This item is the archived peer-reviewed author-version of:

Visual servoing of moving targets through fractional control : a feasibility study

Reference:

Copot Cosmin.- Visual servoing of moving targets through fractional control : a feasibility study
2017 IEEE 21st International Conference on Intelligent Engineering Systems 2017 (INES), 20-23 October, 2017, Budapest, Hungary - ISBN 978-1-4799-7678-2 -
Piscataway, N.J., IEEE, 2017, p. 259-264
Full text (Publisher's DOI): <http://dx.doi.org/doi:10.1109/INES.2017.8118566>

Visual Servoing of Moving Targets through Fractional Control: A feasibility study

Cosmin Copot

University of Antwerp, Department of Electromechanics, Op3Mech
Groenenborgerlaan 171, 2020 Antwerp, Belgium
Email: Cosmin.Copot@UAntwerpen.be

Abstract—In this paper, a control strategy based on fractional order approach for visual servoing systems is investigated. The proposed image-based control strategy is a fractional-order PI which takes into account the point features. In order to evaluate the designed control strategy, a visual servoing simulator consisting of a manipulator robot with 6 degrees of freedom (d.o.f.) and an eye-in-hand camera configuration was considered. The validity of the visual based controllers is shown by the simulation results which demonstrate that the proposed approach based on a fractional-order controller has a stable and convergent behavior when dealing with visual servoing applications. Simulations have been performed and the results revealed that the image-based fractional-order PI controller outperforms the conventional image-based integer-order PI controller.

I. INTRODUCTION

Manipulator robots represent an automated electromechanical system with a flexible functionality through programming according to the environmental conditions in which they operate. The main role of the manipulator robots is to help the human operators in executing repetitive and/or hazardous tasks in industrial habitat. The robot control field studies the structure and functionality of the robot command unit. Based on the dynamic and geometrical model and on the tasks that needs to be performed, dedicated commands are establish. In order to provide these commands to the actuators, the hardware and the software we have to make use of the feedback signals obtained from the sensory system unit [1].

Visual servoing is an active topic with important results in robotics which combine control techniques and image processing methods in order to control a robotic system. Servoing systems are classified in two fundamental structures: position-based visual servoing (PBVS) and image-based visual servoing (IBVS) [2]. Further on, the image-based architecture will be considered. As IBVS does not explicitly solve for the Cartesian pose of the target object, its performance does not depend on the accuracy of a priori models. However, this architecture has also disadvantages, one of it being the convergence of the control law when dealing with large displacement. In order to overcome the downside of the IBVS structure, advanced control strategies are employed [3], [4], [5], [6]. It is well known that the dynamics of a robot motion tracking system is time varying, therefore one of the most challenging task is to design new control methodologies to deal with the nonlinearities of the manipulator robot [2], [7], [8], [9], [10]. Fractional order control strategies seem to be a good candidate

to cope with these challenges. Fractional order or non-integer order system are characterized by dynamical systems whose model structure consist of arbitrary order derivatives and/or integrals [11]. Fractional order control is a tool of Fractional Calculus theory and its potential is given by the fact that it can ensure robustness of the system to be controlled [12].

Potential applications of visual servoing are numerous. It can be used as soon as a vision sensor is available and a task is assigned to a dynamic system to control its motion. Among the applications reporting the use of visual servoing techniques one can identify the following:

- The control of a pan-tilt-zoom camera
- Grasping using a robot arm
- Locomotion and dexterous manipulation with a humanoid robot
- Micro- or nanomanipulation of MEMS or biological cells
- Pipe inspection by an underwater autonomous vehicle
- UAV formation control
- Autonomous navigation of a mobile robot in indoor or outdoor environment
- Aircraft landing
- Autonomous satellite rendezvous
- Biopsy using ultrasound probes or heart motion compensation in medical robotics
- Virtual cinematography in animation

As it can be noticed, most of the visual servoing applications consist of a moving target. In order to cancel the error introduced by the moving target, an integral term is used in the visual control law. In this paper, a fractional-order approach is used to design the controller. The performance of the fractional-order controller was evaluated against the integer-order controller and the results revealed that the image-based fractional-order PI controller outperforms the conventional image-based integer-order PI controller.

The paper is organized as follows: Section II presents the basis of an image-based visual servoing configuration including image-based control architecture and robot modeling. In Section III, the proportional image-based control law is described together with the image-based control law for moving target. In Section IV, the simulation results of the designed controllers are presented. Finally, Section V addresses concluding remarks.

