Optimal prices, package sizes and package compositions in a market with variety seeking and loyal consumers

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Abstract

In this paper we analyze how the presence of loyal versus variety seeking consumers affects manufacturers' package decisions. Package decisions relate to the price of the package, the quantity of the commodity it contains, and the number of different varieties in it. We first study the optimal consumption of packages by the two types of consumers, taking into account their purchasing, transaction and inventory cost. We then derive market equilibria in the monopoly and in the duopoly case. Depending on the variety seekers' brand switching tendency, and on the quality difference between firms, the resulting duopoly Nash equilibrium is (1) a 'driving out' equilibrium in which one firm is able to deter entry by another firm, or (2) a 'market sharing' equilibrium in which the two firms share the market of variety seeking consumers. In either case, selling different package sizes/compositions is shown to be a profitable price discrimination mechanism, allowing the manufacturer to exploit differences in package preferences and willingness to pay between loyals and variety seekers. The model provides a rationale for package size and composition strategies observed in practice, and indicates when quantity discounts or surcharges are likely to prevail.

Key Words: Variety Seeking Behavior, Price Discrimination, Product Policy, Game Theory

1. Introduction.

After an era of proliferation of brand sizes and varieties, manufacturers and retailers find themselves under increased pressure to 'rationalize' their assortments. Within such a setting, the decision on which brands and/or varieties to offer, in which package sizes and at what prices, has become of crucial importance. Recent papers increasingly emphasize that consumers' reactions and price sensitivity may well differ between pack sizes (see, e.g., Kalyanam and Putler 1997, and Kumar and Divakar 1999). Hence, these reactions should be monitored at the brand-size level instead of at the aggregate brand level. Moreover, not only the size of the package, but also its composition matters. Some consumers are more interested in purchasing 'variety packs', while others prefer packages containing just one product variant. As to unit prices, empirical evidence indicates that quantity discounts and surcharges occur simultaneously in many fast moving consumer goods categories (see, e.g., Agrawal 1993). Also, the unit price format (quantity discounts versus surcharges) has a significant impact on purchase behavior (Manning et al 2000). These widely divergent practices and consumer reactions signal the importance, but also the complexity, of the package size and price selection issue.

The problem of package size pricing has received quite some attention in the economic and the marketing literature. See, e.g. Dolan(1987), Wilcox et al (1987), for an overview. Mechanisms behind and reasons for offering price – quantity discounts and surcharges refer to *supply* effects (e.g. the manufacturer wants to take advantage of economies of scale), or to *demand* effects. In the latter case, price-quantity discounts are often viewed as a price discrimination mechanism, which exploits consumer heterogeneity. Gerstner and Hess(1987), for instance, show how price-quantity discounts may help extract surplus from consumers with different use rates or valuations of the product category, coupled with differences in holding or transaction costs. In a more recent paper, Kalyanam and Putler(1997) demonstrate the impact of demographic variables on consumers' package size choice and willingness to pay. Taking a more psychological perspective, Manning et al (2000) have studied consumer perceptions of quantity surcharges. They find these perceptions to depend on prior beliefs and on the magnitude of the surcharge. Still, while the previous contributions provide interesting insights, a complete picture of why consumer reactions differ, seems to be lacking. As is emphasized by Manning et al (2000): 'The challenge to fully understand consumer reactions to quantity surcharges lies ahead'

In this research, we attempt to shed more light on the above issues by examining the impact of an additional consumer characteristic, viz. that of variety seeking. Typical of variety seeking consumers is their 'inherent preference for change' (see, e.g., Pessemier and Mc Alister 1982, and Steenkamp and Baumgartner 1992). In the context of grocery shopping, variety seekers derive utility from purchasing different products (brands, flavor varieties ..) within a category rather than just one. Loyal consumers, in contrast, are characterized by an 'aversion to change', and tend to stick to the same product (brand and flavor) once bought. Variety seeking tendency has been shown to interact with several marketing mix instruments (see e.g. Papatla and Krishnamurthi 1992 and 1996, Kahn and Raju 1991, and Trivedi 1999 for the differential impact of sales promotions on variety seekers versus loyal consumers; Simonson and Winer 1992 for the effect of display format on consumers' preference for variety; and Steenkamp and Baumgartner 1992 for the differential impact of repeated ad exposures on variety seekers and loyals). Yet, to our knowledge, the impact of variety seeking on package size preferences and willingness to pay has not been studied so far.

Our research contributes to the existing literature in two main ways. First, we model the optimal purchase and consumption decisions of variety seeking and loyal consumers, confronted with packages that vary in price, size and composition. Spelling out the cost and utility implications of purchasing and consumption strategies available to both types of consumers improves our understanding of their behaviour. Second, using the insights from this consumer model, we derive the conditions for market equilibria in the monopoly and in the duopoly case. For the duopoly case we derive various types of Nash equilibria. These equilibria differ in the offers (price, quantity and composition) the firms make to the two types of consumers. This analysis also contributes to our understanding of the phenomenon of quantity surcharges. It should be noted that all the market equilibria we analyse are derived under conditions of asymmetric information: firms do not know the true type of each individual consumer.

The paper is organized as follows. Section 2 deals with the consumer's decision problem. Section 3 uses this analysis to derive the optimal offers of a monopolist. In section 4, we extend our analysis to the duopoly case. Section 5 discusses model extensions, while section 6 provides empirical support for the model implications. Finally, conclusions, limitations and future research areas are taken up in section 7.

2. **The consumer's decision problem.**

In this section, we model the decision problem of consumers facing different packages. Each package is defined by its own price, quantity and composition. In turn, we spell out the package preferences of each consumer type, the utilities they derive from various purchasing strategies, the costs of these purchasing strategies, and the resulting surplus expressions and surplus maximizing decisions.

a. *Consumer preferences and consumer types.*

Consider a product category, where products differ in two attributes: brand and variety. Each manufacturer offers a separate brand, which can be available in different varieties. In this context variety may refer to, e.g., flavor (e.g. yellow tea versus raspberry tea) or type (e.g. regular versus low fat yoghurt). We are interested in the consumer's consumption of these products (combinations of brand and variety) over time. We assume that in each period the consumer adopts at most one product, and that the quantity consumed of any product is either zero or one. While this hypothesis of fixed (maximum) category consumption is clearly restrictive and not realistic in some categories, it is perfectly acceptable in others. It has been used on several occasions in the literature (see, e.g., Wansink et al 1998). If we denote the set of products by $I = \{1, 2, ..., n\}$, and if we denote the consumption of product i in period t by c_{it}, then in each period t we must have for all $i \in I$ that $c_{it} = 0$ or 1, and that $\sum_{i \in I} c_{it} \leq 1$.

The utility enjoyed by a consumer in period t is given by the following utility function:

$$u_t = z_t + \sum_{i \in I} \alpha_i \; c_{it} \left\{ B + (1 - B)(\delta^L c_{it-1} + \sum_{j \in I, j \neq i} \delta_{ij}^V c_{jt-1}) \right\} \tag{1}$$

z_t is the quantity of a numéraire commodity consumed in period t. This commodity is a proxy for all the commodities in the economy different from the products in I.

The second term on the RHS of (1) gives the utility derived from the consumption of products in I. This utility depends, first, on the product chosen in period t, c_{it} and, in particular, on the perceived quality this product i , denoted by α_i. Secondly, the utility in period t of consuming a specific product i also depends on the product chosen in period t-1. The way in which the past choice of product affects present utility is crucial for our purposes. It is at the basis of our definition of two types of consumers: loyals[2] and variety seekers.

If $\delta^L = 1$, and if for all $i, j \in I, i \neq j, \delta_{ij}^V = 0$, then the consumer is of the *loyal type*. For such a consumer, the utility derived from putting $c_{it} = 1$ is greater if, in the previous period, the same product i was consumed.

If , in contrast, $\delta^L = 0$ and $0 \langle \delta_{ij}^V \leq 1$ for all $i, j \in I$, $i \neq j$, the consumer is of the *variety seeking type*. For such a consumer, putting $c_{it} = 1$ yields greater utility if in the previous period, a product j was consumed, with $i \neq j$. Moreover, for a variety seeker this increase in utility may be greater when the products i and j belong to different brands than when they belong to the same brand. If i and j belong to different brands, then $\delta_{ij}^V = 1$, while if i and j belong to the same brand $\delta_{ij}^V = \gamma$, with $0 \langle \gamma \leq 1$. For $\gamma \langle 1$, the consumer derives greater utility from switching between brands than within brands. For $\gamma = 1$, he is indifferent between switching within a brand or between brands.

Note that the utility function specified in (1) has dynamics of order *one*: only the choice of product in the *immediately preceding* period is deemed important for current consumption

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2 We use the term 'loyal' to indicate 'aversion to change' rather than 'adherence to a particular product'. Our loyal consumers have a tendency to stay with the same product (both brand and variety) to avoid too much arousal. They do not necessarily have a clear preference for or commitment to a specific product in the category.

utility. In our model, as indicated above, the dummies δ^L and δ^V reflect the *nature* of the preference dynamics. These dummies determine whether a consumer is loyal or variety seeking. The parameter B, with $0 \langle B \langle 1$, indicates the *strength* of the preference dynamics: the smaller (greater) the value of B, the greater the importance of the variety choice of the last (current) period[3].

Our utility function is very similar to that proposed by Givon(1984), and subsequently used by Seetharaman and Chintagunta (1998). In his seminal article, Givon also considers variety seeking tendency as a first order phenomenon, and places variety seekers and loyals along a continuum ranging from extreme variety seeking (in our model: δ^V =1 and B close to zero) over 'zero order' behavior (in our model: B equal to one) to extreme loyalty (in our model: δ^L =1 and B close to zero). Apart from these basic similarities, our utility function extends Givon's model by allowing for differential variety seeking tendencies across product attributes. More recent papers (see, e.g., Trivedi 1999 and Inman 2001) empirically and conceptually support the contention that variety seeking is attribute driven - brand and variety being the dominant dimensions along which variation is sought, and that consumers' preference for change may well differ across attributes. Building upon these insights, our utility function allows for separate brand and variety switching effects.

b. Utilities realized under alternative purchasing strategies

Firms make offers to consumers in terms of packages. A *package* of a given quality is completely specified by the price p of the package, the total quantity q contained in the package, the number n of varieties in the package, and the quality of the product. Such a package will be denoted by the triple (p,q,n), where $p,q \in R_+$, and where n is an integer. If n=1, the firm is offering a *unipack*, containing only one variety. If $n \geq 2$, the firm is offering a *bundle,* containing at least two varieties. We will assume that all varieties in such a bundle are offered in the same quantities, equal to q/n. We will also assume that the quality parameter α_i is the same for all varieties offered by a given brand. It follows that all varieties in a bundle have the same quality α.

A consumer can follow two different purchasing strategies. One strategy involves the *repetitive purchase of the same package.* Under such a strategy the consumer buys a package (p,q,n), consumes one unit of it per period during q periods, and then buys the same package again. The other purchasing strategy involves the *combination of two different packages*. Under this strategy the consumer simultaneously keeps two packages in store. He consumes one unit from one package in one period, and then consumes one unit from the other package in the next period. If a package is fully consumed, it is replaced. As we will see, this may be an attractive purchasing strategy for variety seekers.

Repetitive Purchasing Strategies

Consider a consumer following the strategy of *repetitive purchase* of the same package (p,q,n), with quality index α. If the package is a unipack, the consumer will consume the

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3 Note that our utility function implies the same maximum utility for loyals and variety seekers – provided that different brands with (the same highest)quality α are available.In both cases, this maximum is α, obtained by loyals through always consuming the same product, and by variety seekers through continuously switching between different varieties of different brands. Even though, in practice, variety seekers *may* extract either higher or lower utility from consumption in the category than loyals, our model seems like a reasonable benchmark case.

same variety in each period. The consumer's utility per period (apart from z_t) is then equal to α if the consumer is a loyal, and it is equal to αB if the consumer is a variety seeker.

Suppose now that the package (p, q, n) is a bundle. A loyal consumer could then switch varieties in each period. This would give him a utility equal to αB. However, he could also consume the same variety during q/n periods, then consume another variety during q/n periods, etc. This would give the loyal consumer an average utility of $\alpha - \dfrac{n\alpha(1-B)}{q}$.
For $q > n$, this utility exceeds the value αB

If the consumer is of the variety seeking type, the best he can do is to switch varieties in each period. He then realizes a utility of $\alpha\tilde{B}$, where $\tilde{B}$ is defined as

$$\tilde{B} = B + (1-B)\gamma \tag{2}$$

From the above, it is clear that the loyal consumer's willingness to pay for a package is always greater if this package is a unipack than if it is a bundle. The reverse is true for a variety seeker, who will derive greater utility from repetitive purchasing if the package is a bundle.

Combining Purchasing Strategies

Consider a consumer who follows the strategy of *combining two different packages* $(\overline{p}, \overline{q}, \overline{n})$ and $(\hat{p}, \hat{q}, \hat{n})$, with the quality indices given by $\overline{\alpha}$ and $\hat{\alpha}$. In lemma 1 of the appendix we show that a loyal consumer will never follow this strategy. A variety seeking consumer will consume one unit from one package in one period, and one unit from the other package in the next period. If the two packages belong to the same brand with quality α, he realizes an average utility equal to $\alpha\tilde{B}$. If they belong to different brands, he realizes an average utility equal to $\dfrac{\overline{\alpha}+\hat{\alpha}}{2}$.When will the variety seeking consumer be willing to pay more for combining these two packages than for purchasing $(\overline{p}, \overline{q}, \overline{n})$ repetitively? Even if $(\overline{p}, \overline{q}, \overline{n})$ is a bundle, and assuming that $\overline{\alpha} > \hat{\alpha}$, the combination strategy will be more appealing if

$$\frac{\overline{\alpha}+\hat{\alpha}}{2} > \overline{\alpha}\tilde{B} \;(> \hat{\alpha}\tilde{B}).$$

This is equivalent to

$$\tilde{B} < \frac{1}{2} + \frac{\hat{\alpha}}{2\overline{\alpha}} = \tilde{B}^* \quad .$$

Hence, if $\tilde{B} < \tilde{B}^*$, the utility of combining the two packages always exceeds the utility of purchasing only one package. Moreover, if $\tilde{B} < \dfrac{1}{2}$, the utility of combining the two packages always exceeds, for all values of $\overline{\alpha}, \hat{\alpha}, \overline{\alpha} > \hat{\alpha}$, the utility of repetitively purchasing $(\overline{p}, \overline{q}, \overline{n})$. A consumer with a utility function where $\tilde{B} < \dfrac{1}{2}$ will be called a *strong variety* *seeker*. Note that, in order to have that $\tilde{B} \langle \frac{1}{2}$, the value of γ has to be sufficiently small. This implies that a strong variety seeker always derives more utility from switching between brands than within brands.

c. Costs of alternative purchasing strategies.

Purchasing, handling and storing packages involve costs for the consumer. In line with previous contributions (see, e.g., Gerstner and Hess 1987), we consider three types of costs. First, there is the purchasing cost. This is the price the consumer has to pay for a certain quantity. Second, there is the transaction cost. This cost comprises the cost of visiting the store and of handling the packages. Finally, there is the inventory costs, i.e., the opportunity cost of money invested in inventories.

