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Estimating the Long Rate and its Volatility

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Abstract

We estimate the long rate and its volatility within the Svensson framework. The procedure that best extrapolates the longest observable rate and its volatility is a 2-dimensional grid search conditioned on the ridge regression suggested by Annaert et al. (2013).

Keywords: Long Rate, Nelson-Siegel Model, Svensson Model, Ridge Regression

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1. Introduction

The long rate (i.e. the spot rate of infinite maturity) plays an important role in economic policy making and in the valuation of long term liabilities of pension funds and life insurers imposed by regulators[2]. The long rate, however, is unobservable and has to be estimated or extrapolated from models that describe the term structure of interest rates, which are based on e.g. swap rates up to 10 years. Popular spline methods are bound to fail when used to extrapolate an estimated curve but parsimonious parametric models such as the Nelson-Siegel (1987) and the Svensson (1994) model serve this purpose well since the constant in these models is the long rate.

The Svensson specification of the spot rate function reads

$$r(\tau)=\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{bmatrix}'\begin{bmatrix}1\\ \lambda_1\left(1-e^{-\tau/\lambda_1}\right)/\tau\\ \lambda_1\left(1-e^{-\tau/\lambda_1}\right)/\tau-e^{-\tau/\lambda_1}\\ \lambda_2\left(1-e^{-\tau/\lambda_2}\right)/\tau-e^{-\tau/\lambda_2}\end{bmatrix},$$

where $r(\tau)$ is the continuous spot rate with time to maturity τ and $\beta_0,\beta_1,\beta_2,\beta_3,\lambda_1$ and λ_2 are the model coefficients, with $\lambda_1>0,\lambda_2>0$. β_0 can be interpreted as a level component which estimates the long rate, the exponential models the slope of the curve and the Laguerre functions allows for the presence of a hump/trough. As λ_1 and λ_2 determine the location of the two humps/troughs, they are also known as the shape parameters. The Nelson-Siegel model discards the last term by imposing $\beta_3=0$.

Although these models are heavily used by central banks (Bank of International Settlements, 2005), their estimation copes with serious unreported or often loosely reported difficulties. Both models are highly nonlinear and thus are often linearized by fixing the shape parameter(s). Annaert et al. (2013) show that, depending on the maturities used in the estimation, the explanatory variables in the linearized models may be highly correlated, leading to unstable and inaccurately estimated

[2] The long rate can be useful for discounting long term cash flows since Livingston (1987) has shown that spot rate curves tend to flatten for maturities longer than 10 to 15 years.

parameters. They suggest a conditional ridge regression to cope with the reported problems. At least by using the Nelson-Siegel model, they find a seriously improved fit in the long rate. Whether the Svensson specification and the conditional ridge regression approach improve the long rate estimation remains an open question, which this paper addresses. The estimation method of Annaert et al. (2013) clearly improves estimates of the term structure level, but its performance as regards interest rate volatility has not been investigated. Yet, the results of Díaz et al. (2011) lead to believe that different estimation techniques can have a major impact on interest rate volatility estimates. Our second contribution is to measure the performance of the Nelson-Siegel/Svensson model with different estimation procedures for interest rate volatility modeling.

Using the Euribor rates maturing from 1 week up to 12 months and the Euro swap rates with maturities between 2 and 10 years over the period from January 4, 1999 to May 24, 2011, we show that our estimated long rates excellently fit the 30-year Euro swap rate and its (fitted and forecasted) volatility. All rates were retrieved from Thomson Reuters Datastream®. Following Annaert et al. (2013), we use the smoothed bootstrap procedure to extract spot rates and fit them by using different estimation procedures.[3]

2. Estimation Procedures and Testing Methodology

In the Svensson model, long-term rates are predominantly determined by β_0 as the weights of the slope and two curvature components are close to zero for long maturities. Estimating the long rate boils down to estimating β_0. Since it is merely an extension of the Nelson-Siegel model, the Svensson model also inherits its multicollinearity problems. In this paper, we estimate the long rate and its volatility using five different estimation procedures.

