^{ND}(Q^{ND}) \leq c_L \qquad (13')$$

V. Equilibrium Disclosure Strategies

To derive the equilibrium disclosure strategies for S we first compare the firm's profit under disclosure with the profit under non-disclosure to see which decision results in the highest profit for each possible value of a. If the profit under disclosure for a given value of a is bigger than or equal[10] to the profit under non-disclosure, it is optimal for S to disclose sales, thereby revealing the true value of a. Second, we verify whether the optimal decision for S is consistent with the beliefs that are held by the other players (Bayesian updating) to arrive at the perfect Bayesian equilibrium disclosure strategies. In a perfect Bayesian equilibrium the disclosure strategy is optimal given the other players' beliefs and the beliefs are consistent with the disclosure strategy.

Depending on whether q_S^{ND} is strictly positive or not, a different profit function for S applies. Therefore, we divide the analysis in two parts and start off with the case $q_S^{ND} > 0$.

A. $q_S^{ND} > 0$

Disclosure is preferred to non-disclosure if and only if the profit for S under disclosure is higher than or equal to the profit for S under non-disclosure. We solve the following inequality for a :

$$(a-2c_S+c_L)^2/9b - K^D \geq [3a-\hat{a}-t(s_{ND}-\hat{a})+2c_L-4c_S]^2/36b - K^{ND} \qquad (14)$$

[10] The choice that disclosure is preferred to non-disclosure when the profits are equal is arbitrary but is not

The mathematical roots to expression (14) are :

$$s_1 = \frac{3\hat{a} + 3t(s_{ND} - \hat{a}) + 4c_S - 2c_L - 2\sqrt{(\hat{a} + t(s_{NS} - \hat{a}) - 2c_S + c_L)^2 - 45b(K^D - K^{ND})}}{5}$$

$$s_2 = \frac{3\hat{a} + 3t(s_{ND} - \hat{a}) + 4c_S - 2c_L + 2\sqrt{(\hat{a} + t(s_{ND} - \hat{a}) - 2c_S + c_L)^2 - 45b(K^D - K^{ND})}}{5}$$

The calculations are given in appendix B. s_1 and s_2 can be interpreted as intersections of the profit function for S under disclosure with the profit function for S under non-disclosure if they represent values of *a* that result in a positive output for S. Therefore, we must first have $(\hat{a}+t(s_{ND}-\hat{a})-2c_S+c_L)^2-45b(K^D-K^{ND}) \geq 0$ and second, $s_1 > [\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$ and/or $s_2 > [\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$.

$(\hat{a}+t(s_{ND}-\hat{a})-2c_S+c_L)^2-45b(K^D-K^{ND}) \geq 0$ if and only if :

$$(K^D - K^{ND}) \leq (\hat{a}+t(s_{ND}-\hat{a})-2c_S+c_L)^2/45b \tag{15}$$

Condition (15) implies that when the customers' and creditors' reaction to disclosure is considerably less favourable than their reaction to non-disclosure, full non-disclosure is the optimal strategy. This result is straightforward. If the reaction to disclosure is so strong that it (almost) completely erodes the firm's profit from production, avoiding this additional cost becomes primordial in the disclosure decision.

Second, we verify whether s_1 and/or s_2 result in a positive output for S. The calculations are again given in appendix B. We find that $s_2>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$ is always satisfied but $s_1>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3 \Leftrightarrow (K^D-K^{ND}) > [\hat{a}+t(s_{ND}-\hat{a})-2c_S+c_L]^2/81b$ so s_1 is not always a valid solution to our problem. We extend the analysis to the case $q_S^{ND} = 0$ to look for additional cut off points.

B. $q_S^{ND} = 0$

We compare the profits earned under disclosure with the loss suffered under non-disclosure. Disclosure is preferred to non-disclosure if and only if :

$$[(a-2c_S+c_L)^2/9b] - K^D \geq - K^{ND} \tag{16}$$

essential in the analysis.

Mathematically, there are again two roots to expression (16). The calculations can be found in appendix C.

$$r_1 = -c_L + 2c_S - 3\sqrt{b(K^D - K^{ND})}$$
$$r_2 = -c_L + 2c_S + 3\sqrt{b(K^D - K^{ND})}$$

It is immediately clear that $b(K^D\text{-}K^{ND}) \geq 0$ if and only if $K^D \geq K^{ND}$. So an additional intersection between the profit under disclosure and the profit under non-disclosure can only be found if the customers' and creditors' effect of disclosure is less favourable than their reaction to non-disclosure. It can easily be shown that, for $K^D > K^{ND}$, $r_1 < 2c_S\text{-}c_L$ and $r_2 > 2c_S\text{-}c_L$, so we have one additional cut off point, r_2. For $K^D = K^{ND}$ both roots coincide with $2c_S\text{-}c_L$. This value cannot be interpreted as a threshold value for a because we assumed that $a>2c_S\text{-}c_L$.

Combining these results with what was found in subsection A we summarise the optimal disclosure strategies for S in table 1. In order to be equilibrium strategies they must be consistent with the beliefs held by the other players after disclosure or non-disclosure is observed. This is analysed in the next subsection.

Table 1 : Optimal disclosure strategies for S

Condition	Type of strategy	Threshold values / Intervals
$(K^D\text{-}K^{ND}) > [\hat{a}+t(s_{ND}\text{-}\hat{a})\text{-}2c_S+c_L]^2/45b$	Full non-disclosure	None
$[\hat{a}+t(s_{ND}\text{-}\hat{a})\text{-}2c_S+c_L]^2/45b \geq (K^D\text{-}K^{ND}) > [\hat{a}+t(s_{ND}\text{-}\hat{a})\text{-}2c_S+c_L]^2/81b$	Partial disclosure with two distinct non-disclosure intervals	Disclosure between s_1 and s_2
$[\hat{a}+t(s_{ND}\text{-}\hat{a})\text{-}2c_S+c_L]^2/81b \geq (K^D\text{-}K^{ND}) > 0$	Partial disclosure with two distinct non-disclosure intervals	Disclosure between r_2 and s_2
$(K^D\text{-}K^{ND}) \leq 0$	Partial disclosure with one threshold value	Non-disclosure above s_2