We first consider the cost of a *repetitive purchase of the same package* (p,q,n). The average per period cost to the consumer of this strategy is given by

$$C^r(p,q) = \frac{p}{q} + \frac{T}{q} + \frac{h}{2}\frac{p}{q}(q-1). \tag{3}$$

The first term on the RHS of (3) is the average purchasing cost: it is the price paid for the package, divided by the number of periods the consumer can use the package for his consumption. The second term on the RHS of (3) is the transaction cost per period, T being the total transaction cost incurred to acquire the package. The last term of (3) is the average inventory cost. The average inventory of the varieties in the bundle is $(q-1)/2$. The value of the average inventory is then $p(q-1)/(2q)$. The symbol h represents the cost of keeping one unit of the product in store during one period, expressed as a fraction of the purchase price[4].

We now consider the cost per period of *combining two packages* $(\overline{p},\overline{q},\overline{n})$ and $(\hat{p},\hat{q},\hat{n})$. This cost is given by

$$C^c(\overline{p},\overline{q}) + C^c(\hat{p},\hat{q}) \tag{4}$$

where , for any package (p,q,n), $C^c(p,q)$ is given by

$$C^c(p,q) = \frac{p}{2q} + \frac{T}{2q} + \frac{h}{2}\frac{p}{q}(q-1) \tag{5}$$

$C^c(p,q)$ is the cost attributable to the package (p,q,n), of combining this package with any other package. As this package can now be used during 2q periods, the purchasing cost and the transaction cost per period are now given by $p/2q$ and $T/2q$. The average size of the inventory of this package is $(q-1)/2$, so that the inventory cost equals $hp(q-1)/2q$, just like in expression (3).

Comparing (3) and (5), it is easy to derive the following inequalities

$$\frac{1}{2}C^r(p,q) = C^c(p,q) - \frac{h}{4}\frac{p}{q}(q-1) \langle C^c(p,q) \tag{6}$$

4 Note that our inventory cost expression is similar to that of the classical EOQ model.

$$C^c(p,q) = C^r(p,q) - \frac{p}{2q} - \frac{T}{2q} \langle C^r(p,q) \qquad (7)$$

The interpretation of inequality (6) is as follows. Even though, in the context of a combination strategy, package (p,q,n) accounts for only half of the consumer's consumption, the cost per period associated with this package ($C^c(p,q)$) is more than half the per period cost of consuming this package alone in a repetitive purchasing context ($C^r(p,q)$). The reason is that under a combined consumption strategy, each of the packages adopted by the consumer is depleted at a lower pace, implying that the holding cost for the two packages combined is higher than under repetitive consumption.

At the same time, inequality (7) shows that the per period cost of package (p,q,n), consumed in the context of a combined purchasing strategy ($C^c(p,q)$) , remains lower than the per period cost of consuming that package alone ($C^r(p,q)$), the reason being the lower purchasing and handling cost from needing only half of the quantity of repetitive consumption.

d. Surplus maximization

Being confronted with a number of package offers, the consumer has to decide which offer(s) he will accept, and which purchasing strategy he will follow. Clearly, utility maximization subject to the budget constraint, is equivalent to surplus maximization. By surplus we mean the difference between the consumer's maximal willingness to pay (the second term on the RHS of (1)) and the consumer's cost.

Table 1 summarizes the relevant expressions. The first column of this table states the various purchasing strategies for the two types of consumers. The second column gives the utility for each consumer type, and for each possible purchasing strategy. The final column gives the relevant cost function. The surplus for each consumer can then be calculated as the difference between columns two and three.

Table 1 shows that, unless only bundles are available in the market, repetitive purchasing will entail the *same* costs, but a *lower* consumption utility, for variety seekers than for loyals. Combined purchasing will necessitate variety seekers to incur *higher* costs than loyals (see inequality (6)) for, *at best, the same level* of utility. Differently stated, we observe that ceteris paribus, variety seekers constitute the low valuation consumers. Table 1 suggests that, indeed, if both loyals and variety seekers are offered their optimal package sizes (q) and compositions (unipack or bundle), variety seekers will have a lower willingness to pay per unit than loyals.

Table 1: Utility and cost functions for loyals and variety seekers, from different possible purchasing strategies

Purchasing Strategy	**Maximum utility for the consumer**	**Relevant cost function**
LOYALS		
Purchase one pack		
Unipack $(p,q,1)$	α	$C^r(p,q)$

Bundle (p,q,n)	$\alpha - n\alpha(1-B)/q$	$C^r(p,q)$
Combine two packs		
VARIETY SEEKERS		
Purchase one pack		
Unipack $(p,q,1)$	αB	$C^r(p,q)$
Bundle (p,q,n)	$\alpha\tilde{B}$	$C^r(p,q)$
Combine two packs		
Same brand $(\overline{p},\overline{q},1)$ and $(\hat{p},\hat{q},1)$ $\overline{\alpha}=\hat{\alpha}=\alpha$	$\alpha\tilde{B}$	$C^c(\overline{p},\overline{q})+C^c(\hat{p},\hat{q})$
Different brands $(\overline{p},\overline{q},1)$ and $(\hat{p},\hat{q},1)$	$\dfrac{\overline{\alpha}+\hat{\alpha}}{2}$	$C^c(\overline{p},\overline{q})+C^c(\hat{p},\hat{q})$

3. **Optimal package sizes and prices under monopoly.**

In this paper we consider the quality levels offered by the firm(s) as given, leaving only the package price (p), size (q) and composition (n) as decision variables of the firm(s). In a more general model the determination of these quality levels could also be made endogenous. Following the approach taken by A.Shaked and J.Sutton (1982), one could define a two stage game. In the first stage, firms determine quality levels. In the second stage, firms determine p, q and n, taking quality levels as given. In the present paper we only consider the second stage subgame.

We start with the monopoly case. In this case the complete set of products I is produced by one firm only. From a theoretical point of view this is a useful benchmark case. As all packages offered must belong to the same brand, we must have that for $\forall i,j \in I$, $\alpha_i = \alpha_j = \alpha$.

We now wish to identify the packages (p,q,n) a monopolist should offer if he wants to maximize his profits. For simplicity, we assume that the production cost per unit is the same for all varieties, and that it is constant. Without loss of generality we take it to be equal to zero. The monopolist's optimal offer will, of course, depend on the distribution of the consumers over the two possible types. We first consider the two possible extreme cases : all consumers are loyals, and all consumers are variety seekers. The mixed case, in which both consumer types are present, is taken up next.

Suppose, first, that *all consumers are loyals*. In the appendix we prove that in a profit maximum the monopolist will offer the unipack $(\hat{p}^L,\hat{q}^L,1)$, where $\hat{p}^L$ and $\hat{q}^L$ solve the problem

$$\underset{p^L,q^L}{Max}\ \frac{p^L}{q^L} \tag{8}$$

$$\text{s.t.} \qquad \alpha - C^r(p^L,q^L) \geq 0 \tag{9}$$

The objective function is the monopolist's per period revenue. The constraint requires consumer's surplus to be nonnegative. The structure of the problem is illustrated on Figure 1. Constraint (9) only allows points (p^L, q^L) on or below the curve defined by $\alpha - C^r(p^L, q^L) = 0$. Within the feasible set we then look for the point (p^L, q^L) such that the slope of the straight line connecting that point and the origin is maximal. On the figure the solution of problem (8)-(9) is given by the point $(\hat{p}^L, \hat{q}^L)$. This profit maximum leaves zero surplus to the consumer.

It is clear that the solution of problem (8)-(9) must satisfy the condition

$$\frac{\hat{p}^L}{\hat{q}^L} = -\frac{\partial C^r(\hat{p}^L, \hat{q}^L) / \partial q}{\partial C^r(\hat{p}^L, \hat{q}^L) / \partial p} \tag{10}$$

For any combination (p, q), the expression

$$-\frac{\partial C^r(p,q) / \partial q}{\partial C^r(p,q) / \partial p}$$

represents the slope of the isosurplus curve through the point (p, q). It indicates the consumer's marginal willingness to pay for an increase of the quantity q, given that he is repetitively purchasing the package (p, q, n). Any point (p, q) for which the equality

$$\frac{p}{q} = -\frac{\partial C^r(p,q) / \partial q}{\partial C^r(p,q) / \partial p} \tag{11}$$

holds will be called a C^r*-efficient point*. In such a point, the consumer's marginal willingness to pay for an increase in q - given his strategy of repetitive purchase of the package (p, q, n) - is equal to the actual price the consumer has to pay for a small increase in q. Graphically speaking, a C^r-efficient point is a point where the slope of the straight line through that point and the origin is equal to the slope of the iso-surplus curve through the point. Using (3), one can calculate the derivatives appearing in (11). It is then easy to see that the equation of the set of C^r-efficient points is given by $hpq = 2T$.(p,q). On figure 1, C^r-efficient points are given by the curve EE' Condition (10) can then be interpreted by the property that the point $(\hat{p}^L, \hat{q}^L)$ must be C^r-efficient.

Suppose, now, that *all consumers are variety seekers*. In this case, we prove (see appendix) that in a profit maximum the monopolist will offer a bundle $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, $\hat{n}^{VS} \geq 2$, such that the values of $\hat{p}^{VS}$ and $\hat{q}^{VS}$ solve the problem

$$\underset{p^{VS}, q^{VS}}{Max} \frac{p^{VS}}{q^{VS}} \tag{12}$$

$$\text{s.t.} \quad \alpha\tilde{B} - C^r(p^{VS}, q^{VS}) \geq 0 \tag{13}$$

This solution is also illustrated on figure 1. It is clear that the point $(\hat{p}^{VS}, \hat{q}^{VS})$ is also a C^r-efficient point.

The exact location of the point $(\hat{p}^{VS}, \hat{q}^{VS})$ will, of course, depend on the value of γ. If $\gamma = 1$, so that $\tilde{B} = 1$, it is clear that problems (8)-(9) and (12)-(13) are identical, and that therefore $(\hat{p}^{L}, \hat{q}^{L}) = (\hat{p}^{VS}, \hat{q}^{VS})$. If, on the other hand, $\gamma \langle 1$, the maximal profitability, or the optimal price charged per unit, is smaller for variety seekers than for loyal consumers. In the appendix we prove that $\hat{q}^{VS} \rangle \hat{q}^{L}$. The bundle offered to variety seekers is then larger than the unipack sold to the loyals, and it is sold at a lower price per unit (quantity discount). The reason is that, having to consume products that belong to one and the same brand, variety seekers extract lower consumption utility from the category than loyals ($\alpha\tilde{B} \leq \alpha$), leading to a larger efficient package size, and a lower unit price.

< insert Figure 1>

What happens if there are *both loyals and variety seekers*? The interesting question here is whether the offers $(\hat{p}^{L}, \hat{q}^{L}, 1)$ and $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, with $\hat{n}^{VS} \geq 2$, are incentive compatible. It is easy to show that a variety seeker, if offered the packages $(\hat{p}^{L}, \hat{q}^{L}, 1)$ and $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, will always prefer to repetitively purchase $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$. On the other hand, if a loyal consumer is offered the packages $(\hat{p}^{L}, \hat{q}^{L}, 1)$ and $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, he may prefer to repetitively purchase $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$. However, for $\hat{n}^{VS}$ sufficiently large, we show in the appendix that he prefers to repetitively purchase $(\hat{p}^{L}, \hat{q}^{L}, 1)$.

The above results are summarized in the following theorem.

THEOREM 1.

(a) *If all consumers are loyals, the optimal offer of the monopolist is the unipack* $(\hat{p}^{L}, \hat{q}^{L}, 1)$, *where* $\hat{p}^{L}$ *and* $\hat{q}^{L}$ *are obtained by solving problem (8)-(9).*

(b) *If all consumers are variety seekers, the optimal offer of the monopolist is the bundle* $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, $\hat{n}^{VS} \geq 2$, *where* $\hat{p}^{VS}$ *and* $\hat{q}^{VS}$ *are obtained by solving problem (12)-(13)*

(c) *If there are both loyals and variety seekers, the two offers* $(\hat{p}^{L}, \hat{q}^{L}, 1)$ *and* $(\hat{p}^{VS}, \hat{q}^{VS}, \hat{n}^{VS})$, $\hat{n}^{VS} \geq 2$, *are incentive compatible if* $\hat{n}^{VS}$ *is sufficiently large, i.e., if* $\hat{n}^{VS}$ *satisfies*

$$\hat{n}^{VS} \geq (1-\gamma)\, \hat{q}^{VS}$$

(d) *If* $\gamma = 1$, *then* $\hat{q}^{VS} = \hat{q}^{L}$ *and* $\hat{p}^{VS} = \hat{p}^{L}$. *If* $\gamma \langle 1$, *then* $\hat{q}^{VS} \rangle \hat{q}^{L}$, *and* $\frac{\hat{p}^{VS}}{\hat{q}^{VS}} \langle \frac{\hat{p}^{L}}{\hat{q}^{L}}$.

From theorem 1, we learn that in a market consisting of loyals and variety seekers, the best strategy for the monopolist is to offer unipacks to loyals, and (sufficiently varied) bundles to variety seekers. As indicated in part c of the theorem, the separate market offers to loyals and variety seekers are incentive compatible. Variety seekers, who value change, will have a lower willingness to pay for the unipack than loyals. At the same time, loyals will be turned off by the (sufficiently large) variety in the bundle offered to variety seekers. It follows that consumers self- select the item intended for them, allowing the monopolist to capture the full surplus from both consumer groups. The bundle offered to variety seekers will either be larger than the unipack for loyals (in case the variety seekers derive extra utility from brand switching: $\gamma \langle 1$), or be equal in size ($\gamma = 1$). The former situation implies quantity discounts (lower unit prices for the larger bundle), the latter entails equal unit prices between the bundle and the unipack. This monopoly solution constitutes an interesting benchmark for a more realistic competitive setting, which is discussed next.

4. Equilibrium package sizes and prices in a duopoly

We now assume there are two firms, firm a and firm b. Each firm offers its own set of products, indicated by I_a and I_b. Hence, $I = I_a \cup I_b$. As each firm represents one brand, we have that $\forall i \in I_a, \alpha_i = \alpha_a$ and that $\forall i \in I_b, \alpha_i = \alpha_b$. We will assume that $\alpha_a \rangle \alpha_b$, so that the product offered by firm a has a higher quality than the product offered by firm b.

The two firms are then playing the following game. Each firm decides on the packages it is going to offer to consumers. The two firms make these decisions simultaneously. Given these offers, consumers decide on their purchasing and consumption strategies. These decisions determine the sales and profits of the two firms. We use the Nash equilibrium as the equilibrium concept.

As in the monopoly case, we first study the situation of homogeneous markets. We then turn to the mixed market case.

Case 1: All consumers are loyals.

From lemma 1, we know that loyal consumers never combine packages, and that for given values of (p, q), they always prefer unipacks to bundles. Under these circumstances, and given that $\alpha_a \rangle \alpha_b$, one expects firm a to be able to drive out firm b from the market. This can be accomplished as follows. The maximal surplus firm b can ever offer to consumers is α_b. Firm b can do this by offering the unipack $(0, \infty, 1)$. Firm a should then maximize its profits, subject to the constraint that it offers consumers a surplus at least equal to α_b. More formally, firm a has to solve the following problem :

$$\underset{p_a^L, q_a^L}{Max} \frac{p_a^L}{q_a^L} \tag{14}$$

$$s.t. \ \alpha_a - C^r(p_a^L, q_a^L) \geq \alpha_b - C^r(0, \infty) = \alpha_b \tag{15}$$

This leads to the following theorem:

THEOREM 2 (**Driving out equilibrium**)
Assume all consumers are loyals. Let $(\hat{p}_a^L, \hat{q}_a^L)$ be the solution of problem (14)-(15), and let $(\hat{p}_b^L, \hat{q}_b^L) = (0, \infty)$. Then the combination of strategies $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_b^L, \hat{q}_b^L, 1)$, is a Nash equilibrium.