II. THE BASIC OF IMAGE BASED VISUAL SERVOING

The classical designs for servoing architectures are named image-based visual servoing (IBVS) and pose-based visual servoing (PBVS). In the IBVS approach, a function $\mathbf{f}$ consists of a set of 2D parameters that are directly expressed in the image plane, while in the PBVS approach the function $\mathbf{f}$ consists of a set of 3D parameters related to the pose between the camera and the target. Advanced visual servoing techniques are also available in literature, however, these are out of the scope of this study. In this paper the focus is on image-based visual servoing approach.

Visual servoing can be seen as sensor-based schema control from a vision sensor. Main, if not all visual servoing tasks can be expressed as the regulation to zero of an error $\mathbf{e}(t)$ which is defined by:

$$\mathbf{e}(t) = \mathbf{f}(\mathbf{m}(t), \mathbf{a}) - \mathbf{f}^*(t) \quad (1)$$

where $\mathbf{m}(t)$ is a set of image measurements (e.g., the image coordinates of interest points, or the area, the center of gravity, and other geometric characteristics of an object) and $\mathbf{a}$ is a set of parameters that represent potential additional knowledge about the system (e.g., coarse camera intrinsic parameters or 3D model of objects). The vector $\mathbf{f}^*(t)$ contains the desired value of the features, which can be either constant in the case of a fixed goal either varying if the task consists in following a specified trajectory.

A. Image based Control Architecture

Image-based visual servoing (IBVS) uses direct image measurements as feedback to control the motion of a robot. The robots positioning task is expressed as an image-based error function to be minimized using a suitable control law. As IBVS does not explicitly solve for the Cartesian pose of the target object, its performance does not depend on the accuracy of a priori models. However, since the domain of the control law is in the image space, there is no direct control over the Cartesian or joint-space trajectory of the robot end-effector.

Generally, an image-based servoing architecture consists of: a manipulator robot, a visual sensor and a image-based controller. In Figure 1 a schematic of a image-based control architecture of manipulator robots with 6 degrees of freedom is being illustrated. The core of the architecture is represented by the image-based controller which requires apriori information related to the system behavior in order to minimize the error between the current configuration of the visual features $\mathbf{f}$ and the desired configuration of the features $\mathbf{f}^*$.

Let $\mathbf{p} = (X, Y, Z)$ be a 3D point defined in the camera space, its projection in the image plane (Figure 2) is the point $\mathbf{x}$ of coordinates (x, y) , where:

$$\begin{cases} x = (X/Z)\iota \\ y = (Y/Z)\iota \end{cases} \quad (2)$$

and ι represents the focal length, with $\iota = 1$. The time derivative of a 2D point is defined as:

$$\begin{cases} \dot{x} = \dot{X}/Z - x\dot{Z}/Z \\ \dot{y} = \dot{Y}/Z - y\dot{Z}/Z \end{cases} \quad (3)$$

Regardless of the visual sensor configuration, eye-in-hand (the camera is mounted on the effector of the robot) or eye-to-hand (the camera is fixed in the working space), the derivative of the point $\mathbf{p}$ with respect to the camera velocity $\mathbf{v}_c$ can be calculated based on the fundamental kinematics equation:

$$\dot{\mathbf{p}} = -v - \omega \times \mathbf{p} = -v + [\mathbf{p}]_{\times} \omega \quad (4)$$

where $\mathbf{v}_c = [v^T, \omega^T]^T = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^T$ is the camera velocity with v and ω the linear and angular velocity respectively, $[\mathbf{p}]_{\times}$ is the antisimetric matrix defined as:

$$[\mathbf{p}]_{\times} = \begin{bmatrix} 0 & -Z & Y \\ Z & 0 & -X \\ -Y & X & 0 \end{bmatrix} \quad (5)$$

Substituting (5) in (4), the derivative of a point $\mathbf{p}$ with respect to the visual sensor is:

$$\begin{aligned} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} &= - \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & -Z & Y \\ Z & 0 & -X \\ -Y & X & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \\ &= - \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} -\omega_y Z + \omega_z Y \\ \omega_x Z - \omega_z X \\ -\omega_x Y + \omega_y X \end{bmatrix} \end{aligned} \quad (6)$$

thus

$$\begin{cases} \dot{X} = -v_x - \omega_y Z + \omega_z Y \\ \dot{Y} = -v_y + \omega_x Z - \omega_z X \\ \dot{Z} = -v_z - \omega_x Y + \omega_y X \end{cases} \quad (7)$$