C. Updating beliefs and equilibrium strategies

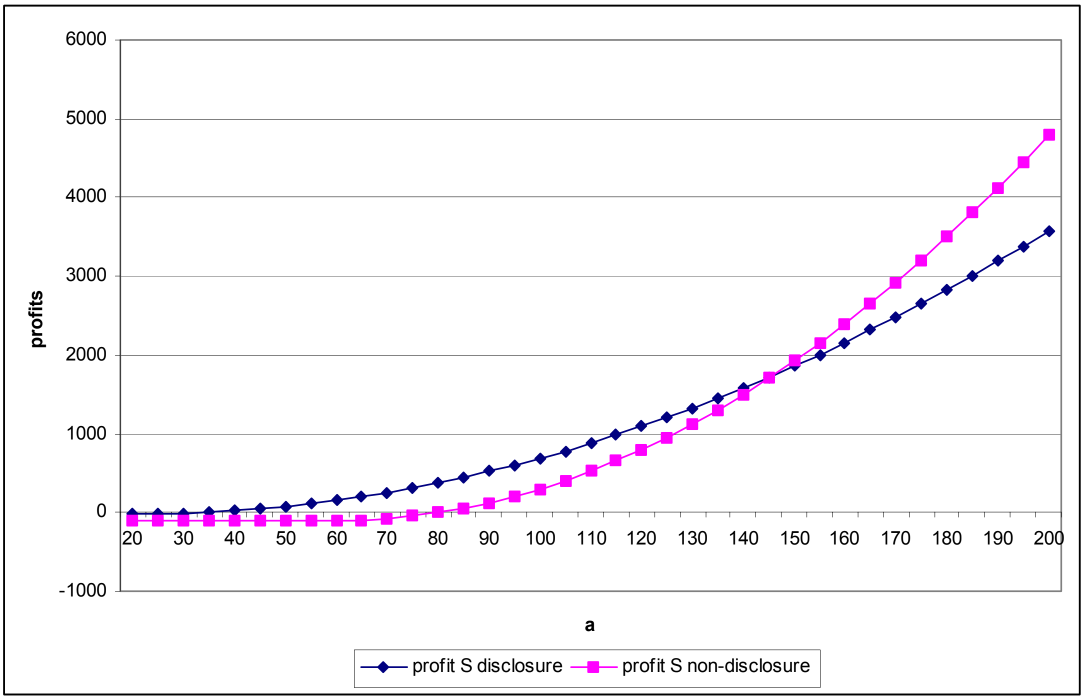
The other players update their beliefs about the true value of *a* when non-disclosure is observed if this non-disclosure signal contains information about *a*. Depending on the relative value of the parameters, three different types of disclosure strategies can be found. The informative value of the non-disclosure signal in each of these strategies is reflected in parameter *t* that ranges from 0 to 1, indicating that non-disclosure is not informative at all to perfectly informative. The non-disclosure signal, s_{ND}, is the true value of *a* plus some noise ε. The variance of the error term $V(\varepsilon)$ and the variance of the prior distribution of *a*, $V(a)$ determine *t*. Under full non-disclosure, the receivers of the non-disclosure signal can make no inferences about *a* ($t=0$). When the non-disclosure signal is part of a partial disclosure strategy with two distinct non-disclosure intervals, the receivers cannot see the difference between the lower and the upper non-disclosure interval. However, they know that the true value of *a* lies within one of these intervals so non-disclosure does contain some information, be it with a lot of noise ($0<t<<1$). When there is only one threshold value that separates disclosure and non-disclosure, observation of a non-disclosure signal clearly means that the true value of *a* is higher than this threshold value so there is a lot of information in the non-disclosure signal, but still with some noise ($0<<t<1$).

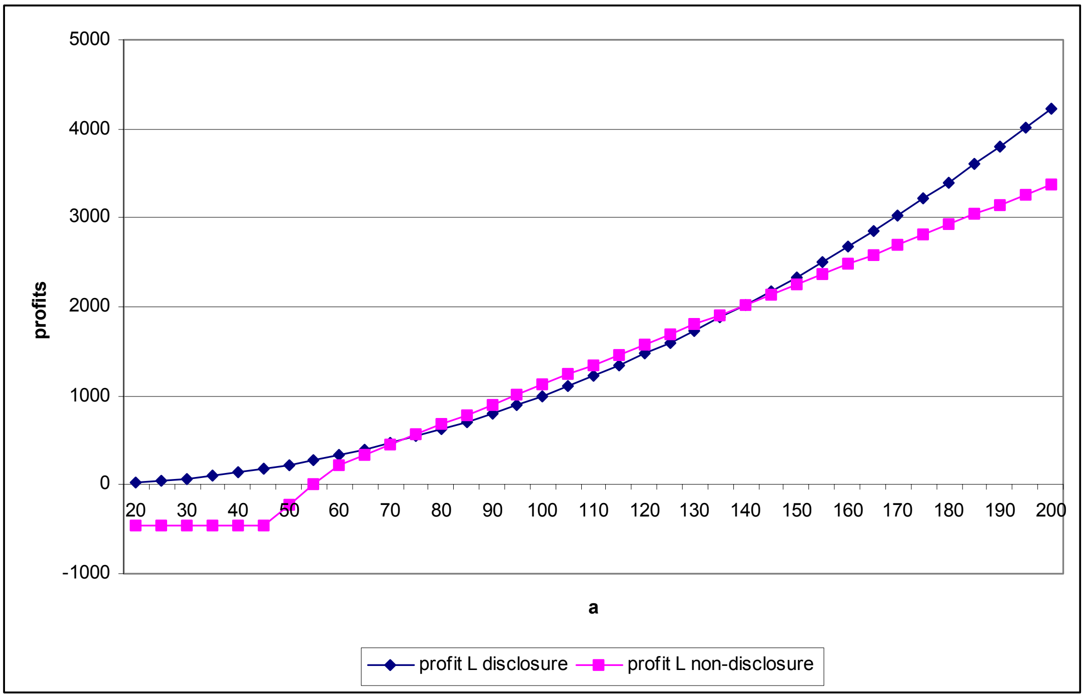
After observing non-disclosure, the players may update their beliefs about the true value of *a*. The disclosure strategies in table 1 need to be consistent with these updated, posterior beliefs, to be equilibrium strategies. The conditions of the strategies and the (non-)disclosure intervals or threshold values that describe them are applied to the endogenously determined updated values of *t* and s_{ND}. They can only be equilibrium strategies if none of the players have an incentive to move away from them. We therefore need to look at the other players' preferences about disclosure. It can easily be argued that the creditors and customers always prefer disclosure because it allows them to make more accurate decisions or gives them more bargaining power respectively. There is no advantage for them in not being informed. The competitor's preference however, is not straightforward. Under non-disclosure the sequential game of output competition provides the competitor with a first mover advantage. The competitor prefers disclosure if and only if :

$$(a-2c_L+c_S)^2/9b \geq [\hat{a}+t(s_{ND}-\hat{a})-2c_L+c_S][3a-\hat{a}-t(s_{ND}-\hat{a})-4c_L+2c_S]/18b \qquad (17)$$

Figure 2 : Full disclosure equilibrium

panel a : prior beliefs (t=0)