In this equilibrium all consumers buy the unipack from firm a. Firm b effectively disappears from the market. The situation here is very reminiscent of the classical duopoly Bertrand game in which two firms sell an identical product, but the cost of one firm is lower than the cost of the other firm. The low cost firm will then be able to drive out the high cost firm.

If firm a would be a monopolist, constraint (15) would be replaced by constraint (9). The implication is that the duopoly price per unit is lower than the monopoly price, and that the duopoly quantity is greater than the monopoly quantity. Also, in de duopoly solution consumers enjoy the surplus α_b, while in the monopoly solution they enjoy no surplus.

Case 2: All consumers are variety seekers.

A natural question that arises here is whether there still exists a Nash equilibrium in which firm a drives out firm b, i.e., a Nash equilibrium of the type of theorem 2. As $\alpha_a \rangle \alpha_b$ it seems easy for firm a to design a bundle such that all consumers prefer this bundle to whatever bundle offered by firm b. However, the situation is not so simple. Unlike loyals, variety seekers may find it attractive to combine packages from different brands. This leads to a new type of Nash equilibrium , different from the type studied in theorem 2.

We will now analyze these two types of Nash equilibria. In the first type, firm a drives firm b out of the market. This is a *driving out equilibrium*. In the second type, consumers combine packages from both firms, and these firms then share the market. We call this a *market sharing equilibrium.*

Driving out equilibrium in a variety seekers' market

If firm a wants to drive firm b out of the market, and if it wants to do this in an optimal way, it has to find the bundle $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, with $\hat{n}_a^{VS} \geq 2$, such that $\hat{p}_a^{VS}, \hat{q}_a^{VS}$ solve the following problem:

$$\underset{p_a^{VS}, q_a^{VS}}{Max} \frac{p_a^{VS}}{q_a^{VS}} \tag{16}$$

s.t. $$\alpha_a \tilde{B} - C^r(p_a^{VS}, q_a^{VS}) \geq \frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(0, \infty) \tag{17}$$

$$\alpha_a \tilde{B} - C^r(p_a^{VS}, q_a^{VS}) \geq \alpha_b \tilde{B} - C^r(0, \infty) \tag{18}$$

Constraint (17) requires that combining two packages $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ and $(p_b^{VS}, q_b^{VS}, n_b^{VS})$ is never more attractive to consumers than purchasing only package $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, even if $(p_b^{VS}, q_b^{VS}) = (0, \infty)$ and $C^c(p_b^{VS}, q_b^{VS}) = 0$. Constraint (18) requires that repetitively purchasing a bundle $(p_b^{VS}, q_b^{VS}, n_b^{VS})$ is also unattractive, even when $(p_b^{VS}, q_b^{VS}) = (0, \infty)$ and $C^r(p_b^{VS}, q_b^{VS}) = 0$.

Using (7) we easily derive from (17) that

$$\alpha_a \tilde{B} - \frac{\alpha_a + \alpha_b}{2} \geq C^r(p_a^{VS}, q_a^{VS}) - C^c(p_a^{VS}, q_a^{VS}) \rangle\, 0$$

It follows that

$$\tilde{B} \geq \tilde{B}^* = \frac{1}{2} + \frac{\alpha_b}{2\alpha_a} \qquad (19)$$

Therefore, firm a is able to drive out firm b (or, equivalently, the feasible set defined by (17)-(18) is nonempty) if only if consumers are sufficiently weak variety seekers. If inequality (19) holds, firm a is able to offer such a bundle that no consumer is interested in a package offered by firm b, not even the extreme package $(p_b^{VS}, q_b^{VS}, n_b^{VS}) = (0, \infty, n_b^{VS})$ with $n_b^{VS} \geq 2$ .There is then no market left for firm b.

However, the fact that firm a is *able* to do this, does not mean that it is *optimal* for firm a to do this. An alternative for firm a is to give consumers an incentive to combine the offers $(p_a^{VS}, q_a^{VS}, n_a^{VS})$ and $(p_b^{VS}, q_b^{VS}, n_b^{VS})$, and to share the market with firm b. From firm a's point of view, the most favourable conditions for such market sharing are those where firm b is offering the extreme package $(p_b^{VS}, q_b^{VS}, n_b^{VS}) = (0, \infty, n_b^{VS})$. For the driving out strategy to be optimal, it must thus be more profitable than this (most favourable) alternative in which firm a would solve the problem:

$$\underset{p_a^{VS}, q_a^{VS}}{Max} \frac{p_a^{VS}}{2q_a^{VS}} \qquad (20)$$

s.t. $$\alpha_a \tilde{B} - C^r(p_a^{VS}, q_a^{VS}) \leq \frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(0, \infty) \qquad (21)$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(0, \infty) \geq \alpha_b \tilde{B} - C^r(0, \infty) \qquad (22)$$

If, then, the maximal value of (16) exceeds the maximal value of (20), the optimal strategy of firm a is to leave no market for firm b, and to effectively drive out that firm. The following theorem states conditions under which this will be the case.

THEOREM 3a **(Driving out equilibrium)**

Assume all consumers are variety seekers. Let $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ *be the solution of problem (16)-(18). If the inequality*

$$\frac{1}{2} + \frac{\alpha_b}{2\alpha_a} + \frac{T}{2\alpha_a} \leq \tilde{B} \langle 1 \qquad (23)$$

holds, and if (17) is not binding in $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$, *then the maximal value of (16) exceeds the maximal value of (20). The pair of strategies* $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS}) = (0, \infty, \hat{n}_b^{VS})$ *is then a Nash equilibrium for any value* $\hat{n}_a^{VS}$, $\hat{n}_a^{VS} \geq 2$, *and for any value* $\hat{n}_b^{VS}$ *,.*

The proof is given in the appendix. The appendix also indicates sufficient parameter restrictions for (17) not to be binding in $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$.

Theorem 3a shows that, even if all consumers are variety seeking, it may be optimal for firm a - the high quality firm - to drive its competitor out of the market by offering a bundle at a sufficiently low unit price. Consumers will repetitively purchase this bundle, and consume the varieties in it alternatingly. Such a driving out strategy can only be feasible for firm a if consumers are sufficiently 'weak' variety seekers, i.e. if they derive little additional utility from brand switching over variety switching ($\tilde{B}$ sufficiently high). The reason is clear: strong variety seekers (with low levels of $\tilde{B}$) would lose out on too much consumption utility when confined to only one brand. This would lower their willingness to pay, and hence also their profitability, for the driving out brand. Moreover, as indicated in (23) and the Appendix, the driving out equilibrium is more likely to apply, and becomes more profitable, as the quality premium of brand a over brand b (α_a-α_b) increases.

Market Sharing Equilibrium in a variety seekers' market

If inequality (19) does not hold, driving out firm b is no feasible option for firm a. Sharing the market with firm b may then be an attractive possibility. This is especially so if all variety seekers are strong variety seekers, with $\tilde{B} \langle 1/2$, interested in switching between brands. To see this more clearly, consider the following situation. Let firm a offer $(p_a^{VS}, q_a^{VS}, n_a^{VS})$, and let firm b offer $(p_b^{VS}, q_b^{VS}, n_b^{VS})$. If variety seeking consumers combine these two packages, they get a utility of $(\alpha_a + \alpha_b)/2$. For $\tilde{B} \langle 1/2$, this utility is greater than the utilities attainable by buying either package separately, viz. $\alpha_a \tilde{B}$ for $(p_a^{VS}, q_a^{VS}, n_a^{VS})$ or $\alpha_b \tilde{B}$ for $(p_b^{VS}, q_b^{VS}, n_b^{VS})$. By offering consumers the possibility to combine packages, firms can then capture the consumers' higher willingness to pay. At the same time, firms will have to share the market, and – as shown below - can do so in a number of ways. We now describe the nature of this Nash equilibrium in more detail.

Formally, define the set

$$S = \left\{ (s_a, s_b) \ such \ that \ s_a \geq \alpha_a \tilde{B}, \, s_b \geq \alpha_b \tilde{B}, \, s_a + s_b = \frac{\alpha_a + \alpha_b}{2} \right\} \tag{24}$$

where s_a and s_b can be interpreted as firm a and b's share of the consumers' total willingness to pay $(\alpha_a + \alpha_b)/2$.

For each firm i=a,b we can then consider the following problem.

$$\underset{p_i^{VS}, q_i^{VS}}{Max} \frac{1}{2} \frac{p_i^{VS}}{q_i^{VS}} \tag{25}$$

$$\text{s.t. } s_i - C^c(p_i^{VS}, q_i^{VS}) \geq 0 \tag{26}$$

where $(s_a, s_b) \in S$ is given.

In the appendix we prove the following theorem.

THEOREM 3b **(Market sharing equilibrium)**

Let all consumers be strong variety seekers. Let, for any $(s_a, s_b) \in S$, $(\hat{p}_i^{VS}, \hat{q}_i^{VS})$ *be the solution of problem (25)-(26) for i = a,b. Then, for all values of* $\hat{n}_a^{VS}$ *and* $\hat{n}_b^{VS}$ *, the pair of strategies* $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ *and* $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ *is a Nash equilibrium.*

Theorem 3b describes an interesting situation. It shows that, in a market where consumers are strong variety seekers ($\tilde{B} \langle 1/2$) who derive extra utility from brand switching, firms can be better off with than without the presence of a competitor. Even in the absence of any formal agreements or cooperation, they find it optimal to share the market. The intuition is that, by allowing consumers to switch brands, firms (jointly) benefit from the consumers' increased willingness to pay. Each firm offers a unipack or bundle, which is bought by every consumer. Yet, as each consumer combines the packages $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$, each firm captures only half of the total market consumption.

It is interesting to note that the package size offered by firm a in the market sharing situation (Theorem 3b) is substantially smaller than that sold to variety seekers in the driving out case (Theorem 3a) and than the package sold in a loyals' market (Theorem 2). This follows immediately from Lemma 2 and expression (6). The reason is that in a combined purchasing strategy, consumers need to keep *two* packages in store at the same time and, to prevent holding costs from becoming prohibitive, prefer each pack to be smaller than in a repetitive purchasing scenario.

As the set S defined by (24) contains an infinite number of combinations (s_a, s_b), it follows that there are an infinite number of Nash equilibria of the type of theorem 3b, one equilibrium for each combination (s_a, s_b). One way to solve this indeterminacy is to assume that the two firms engage in a bargaining process which will lead to a unique value of (s_a, s_b)[5]. Alternatively, one could argue that one should choose the combination $(s_a, s_b) = (\frac{\alpha_a}{2}, \frac{\alpha_b}{2})$. This is, indeed, a reasonable choice. Each firm's share is exactly equal to its contribution to consumption utility, so that each firm then gets its 'fair' share of the market profits. In what follows, we will assume that - for cases where the market sharing is not unique- firms adopt the 'fair share' $(s_a, s_b) = (\frac{\alpha_a}{2}, \frac{\alpha_b}{2})$. This will be the case in theorem 4b6. We will also show that in some interesting cases (see theorem 4c) the values of s_a and s_b are determined endogenously.

Case 3: Mixed markets

Finally, we consider a mixed market in which a fraction ω^L of the consumers are loyals, while a fraction ω^{VS} of the consumers are variety seekers. A first question is whether the Nash equilibria obtained in the homogeneous market cases can be combined into a Nash equilibrium in the mixed market. More specifically, we first investigate the possibility of

5 To formalize this, one could use the approach developed by Binmore, Rubinstein and Wolinsky (1986).

6 This assumption is mainly introduced for ease of exposition. Allowing for other combinations (s_a, s_b) from the set S does not alter the nature of the equilibria, but slightly changes the expressions for the theorem's necessary conditions.

combining the 'driving out' equilibria of theorems 2 and 3a, into a Nash equilibrium in the mixed market case. Next, we consider the combination of the 'driving out' equilibrium of theorem 2 (loyals' market) with the 'market sharing' equilibrium of theorem 3b (variety seekers' segment).

The following theorem shows that combining the driving out equilibria of theorems 2 and 3a always leads to a Nash equilibrium in a mixed market.

THEOREM 4a **(Driving out equilibrium)**

Consider a mixed market. Let $(\hat{p}_a^L, \hat{q}_a^L)$ *be the solution of problem (14)-(15), and let* $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ *be the solution of problem (16)-(18). Assume that constraint (17) is not binding in* $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$*, and that inequality (23) holds. Then:*

(a) The pair of strategies

<u>for firm a</u>: $(\hat{p}_a^L, \hat{q}_a^L, 1)$ *and* $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ *with*

$$\hat{n}_a^{VS} \geq (1-\gamma)\left(\frac{\alpha_a - \alpha_b}{\alpha_a}\right)\hat{q}_a^{VS} \tag{27}$$

<u>for firm b:</u> $(\hat{p}_b, \hat{q}_b, \hat{n}_b) = (0, \infty, \hat{n}_b)$ *for any integer value of* $\hat{n}_b$

is a Nash equilibrium.

(b) If $\gamma = 1$*, then* $\hat{q}_a^{VS} = \hat{q}_a^L$ *and* $\hat{p}_a^{VS} = \hat{p}_a^L$*. If* $\gamma < 1$*, then* $\hat{q}_a^{VS} > \hat{q}_a^L$*, and* $\frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}} < \frac{\hat{p}_a^L}{\hat{q}_a^L}$.

The proof is given in the appendix. The parameter restrictions for which (17) is not binding are identical to those in the homogeneous market (theorem 3a). Note that (27) certainly holds if $\hat{n}_a^{VS} = \hat{q}_a^{VS}$.

In the equilibrium described in Theorem 4a, firm a – the high quality firm - preempts both markets by offering both a unipack (to loyals) and a bundle (to variety seekers) at entry deterring conditions. If $\gamma < 1$, the bundle offered to variety seekers is larger than the unipack offered to the loyals, and the larger package is offered at lower unit prices. Hence, quantity discounts prevail.

Like Theorem 3a, Theorem 4a still requires (see condition (23)) that the value of $\tilde{B}$ be sufficiently high. For lower values of $\tilde{B}$ (stronger brand switching tendencies among variety seekers), driving firm b out of the variety seeking segment is not profitable for firm a. This firm will be better off sharing the variety seeking segment with firm b. At the same time, however, it wants to keep firm b out of the loyal segment. It is then possible that a combination of theorem 2 (on the loyals' segment) and of theorem 3b (on the variety seekers' segment) constitutes a Nash equilibrium on the mixed market. This leads to theorem 4b.

THEOREM 4b **(Market sharing equilibrium)**

Consider a mixed market with strong variety seekers. Let $(\hat{p}_a^L, \hat{q}_a^L)$ *be the solution of problem (14)-(15), and let* $(\hat{p}_i^{VS}, \hat{q}_i^{VS})$ *be the solution of (25)-(26), with* $s_i = \alpha_i/2$*, for i=a,b. Assume that inequalities*

$$h(\frac{\alpha_a}{4} - \alpha_b)(\frac{2T}{\alpha_a} - 1) \geq \alpha_b \text{ and } \frac{2T}{\alpha_a} > 1 \quad (28)$$

hold. Then:

(a) the pair of strategies

for firm a : $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ with

$$\hat{n}_a^{VS} \geq \frac{(\alpha_a - 2\alpha_b)\hat{q}_a^{VS}}{2\alpha_a(1-B)} \quad (29)$$

for firm b : $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ for any integer $\hat{n}_b^{VS}$

is a Nash equilibrium.

(b) In this equilibrium, $\frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}} < \frac{\hat{p}_a^L}{\hat{q}_a^L}$ *with* $\hat{q}_a^{VS} < \hat{q}_a^L$.