First, we estimate the Nelson-Siegel specification using the Nelson and Siegel (1987) grid search (GS). Fixing the shape parameter linearizes the model, which allows for OLS estimation. Next, the estimated parameters based on the shape parameter that results in the lowest sum of squared errors are deemed to be optimal. Analogously, the Svensson model can be linearized by first fixing *both*

[3] Due to the lack of swap rates with maturities of 11 years, 13 years, 14 years and so on, it is not possible to bootstrap zero rates with a maturity longer than 10 years.

shape parameters as in Ferenczi and Werner (2006). We shall refer to this second method as a two-dimensional grid search (2dimGS). Notice that in the two-dimensional grid λ_1 can be chosen to be equal to λ_2 leading the Svensson model to collapse to the Nelson-Siegel model. In our empirical work, the grid on the shape parameters used spans the interval (0, 10]. Annaert et al. (2013) have shown that using ridge regression instead of OLS whenever the condition number of the linear model exceeds a threshold, improves the GS estimates. We apply the same cut-off criterion on the condition-number as Annaert et al. (2013) to the GS and the 2dimGS procedures. We label them GS-RR and 2dimGS-RR. Although the grid search based Svensson model (2dimGS) will always yield a lower sum of squared errors (SSE) than that obtained from the GS, this need not be the case for the procedures augmented with ridge regression. As the Svensson model may experience more multicollinearity problems (compared to the Nelson-Siegel model), more bias has to be added into the estimates in order to lower their variance. Therefore, to determine whether the Svensson model's extra hump/trough is desirable, we finally compare the in-sample fitting errors produced by GS-RR with the ones based on the 2dimGS-RR. Whichever model produces the lower sum of in-sample errors is favored. We refer to this estimation procedure as 2dimGS/GS-RR.

In order to evaluate the usefulness of the suggested procedures, we use three evaluation criteria. First, we fit a Svensson-type term structure model for the spot rate specification for each day in our sample. In order to judge the procedure's extrapolation quality, and hence their ability to estimate the long rate, we focus on the longest observable rate i.e. the 30-year swap rate. The procedure that renders the lowest mean absolute error (MAE) between the extrapolated 30-year rate and the contemporaneous actual 30-year swap rate, is considered to be the preferred procedure. Next, we compute the MAE between the conditional volatility of the 30-year swap rates and the same models fitted through the extrapolated rates. We fit a GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) with both Gaussian and Student-*t* innovations to the time series of actual 30-year swap rates. Since the EGARCH(1,1) model with Student-*t* innovations provides the best fit for all estimation methods in our paper, we only report the results vis-à-vis this specification. We then fit the same model through the time series of the extrapolated 30-year swap rates. By computing the MAE between the conditional volatility of the observed rates and that of the extrapolated rates, we test the ability of the estimation procedures to capture the volatility dynamics at the long end of the yield curve. The model with the lowest MAE is considered the most accurate to describe conditional volatility. Finally, we forecast the conditional volatility of the 30-year swap rates with the same EGARCH model. To generate forecasts, we start with a training period of 2000 days and predict the one-day-ahead conditional volatility. We re-estimate the EGARCH model for every day in our sample using an expanding estimation window. Since we have 3174 days in our sample, we can make 1173 forecasts. We then compute the difference between the forecasted volatility based on the extrapolated and observed rates on the same day. Again, the procedure with the lowest MAE is considered as the most appropriate one to forecast the term structure of volatilities.

To test whether two MAEs are statistically different from each other, we compute the MAE ratio, α, as $MAE_{Method1} / MAE_{Method2}$. If α is significantly greater than 1, then we consider *Method 1* to be worse than *Method 2*. Since the methods that we propose are not independent from each other and since we observe clustering in the MAE, we block-bootstrap the absolute errors in pairs to calculate the Ashley test statistic.[4] To do so, we first divide the block-resampled time series from *Method 1* and *Method 2* by $MAE_{Method1}$ and $MAE_{Method2}$. Then we compute the MAE ratios. We resample 50 000 times and draw the distribution of the MAE ratio to compute the empirical *p*-value.