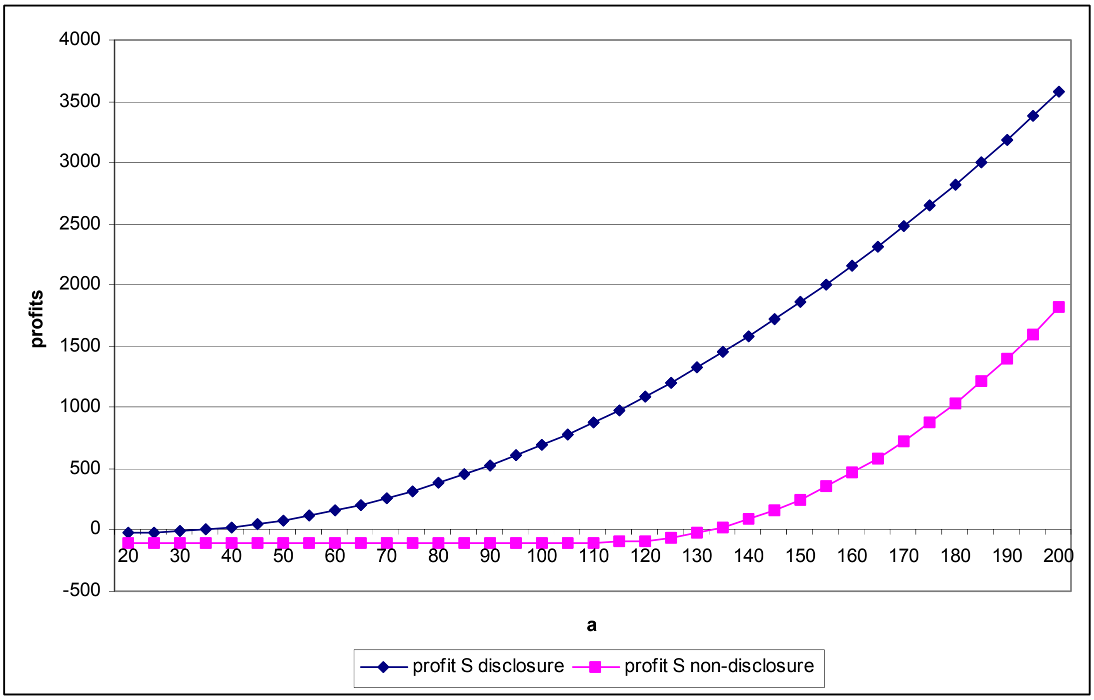


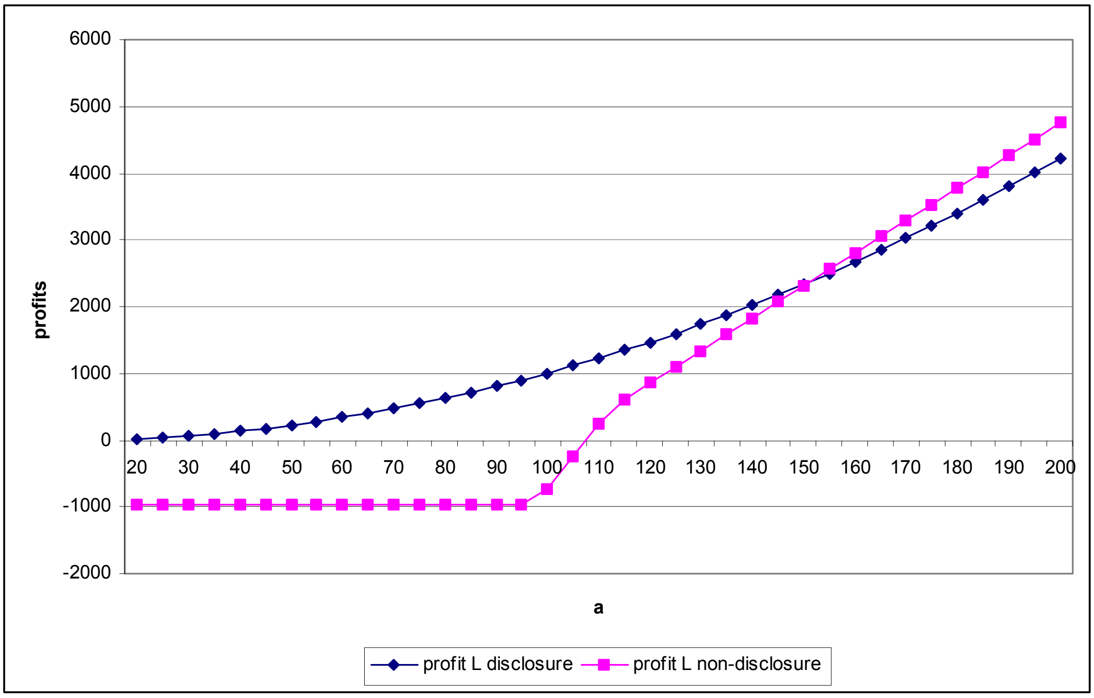


$\hat{a}=140$; $b=1$; $c_S=15$; $c_L=10$; $K^D=20$; $K^{ND}=100$; $A=300$

Figure 2 (continued)

panel b : posterior beliefs (t=0.75; s_{ND}=350)





$\hat{a}$=140; b=1; c_S=15; c_L=10; K^D=20; K^{ND}=100; A=300

Rearranging (17) and solving for *a* we find that the competitor prefers non-disclosure to disclosure for any value of *a* between $(\hat{a}+t(s_{ND}-\hat{a})+2c_L-c_S)/2$ and $\hat{a}+t(s_{ND}-\hat{a})$. For $K^D \leq K^{ND}$ it can be shown that the threshold value of non-disclosure, s_2, is bigger than the upper boundary of the interval over which L prefers non-disclosure. L therefore has an incentive to revise his beliefs upward, so that the threshold value approaches the maximum value of the demand intercept A and the partial disclosure strategy is unravelled into a full disclosure equilibrium.

Proposition 1 : If $K^D \leq K^{ND}$ then full disclosure is the equilibrium strategy.

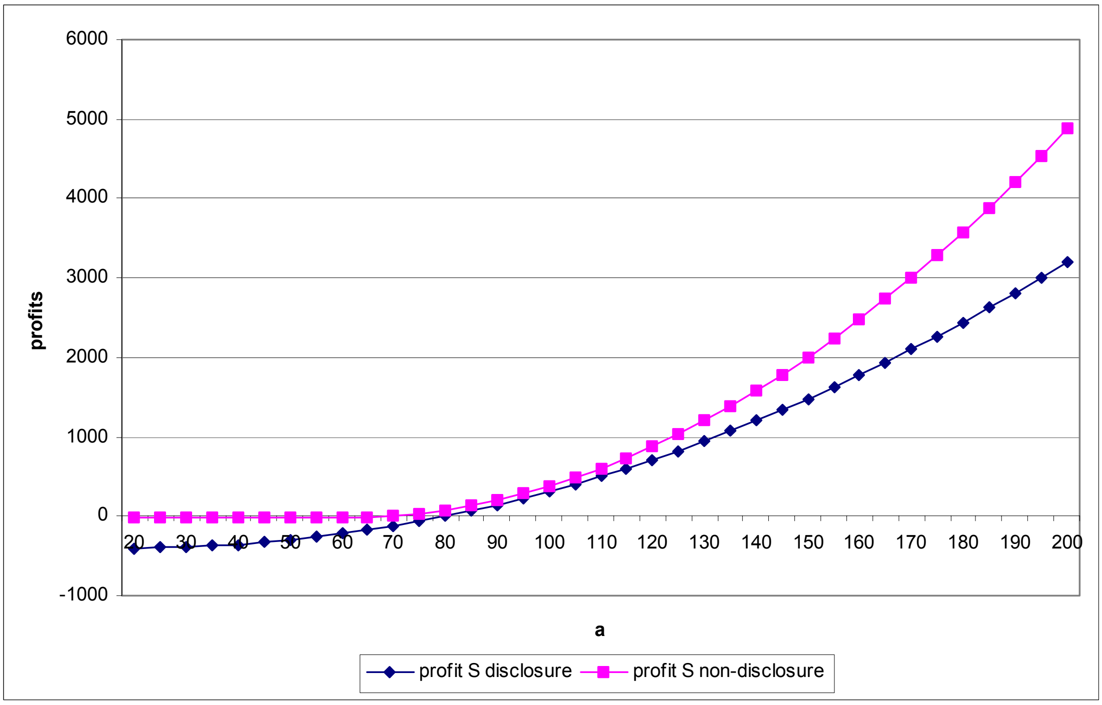
This is illustrated in figure 2. To make the graphs more clear we only show the profit functions over a limited range of values for *a*. In panel a of figure 2 we see that the optimal disclosure strategy would be a partial disclosure strategy with one threshold value if the competitor would not make any inferences about the true value of *a* when non-disclosure is observed, so if he would make his output decision based on his prior beliefs about *a*.