The proof is given in the appendix. Note that (29) certainly holds for $\hat{n}_a^{VS} = \hat{q}_a^{VS}$.

In this Nash equilibrium, the high quality firm a offers two packages. It sells a large unipack to loyals, at an entry deterring unit price. At the same time, it offers a small bundle to variety seekers at a price that leaves room for firm b to profitably enter the variety seeking segment.

The equilibrium in Theorem 4b represents an interesting situation. It describes a case with quantity *surcharges* on the offerings of firm a: $\hat{q}_a^L$ exceeds $\hat{q}_a^{VS}$, while $\frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}} < \frac{\hat{p}_a^L}{\hat{q}_a^L}$. Such quantity surcharges can only prevail thanks to the restrictive conditions specified in (28). Under condition (28), the quality difference between firm a and b is so large that firm a can charge *high unit prices to loyals* and still keep firm b out of the loyal segment. Moreover, condition (28) implies *low enough unit prices in the variety seeking segment* to make the large pack unit price *exceed* that of the smaller package offered to variety seekers[7].

Furthermore, in theorem 4b, the 'driving out' offer of firm a in the loyals' market (theorem 2), and the 'market sharing' offers of firms a and b in the variety seeking market (theorem 3b), are simply put together in the mixed market, and are incentive compatible. Loyal consumers will repetitively purchase the large unipack of firm a. They will certainly prefer this large pack over the small bundle that firm a offers to variety seekers, because - based on (29) - the latter package would force them to consume different varieties instead of just one.

7 A different way of looking at this is that, despite the need to deter entry in the loyal segment, loyal consumers remain the 'high valuation' consumers if condition (28) is fulfilled, and can be charged higher unit prices. See also section 2.

Variety seekers will combine the small bundle offered by firm a, with the small package offered by firm b. A necessary condition for this to apply is that $\alpha_a \rangle 2\alpha_b$. If this inequality does not hold, variety seekers prefer combining $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ to combining $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$. This is proved in lemma 4 of the appendix. If $\alpha_a \langle 2\alpha_b$ the drive out price $\frac{\hat{p}_a^L}{\hat{q}_a^L}$ firm a charges to keep firm b out of the loyal segment is so low that variety seekers will prefer the pack intended for loyals over the pack intended for them. Clearly, theorem 4b can then no longer be valid.

The situation where $\alpha_a \langle 2\alpha_b$ is illustrated on figure 2. A variety seeking consumer, purchasing $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$, will prefer to combine this package with unipack B rather than with package A .To solve this incentive compatibility problem firm a could decrease the price $\frac{p_a^{VS}}{q_a^{VS}}$ to the price implied by point D, indicated on figure 2. The variety seekers' iso-surplus curve (indicated by the dotted line on figure 2) then goes through the point B. Variety seekers are then indifferent between combining $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ with D or B. This solves the incentive compatibility problem. However, it also gives variety seekers a positive net surplus which could be captured by firm b. Firm a can do better by increasing the unit price $\frac{p_a^{VS}}{q_a^{VS}}$. To keep satisfying the incentive compatibility requirement, firm a will have to make the loyals' package less appealing to variety seekers, by increasing its size. Yet, to keep loyals interested in this larger package (which now moves away from their efficient size), it has to decrease the price $\frac{p_a^L}{q_a^L}$. The problem then is to find an an optimal balance between giving up profits on the loyals' market (by decreasing $\frac{p_a^L}{q_a^L}$, starting from B), and increasing profits on the variety seeker's market (by increasing $\frac{p_a^{VS}}{q_a^{VS}}$, starting from D). On figure 2 the optimal points are given by C and E.

The formal statement of firm a's problem is

$$\underset{p_a^L, q_a^L, p_a^{VS}, q_a^{VS}}{Max} \quad \omega^L \frac{p_a^L}{q_a^L} + \omega^{VS} \frac{1}{2} \frac{p_a^{VS}}{q_a^{VS}} \tag{30}$$

$$\text{s.t.} \qquad \alpha_a - C^r(p_a^L, q_a^L) \geq \alpha_b \tag{31}$$

$$C^c(p_a^L, q_a^L) \geq C^c(p_a^{VS}, q_a^{VS}) \tag{32}$$

Note that inequality (32) is equivalent to

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(p_b^{VS}, q_b^{VS}) \geq \frac{\alpha_a + \alpha_b}{2} - C^c(p_a^L, q_a^L) - C^c(p_b^{VS}, q_b^{VS})$$

for any choice of (p_b^{VS}, q_b^{VS}).

<insert Figure 2>

Consider, also, the following problem for firm b.

$$\underset{p_b^{VS}, q_b^{VS}}{Max} \;\; \omega^{VS} \frac{1}{2} \frac{q_b^{VS}}{p_b^{VS}} \tag{33}$$

$$\text{s.t.} \qquad \frac{\alpha_a + \alpha_b}{2} - C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS}) - C^c(p_b^{VS}, q_b^{VS}) \geq 0 \tag{34}$$

where $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ is, together with $(\hat{p}_a^{L}, \hat{q}_a^{L})$, the solution of (30)-(32).

We can now state the following theorem.

THEOREM 4.c.(**Market sharing equilibrium)**
Consider a mixed market. Let $(\hat{p}_a^{L}, \hat{q}_a^{L})$ *and* $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ *be the solution of problem (30)-(32), and let* $(\hat{p}_b^{VS}, \hat{q}_b^{VS})$ *be the solution of problem (33)-(34). If then*

$$\tilde{B} \langle 1 - \frac{\alpha_b}{\alpha_a} \langle \frac{1}{2} \tag{35}$$

then :

(a) the pair of strategies

<u>for firm a:</u> $(\hat{p}_a^{L}, \hat{q}_a^{L}, 1)$ *and* $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$*, for any integer value of* $\hat{n}_a^{VS}$

<u>for firm b:,</u> $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ *for any integer value of* $\hat{n}_b^{VS}$

is a Nash equilibrium.

(b)In this solution, $\hat{q}_a{}^{VS} < \hat{q}_a{}^{L}$ *and* $\dfrac{\hat{p}_a{}^{VS}}{\hat{q}_a{}^{VS}} > \dfrac{\hat{p}_a{}^{L}}{\hat{q}_a{}^{L}}$

In the equilibrium of theorem 4c, the high quality firm a still offers two packages: a large unipack to loyals, and a small unipack or bundle to variety seekers. The unit price of the small package exceeds that of the large package, so that quantity discounts prevail. The lower quality firm, b, is priced out of the loyal segment, but shares the variety seekers' market with firm a. A *sufficient* condition for this equilibrium to hold is that (see (35)) the variety seekers segment consists of 'strong variety seekers' and that quality differences between firms are small ($\alpha_a < 2\alpha_b$).

In this equilibrium, prices and quantities are set in such a way that loyal consumers repetitively purchase the large unipack of firm a, while variety seekers combine the small packages offered by firms a and b. Closer analysis of the Kuhn Tucker conditions for problem (30)-(32) sheds further light on the nature of the solution. First, it is easy to see that in the optimum both inequalities are binding. Moreover, eliminating the Lagrange multipliers leads to the following two optimality conditions

$$\frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}} = -\frac{\partial C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS}) / \partial q_a^{VS}}{\partial C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS}) / \partial p_a^{VS}} \tag{36}$$

$$\frac{\hat{p}_a^L}{\hat{q}_a^L} = -\frac{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial q_a^L}{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial p_a^L} + \frac{\hat{q}_a^L}{2\hat{q}_a^{VS}} \frac{\omega^{VS}}{\omega^L} [\frac{\partial C^c(\hat{p}_a^L, \hat{q}_a^L) / \partial p_a^L}{\partial C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS}) / \partial p_a^{VS}}].[\frac{\partial C^c(\hat{p}_a^L, \hat{q}_a^L) / \partial q_a^L}{\partial C^c(\hat{p}_a^L, \hat{q}_a^L) / \partial p_a^L} - \frac{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial q_a^L}{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial p_a^L}] \tag{37}$$

Condition (36) tells us that the solution $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ is C^c-efficient. It is illustrated on figure 2. This condition can be compared with condition (11) which defines C^r-efficiency. It is easy to derive that the locus of C^c-efficient points is given by the equation $hpq = T$. The two loci of C^r- and of C^c-efficient points are drawn on figure 2. ,

Before interpreting condition (37) let us first note that , for all (p, q) we must have

$$-\frac{\frac{\partial C^c(p,q)}{\partial q}}{\frac{\partial C^c(p,q)}{\partial p}} \langle -\frac{\frac{\partial C^r(p,q)}{\partial q}}{\frac{\partial C^r(p,q)}{\partial p}} \tag{38}$$

This inequality follows easily from (7). Indeed, equality (7) allows us to write derivatives (with respect to p and q) of C^c in terms of the corresponding derivatives of C^r. Property (38) is the *single crossing condition* or the *sorting condition* (see, e.g., Maskin and Riley 1984, and Salanié 1997). It means that, given any package (p, q, n), the loyal consumer, using this package in a strategy of repetitive purchases, has a marginal willingness to pay for a small increase in q which is greater than the marginal willingness to pay of the variety seeking consumer who is combining this package with another package. Graphically it means that on figure 2 the iso-surplus curve through the point (p, q) of the loyal consumer cuts the iso-surplus curve through the same point of the variety seeking consumer from below. This property allows firm a to sort the two types of consumers.

Using (35) in (34), we see that

$$\frac{\hat{p}_a^L}{\hat{q}_a^L} \rangle -\frac{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial q_a^L}{\partial C^r(\hat{p}_a^L, \hat{q}_a^L) / \partial p_a^L} \tag{39}$$

This means that the point $(\hat{p}_a^L, \hat{q}_a^L)$ is not C^r-efficient. This is also clear on figure 2.

Conditions (36) and (37) show that the optimal solution in theorem 4c depends on the relative importance of the loyal (ω^L) and the variety seeking segment (ω^{VS}). As the variety seeking segment becomes smaller, the second term on the RHS of (37) decreases. In the extreme case where ω^{VS} is zero, expression (37) reduces to the optimum described in theorem 2. Loyal consumers get their efficient package at the highest possible (entry deterring) price. Variety seekers will still be offered an efficient package (equation (36)), but one that does not fully capture their willingness to pay, i.e. one for which the strict inequality $C^c(p_a^{VS}, q_a^{VS}) < \alpha_a / 2$ holds. Graphically, this is the combination E and C on figure 2. In this case, to preserve incentive compatibility, the small pack is being made more attractive to variety seekers through lower prices, the large pack remaining the same as in the separate loyal market. As the variety seeking segment grows larger, the solution for loyals will move away from their separate efficiency condition (loyals being offered larger than efficient packs), and the variety seeking solution will come closer to that of the variety seeking segment separately (theorem 3b). In any case, as long as $\omega^L > 0$, the fact that firm a wants to keep b out of the loyal segment makes it pay some price in terms of profitability in the variety seeking segment. Firm b, in turn, benefits from this by reaping extra profit from the variety seekers.

It is easy to see that, in theorem 4c, quantity discounts must prevail. Based on condition (32) in the theorem, the variety seekers are indifferent between the large and the small pack, despite the fact that the large pack size is not 'optimal' or efficient for them. This is only possible if the unit price of the large pack is lower than that of the (optimally sized) small pack!

Table 2 summarizes the Nash equilibria obtained in a mixed market. Note that firm a and b will get the same profit if they replace the unipacks offered to variety seekers by bundles. However, it is logical to assume that in practice, firms will not offer more varieties (bundles instead of unipacks) unless this generates higher profits, hence the description of equilibrium packs in Table 2.

Table 2:Summary table with findings: Mixed Market

Equilibrium Packs	Unit pricing	Condition: Consumers	Condition: Firms	Theorem
High Quality brand offers large unipack to L, and bundle that is either (i) equally large or (ii) larger to VS	(i) Same unit price or (ii) Discount	VS have weak brand switching tendency*		4a
High Quality brand offers large unipack to L, and small bundle to VS Low Quality brand offers small bundle to VS	Surcharge	VS have strong brand switching tendency**	Large quality differences**	4b
High Quality brand offers large unipack to L, and small unipack to VS Low Quality brand offers small unipack to VS	Discount		Small Quality differences	4c

*With additional conditions, see theorem 4a
** With additional conditions, see theorem 4b

Numerical illustrations for each of these cases (theorems 4a, 4b, and 4c) are provided in appendix.

5. Model extensions

The developments above rest on a number of simplifying assumptions. The question arises whether our findings are robust against deviations from those assumptions. In turn, we discuss extensions on the demand side (consumer model) and on the supply side (firms'costs and competition).

5.1. Demand side extensions

Consumer Purchasing and Consumption strategies

In the developments above, we considered only two possible purchasing strategies: repetitive and combined purchasing. In reality, consumers may consider a third option, which we refer to as the 'alternating purchasing strategy' hereafter. When offered the two packages $(\overline{p},\overline{q},\overline{n})$ and $(\hat{p},\hat{q},\hat{n})$, the consumer can first buy and consume the package $(\overline{p},\overline{q},\overline{n})$. The stock of this package is exhausted after $\overline{q}$ periods. He can then buy and consume the package $(\hat{p},\hat{q},\hat{n})$. After $\hat{q}$ periods he will again buy package $(\overline{p},\overline{q},\overline{n})$, etc. The difference with a 'repetitive purchasing' strategy is that *different* packs are bought. The difference with a 'combined purchasing' strategy is that packs are bought *subsequently*, and never available to the consumer simultaneously.

The average cost per period of the alternating purchasing strategy is then simply a weighted average of the costs of the two separate packages:

$$\frac{\overline{q}}{\overline{q}+\hat{q}}C^{r}(\overline{p},\overline{q})+\frac{\hat{q}}{\overline{q}+\hat{q}}C^{r}(\hat{p},\hat{q}) \qquad (40)$$

While allowing for alternating purchasing may make the conditions for some of the theorems more restrictive, it does not invalidate the type of equilibria derived above. More detailed developments can be obtained from the authors.

Customer heterogeneity. The developments above showed that, even if consumers only differ in their variety seeking tendencies, offering different packages may be a profitable price discrimination mechanism. In practice, consumers are heterogeneous on other dimensions, and this heterogeneity may somewhat alter (reinforce or downplay) the propositions above. We discuss four types of heterogeneity, and their implications, below.

*Firs*t, differences in category *use rates* have been known to constitute a rationale for offering various package sizes (see, e.g., Gerstner and Hess 1987), where heavy users are offered large packs (often at lower unit prices), and light users are targeted with smaller sizes. Use rate differences may enhance or diminish the differences in pack size preference between loyals and variety seekers. If heavy (light) users are also those with a weaker (stronger) need for variation, the preference gap and, hence, the potential benefits from package-based price discrimination, becomes more outspoken. Conversely, if use rate and variety seeking tendency are positively correlated, the rationale for offering different package sizes will diminish. Whether positive or negative correlations prevail is not clear a priori, and probably idiosyncratic to categories.

Second, the developments above rely on the premise that all consumers have *identical price/quality sensitivities*. As a consequence, the competitive interplay pictured above is only valid for firms competing in a given price/quality tier – that is: potentially acceptable to one and the same customer group. It may not be valid, for instance, to describe the behaviour of brands at the high end of the market (premium national brands) vis-à-vis brands situated at the very low end of the market (e.g. generics).