If we substitute (7) in (3), the velocity of a 2D image feature $\dot{\mathbf{x}} = (\dot{x}, \dot{y})^T$ can be written as:

$$\begin{cases} \dot{x} = -v_x/Z + xv_z/Z + xy\omega_x - (1+x^2)\omega_y + y\omega_z \\ \dot{y} = -v_y/Z + yv_z/Z + (1+y^2)\omega_x - xy\omega_y - x\omega_z \end{cases} \quad (8)$$

By writing previous equation in a matrix form, the following is obtained:

$$\dot{\mathbf{x}} = \mathbf{L}_x \mathbf{v}_c \quad (9)$$

Fig. 1. Visual servoing architecture to control manipulator robots

Fig. 2. Perspective projection of a 3D point

where $\mathbf{L}_x$ is the interaction matrix related to $\mathbf{x}$. Combining (8) and (9), the interaction matrix for a 2D point $\mathbf{x}$ of coordinates (x, y) can be calculated by:

$$\mathbf{L}_x = \begin{bmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix} \quad (10)$$

where Z is the depth of the point relative to the camera frame. For a set of n point features, the interaction matrix can be computed as:

$$\mathbf{L}_n = \begin{bmatrix} \mathbf{L}_{x1} \\ \mathbf{L}_{x2} \\ \vdots \\ \mathbf{L}_{xn} \end{bmatrix} \quad (11)$$

B. The robot model - VCMD MODEL

In order to model the dynamic of the system in open loop the two entities which have to be analyzed are: manipulator robot and visual sensor. Further, an eye-in-hand configuration of the ensemble robot-visual sensor is being considered.

There are two possibilities to model the robot: kinematic and dynamic modeling. From these two, robot dynamics represents a key role with respect to the performance of the visual servoing system. For servoing applications the manipulator robot can be modeled as a VCMD [13]. As it can be noticed in Figure 1 the input signal for the VCMD is the output for the visual based controller $\mathbf{v}_c^*$. This signal is the reference velocity of the camera and it has the following form $\mathbf{v}_c^* = [v^* \omega^*]^T$, where $v^* = [v_x^* \ v_y^* \ v_z^*]^T$ is the linear velocity and $\omega^* = [\omega_x^* \ \omega_y^* \ \omega_z^*]^T$ is the angular velocity. The input signal $\mathbf{v}_c^*$ is represented in Cartesian space and in order to be applied to the manipulator robot a homogeneous transformation is needed. If we denote with $[t_1, t_2, t_3, t_4, t_5, t_6]^T$ the pose obtained by integrating $\mathbf{v}_c^*$ then the Jacobian of the robot can be defined as:

$$\mathbf{J}_r = \begin{bmatrix} \frac{\partial t_1}{\partial q_1} & \dots & \frac{\partial t_1}{\partial q_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial t_6}{\partial q_1} & \dots & \frac{\partial t_6}{\partial q_6} \end{bmatrix} \quad (12)$$

where: q_i , $i = \overline{1,6}$ represent the states of the robot joints. Therefore, transformation of $\mathbf{v}_c^*$ from Cartesian space in the robot space joint can be obtained via $\mathbf{J}_r^{-1}$.

In [13] a linearized model for a MIMO VCMD is proposed. Starting from the hypothesis that each joint has its own velocity control loop (see figure 3). The controllers C_i , $i = \overline{1,6}$ corresponding to the control loops are designed in such way that the joints decoupling is ensured (this being valid for the most manipulator robots). Due to Jacobian matrix structure related to the robot and due to the dynamic of the robot with respect to the joint position, the result is a non-linear VCMD model. For linearization purposes the following assumptions are being made: the non-linear effect generated by Coriolis force, the inertial force and gravitational torque have a insignificant disturbance effect on the control loop. This can be rejected by an adequate design of the controllers C_i , $i = \overline{1,6}$.

The nonlinearities generated by all these forces become significant when the robot is moving with high speed. Due to the fact that the control loop for the joints are very fast, this can be designed in continuous time and therefore a ZOH bloc has to be included as it is shown in Figure 3. To apply the $\mathbf{v}_c^*$ speed in the robot joint space the inverse of the robot Jacobian $\mathbf{J}_r^{-1}$ is used.