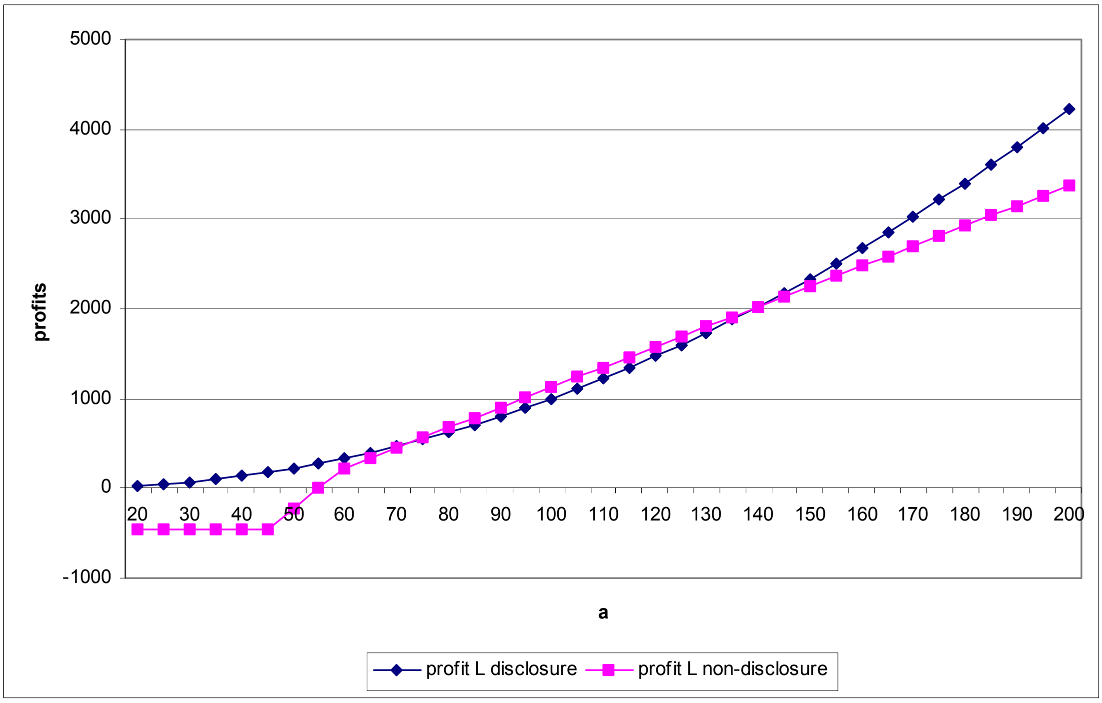
However, the threshold value is bigger than $\hat{a}$, the prior belief. The observation of non-disclosure therefore informs player L that the true value of *a* is higher than his prior belief which induces him to revise his beliefs upwards. The higher the posterior belief about *a*, the higher the threshold value s_2 so the smaller the non-disclosure interval. Given that it is to the advantage of player L that the non-disclosure interval is as small as possible, it is optimal that the posterior belief about *a* is very high so that the partial disclosure strategy unravels into a full disclosure equilibrium. This is shown in panel b.

Note that figure 2 shows that there is an area where the profit for L is highest under non-disclosure. This may somehow seem counterintuitive. From the timing of the game, however, we know that under non-disclosure the profit maximising outputs are implicitly chosen sequentially. The reason is that S has to take the profit maximising output chosen by L into account in order to maximise its own profit. This implies that to some extent L is granted a 'first mover advantage'. Under disclosure both firms move simultaneously. This explains why non-disclosure leads to higher profits for L for certain values of *a*. The specific shape of the profit function for L under non-disclosure for the lower values of *a* is determined by the fact that S produces no output under these conditions so that equation (13') applies and L suffers a loss.

Proposition 2 : If $(K^D-K^{ND}) > [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b$ then full non-disclosure is the equilibrium strategy.

Figure 3 : Non-disclosure equilibrium





$\hat{a}=140$; $b=1$; $c_S=15$; $c_L=10$; $K^D=400$; $K^{ND}=20$; $A=300$

For $(K^D-K^{ND}) > [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b$ full non-disclosure is the equilibrium strategy. The non-disclosure signal contains no information so the competitor maximises his expected profit based on the prior distribution of *a*. Figure 3 illustrates proposition 2.

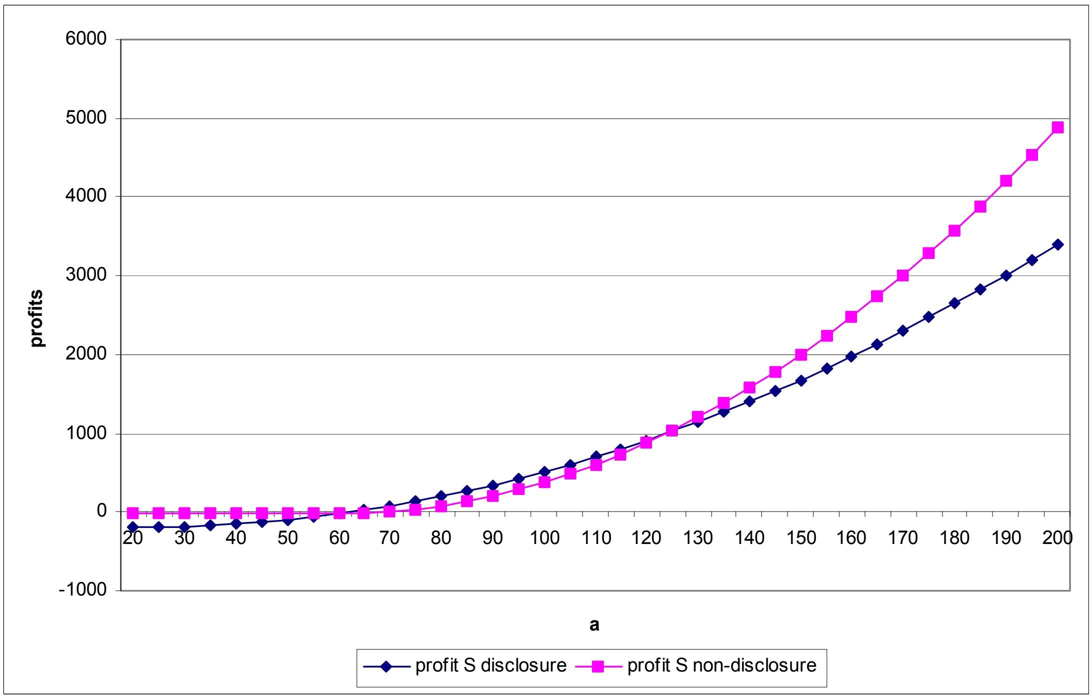
For $(K^D-K^{ND}) \leq [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b$ and $K^D>K^{ND}$ we find a partial disclosure strategy with two distinct non-disclosure intervals, one at the lower end and one at the upper end of the range of possible values for *a*. When observing non-disclosure in this case, L uses the prior distribution of *a* and the boundaries of the non-disclosure intervals to update his beliefs about *a*. Under the assumption of a normal distribution for *a*, L interprets the observation of non-disclosure as a signal that the true value of *a* lies within the upper non-disclosure interval[11]. The prior means $\hat{a}$ is part of this upper non-disclosure interval for $(K^D-K^{ND}) \leq [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b$ and $K^D>K^{ND}$. This interval also partially coincides with the range of values of *a* for which L prefers non-disclosure. This is shown in figure 4. The competitor therefore has no further incentive to induce disclosure of more information and the partial disclosure strategy with two distinct non-disclosure intervals is an equilibrium strategy.