Third, even though consumers may find all brands in a given quality tier acceptable, they may still *differ in their quality perceptions* of – and hence, preference for – *these brands*. Relaxing the assumption of preference homogeneity in our model does not jeopardize the basic propositions, but may alter the competitive outcome – and render it more realistic. Suppose, for instance, that the market consists of two submarkets (each submarket comprising loyals *and* strong variety seekers), where firm a is perceived as more appealing than firm b in one submarket but not in the other. It can be shown that, in such conditions, the situation where both firms a and b offer a large and a small unipack may constitute a Nash equilibrium. Each firm exclusively sells its large pack to loyal consumers in the submarket where it is perceived to be most appealing. Moreover, both firms sell their small package to variety seekers in both submarkets (they share the variety seeking consumers in both submarkets). The proof is similar to theorem 4c, and can be obtained from the authors. As these developments illustrate, preference heterogeneity across brands does not invalidate the type of Nash equilibria derived from our model.

Finally, consumers are likely to differ in their preferences for varieties. Of particular interest here is the distinction between *regular* versus *exotic varieties*. Regular varieties correspond to 'normal' or base versions of the category. Examples are milk chocolate bars, salt potato chips, and yellow tea. Exotic varieties are more 'out of the ordinary' versions, such as – for instance, chocolate bars with banana liquor, potato chips with curry taste, and raspberry tea. By nature, regular varieties are more likely to be consumed on a 'repetitive' basis, and selected in the context of 'repetitive purchasing strategies' by loyal consumers. Exotic varieties, in contrast, tend to be consumed 'for a change', and fit into combined purchasing strategies of strong variety seekers. Since our equilibria involve targeting different packages to loyals and to variety seekers, the implications of this type of preference heterogeneity are quite straightforward. More specifically, for markets consisting of a mixture of loyals and strong variety seekers, theorems 4b and 4c imply that exotic varieties (primarily directed at variety seekers) should be offered in fewer package sizes, and be more predominantly offered in smaller packs, than regular varieties.

Order of the utility function - dynamics. The utility function used above has dynamics of order one: the only 'relevant' component from the past being the immediately preceding period. Increasing the order of the dynamics will have two major implications for competitive equilibria. First, firms aiming to deter competition in the variety seeking segment (see theorems 3a and 4a) will need to increase the number of varieties they include in their bundle offers from 'two' to 'one plus the order of the utility dynamics'[8]. Second, if brand switching tendencies are strong and market sharing is called for (conditions described in theorems 3b, 4b and 4c), such market sharing may involve more than two competitors. Supply in the variety seeking segment is likely to become more fragmented, and even weaker firms will find it easier to secure themselves a piece of the pie (see also below).

5.2. Supply side extensions

Cost of additional varieties.

The cost functions above assume that unit costs for a firm are independent of (1) the package size, (2) the package assortment (number of package sizes offered by the firm), (3) the variety considered and (4) the variety assortment (number of varieties offered by the firm). Even though relaxing these assumptions is not likely to invalidate our equilibria, it may alter the conditions under which different equilibria apply as well as the optimal values of the decision variables (package sizes, prices and number of varieties per pack). For instance, if adding a variety is more costly than adding a package size or producing in smaller quantities, the market sharing equilibrium described in theorem 4c becomes relatively more appealing than the entry deterring equilibrium described in theorem 4a – at least, if quality differences are sufficiently low. Such cost considerations, therefore, have to be 'added' to the demand side mechanisms described in our model, and may render them more complex.

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8 Note that in the developments above, offering a third, fourth, … variety did not increase the consumption utility of a bundle to variety seekers.

Number of competitors. In the variety seeking segment, consumers' strong brand switching tendency makes the duopolist firms complementary rather than rival such that, in the 'shared market' solution, they 'jointly' reap monopolistic benefits. Adding a third competitor will alter this situation, and call for entry deterrence in the loyal segment (firm a deters b) as well as the variety seeking segment (firms a and b deter firm c). Yet, to the extent that this 'third' competitor (c) has lower market appeal than the first (a) and the second (b) market player, each of the Nash equilibria described above may still occur - be it under slightly modified conditions. It is easy to see, for instance, that the introduction of a third firm only requires that, in Theorem 4c, α_a is replaced by $\alpha_a - \alpha_c$, and α_b is replaced by $\alpha_b - \alpha_c$. Proof for this theorem is similar to the developments for Theorem 4c.

6. Empirical validation

While based on a 'stylized' framework, the developments above lead to a number of verifiable implications. First, they underscore that, in mixed markets (with loyal consumers and strong brand switchers), a rational, profit maximizing manufacturer may offer his products at quantity discounts as well as surcharges – be it that the latter occur under quite restrictive conditions. Second, our derivations suggest that different package size strategies may prevail for *higher quality (premium) brands compared to lower quality brands,* and *for regular varieties compared to exotic varieties*. Following the developments above, higher quality brands and regular varieties would be more often available (i) in multiple sizes and (ii) in large packs, provided both firms are active.

Table 3: Price-Quantity elasticities[a] in categories with loyalty and variety seeking

Category	**Price-Quantity elasticity: Median (Range)**
Ice Cream	0 (-0.01,.1024)
Canned Fruit	-.226 (-.392,.027)
Chocolate Spread	-.1587(-.181,-,132)
Cereals	-.167 (-.539,-.083)
Jam	-.074 (-.094,-.073)
Chocolate Bars	-.045 (-.096,-.010)
Coffee	-.032 (-.310,0)
Tea	-.050 (-.059,-.045)
Canned Dog Food	-.135 (-.220,.345)
Potato Chips	-.110 (-.616,.605)

[a] For each pair of products that differ only in package size, the price-quantity elasticity is computed as: $\frac{(Unit\ price\ large - Unit\ price\ small)/Unit\ price\ small}{(Size\ large - Size\ small)/Size\ small}$

To check these propositions, we collected price and package assortment information in different product categories, from a large retail chain. Since our propositions are derived for a 'mixed market' setting, only categories with variety seeking as well as loyal behavior were considered. Indications on such mixed behavior were obtained from Givon(1984), and Campo (1993). Brands in each product category were classified into 4 brand types: A-brands (Premium, National brands), B-brands (Secondary, often Regional Brands), Private Labels and Hard Discounters/Generics. Moreover, the varieties offered in each category were classified as 'regular' versus 'exotic[9]'.

[9] For both the brand and the type dimension, classifications were obtained from two independent

Table 3 provides information on unit pricing. For each product category, it reports the average and the range of price-quantity elasticities for different product categories. Within each category, the price-quantity elasticity per pair of products that differ only in pack size, is computed as the percentage increase in unit price between the small and the large pack, relative to the percentage increase in pack size. These elasticities allow for meaningful aggregation over product pairs within categories. They also allow for comparison across categories. Negative price-quantity elasticities point to quantity discounts, positive values indicate surcharges. The table shows that surcharges *do* occur, but less frequently than discounts – a result consistent with our derivations, as well as with previously reported empirical findings (Agrawal 1993, Manning et al 2000).

Table 4: % of products offered in multiple packs for A and B brands, in categories with loyalty and variety seeking

Category	A brands		B brands	
	All[a]	Regular	All	Regular
Ice Cream	.176	.231	0	0
Canned Fruit	.25	.25	0	0
Chocolate Spread	.385	.500	.133	.333
Cereals	.625	-	0	-
Jam	0	0	0	0
Chocolate Bars	.270	.555	0	0
Coffee	.161	.250	0	0
Tea	.083	1	.166	1
Canned Dog Food	.369	.666	.25	0
Potato Chips	.2	1	0	0

[a]: The denominator comprises the total number of <u>different</u> products (each product being a unique combination of brand and variety) offered, the numerator how many of these are available in at least two pack sizes.

Table 4 provides indications on the % of products[10] offered in multiple packs, as a function of brand type, and for all products (regular + exotic) in the category - as opposed to regular-type products only. To verify our propositions, we concentrate on the comparison of A and B brands[11]. The table clearly supports the premise that high quality brands (A-brands) engage more often in a multiple pack strategy than lower quality brands (B-brands). An exception is the Tea category, where the % of B-brand products offered in several packs exceeds that of A-brands. Closer inspection of the data reveals that this is true because, in contrast to B-brands that only comprise regular-type products, A-brands also offer a number of exotic tea variants, available in only one size. This is indeed a second finding from table 4: that regular-type products are offered in more sizes than exotic variants. For any brand type, the % of regular-type products available in at least two pack sizes is always larger than the % of all products pooled together. Table 4 therefore confirms the proposition that a multiple pack

experts.The matching coefficient of classifications over all categories served as a reliability check. For brands, this coefficient amounts to .93, indicating that brand assignments are reliable. For type classification (regular versus exotic) the matching coefficient is somewhat lower (.87), but still acceptable. As a further check, we conducted the analysis excluding the regular/exotic varieties on which experts disagreed. This did not alter the conclusions, suggesting that our findings are robust.

[10] Each product being a unique combination of brand and variety.

[11] The private label and generic brands were treated separately for a number of reasons. First, being marketed by the retail chain itself, the retailer may be more inclined to allocate more different sizes and shelf space to these brands. Second, because f heir different price/quality positioning, these brands may not compete for the same customers as National brands (see, e.g., Ailawadi et al, 2001). Since our propositions are derived for a framework with homogeneous consumers in terms of price/quality perceptions and price/quality trade-offs, we do not necessarily expect them to hold for brands competing for consumers that strongly differ in these respects.

strategy is more likely to be adopted for A brands (compared to B-brands), and for regular-type products (compared to exotic variants).
As mentioned above, our derivations imply that A-brands and regular varieties are more likely to be found in large packs, than competing B-brands or exotic products. To check this, package sizes in each product category are classified into two (small versus large) or three (small, medium and large) levels[12].

Table 5a reports, for each brand type and package size, the presence of this brand type in the considered size, relative to its presence in the category as a whole[13]. If this ratio- referred to as the index of availability in a package size - exceeds one, this indicates that the brand type is more predominantly available in that package size; if it is below one, the brand type offers relatively fewer products in that size. Again, we report the results for national (A and B) brands. The table shows that, as expected, A-brands are relatively less available in the small size, and more so in the large packs, than B brands. The Tea and Jam categories constitute an exception: here, the ratio for A in the large size is below 1. This is caused by two factors: the large number of (small sized) exotic products offered by A-brands, and the wide availability of the private label in the large packs. Note also that, since the number of products in this category is very small, the results for tea should be treated with caution.
Table 5b offers information on the size availability for regular compared to exotic items. Overall, it is found that regular-type products are relatively more present in larger sizes, while exotic variants tend to be offered in the small size. This confirms our propositions.

Of course, while the tables support the propositions derived from the model, the underlying rationale may be different from that provided in the model. As a stronger test, we therefore compare the figures above, to the package size patterns[14] for categories with no variety seeking (Mayonnaise, Milk, Toilet Paper, Washing Powder, Margarine, and Diapers, see Table 6). In those categories, different packs are probably targeted at consumers with various use rates rather than differing needs for variation. Interestingly, the pattern of findings in categories with no variety seeking is indeed different. Although these categories also point to a low presence of B-brands in the large size, B-brands are equally under-represented in the smallest size. It seems that, in these categories, A-brands simply offer more packs and more extreme (small and large) sizes, while B-brands limit themselves to offering an intermediate size[15].

Table 5a : Index of brand availability in small, medium and large pack in categories with loyalty and variety seeking

Category	A brands			B brands		
	Small	Medium	Large	Small	Medium	Large
Ice Cream	.821	-	1.917	1.269	-	.289
Canned Fruit	.607	1.080	.530	1.640	.875	.398
Chocolate Spread	.775	-	1.869	1.177	-	.322
Cereals	.99	-	1.06	1.12	-	0
Jam	1.10	-	0	.63	-	3.70
Chocolate Bars	.52	1.50	1.51	1.77	.32	0
Coffee	.97	-	1.07	1	-	1

12 Only for washing powder, a few products were offered in four different sizes. For simplicity of exposition, we report the results for the two largest sizes together.

13 Specifically, the index is defined as [(number of products offered by brand in size)/(total number of products available in that size)]/[(number of products offered by brand)/(total number of products available)]

14 Since these categories hardly contain exotic items, we concentrate on the package sizes available for *brand types*.

15 The willingness of A-brands to offer the small pack can be interpreted as a 'service' to the low use rate segment and/or an attempt to pre-empt the market.

Tea	1.08	-	.52	.60	-	1.40
Canned Dog Food	.76	1.36	1.07	1.84	.92	0
Potato Chips	.71	-	1.21	1.46	-	0

Table 5b : Index of Product Type availability in small, medium and large pack in categories with loyalty and variety seeking

Category	Regular items			Exotic items		
	Small	Medium	Large	Small	Medium	Large
Ice Cream	.774	-	2.00	1.225	-	0
Canned Fruit	.888	.957	1.383	1.215	1.083	.265
Chocolate Spread	.839	-	1.618	1.260	-	0
Jam	.93	-	1.72	1.10	-	0
Chocolate Bars	.55	1.28	2.00	1.45	.72	0
Coffee	.92	-	1.40	1.15	-	.30
Tea	.49	-	3.81	1.13	-	.25
Canned Dog Food	.83	1.10	1.16	1.15	.96	.87
Potato Chips	1.25	-	.84	.78	-	1.14

[a] only categories with regular versus exotic products are included in this table

Table 6: Index of brand availability in small, medium and large pack in categories with little variety seeking

Category	A brands			B brands		
	Small	Medium	Large	Small	Medium	Large
Milk	1.51	-	.84	0	-	1.32
Margarine	1.20	.83	1.33	0	1.67	0
Mayonnaise	.91	.86	1.06	0	1.79	0
Baby Diapers	1.43	.88	1.07	0	1.90	0
Washing Powder	.97	1.21	.50	1.56	1.56	.34
Toilet Tissue	1.20	.68	1.50	0	1.64	0

7. Conclusions and Future Research

This paper analysed how the presence of consumers with different preference dynamics (loyals versus variety seekers) affects manufacturers' package size pricing strategies.
Our analysis shows that selling *different packs* at different prices can be a profitable price discrimination mechanism, exploiting differences in package (size) preference and willingness to pay between variety seeking and loyal consumers. Offering a uni-pack (to loyals) next to a bundle pack with several varieties (to variety seekers) may allow the high quality brand manufacturer to maximally extract surplus from both consumer types, while keeping competitors at bay (Theorem 4a). Yet, if variety seekers exhibit a strong brand switching tendency, such a strategy is not optimal, and the manufacturer may actually *benefit* from the presence of a competitor in the variety seeking segment. Indeed, the availability of alternative brands may increase the variety seekers' consumption utility in the category, and hence their willingness to pay. As a result, the high quality brand manufacturer may find it more profitable to share the variety seeking segment with his competitor. In the resulting Nash equilibrium (Theorem 4c), the higher quality brand will typically offer only *one variety, but in different sizes*. The smaller size is tailored to the needs of variety seekers, while the large size pack is offered to loyal consumers. The lower quality brand will supply a small uni-pack only, targeted at the variety seekers. Exceptionally, for large quality differences, both firms will replace their small pack to variety seekers by a small bundle, thereby discouraging loyals – whose entry deterring prices are still fairly high because of the quality difference – from purchasing the offer to VS (Theorem 4b).

Our model equally sheds light on whether *quantity discounts or surcharges* are likely to prevail. In the drive out equilibrium (Theorem 4a), the large bundles offered to variety seekers are sold at a lower unit price than the (somewhat smaller) unipacks made available to loyals, hence quantity discounts prevail. In the market sharing equilibria, two situations may arise. If the quality difference with his competitor is limited (Theorem 4c), the high quality manufacturer will offer his large pack (to loyals) at a discount. The underlying rationale is that a low unit price is required to seal the loyal segment against competition. Only if the quality difference is large, deterring entry from the loyal segment is possible while keeping unit prices high. In such a case, the loyals *may* (under restrictive conditions, see Theorem 4b) even be charged a higher unit price for their large packs than the variety seekers (quantity surcharges), given their potential to extract higher consumption utility from uni-packs at lower costs. Not only do these insights improve our understanding, they also have managerial relevance and can be valuable to manufacturers and retailers) seeking to 'optimize' their assortment of package sizes and corresponding prices.