Fig. 3. The VCMD model of a manipulator robot

Taking into account that the joint inertia is varying very less with respect to the robot position $\mathbf{x}_q$, the inertia matrix can be considered constant around a given position of the robot. For servoing systems based on visual feedback speed joints are controlled individually and the controllers C_i , $i = \overline{1,6}$ are designed to reject the nonlinear effect produced by the disturbances. Starting from these assumptions and considering that the dynamic model of the robots but also the Jacobian $\mathbf{J}_r$ are constant in a given position of the robot $\mathbf{x}_q$, the control loops of the robot can be modeled using the transfer function $G(s)$:

$$\mathbf{v}_q(s) = G(s)\mathbf{v}_q^*(s) \quad (13)$$

The discrete linear model of the VCMD is illustrated in Figure 4 and is described by:

$$G_{VCMD}(z^{-1}) = (1 - z^{-1}) \mathbb{Z} \left(\frac{\mathbf{J}_r G(s) \mathbf{J}_r^{-1}}{s^2} \right) \quad (14)$$

where $\mathbb{Z}$ represents the z transform.

Fig. 4. The linearised model for VCMD

In [13] the dynamics of the visual sensor is modeled as pure delays. These delays are generated by: the time necessary for image acquisition and image formation, the time needed to transfer the image from the camera to the computer and the time required for image processing.

In order to determine the model of the visual sensor we can assume that the time required for the acquisition and formation of the image, T_{ach} , is equal to the sampling period T_s , the time necessary to transfer the image, T_{trans} , is very small in comparison with the sampling period and the time for image processing, T_{proc} , is equal to the sampling period. Thus, the visual sensor can be modeled as dead-time element equal to $2T_s$. Using the camera model together with the VCMD model and knowing the initial pose of the camera denoted with $\mathbf{x}_0$,

we can develop a control structure as in Figure 5 which can then be successfully applied in real-time servoing applications. Assuming that $G(s)$ is diagonal, then the discrete transfer matrix can be written as:

$$G_p(z^{-1}) = z^{-2} (1 - z^{-1}) J_r \mathbb{Z} \left(\frac{G(s)}{s^2} \right) J_r^{-1} = z^{-2} J_r \text{diag} \left\{ \frac{b_i(z^{-1})}{a_i(z^{-1})} \right\} J_r^{-1}, \quad i = \overline{1, 6} \quad (15)$$

The ARIMAX model has the following deterministic form:

$$z^{-1} A^{-1}(z^{-1}) B(z^{-1}) = G_p(z) = z^{-2} J_r \text{diag} \left\{ \frac{b_i(z^{-1})}{a_i(z^{-1})} \right\} J_r^{-1} \quad (16)$$

where

$$\begin{aligned} A(z^{-1}) &= \text{diag} \{ a_i(z^{-1}) \} J_r^{-1} \\ B(z^{-1}) &= z^{-1} \text{diag} \{ b_i(z^{-1}) \} J_r^{-1} \end{aligned} \quad (17)$$

Fig. 5. Control architecture for a servoing system using VCMD model

Another method that can be used to describe the dynamics modeling of a manipulator robot is presented in [14].

III. CONTROLLERS FOR MOVING TARGETS

A. Classical image-based control law

The main objective of the IBVS architecture is to steer a manipulator robot using visual information acquired by the camera. In order to fulfill the task, a trajectory for the visual sensor need to be designed. This trajectory can be computed by the integration of the camera velocity which is given through the minimization of the error between the current and the desired visual features. Next, was considered that the object is characterized by n point features of coordinates $(u_i, v_i), i = \overline{1, n}$ defined by:

$$\mathbf{f} = [\mathbf{f}_1^T \dots \mathbf{f}_n^T]. \quad (18)$$

Let $\mathbf{v}_c = [\mathbf{v}, \omega]$ be the camera velocity vector and assuming that the structure of a point feature is $\mathbf{f}_i = (u_i, v_i) = \mathbf{K} \cdot [x_i, y_i, 1]^T, i = \overline{1, n}$, where $\mathbf{K}$ is the intrinsic parameter matrix, the link between the point features motion in the image plane and the camera motion in the Cartesian space, $\mathbf{v}_c$, is given by:

$$\dot{\mathbf{f}}_i = \mathbf{L}_{\mathbf{f}_i} \mathbf{v}_c, \quad i = \overline{1, n}. \quad (19)$$