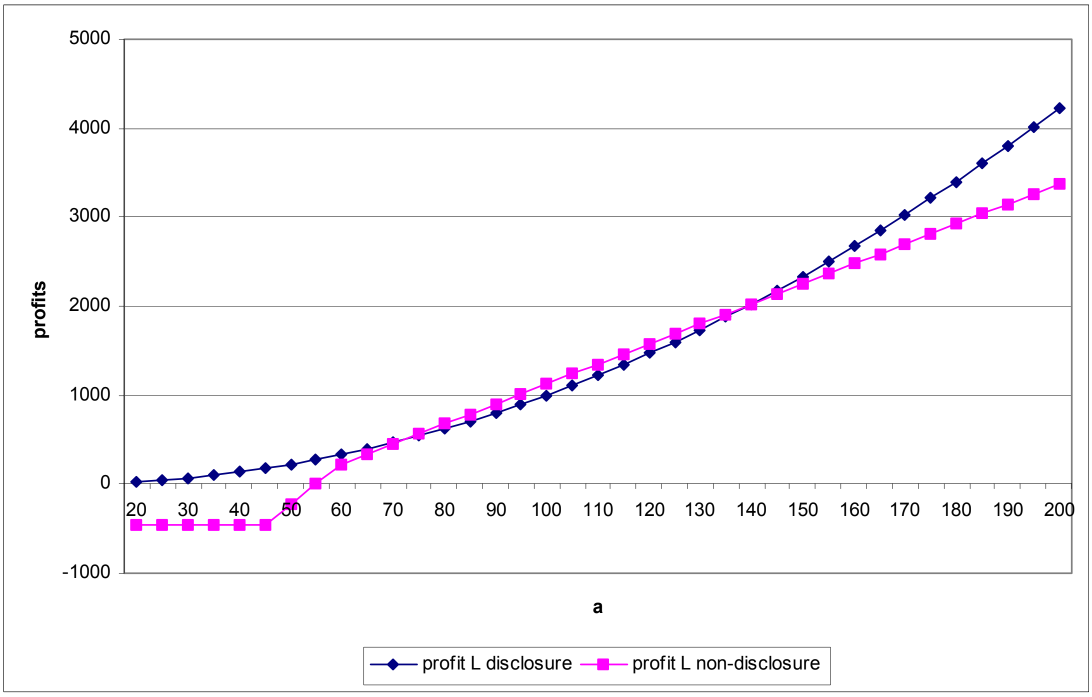
Proposition 3 : If $(K^D-K^{ND}) \leq [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b$ and $K^D>K^{ND}$ then a partial disclosure strategy with two distinct non-disclosure intervals is the equilibrium strategy; for $[\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/45b \geq (K^D-K^{ND}) > [\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/81b$ disclosure is optimal over the interval $[s_1,s_2]$; for $[\hat{a}+t(s_{ND}-\hat{a})+c_L-2c_S]^2/81b \geq (K^D-K^{ND}) > 0$ disclosure is optimal over the interval $[r_2,s_2]$.

[11] Obviously the distribution of *a* is important, especially in this part of the analysis. If the intercept of the demand function would, for example, follow a chi-square distribution with a prior means towards the lower end of the range, then the observation of non-disclosure would most likely be interpreted as a signal for a low value of *a*.

Figure 4 : Partial disclosure equilibrium

panel a : prior beliefs (t=0)