While the model framework is stylized, empirical evidence appears to support the analytical results. Our findings are consistent with the observation that, whereas both quantity discounts and surcharges may occur, the latter are the exception rather than the rule, especially in product categories where close competitors (with small quality differences) are present. The data also confirm the proposition that, in categories where some consumers are loyal and others are (strongly) variety seeking (brand switching), premium (A) brands are typically available in small and large sizes, while lower quality (B) brands stick to offering smaller sizes. Our propositions are also consistent with the empirical observation that 'regular' product variants – more likely to be consumed by loyal consumers – are more widely available in large packs than 'exotic' variants - addressed at variety seekers.

From a competition perspective, our findings are consistent with Inman (2001)'s statement that, in cases where 'variety switching' (switching along the flavor:type dimension) prevails, firms find it worthwhile to 'tie' variety seekers by offering a complete assortment of varieties within their brand line. Our 'drive out-equilibrium' typifies such situations. At the same time, our findings confirm the statement of Feinberg et. al. (1992) that the profit potential and rate of success of lower qualitative brands increase with the degree of brand switching in the market. As the consumer's brand switching tendency increases, brands have a higher incentive for 'co-habitation' with competitors. While this implies that they have to settle for only a portion of consumer purchases, such market sharing also allows them to charge higher unit prices than in a drive out strategy. Note that, in our model, these observations only apply to products that are still sufficiently comparable in quality to have a large overlap in customer base. Generic products or hard discounters may have a quality and a price level so out of line with competitors, that they appeal to a 'separate' segment of extremely price sensitive consumers. For those consumers, (low) price is the predominant driver of consumer surplus and, hence, product choice. The impact of variety seeking on consumption utility will tend to be negligible for these consumers, who are mainly seduced by the lower unit price of larger packs.

Clearly, our analysis exhibits a number of limitations. Consumers are assumed to be rational, and perfectly informed about the market offer. Though these assumptions have often been used in marketing contributions, they might affect the realism of our results in some (complex, price and quality sensitive) product categories.

Another limitation is that of constant category consumption rates per consumer: as long as consumers get non-negative surplus, altering the packages and prices does not lead to category expansion or contraction effects. While this assumption seems perfectly reasonable in some categories, it is unlikely to hold in others (Wansink et al 1998). Intuitively, we do not expect deviations from constant consumption rates to invalidate the mechanisms derived from our model, but a more formal analysis is called for. Like many authors before us, we concentrate on purchase decisions in a single product category, aggregating other categories into a ‘numeraire commodity’. Future research may entail a more comprehensive study on package size pricing and variety seeking, incorporating multiple category purchases and consumer budget constraints.

In our consumer decision problem, transaction costs are incurred for each package acquired by the consumer. Future extensions could distinguish between shopping costs (that is, a fixed amount to be paid whenever a purchase is made independent of how many units are bought, for instance: the cost of a store visit) and package handling costs. To the extent that the cost of a store visit remains small compared to other cost levels, it is not likely to greatly affect the model outcomes. Given that most shopping trips imply purchases in many product classes simultaneously, store visit costs may not be too relevant for product classes where consumption of one pack takes longer than the average inter-shopping time. Even in cases where store visit costs are important and relevant, the equilibria derived above may remain valid as long as the holding cost of opened packages (which is the type of inventory cost encountered in our present calculations) is substantially larger than that of unopened packs. Even so, future research may concentrate on a more formal analysis of the impact of shopping versus package handling cost on optimal package sizes and prices.

Finally, in our model, each firm offers only one brand. In reality, firms may offer multiple brands in the same market and – in doing so – avoid the need for market sharing while catering to consumers’ brand switching tendencies. In practice, there are limitations to the number of brands a firm can offer, either because of increasing costs linked with marketing an additional brand (costs of internal coordination, and of ‘convincing’ retailers to allocate their shelf space to products from one and the same firm), or because of ‘legal’ limitations (regulations against monopolies). Nevertheless, extending the analysis to multibrand firms may be a relevant endeavor for future research.

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Appendix

Lemma 1: *Loyal consumers never combine different packages.*

Proof

Consider two packages $(\overline{p}, \overline{q}, \overline{n})$ and $(\hat{p}, \hat{q}, \hat{n})$, with qualities given by $\overline{\alpha}$ and $\hat{\alpha}$, and with $\overline{n} \geq 1$ and $\hat{n} \geq 1$. If a loyal consumer repetitively purchases the package $(\overline{p}, \overline{q}, \overline{n})$, he realizes a surplus

$$\overline{\alpha} - C^r(\overline{p}, \overline{q}) \tag{A.1}$$

if $\overline{n} = 1$, and a surplus

$$\overline{\alpha} - \frac{\overline{n}\,\overline{\alpha}(1-B)}{\overline{q}} - C^r(\overline{p}, \overline{q}) \tag{A.2}$$

if $\overline{n} \geq 2$.

Similarly, if he repetitively purchases the package $(\hat{p}, \hat{q}, \hat{n})$, he realizes a surplus

$$\hat{\alpha} - C^r(\hat{p}, \hat{q}) \tag{A.3}$$

if $\hat{n} = 1$, and a surplus

$$\hat{\alpha} - \frac{\hat{n}\hat{\alpha}(1-B)}{\hat{q}} - C^r(\hat{p}, \hat{q}) \tag{A.4}$$

if $\hat{n} \geq 2$.

If a loyal consumer combines the packages $(\overline{p}, \overline{q}, \overline{n})$ and $(\hat{p}, \hat{q}, \hat{n})$, and if in each period he switches consumption from one package to the other, he realizes a surplus

$$\frac{\overline{\alpha} + \hat{\alpha}}{2} B - C^c(\overline{p}, \overline{q}) - C^c(\hat{p}, \hat{q})$$

$$= \frac{1}{2}\left[\overline{\alpha}B - 2C^c(\overline{p}, \overline{q})\right] + \frac{1}{2}\left[\hat{\alpha}B - 2C^c(\hat{p}, \hat{q})\right] \quad \text{(A.5)}$$

By (6) this is smaller than

$$\frac{1}{2}[\overline{\alpha}B - C^r(\overline{p}, \overline{q})] + \frac{1}{2}[\hat{\alpha}B - C^r(\hat{p}, \hat{q})] \quad \text{(A.6)}$$

As

$$\overline{\alpha}B \langle \overline{\alpha} \qquad \text{and} \qquad \overline{\alpha}B \leq \overline{\alpha} - \frac{\overline{n}\overline{\alpha}(1-B)}{\overline{q}}$$

$$\hat{\alpha}B \langle \hat{\alpha} \qquad \text{and} \qquad \hat{\alpha}B \leq \hat{\alpha} - \frac{\hat{n}\hat{\alpha}(1-B)}{\hat{q}}$$

this implies that (A.6), and hence also (A.5), is smaller than the maximum of (A.1), (A.2), (A.3) and (A.4). •

Lemma 2: *Let A be any positive constant, and consider the following problem*

$$\underset{p,q}{Max} \frac{p}{q} \quad \text{(A.7)}$$

$$s.t. \quad A - C^r(p,q) \geq 0 \quad \text{(A.8)}$$

Then the values

$$\hat{q} = \frac{T}{A} + \left[(\frac{T}{A})^2 + (\frac{T}{A})(\frac{2}{h} - 1) \right]^{1/2} \quad \text{(A.9)}$$

$$\hat{p} = \frac{A\hat{q} - T}{1 + \frac{h}{2}(\hat{q} - 1)} \quad \text{(A.10)}$$

solve this problem. Moreover, if F(A) is the maximal value function of problem (A.7)-(A.8), F is increasing in A.

Proof

The reader can easiliy check that the values (A.9) and (A.10) solve (A.7)-(A.8). Application of the envelope theorem shows that the function F is increasing. ●

Proof of Theorem 1.

Part (a):

Suppose, first, that the monopolist offers a unipack $(p, q, 1)$ with quality level α. The most profitable unipack is then given by the solution $(\hat{q}^L, \hat{p}^L)$ of problem (8)-(9). Maximal profits are $F(\alpha)$.

Suppose that the monopolist considers offering a bundle (p, q, n), $n \geq 2$. The best bundle it can offer is obtained by solving

$$\underset{p,q}{Max} \frac{p}{q} \tag{A.11}$$

$$\text{s.t.} \qquad \alpha - \frac{n\alpha(1-B)}{q} - C^r(p,q) \geq 0 \tag{A.12}$$

It is clear that the feasible set defined by (A.12) is a proper subset of the feasible set defined by (9). Hence, the maximal value of problem (A.11)-(A.12) is smaller than $F(\alpha)$.

By lemma 1, it is not interesting for the monopolist to offer several packages, as loyal consumers will buy only one of them.

Part (b):

Suppose that the monopolist offers a bundle (p, q, n), with $n \geq 2$. The best such bundle he can offer is obtained by solving problem (12)-(13). Maximal profits are then $F(\alpha\tilde{B})$.

Suppose the monopolist offers one unipack $(p, q, 1)$. The best unipack he can offer variety seekers is given by the solution of

$$\underset{p,q}{Max} \frac{p}{q}$$

$$\text{s.t.} \qquad \alpha B - C^r(p,q) \geq 0$$

This gives maximal profits $F(\alpha B)$. As F is increasing, and as $B \langle \tilde{B}$, we have that $F(\alpha\tilde{B}) \rangle F(\alpha B)$.

Suppose, finally, that the monopolist offers two packages $(\overline{p}, \overline{q}, \overline{n})$ and $(\hat{p}, \hat{q}, \hat{n})$, both of quality α, and with $\overline{n}$, $\hat{n} \geq 1$. To the extent that variety seeking consumers prefer to combine these packages, rather than to purchase only one of them repetitively, the monopolist's maximal profits are obtained by solving

$$\underset{\overline{p},\overline{q},\hat{p},\hat{q}}{Max} \frac{1}{2}\frac{\overline{p}}{\overline{q}} + \frac{1}{2}\frac{\hat{p}}{\hat{q}}$$

$$\text{s.t. } \alpha\tilde{B} - C^c(\overline{p},\overline{q}) - C^c(\hat{p},\hat{q}) \geq 0$$

As this problem is symmetric in $(\overline{p},\overline{q})$ and $(\hat{p},\hat{q})$, it is clearly equivalent to

$$\underset{p,q}{Max}\ \frac{p}{q} \tag{A.13}$$

$$\text{s.t. } \alpha\tilde{B}-2C^{c}(p,q)\geq 0 \tag{A.14}$$

Using inequality (6), it is clear that the feasible set of this problem is a proper subset of the feasible set defined by (13). Hence , the maximal value of problem (A.13)-(A.14) must be smaller than the maximal value of problem (12)-(13), viz., $F(\alpha\tilde{B})$.

Part (c) :

Suppose a *variety seeker* is offered the packages $(\hat{p}^{L},\hat{q}^{L},1)$ and $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$ with $\hat{n}^{VS}\geq 2$. It is easy to see that for a variety seeker repetitively purchasing $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$ yields a higher surplus than repetitively purchasing $(\hat{p}^{L},\hat{q}^{L},1)$. This follows from

$$\alpha\tilde{B}-C^{r}(\hat{p}^{VS},\hat{q}^{VS})=0=\alpha-C^{r}(\hat{p}^{L},\hat{q}^{L})\ \rangle\ \alpha B-C^{r}(\hat{p}^{L},\hat{q}^{L})$$

Moreover, repetitively purchasing $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$ also yields a higher surplus than combining $(\hat{p}^{L},\hat{q}^{L},1)$ and $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$. Indeed, using (5) and (7), the inequality

$$\alpha\tilde{B}-C^{r}(\hat{p}^{VS},\hat{q}^{VS})\geq\alpha\tilde{B}-C^{c}(\hat{p}^{L},\hat{q}^{L})-C^{c}(\hat{p}^{VS},\hat{q}^{VS})$$

is equivalent to

$$\frac{h}{2}\frac{\hat{p}^{L}}{\hat{q}^{L}}(\hat{q}^{L}-1)\geq\frac{1}{2}\left(\frac{\hat{p}^{VS}}{\hat{q}^{VS}}-\frac{\hat{p}^{L}}{\hat{q}^{L}}\right)+\frac{T}{2}\left(\frac{1}{\hat{q}^{VS}}-\frac{1}{\hat{q}^{L}}\right) \tag{A.15}$$

By part (d) of the theorem, the two terms on the RHS of (A.15) are smaller than or equal to zero. Hence, inequality (A.15) must certainly hold.

Suppose now that the packages $(\hat{p}^{L},\hat{q}^{L},1)$ and $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$ are offered to a *loyal consumer*. From lemma 1 we know that he will never combine them. Repetitively purchasing $(\hat{p}^{L},\hat{q}^{L},1)$ now yields a higher surplus than repetitively purchasing $(\hat{p}^{VS},\hat{q}^{VS},\hat{n}^{VS})$ if

$$\alpha-C^{r}(\hat{p}^{L},\hat{q}^{L})=0=\alpha\tilde{B}-C^{r}(\hat{p}^{VS},\hat{q}^{VS})\geq\alpha-\frac{\hat{n}^{VS}\alpha(1-B)}{\hat{q}^{VS}}-C^{r}(\hat{p}^{VS},\hat{q}^{VS})$$

The last inequality will hold if and only if

$$\hat{n}^{VS}\geq(1-\gamma)\,\hat{q}^{VS}.$$

E.g., if $\hat{n}^{VS}=\hat{q}^{VS}$, this inequality certainly holds.

Part (d):

In case $\gamma = 1$, the problems (8)-(9) and and (12)-(13) are identical. In case $\gamma \langle 1$, we have that $\alpha\tilde{B} \langle \alpha$. The inequality $\hat{q}^{VS} \rangle \hat{q}^{L}$ then follows from the fact that $\hat{q}$ in (A.9) is a decreasing function of A. Also, if $\alpha\tilde{B} \langle \alpha$, we have that $F(\alpha\tilde{B}) \langle F(\alpha)$. •

Proof of theorem 2.

First take the point of view of firm a. This firm takes as given that firm b offers consumers the unipack $(\hat{p}_b^L, \hat{q}_b^L, 1) = (0, \infty, 1)$. This unipack gives consumers a surplus of α_b. As loyal consumers never combine packages (see lemma 1), firm a wants to offer a unipack which consumers prefer to $(\hat{p}_b^L, \hat{q}_b^L, 1) = (0, \infty, 1)$.The best such unipack is the unipack that solves problem (14)-(15).

Now take the point of view of firm b. This firm takes as given that firm a offers the unipack $(\hat{p}_a^L, \hat{q}_a^L, 1)$. Consumers then enjoy a surplus of α_b. Clearly, the best firm b can do is also to offer a unipack that gives consumers a surplus at least equal to α_b. The only unipack that satisfies this requirement is $(\hat{p}_b^L, \hat{q}_b^L, 1) = (0, \infty, 1)$. •

Proof of theorem 3a.

We first show that, if (17) is not binding in the optimum of problem (16)-(18), and if inequality (23) holds, the maximal value of (16) must exceed the maximal value of (20).