3. Empirical Results

Table 1 shows that the MAEs in the extrapolated 30-year swap rates vary between 11.61 bps (2dimGS/GS-RR) and 27.01 bps (GS). Augmenting GS with the ridge regression procedure almost halves the MAEs. Also from an economic point of view, a reduction of 13.5 bps in MAE is huge. The inclusion of an extra hump factor in the Nelson-Siegel model (GS vis-à-vis 2dimGS) generally pays off in terms of MAE as well. Augmenting the 2dimGS with the ridge regression does not. Notice that the 2dimGS/GS-RR procedure has the lowest MAE of a mere 11.61 bps.

Table 1 Long Rate Mean Absolute Errors (in bps) [5]

Model	GS	GS-RR	2dimGS	2dimGS-RR	2dimGS/ GS-RR

[4] The bootstrapped results are robust using various block sizes ranging from 15 up to 120. The optimal block size based on the algorithm of Politis and White (2004) varies from 80 to 120. Here we report results based on a block size of 50.
[5] We also estimated the Svensson model with nonlinear least squares. The estimated MAEs (60.58 bps, 87.37 bps and 83.66 bps respectively for long rate extrapolation, volatility fitting and forecasting) are significantly higher than any method listed in Table 1 based on a bootstrapped Ashley test at the 5% level.

Long Rate Extrapolation	27.01	13.51	18.12	20.13	11.61
Volatility Fitting	20.14	12.06	25.18	24.45	9.41
Volatility Forecasting	30.39	20.62	28.15	28.00	13.85

Note: Table 1 reports the mean absolute error (MAE) between (1) the actual 30-year swap rates and those extrapolated from a Nelson-Siegel/Svensson term structure specification using several estimation methods, (2) the Student-t-EGARCH(1,1) conditional volatilities estimated on extrapolated 30-year swap rates and their empirical counterparts and (3) one-day-ahead conditional volatility forecasts estimated on extrapolated 30-year rates and their empirical counterparts. The estimation methods are described in Section 2.

With respect to volatility fitting and forecasting, the MAEs confirm the superiority of the 2dimGS/GS-RR procedure. GS shows a fitting error of 20.14 bps and a forecasting error of 30.39 bps. GS-RR improves the fitting and forecasting ability by approximately 8 and 10 bps respectively. The Svensson specification does in general not lead to improved MAEs unless the 2dimGS/GS-RR procedure is applied. It has the lowest fitting errors with an MAE of only 9.41 bps and the lowest average forecasting error of 13.85 bps.

Table 2 Bootstrapped Ashley test (*p*-values in percentage, Spread in bps)

Challenger	GS	GS-RR	2dimGS	2dimGS-RR
Long Rate Extrapolation				
α	2.33	1.16	1.56	1.73
p-value	(0.00)	(1.82)	(0.00)	(0.00)
Volatility Fitting				
α	2.14	1.28	2.68	2.60
p-value	(0.00)	(0.00)	(0.00)	(0.00)
Volatility Forecasting				
α	2.19	1.49	2.03	2.02
p-value	(0.01)	(0.09)	(0.00)	(0.00)

Note: We perform a blocked bootstrap (block size = 50) with 50 000 runs. The first row reports the α *as defined in Section 2, and the empirical p-values (in percentage) are reported in brackets.*

For Nelson-Siegel/Svensson models, the 2dimGS/GS-RR procedure improves the estimation quality of the long rate substantially. In order to avoid eyeballing, we also formally test the difference between the MAEs following Ashley (1998). In Table 2, an α in excess of 1 indicates that the 2dimGS/GS-RR method has a lower MAE than the 'challenger'. The results demonstrate that 2dimGS/GS-RR significantly dominates all other models. Hence, we advocate its use for long rate estimation.

4. Conclusion

This paper explores various methods to estimate the long rate and its volatility. We find that, the ridge regression conditioned on both the Nelson-Siegel and the Svensson specification (2dimGS/GS-RR) provides the best fit to both the long rate and its (fitted and forecasted) volatility.

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