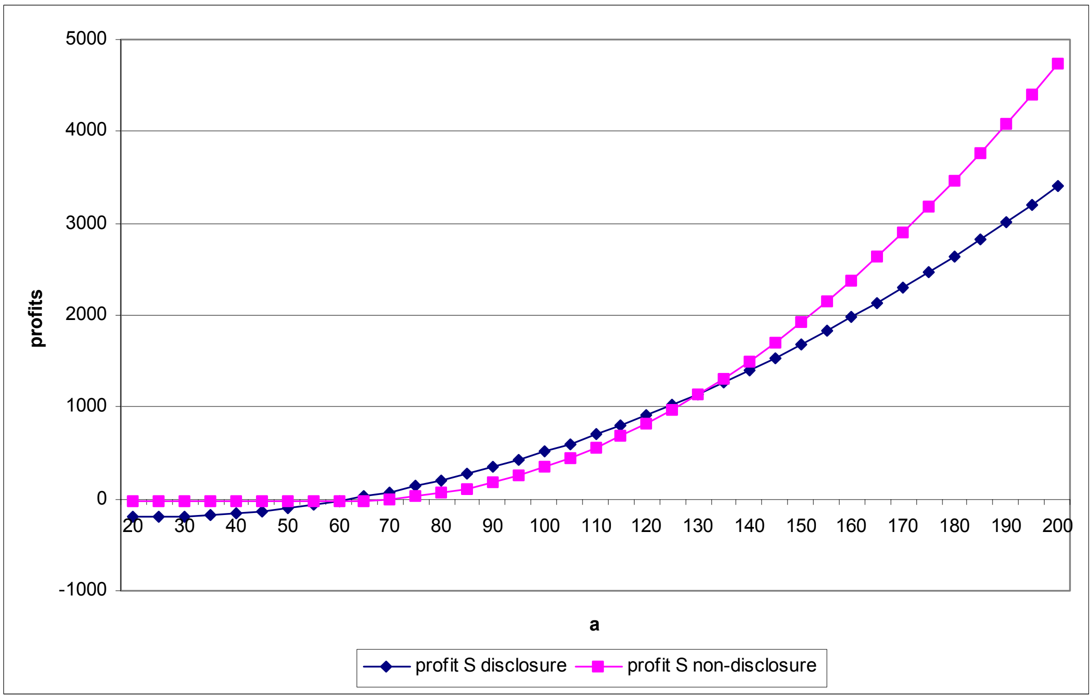


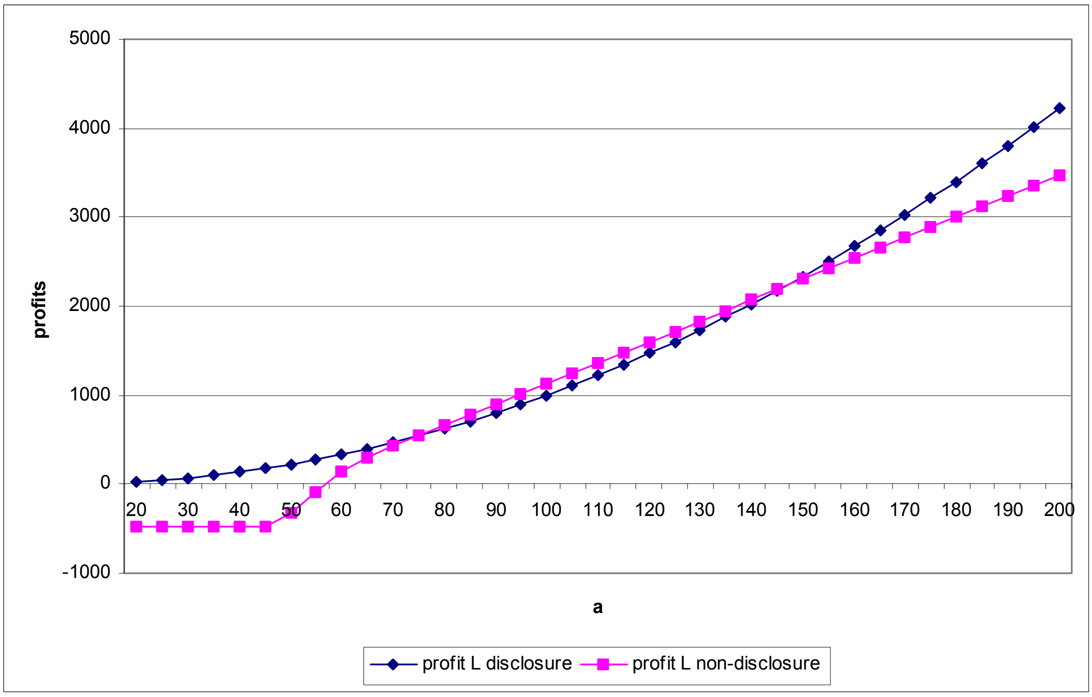


$\hat{a}=140$; $b=1$; $c_S=15$; $c_L=10$; $K^D=200$; $K^{ND}=20$; $A=300$

Figure 4 (continued)

panel b : posterior beliefs (s_{ND}=160; t=0.3)





$\hat{a}$=140; b=1; c_S=15; c_L=10; K^D=200; K^{ND}=20; A=300

D. Discussion of results

Which equilibrium eventually arises depends on the relative values of the parameters. Also the cut off points are smaller or larger depending on the value of the parameters. This has interesting implications for empirical research. Some of these parameters, like the price elasticity of demand, are industry specific. They might explain why in certain industries disclosure is observed more or less often than in others. Other parameters, like the creditors' and customers' reaction to disclosure or non-disclosure are firm specific, so they might indicate which firms within an industry are more likely to disclose sales than others. The parameters that are captured by the shift in the profit function are the interest cost, which is a function of the amount of outstanding debt, and the risk that an important customer uses his bargaining power to negotiate lower prices which reduces the SME's profit. So the difference between disclosure and non-disclosure becomes more important as the SME has more outstanding debt and is more dependent on one major customer.

Although the combination of the competitive effect of disclosure on the one hand and the customers' and creditors' effect of disclosure on the other hand provides several incentives for the SME to keep information private, we find a full disclosure and a partial disclosure equilibrium. Only when the shift in the profit function under disclosure is considerably stronger than the shift under non-disclosure, full non-disclosure can be an equilibrium strategy. This result is in sharp contrast to the general opinion, often heard from practitioners, that it is in the best interest of an SME not to disclose any information unless it is obliged to. Moreover, it contradicts the information sharing literature that was discussed in the introduction. In the models by Ponssard (1979), Novshek and Sonnenschein (1982), Clarke (1983) and Gal-Or (1985) no information sharing is proven to be the only equilibrium strategy for firms that have private information about demand, compete in outputs and produce homogeneous goods. The reason for this striking difference should be found in the basic assumptions underlying the models. In our model it is only the SME that has to make the disclosure decision because for the competitor, which is a large firm, disclosure of sales is mandatory. So it is not really a model of mutual information *sharing* but rather of unilateral information *disclosure*. Second, next to the game of output competition, the SME has to take into account other effects of disclosure, which are introduced as a shift in the profit function which is different under disclosure and non-disclosure. If the shift is stronger under non-disclosure, a full disclosure equilibrium is possible. Third, and perhaps most important, in information sharing models it is usually assumed that firms must precommit to either sharing or no sharing. Translated to our model this would imply that the SME must make the disclosure decision before knowing the actual value of a and the receivers of the information would know this. It is not unlikely that this type of assumption might completely change the results. In Darrough (1993) both ex ante and ex post disclosure about demand are analysed. She also finds that ex ante non-disclosure is the only equilibrium but ex post partial disclosure with two distinct non-disclosure intervals is an equilibrium strategy.

The results here are very similar to the analytical disclosure models we find in the accounting literature, that are built on the same major assumptions about truthful disclosure, endowment of private information and (proprietary) costs of disclosure. Our model combines both an endogenous cost of disclosure, through the effect of the disclosure decision on the output set by the competitor, and an exogenous cost of disclosure, that represents the reaction of creditors and customers. Therefore we find results that are comparable to e.g. both Verrecchia (1983) who proves the existence of a threshold value of disclosure for an exogenous proprietary cost and Wagenhofer (1990) who shows that for a firm faced with a potential entrant, modelled as an endogenous cost of disclosure, a partial disclosure equilibrium with two distinct non-disclosure intervals exists. While in Verrecchia (1983) full non-disclosure is theoretically possible, the model in Wagenhofer (1990) does not allow for a full non-disclosure equilibrium. The potential entrant in Wagenhofer (1990) can enforce full disclosure. In Verrecchia (1983) a full disclosure equilibrium can only be found when the exogenous proprietary cost is zero.

The combined impact of both an exogenous and an endogenous cost of disclosure is also studied by Suijs (1999). He adds a positive exogenous cost of disclosure to the model in Wagenhofer (1990) and finds additional incentives to keep information private. Only full non-disclosure and partial disclosure with two distinct non-disclosure intervals are equilibrium strategies. The reason why we also find a full disclosure equilibrium is that in our model $K^{D} \leq K^{ND}$ is possible. This coincides with a negative exogenous cost of disclosure, whereas in Suijs (1999) only a strictly positive exogenous cost of disclosure is allowed.