Consider the behaviour of the objective function (16), subject only to constraint (17). Using (7), this constraint can be written as

$$p_a \leq 2\,(\alpha_a \tilde{B} - \frac{\alpha_a + \alpha_b}{2})\, q_a - T$$

The value of (16) would then be unbounded from above. (Note that restriction (23) implies that $\alpha_a \tilde{B} - \dfrac{\alpha_a + \alpha_b}{2}$ is positive.) These values of p_a / q_a would then violate (18). This constraint must, therefore, be binding in the optimum. If (18) holds with an equality, it can be written as

$$\frac{p_a}{q_a} = \frac{(\alpha_a - \alpha_b)\tilde{B} - \dfrac{T}{q_a}}{1 + \dfrac{h}{2}(q_a - 1)} = f^{do}(q_a)$$

In a similar way we now show that constraint (22) must also be binding in the optimum of problem(20)-(22). Consider the behaviour of objective function (20), subject only to constraint (21). It is clear that under this constraint the value of the objective function (20) is unbounded from above. These values of p_a / q_a must then violate (22). Hence, constraint (22) must be binding in the optimum of problem (20)-(22). If (22) holds with equality, it can be written as

$$\frac{1}{2}\frac{p_a}{q_a}=\frac{\left[\frac{\alpha_a+\alpha_b}{2}-\alpha_b\tilde{B}\right]-\frac{T}{2q_a}}{1+h(q_a-1)}=f^{ms}(q_a)$$

If inequality (23) holds, it is easy to see that the numerator of the function $f^{do}(q_a)$ exceeds the numerator of the function $f^{ms}(q_a)$. As the reverse is true for the denominators of the two expressions, it follows that

$$\forall q_a \geq 1,\ f^{do}(q_a)\geq f^{ms}(q_a)$$

It then follows that

$$Max\left\{f^{do}(q_a)\ s.t.\ q_a\geq 1\right\}\geq Max\left\{f^{ms}(q_a)\ s.t.\ q_a\geq 1\right\}\geq$$

$$Max\left\{f^{ms}(q_a)\ s.t.\ q_a\geq 1, and\ (21)\right\}$$

This shows that, if (17) is not binding in the optimum of problem (16)-(18), the maximal value of (16) must exceed the maximal value of (20), provided inequality (23) holds.

It is now easy to see that the pair of strategies $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS}),(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})=(0,\infty,\hat{n}_b^{VS})$ is a Nash equilibrium for all $\hat{n}_a^{VS}\geq 2$ and $\hat{n}_b^{VS}\geq 1$. If firm b follows the strategy $(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})=(0,\infty,\hat{n}_b^{VS})$, then $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$ is the best reply by firm a, assuming inequality (23), and assuming that (17) is not binding in $(\hat{p}_a^{VS},\hat{q}_a^{VS})$. Conversely, if firm a follows the strategy $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$, there are no profitable actions left for b : it cannot give consumers an incentive to purchase only its own package, nor can it give consumers an incentive to combine its package with that of firm a. ●

Using (A.9) it is easy to see that constraint (17) is not binding if and only if

$$\tilde{B}\rangle\frac{\alpha_a+\alpha_b+\frac{h}{2}(\hat{q}_a^L-1)(\alpha_a+\alpha_b+\frac{T}{\hat{q}_a^L})}{\alpha_a+\alpha_b+\frac{h}{2}(\hat{q}_a^L-1)2\alpha_a}$$

where

$$\hat{q}_a^L=X+\left[X^2+X(\frac{2}{h}-1)\right]^{1/2}$$

and

$$X = \frac{T}{(\alpha_a - \alpha_b)\tilde{B}}$$

Proof of theorem 3b.

We will prove that, from firm a 's point of view, the package $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ is its best reply to firm b's offer $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$. The proof from firm b's point of view is, of course, very similar.

First note that, for any choice (p_a, q_a, n_a) with $n_a \geq 1$, consumers find that combining (p_a, q_a, n_a) and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ is always better than repetitively purchasing (p_a, q_a, n_a). This follows from

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a, q_a) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) = \frac{\alpha_a + \alpha_b}{2} - C^c(p_a, q_a) - s_b =$$

$$\frac{\alpha_a + \alpha_b}{2} - C^r(p_a, q_a) + \frac{p_a + T}{2q_a} - s_b =$$

$$s_a - C^r(p_a, q_a) + \frac{p_a + T}{2q_a} \geq \alpha_a \tilde{B} - C^r(p_a, q_a) + \frac{p_a + T}{2q_a} \rangle$$

$$\alpha_a \tilde{B} - C^r(p_a, q_a) \geq \alpha_b B - C^r(p_a, q_a)$$

Here we made use of (26) -holding with equality- and of (7).

Secondly, we easily derive that variety seeking consumers will never repetitively purchase $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$. Indeed, if $\hat{n}_b^{VS} \geq 2$

$$\alpha_b \tilde{B} - C^r(\hat{p}_b^{VS}, \hat{q}_b^{VS}) = \alpha_b \tilde{B} - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) - \frac{\hat{p}_b^{VS} + T}{2\hat{q}_b^{VS}} =$$

$$\alpha_b \tilde{B} - s_b - \frac{\hat{p}_b^{VS} + T}{2\hat{q}_b^{VS}} \langle\ 0$$

(In case $\hat{n}_b^{VS} = 1$, $\tilde{B}$ should be replaced by B in the last expression.) Hence, the best firm a can do is to maximize (25), subject to

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a, q_a) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) = s_a - C^c(p_a, q_a) \geq 0\,.$$

This is exactly problem (25)-(26).●

Proof of theorem 4a

Part (a):

We only have to check incentive compatibility. If a *loyal consumer* is offered the choice between $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, incentive compatibility requires that

$$\alpha_a - C^r(\hat{p}_a^L, \hat{q}_a^L) \geq \alpha_a - \frac{\hat{n}_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}} - C^r(\hat{p}_a^{VS}, \hat{q}_a^{VS}) \tag{A.18}$$

Making use of the fact that constraints (15) and (18) hold with equalities, the foregoing inequality reduces to the inequality

$$\hat{n}_a^{VS} \geq (1-\gamma)(\frac{\alpha_a - \alpha_b}{\alpha_a})\,\hat{q}_a^{VS}$$

which is inequality (27).

If a *variety seeking consumer* is offered the packages $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, with $\hat{n}_a^{VS} \geq 2$, repetitively purchasing $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ will be better than repetitively purchasing $(\hat{p}_a^L, \hat{q}_a^L, 1)$ if

$$\alpha_a\tilde{B} - C^r(\hat{p}_a^{VS}, \hat{q}_a^{VS}) \geq \alpha_a B - C^r(\hat{p}_a^L, \hat{q}_a^L) \tag{A.19}$$

Again using the fact that constraints (15) and (18) hold with equalities, it follows that inequality (A.19) always holds.

Moreover, repetitively purchasing $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ also yields a higher surplus than combining $(\hat{p}_a^L, \hat{q}_a^L, 1)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$. Indeed, using (5) and (7), the inequality

$$\alpha\tilde{B} - C^r(\hat{p}_a^{VS}, \hat{q}_a^{VS}) \geq \alpha\tilde{B} - C^c(\hat{p}_a^L, \hat{q}_a^L) - C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS})$$

is equivalent to

$$\frac{h}{2}\frac{\hat{p}_a^L}{\hat{q}_a^L}(\hat{q}_a^L - 1) \geq \frac{1}{2}\left(\frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}} - \frac{\hat{p}_a^L}{\hat{q}_a^L}\right) + \frac{T}{2}\left(\frac{1}{\hat{q}_a^{VS}} - \frac{1}{\hat{q}_a^L}\right) \tag{A.20}$$

In part (b) of the theorem, we prove that the two terms on the RHS of (A.20) are smaller than or equal to zero. Hence, inequality (A.20) must certainly hold.

Part (b):

If $\gamma = 1$, then $\tilde{B} = 1$, and $\alpha_a - \alpha_b = (\alpha_a - \alpha_b)\tilde{B}$. We must then have that $\hat{q}_a^L = \hat{q}_a^{VS}$, and that $F(\alpha_a - \alpha_b) = F((\alpha_a - \alpha_b)\tilde{B})$. If $\gamma < 1$, then $\tilde{B} < 1$, so that $\alpha_a - \alpha_b > (\alpha_a - \alpha_b)\tilde{B}$. From lemma 2 we then obtain that $\hat{q}_a^L < \hat{q}_a^{VS}$ and that $F(\alpha_a - \alpha_b) > F((\alpha_a - \alpha_b)\tilde{B})$. •

Proof of Theorem 4b.

As a preliminary step, consider the following result. If

$$\frac{2T}{\alpha_a} \rangle 1 \tag{A.21}$$

then

$$\forall q_a^{VS} \geq \frac{2T}{\alpha_a}, \alpha_b \langle (\alpha_a - 2\alpha_b - \frac{T}{q_a^{VS}})\frac{h}{2}(q_a^{VS} - 1) \tag{A.22}$$

is equivalent to

$$\alpha_b \langle (\frac{\alpha_a}{4} - \alpha_b)\, h\, (\frac{2T}{\alpha_a} - 1) \tag{A.23}$$

It is clear that the RHS of the inequality in (A.22) is increasing in q_a^{VS}. Property (A.22) is, therefore, equivalent to the property that the inequality holds for $q_a^{VS} = \frac{2T}{\alpha_a}$. Evaluating (A.22) in this point gives exactly (A.23).

Part (a):

We only have to check incentive compatibility.

For *loyal consumers* this requires:

$$\alpha_a - C^r(\hat{p}_a^L, \hat{q}_a^L) \geq \alpha_a - \frac{\hat{n}_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}} - C^r(\hat{p}_a^{VS}, \hat{q}_a^{VS}) \tag{A.24}$$

and

$$\alpha_a - C^r(\hat{p}_a^L, \hat{q}_a^L) \geq \alpha_b - \frac{\hat{n}_b^{VS}\alpha_b(1-B)}{\hat{q}_b^{VS}} - C^r(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \tag{A.25}$$

These inequalities require that, for loyal consumers, repetitive purchases of $(\hat{p}_a^L, \hat{q}_a^L, 1)$ are at least as good as repetitive purchases of $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$, or of $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$.

Consider first (A.24). By (15), the LHS of (A.24) is equal to α_b. Using (7) and (26), the RHS of (A.24) can be written as

$$\frac{\alpha_a}{2} - \frac{\hat{n}_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}} - \frac{\hat{p}_a^{VS} + T}{2\hat{q}_a^{VS}} \tag{A.26}$$

From (26) we know that

$$\hat{p}_a^{VS} = \frac{\alpha_a \hat{q}_a^{VS} - T}{1 + h(\hat{q}_a^{VS} - 1)}$$

Using this equality in (A.26), we can finally rewrite inequality (A.24) as

$$(\frac{1}{2}(\alpha_a - \frac{T}{\hat{q}_a^{VS}}) - \alpha_b - \frac{\hat{n}_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}})h(\hat{q}_a^{VS}-1) \leq (\alpha_b + \frac{\hat{n}_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}}) \quad \text{(A.27)}$$

Given that the right side of (A.27) is positive, and that the left side is negative for $\hat{n}_a^{VS}$ satisfying (29), this inequality holds.

Consider now inequality (A.25). As the LHS of (A.25) equals α_b, this inequality follows immediately.

For a *variety seeking consumer*, incentive compatibility requires

$$\frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \geq \frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{L},\hat{q}_a^{L}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \quad \text{(A.28)}$$

$$\frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \geq \alpha_a\tilde{B} - C^c(\hat{p}_a^{L},\hat{q}_a^{L}) - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) \quad \text{(A.29)}$$

$$\frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \geq \alpha_a B - C^r(\hat{p}_a^{L},\hat{q}_a^{L}) \quad \text{(A.30)}$$

$$\frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \geq \alpha_a B - C^r(\hat{p}_a^{VS},\hat{q}_a^{VS}) \quad \text{(A.31)}$$

$$\frac{\alpha_a+\alpha_b}{2} - C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) - C^c(\hat{p}_b^{VS},\hat{q}_b^{VS}) \geq \alpha_b B - C^r(\hat{p}_b^{VS},\hat{q}_b^{VS}) \quad \text{(A.32)}$$

These inequalities require that, for variety seeking consumers, combining $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$ and $(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})$ is at least as good as combining $(\hat{p}_a^{L},\hat{q}_a^{L},1)$ and $(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})$, or $(\hat{p}_a^{L},\hat{q}_a^{L},1)$ and $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$. Furthermore, combining $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$ and $(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})$ should be at least as good as repetitively purchasing $(\hat{p}_a^{L},\hat{q}_a^{L},1)$, $(\hat{p}_a^{VS},\hat{q}_a^{VS},\hat{n}_a^{VS})$, or $(\hat{p}_b^{VS},\hat{q}_b^{VS},\hat{n}_b^{VS})$.

Inequality (A.28) is the more difficult to prove. It is equivalent to

$$C^c(\hat{p}_a^{VS},\hat{q}_a^{VS}) \leq C^c(\hat{p}_a^{L},\hat{q}_a^{L}) \quad \text{(A.33)}$$

By (26), the LHS of (A.33) equals $\frac{\alpha_a}{2}$. Using (7) and (15), the RHS of (A.33) equals

$$\alpha_a - \alpha_b - \frac{\hat{p}_a^L + T}{2\hat{q}_a^L} \quad \text{(A.34)}$$

From (15) we know that

$$\frac{\hat{p}_a^L}{\hat{q}_a^L} = \frac{\alpha_a - \alpha_b - \dfrac{T}{\hat{q}_a^L}}{1 + \dfrac{h}{2}(\hat{q}_a^L - 1)}$$

Using this expression in (A.34), inequality (A.33) can finally be rewritten as

$$(\alpha_a - 2\alpha_b - \frac{T}{\hat{q}_a^L})\frac{h}{2}(\hat{q}_a^L - 1) \geq \alpha_b$$

By (A.22), this inequality must hold if $\hat{q}_a^L > T/2\alpha_a$. In part (b) of the theorem we prove that $\hat{q}_a^L > \hat{q}_a^{VS} > T/2\alpha_a$, so that the inequality $\hat{q}_a^L > T/2\alpha_a$ certainly holds.

We now prove (A.29). By (26) the RHS of (A.29) can be written as

$$\alpha_a(\tilde{B} - \frac{1}{2}) - C^c(\hat{p}_a^L, \hat{q}_a^L)$$

which is clearly negative. As the LHS of (A.29) is zero, this inequality certainly holds. Using (15), inequality (A.30) holds, given that $B < 1/2$. Using again the assumption that $B < 1/2$, and using (7) and (26), the inequalities (A.31) and (A.32) easily follow.

Part (b):

Applying (A.9) to problem (14)-(15) we obtain

$$\hat{q}_a^L = \frac{T}{\alpha_a - \alpha_b} + \left[(\frac{T}{\alpha_a - \alpha_b})^2 + (\frac{T}{\alpha_a - \alpha_b})(\frac{2}{h} - 1) \right]^{1/2}$$

Applying again (A.9) to (25)-(26) we obtain

$$\hat{q}_a^{VS} = \frac{T}{\alpha_a} + \left[(\frac{T}{\alpha_a})^2 + (\frac{T}{\alpha_a})(\frac{1}{h} - 1) \right]^{1/2}$$

It then follows easily that $\hat{q}_a^L > \hat{q}_a^{VS}$.