In (19) the interaction matrix for a point feature $\mathbf{L}_{\mathbf{f}_i}$ can be computed using [2]:

$$\mathbf{L}_{\mathbf{f}_i} = \begin{bmatrix} -\frac{1}{Z_i} & 0 & \frac{x_i}{Z_i} & x_i y_i & -(1 + x_i^2) & y_i \\ 0 & -\frac{1}{Z_i} & \frac{y_i}{Z_i} & 1 + y_i^2 & -x_i y_i & -x_i \end{bmatrix}, \quad (20)$$

where Z_i is the depth of the corresponding projection point in the Cartesian space related to the camera frame. While point

features are considered as visual features, the error that has to be minimized is:

$$\mathbf{e}(k) = \mathbf{f}(k) - \mathbf{f}^*, \quad (21)$$

where $\mathbf{f}(k)$ represents the point features configuration at the iteration k and $\mathbf{f}^*$ is the desired vector configuration. This error represents the input signal of the image-based controller, while the output is the desired camera velocity $\mathbf{v}_c^*$.

The connection between the point features error and camera velocity is computed by:

$$\dot{\mathbf{e}} = \mathbf{L}_{\mathbf{f}} \mathbf{v}_c \quad (22)$$

Tacking into account the robot dynamics, the previous equation become:

$$\dot{\mathbf{e}} = \mathbf{L}_{\mathbf{f}} G(z^{-1}) \mathbf{v}_c^* \quad (23)$$

In order to have an exponential decrease of the error:

$$\dot{\mathbf{e}} = -\lambda \mathbf{e} \quad (24)$$

and considering $\mathbf{v}_c^*$ as the command signal for the robot controller, the following control law is obtained:

$$\mathbf{v}_c^* = -\lambda G^{-1}(z^{-1}) \mathbf{L}_{\mathbf{f}}^+ (\mathbf{f}(t) - \mathbf{f}^*) \quad (25)$$

where $\mathbf{L}_{\mathbf{f}}^+$ is the pseudo-inverse of the $\mathbf{L}_{\mathbf{f}}$ interaction matrix and is defined as:

$$\mathbf{L}_{\mathbf{f}}^+ = \left(\mathbf{L}_{\mathbf{f}}^T \mathbf{L}_{\mathbf{f}} \right)^{-1} \mathbf{L}_{\mathbf{f}}^T \quad (26)$$

The control systems based on the visual feedback are nonlinear and have coupled dynamics. The stability of these type of system can be analysed by using Lyapunov function. A detailed analysis of the stability of the visual serving systems is presented in [15].

B. Image-based control law for moving targets

Considering the general case of a servoing application with a moving target, the time variation of the error from (22) can be written as [16]:

$$\dot{\mathbf{e}} = \mathbf{L}_{\mathbf{f}} \mathbf{v}_c + \frac{\partial \mathbf{e}}{\partial t} \quad (27)$$

where $\frac{\partial \mathbf{e}}{\partial t}$ represents the time variation of the error with respect to the target motion (for static object $\frac{\partial \mathbf{e}}{\partial t} = 0$). By imposing an exponential decrease of the error (24), the designed image-based control law is given by:

$$\mathbf{v}_c = -\lambda \mathbf{L}_{\mathbf{f}}^+ \mathbf{e} - \mathbf{L}_{\mathbf{f}}^+ \widehat{\frac{\partial \mathbf{e}}{\partial t}} \quad (28)$$

with $\widehat{\frac{\partial \mathbf{e}}{\partial t}}$ an approximation of $\frac{\partial \mathbf{e}}{\partial t}$ which was considered in order to overcome the errors introduced by the moving target. By substituting (28) in (27), the behavior of the closed-loop system can be expressed as:

$$\dot{\mathbf{e}} = -\lambda \mathbf{L}_{\mathbf{f}} \mathbf{L}_{\mathbf{f}}^+ \mathbf{e} - \mathbf{L}_{\mathbf{f}} \mathbf{L}_{\mathbf{f}}^+ \widehat{\frac{\partial \mathbf{e}}{\partial t}} + \frac{\partial \mathbf{e}}{\partial t} \quad (29)$$

For the case of a static object where $\frac{\partial \mathbf{e}}{\partial t} = 0$ and implicit $\widehat{\frac{\partial \mathbf{e}}{\partial t}} = 0$, the system is stable and the error converges to zero

if $\mathbf{L}_f \mathbf{L}_f^+ > 0$. However, in the case of a moving target, in order to compensate tracking errors, an accurate estimation of $\mathbf{L}_f \mathbf{L}_f^+$ is necessary, such as:

$$\mathbf{L}_f \mathbf{L}_f^+ \frac{\widehat{\partial e}}{\partial t} = \frac{\partial e}{\partial t} \quad (30)$$

Another approach widely used in automatic control to cancel the steady-state error implies the use of an integral term in the control law. Here, the integral action can be written as:

$$\left. \frac{\widehat{\partial e}}{\partial t} \right|_{k+1} = \left. \frac{\widehat{\partial e}}{\partial t} \right|_k + \mu \mathbf{e}(k) \quad (31)$$

where $\left. \frac{\widehat{\partial e}}{\partial t} \right|_k$ is the estimation of $\frac{\partial e}{\partial t}$ at moment k , $\mathbf{e}(k)$ is the error at moment k and μ is the gain of the integral action.