VI. Conclusion

In this paper we showed that even when a firm faces different types of disclosure costs and therefore has several incentives to keep information private, partial and even full disclosure of sales are the prevailing equilibrium strategies. The model analyses the equilibrium disclosure decision for an SME that has private information about demand, signalled through sales and competes in output with a large firm for which disclosure of sales is mandatory. Both firms produce homogeneous goods. The reactions of other players like the firm's creditors and/or customers are included in the model as a shift in the profit function. The trade off caused by these different effects of disclosure is often decided in favour of disclosure. Only when the customers' and creditors' reaction to disclosure is considerably less favourable than their reaction to non-disclosure, a full non-disclosure equilibrium exists. When the shift in the profit function under disclosure is still stronger than the shift under non-disclosure but the difference is smaller, we find a partial disclosure equilibrium with two distinct non-disclosure intervals. When the customers' and creditors' reaction to disclosure is more favourable than their reaction to non-disclosure, full disclosure is the equilibrium strategy.

The model presented here is closely related to the voluntary disclosure models in the accounting literature, especially the work by Verrecchia (1983) and Wagenhofer (1990). We combine both an endogenous cost of disclosure, which is the output decision of a large competitor, and an exogenous cost of disclosure which represents the reactions of creditors and customers in the very specific environment of an unlisted SME. In the voluntary disclosure literature usually only one type of disclosure cost is taken into account, while the main effect of disclosure is the reaction of the capital markets. Nevertheless, our results are compatible with the accounting literature and show that even with different types of disclosure costs, disclosure of information can be optimal.

The specific setting of a Cournot duopoly with homogeneous goods and private information about the intercept of the demand function, is inspired by the information sharing literature. However, one of the main assumptions in information sharing is usually a precommitment to share information through a trade association. Moreover, it is usually assumed that both firms receive a (noisy) private signal about demand so that information sharing is a mutual decision. This is the reason why the conclusion in information sharing is that the only equilibrium strategy is not to share any information, while in our model the opposite is found in most cases.

The conclusion that disclosure is an equilibrium strategy in many cases, is maybe not what might have been expected intuitively. However, in practice we see that between 30% and 40% of all Belgian SMEs does disclose sales in the abridged profit and loss account. Our analysis proves that there is indeed an incentive to voluntarily disclose private information, even when the information is about a strategically important variable like sales and is observed by different parties.

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APPENDIX A : Profit Maximising Outputs – Profit Functions

DISCLOSURE :

Standard Cournot game with linear demand and cost functions, homogeneous products and different marginal costs.

NON – DISCLOSURE :

$$\pi_L^{ND} = \underset{q_L^{ND}}{\text{Max}} \left[(1\text{-}t)\hat{a} + ts_{ND} - b(q_S^{ND}+q_L^{ND}) - c_L \right] q_L^{ND}$$

First order condition : $\delta\pi_L^{ND}/\delta q_L^{ND} = 0$

$$\mathbf{q_L^{ND} = [\ (1\text{-}t)\hat{a} + ts_{ND} - bq_S^{ND} - c_L\ ]\ /\ 2b} \qquad \text{Reaction curve firm L} \qquad \text{(A1)}$$

$$\pi_S^{ND} = \underset{q_S^{ND}}{\text{Max}} \left[a - b(q_S^{ND}+q_L^{ND}) - c_S \right] q_S^{ND}$$

First order condition : $\delta\pi_S^{ND}/\delta q_S^{ND} = 0$

$$\mathbf{q_S^{ND} = [a - bq_L^{ND} - c_S\ ]\ /\ 2b} \qquad \text{Reaction curve firm S} \qquad \text{(A2)}$$

Substitute (A2) in (A1) :

$$q_L^{ND} = (1/2b)\ [\ (1\text{-}t)\hat{a} + ts_{ND} - (1/2)(a\text{-}bq_L^{ND}\text{-}c_S) - c_L]$$

$$q_L^{ND} = (1/2b)(1\text{-}t)\hat{a} + (1/2b)ts_{ND} - (1/4b)a + (1/4)q_L^{ND} + (1/4b)c_S - (1/2b)c_L$$

Set $a = E(a|ND) = (1\text{-}t)\hat{a} + ts_{ND}$

$$(3/4)q_L^{ND} = (1/4b)[(1\text{-}t)\hat{a} + ts_{ND} + c_S - 2c_L]$$

$$\mathbf{q_L^{ND} = \ [\ \hat{a} + t\ (s_{ND} - \hat{a}) + c_S - 2c_L\ ]\ /\ 3b} \qquad \text{Profit maximising output L} \qquad \text{(A3)}$$

Substitute (A3) in (A2) :

$$q_S^{ND} = (1/2b)\ [\ a - (1/3)\ [\ \hat{a} + t(s_{ND} - \hat{a}) + c_S - 2c_L] - c_S\ ]$$

$$q_S^{ND} = (1/2b)a - (1/6b)\hat{a} - (1/6b)t(s_{ND}\text{-}\hat{a}) - (1/6b)c_S+(1/6b)2c_L - (1/2b)c_S$$

$$\mathbf{q_S^{ND} = [\ 3a - \hat{a} - t\ (s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L\ ]\ /\ 6b} \qquad \text{Profit maximising output S} \qquad \text{(A4)}$$

$$\pi_S^{ND} = \ [a - b[(\ \hat{a} + t(s_{ND}\text{-}\hat{a}) + c_S - 2c_L)\ /\ 3b + (\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L)\ /\ 6b] - c_S\ ]$$
$$*\ [\ (\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L)\ /\ 6b\ ] - K^{ND}$$

$$\pi_S^{ND} = \ (1/6)[\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L\ ] * (1/6b)[\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L] - K^{ND}$$

$\boldsymbol{\pi_S^{ND} = [\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L\ ]^2 / 36b - K^{ND}}$ Profit function S (A5)

$\pi_L^{ND} = [a - b[(\ \hat{a} + t(s_{ND}\text{-}\hat{a}) + c_S - 2c_L\ ) / 3b + (\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_S + 2c_L) / 6b] - c_L\ ]$

$* [\ (\ \hat{a} + t(s_{ND}\text{-}\hat{a}) + c_S - 2c_L\ ) / 3b\ ]$

$\pi_L^{ND} = (1/6)[\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_L + 2c_S\ ] * (1/3b)[\hat{a} + t(s_{ND}\text{-}\hat{a}) + c_S - 2c_L]$

$\boldsymbol{\pi_L^{ND} = [\ 3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) - 4c_L + 2c_S\ ][\ \hat{a} + t(s_{ND}\text{-}\hat{a}) + c_S\ - 2c_L] / 18b}$

Profit function L (A6)

APPENDIX B : Disclosure Strategy for $q_S^{ND} > 0$ – Calculations

S prefers disclosure to non-disclosure if and only if :

$(a\text{-}2c_S+c_L)^2/9b - K^D \geq [3a - \hat{a} - t(s_{ND}\text{-}\hat{a}) + 2c_L - 4c_S]^2/36b - K^{ND}$

$4a^2+16c_S^2+4c_L^2\text{-}16ac_S+8ac_L\text{-}16c_Sc_L\text{-}36bK^D\text{-}9a^2\text{-}\hat{a}^2\text{-}t^2(s_{ND}\text{-}\hat{a})^2\text{-}16c_S^2\text{-}4c_L^2+6a\hat{a}+6t(s_{ND}\text{-}\hat{a})a+24ac_S\text{-}$
$12ac_L\text{-}2t(s_{ND}\text{-}\hat{a})\hat{a}\text{-}8\hat{a}c_S+4\hat{a}c_L\text{-}8t(s_{ND}\text{-}\hat{a})c_S+4t(s_{ND}\text{-}\hat{a})c_L+16c_Sc_L+36bK^{ND} \geq 0$