We now prove that $\frac{\hat{p}_a^L}{\hat{q}_a^L} > \frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}}$. If (15) holds with an equality, we have that

$$\frac{p_a^L}{q_a^L} = \frac{(\alpha_a - \alpha_b - \dfrac{T}{q_a^L})}{1 + \dfrac{h}{2}(q_a^L - 1)} = f^L(q_a^L) \qquad \text{(A.35)}$$

If (26) holds with an equality, we have

$$\frac{p_a^{VS}}{q_a^{VS}} = \frac{(\alpha_a - \frac{T}{q_a^{VS}})}{1 + h(q_a^{VS} - 1)} = f^{VS}(q_a^{VS}) \tag{A.36}$$

Under (A.22), it is clear that

$$\forall q_a \geq \frac{2T}{\alpha_a} \rangle 1, \; f^L(q_a) \rangle f^{VS}(q_a)$$

It then easily follws that $\frac{\hat{p}_a^L}{\hat{q}_a^L} \rangle \frac{\hat{p}_a^{VS}}{\hat{q}_a^{VS}}$. •

Lemma 3:*For any combination* (p_a^L, q_a^L) *and* (p_a^{VS}, q_a^{VS}) *that satisfies*

$$C^r(p_a^L, q_a^L) = \alpha_a - \alpha_b \quad \textit{and} \; C^c(p_a^{VS}, q_a^{VS}) = \frac{\alpha_a}{2} \tag{A.37}$$

the following implication holds

$$\alpha_a \langle 2\alpha_b \Rightarrow C^c(p_a^L, q_a^L) \langle C^c(p_a^{VS}, q_a^{VS}) \tag{A.38}$$

Proof:

Using (7), and (A.37) we know that

$$C^c(p_a^L, q_a^L) - C^c(p_a^{VS}, q_a^{VS}) = C^r(p_a^L, q_a^L) - \frac{p_a^L + T}{2q_a^L} - \frac{\alpha_a}{2} =$$

$$(\frac{\alpha_a}{2} - \alpha_b) - \frac{p_a^L + T}{2q_a^L} \langle 0 \, .$$ •

Proof of theorem 4c.

Part (a):

Let us first take the *point of view of firm a*. Given firm b's strategy $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$, the best firm a can do is to solve the following problem:

$$\underset{p_a^L, q_a^L, p_a^{VS}, q_a^{VS}}{Max} \; \omega^L \frac{p_a^L}{q_a^L} + \omega^{VS} \frac{1}{2} \frac{p_a^{VS}}{q_a^{VS}} \tag{A.39}$$

s.t. $\qquad \alpha_a - C^r(p_a^L, q_a^L) \geq \alpha_b \qquad$ (A.40)

$$\alpha_a - C^r(p_a^L, q_a^L) \geq \alpha_a - \frac{n_a^{VS} \alpha_a (1-B)}{q_a^{VS}} - C^r(p_a^{VS}, q_a^{VS}) \tag{A.41}$$

$$\alpha_a - C^r(p_a^L, q_a^L) \geq \alpha_b - C^r(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \tag{A.42}$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \geq 0 \tag{A.43}$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \geq \frac{\alpha_a + \alpha_b}{2} - C^c(p_a^L, q_a^L) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \tag{A.44}$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \geq \alpha_a B - C^r(p_a^L, q_a^L) \tag{A.45}$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \geq \alpha_a \tilde{B} - C^r(p_a^{VS}, q_a^{VS}) \tag{A.46}$$

$$\frac{\alpha_a + \alpha_b}{2} - C^c(p_a^{VS}, q_a^{VS}) - C^c(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \geq \alpha_b \tilde{B} - C^r(\hat{p}_b^{VS}, \hat{q}_b^{VS}) \tag{A.47}$$

Constraints (A.40)-(A.42) refer to loyal consumers. Repetitively purchasing $(p_a^L, q_a^L, 1)$ should give loyal consumers a surplus of at least α_b, and it should be at least as good as repetitively purchasing $(p_a^{VS}, q_a^{VS}, n_a^{VS})$ or $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$. Constraints (A.43)-(A.47) refer to variety seekers. Combining $(p_a^{VS}, q_a^{VS}, n_a^{VS})$ and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$ should yield a nonnegative surplus. It should also be at least as good as combining $(p_a^L, q_a^L, 1)$ and $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$. It should also be at least as good as repetitively purchasing $(p_a^L, q_a^L, 1)$, $(p_a^{VS}, q_a^{VS}, n_a^{VS})$ or $(\hat{p}_b^{VS}, \hat{q}_b^{VS}, \hat{n}_b^{VS})$.

It is clear that the feasible set of problem (A.39)-(A.47) is a subset of the feasible set of problem (30)-(32). If we can then show that $(\hat{p}_a^L, \hat{q}_a^L)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS})$ is feasible in problem (A.39)-(A.47), these values must also solve this latter problem.

Clearly, constraints (31) and (A.40) are identical. Now turn to constraint (A.41). Using (7), and using

$$\frac{\alpha_a}{2} - C^c(\hat{p}_a^{VS}, \hat{q}_a^{VS}) \geq 0 \tag{A.48}$$

constraint (A.41) implies that

$$\frac{n_a^{VS}\alpha_a(1-B)}{\hat{q}_a^{VS}} \geq \frac{\alpha_a}{2} - \alpha_b - \frac{\hat{p}_a^{VS} + T}{2\hat{q}_a^{VS}}$$

This inequality is certainly satisfied if (35) holds.

(A.42) is implied by (A.40). (A.43) is satisfied by (34). (A.44) is equivalent to (32). Using (31) and (35), it follows that (A.45) is satisfied. Using (7) and (A.48), it follows that (A.46) is satisfied. Using (7), (A.48) and (A.43) shows that (A.47) is satisfied.

Let us now take the *point of view of firm b*. By (31) firm b is driven out of the loyals' market. Given the offer of $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ by firm a, the best it can do is to solve problem (33)-(34).

Part (b):

Prices: Using the fact that (32) holds with an equality, we know that, for variety seekers, the packages $(\hat{p}_a^{L}, \hat{q}_a^{L}, 1)$ and $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ must lie on the same iso-surplus curve, involving C^c. Yet, since $(\hat{p}_a^{VS}, \hat{q}_a^{VS}, \hat{n}_a^{VS})$ is C^c-efficient (its size being optimally adjusted to variety seekers), it *must* be sold at a higher unit price.

Quantities: Consider the (extreme) benchmark case where $(\hat{p}_a^{L}, \hat{q}_a^{L}, 1)$ would be C^r -efficient. This is the same solution as in the sealed loyals market. It only holds when . Even in that case, the optimal package size $\hat{q}_a^{VS}$ would remain smaller than $\hat{q}_a^{L}$. The reason is that it leads to the same surplus *and* it is C^c -efficient, while $\hat{q}_a^{L}$ is not. The latter is C^r -efficient, and thus places less weight on holding cost.
In less extreme solutions, $\hat{q}_a^{L}$ will be higher than in the benchmark case - so as to become less appealing to variety seekers - while $\hat{q}_a^{VS}$ will be lower (because it remains C^c -efficient, but can now aim for a higher surplus). It follows that for *any* solution of Theorem 4c (any set of weights ω^L and ω^{VS}), $\hat{q}_a^{VS}$ is smaller than $\hat{q}_a^{L}$. •

The following table gives some numerical illustrations for the three types of Nash equilibria we derived in the mixed market case.

Illustrations: Mixed Market

Case		$\hat{q}_a^{VS}$	$\hat{q}_a^{L}$	$\hat{p}_a^{VS} / \hat{q}_a^{VS}$	$\hat{p}_a^{L} / \hat{q}_a^{L}$	Profit firm a
Theorem 4a	α_a=3.5 α_b=1 T=2 H=0.5 $\tilde{B}$=0.93 ω^L=0.8, ω^{VS}=0.2	2.544	2.682	1.12	1.237	1.2128
Theorem 4b	α_a=6 α_b=1 T=15 H=0.5 $\tilde{B}$=0.4 ω^L=0.8, ω^{VS}=0.2	5.46	7.24	1.007	1.144	1.016

Theorem 4c	α_a=3 α_b=2 T=2 H=0.5 $\tilde{B}$=0.2	ω^L=1 ω^{VS}=0	-	5.2	-	.30	.24
		ω^L=0.8, ω^{VS}=0.2	3.43	5.5	.34	.299	.274
		ω^L=0.5, ω^{VS}=0.5	3.22	7.1	.385	.284	.239
		ω^L=0 ω^{VS}=1	1.72	-	1.35	-	.27

We can also use these examples to illustrate the lower bounds (27) and (29).

For theorem 4.a. : Consider the case where $\tilde{B} = .93$. Then, if B=.6 and $\gamma = .825$, it is necessary that $\hat{n}_a^{VS} \geq 1$. Or, if B=.9 and $\gamma = .3$, then $\hat{n}_a^{VS} \geq 3$.
For theorem 4.b. : Consider the case where $\tilde{B} = .4$. Then, if B=.3 and $\gamma = 0.143$, we must have $\hat{n}_a^{VS} \geq 3$. Or, if B=.05 and $\gamma = .368$, then $\hat{n}_a^{VS} \geq 3$.

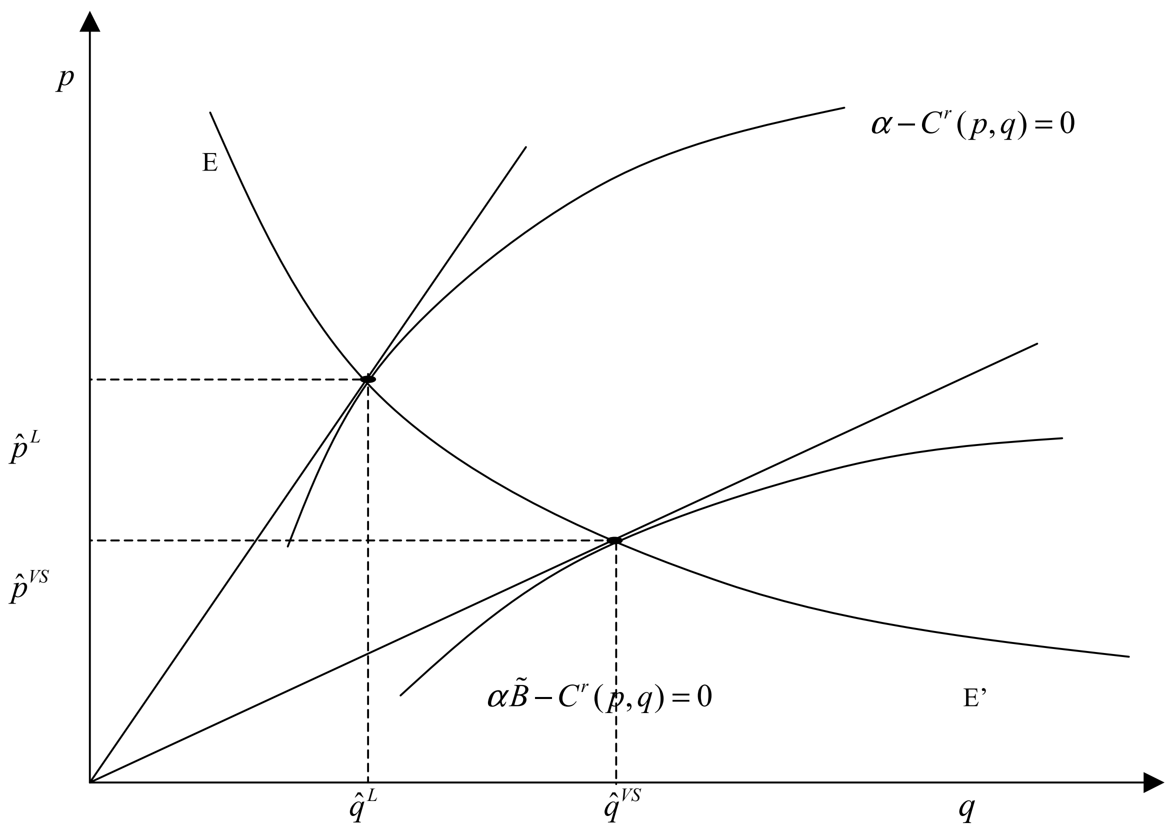

Figure 1

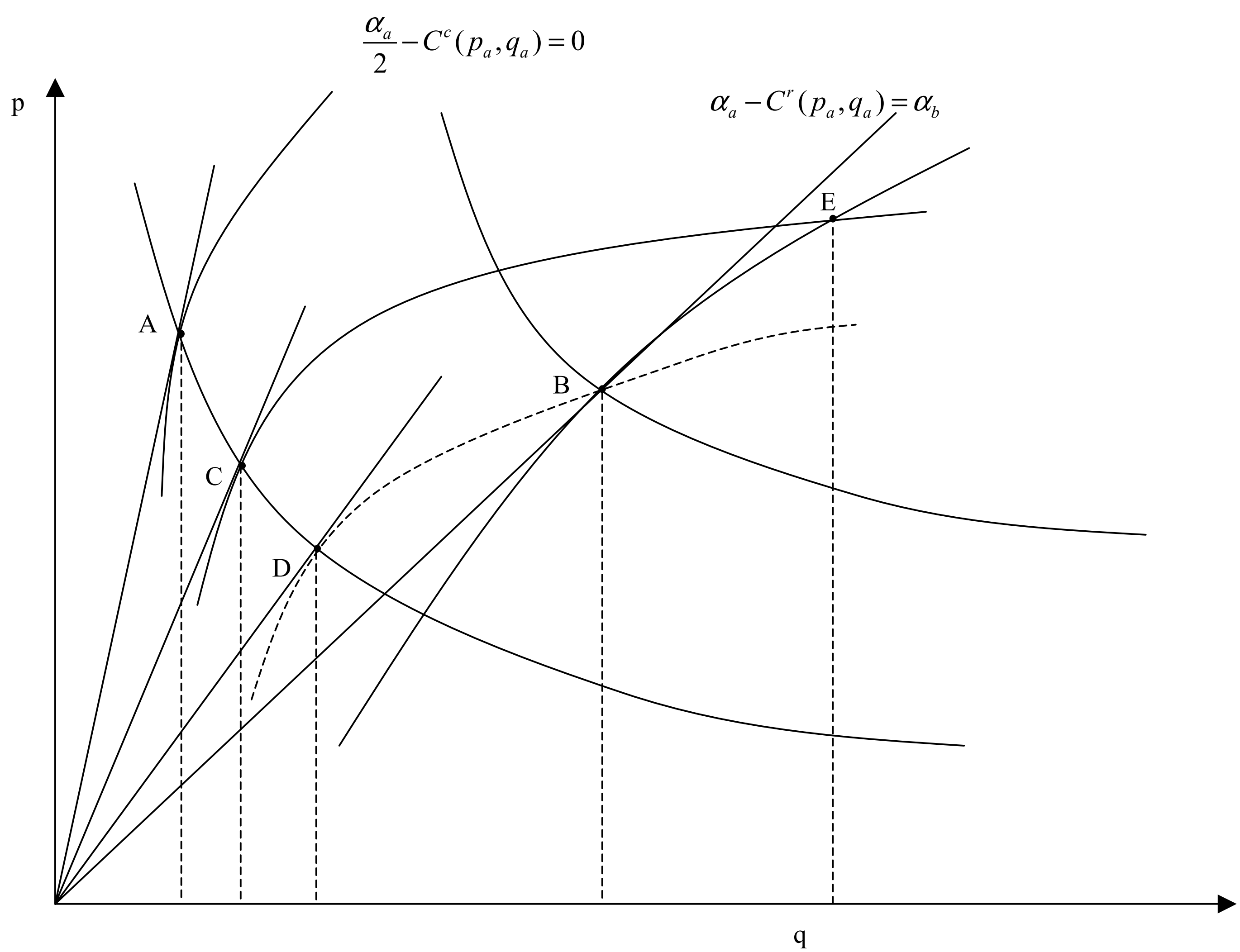

Figure 2