$-5a^2 + [6\hat{a}+8c_S\text{-}4c_L+6t(s_{ND}\text{-}\hat{a})]a - 36b(K^D\text{-}K^{ND}) - \hat{a}^2 - 8\hat{a}c_S + 4\hat{a}c_L - t^2(s_{ND}\text{-}\hat{a})^2 - 2t(s_{ND}\text{-}\hat{a})(\hat{a}+4c_S\text{-}$
$2c_L) \geq 0$

d = $[6\hat{a}+8c_S\text{-}4c_L+6t(s_{ND}\text{-}\hat{a})]^2$
$+ 20\ [- 36b(K^D\text{-}K^{ND}) - \hat{a}^2 - 8\hat{a}c_S + 4\hat{a}c_L - t^2(s_{ND}\text{-}\hat{a})^2 - 2t(s_{ND}\text{-}\hat{a})(\hat{a}+4c_S\text{-}2c_L)\ ]$

d = $36\hat{a}^2+64c_S^2+16c_L^2+36t^2(s_{ND}\text{-}\hat{a})^2\text{-}720b(K^D\text{-}K^{ND})+96\hat{a}c_S\text{-}48\hat{a}c_L+72t(s_{ND}\text{-}\hat{a})\hat{a}\text{-}$
$64c_Sc_L+96t(s_{ND}\text{-}\hat{a})c_S\text{-}48t(s_{ND}\text{-}\hat{a})c_L\text{-}20\hat{a}^2\text{-}160\hat{a}c_S+80\hat{a}c_L\text{-}20t^2(s_{ND}\text{-}\hat{a})^2\text{-}$
$40t(s_{ND}\text{-}\hat{a})(\hat{a}+4c_S\text{-}2c_L)$

d = $16\hat{a}^2+64c_S^2+16c_L^2+16t^2(s_{ND}\text{-}\hat{a})^2\text{-}64\hat{a}c_S+32\hat{a}c_L\text{-}64c_Sc_L+32t(s_{ND}\text{-}\hat{a})(\hat{a}\text{-}2c_S+c_L)\text{-}$
$720b(K^D\text{-}K^{ND})$

d = $16[(\hat{a}\text{-}2c_S+c_L+t(s_{ND}\text{-}\hat{a}))^2\text{-}45b(K^D\text{-}K^{ND})]$

$$s_1 = \frac{-6\hat{a} - 8c_S + 4c_L - 6t(s_{ND} - \hat{a}) + 4\sqrt{(\hat{a} - 2c_S + c_L + t(s_{ND} - \hat{a}))^2 - 45b(K^D - K^{ND})}}{-10}$$

$$\mathbf{s_1} = \frac{3\hat{a} + 4c_S - 2c_L + 3t(s_{ND} - \hat{a}) - 2\sqrt{(\hat{a} - 2c_S + c_L + t(s_{ND} - \hat{a}))^2 - 45b(K^D - K^{ND})}}{5}$$

$$\mathbf{s_2} = \frac{3\hat{a} + 4c_S - 2c_L + 3t(s_{ND} - \hat{a}) + 2\sqrt{(\hat{a} - 2c_S + c_L + t(s_{ND} - \hat{a}))^2 - 45b(K^D - K^{ND})}}{5}$$

Interpretation of s_1 and s_2 as intersections of the profit functions under disclosure and non-disclosure if and only if $s_1>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$ and/or $s_2>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$.

$$\frac{3\hat{a} + 4c_S - 2c_L + 3t(s_{ND} - \hat{a}) - 2\sqrt{(\hat{a} - 2c_S + c_L + t(s_{ND} - \hat{a}))^2 - 45b(K^D - K^{ND})}}{5} >$$

$[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$

if and only if :

$9\hat{a}+12c_S-6c_L+9t(s_{ND}-\hat{a})-6\sqrt{(\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a}))^2-45b(K^D-K^{ND})} > 5\hat{a}+5t(s_{ND}-\hat{a})+20c_S-10c_L$

$2[\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})] > 3\sqrt{(\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a}))^2-45b(K^D-K^{ND})}$

$4[\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})]^2 > 9[\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})]^2 - 405b(K^D-K^{ND})$

$[\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})]^2 < 81b(K^D-K^{ND})$

$s_1>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3 \Leftrightarrow (K^D - K^{ND}) > [\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})]^2 / 81b$

$$\frac{3\hat{a} + 4c_S - 2c_L + 3t(s_{ND} - \hat{a}) + 2\sqrt{(\hat{a} - 2c_S + c_L + 2t(s_{ND} - \hat{a}))^2 - 45b(K^D - K^{ND})}}{5} >$$

$[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$

if and only if :

$9\hat{a}+12c_S-6c_L+9t(s_{ND}-\hat{a})+6\sqrt{(\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a}))^2-45b(K^D-K^{ND})} > 5\hat{a}+5t(s_{ND}-\hat{a})+20c_S-10c_L$

$2[\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a})] > -3\sqrt{(\hat{a}-2c_S+c_L+t(s_{ND}-\hat{a}))^2-45b(K^D-K^{ND})}$

which is always satisfied because the left hand term is positive

$s_2>[\hat{a}+t(s_{ND}-\hat{a})+4c_S-2c_L]/3$

APPENDIX C : Disclosure Strategy for $q_S^{ND} = 0$ – Calculations

S prefers disclosure to non-disclosure if and only if :

$[(a-2c_S+c_L)^2/9b] - K^D \geq -K^{ND}$

$(a-2c_S+c_L)^2 - 9b\,(K^D - K^{ND}) \geq 0$

$a^2 + 2(c_L-2c_S)a + (c_L-2c_S)^2 - 9b(K^D - K^{ND}) \geq 0$

$d = 4(c_L-2c_S)^2 - 4(c_L-2c_S)^2 + 36b(K^D - K^{ND})$

$d = 36b(K^D - K^{ND})$

$$r_1 = \frac{-2(c_L - 2c_S) - 6\sqrt{b(K^D - K^{ND})}}{2}$$

$$r_1 = -c_L + 2c_S - 3\sqrt{b(K^D - K^{ND})}$$

$$r_2 = -c_L + 2c_S + 3\sqrt{b(K^D - K^{ND})}$$

Interpretation of r_1 and r_2 as intersections of the profit functions under disclosure and non-disclosure if and only if $r_1 > 2c_S - c_L$ and/or $r_2 > 2c_S - c_L$.

$-c_L + 2c_S - 3\sqrt{b(K^D - K^{ND})} > 2c_S - c_L$

if and only if :

$-3\sqrt{b(K^D - K^{ND})} > 0$ which is impossible $\Rightarrow$ **$r_1 \leq 2c_S - c_L$**

$-c_L + 2c_S + 3\sqrt{b(K^D - K^{ND})} > 2c_S - c_L$

if and only if :

$3\sqrt{b(K^D - K^{ND})} > 0$ which is always satisfied for $K^D > K^{ND}$ $\Rightarrow$ **$r_2 > 2c_S - c_L \Leftrightarrow K^D > K^{ND}$**