C. Fractional order controller

Due to its advantages [17], [18], further-on fractional control is used to design the integral action. The continuous fractional-order (derivative or integral of non-integer order) operator, denoted ${}_a D_t^\alpha$, where a represents the initial conditions, t is the time of evaluation and $\alpha \in \mathbb{R}$ represents a generic order, is given by [19], [20]:

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_0^t (d\tau)^{-\alpha}, & \alpha < 0 \end{cases} \quad (32)$$

Three definitions are generally used for the fractional differintegral: the Grunwald-Letnikov definition, the Riemann-Liouville and the Caputo definition [21]. The notion of fractional-order integral of order $\alpha > 0$ is a natural consequence of Cauchy's formula for repeated integrals, expressed as [17]:

$${}_a D_t^{-\alpha} f(t) = \frac{1}{(\alpha-1)!} \int_a^t (t-\tau)^{(\alpha-1)} f(\tau) d\tau, \quad (33)$$

$\tau > a, \alpha \in \mathbb{Z}^+$

Extending (33) to $\alpha \in \mathbb{R}^+$, the Riemann-Liouville definition of a fractional-order integral is obtained as:

$${}_a D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{(\alpha-1)} f(\tau) d\tau \quad (34)$$

where $0 < \alpha < 1$ and $\Gamma(\alpha)$ is the Euler's Gamma function which is a generalization of a factorial:

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt \quad (35)$$

Let $a = 0$ in (34), the notation of the fractional-order integral is simplified to:

$$D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{(\alpha-1)} f(\tau) d\tau \quad (36)$$

There are other definition of the fractional-order derivative, but is out of the scope of this paper. Based on the fractional-order approach, the time variation of the error from (27) becomes:

$$\dot{\mathbf{e}} = \mathbf{L}_f \mathbf{v}_c + \frac{\widehat{\partial^\alpha e}}{\partial t^\alpha} \quad (37)$$

Following the same principle as before, the image-based control law is defined as:

$$\mathbf{v}_c = -\lambda \mathbf{L}_f^+ \mathbf{e} - \mathbf{L}_f^+ \frac{\widehat{\partial^\alpha e}}{\partial t^\alpha} \quad (38)$$

where $\frac{\widehat{\partial^\alpha e}}{\partial t^\alpha}$ was calculated based on (36) and represents the estimated value of the time variation of the error $\frac{\partial e}{\partial t}$.

IV. SIMULATION RESULTS

The image-based control law of a visual servoing architecture based on fractional-order approach was implemented, tested and validated using Matlab[®] framework. The simulator used to perform the experiments was partially developed by the author as an extension of the simulator presented in [22]. The performance of the fractional-order image-based control law was compared with the one obtained using classical inter-order approach. The parameter of the fractional-order PI^α controller were chosen as follow: $\lambda = 0.73$, $\mu = 0.27$ and a fractional-order of $\alpha = 0.45$, while the parameters of the integer-order PI controller are given as: $\lambda = 0.9$ and $\mu = 0.069$. Multiple simulation were performed in order to validate the image-based control law and one of the visual servoing task configuration is presented in Figure 6. Note that the desired image feature configuration is not static but dynamic with moving target. The evolution of the camera velocity vectors for this configuration is illustrated in Figures 7 and 9. The error of the image feature can be visualized in Figures 8 and 10.

Fig. 6. Point feature configuration for the considered servoing scenario

As can be observed in the considered scenario, the simulation results show a faster convergence of the servoing system when dealing with fractional-order approach in comparison with integer-order controller. Due to flexibility introduced by

Fig. 7. Evolution of the camera velocities using fractional-order approach

Fig. 8. Image feature error when fractional-order control law was considered

Fig. 9. Evolution of the camera velocities using integer-order approach

Fig. 10. Image feature error when integer-order control law was considered

the fractional control law (an extra parameter) the servoing system can fulfill easier the specifications/tasks.

V. CONCLUSION

In this paper, an application of the fractional-order controller over a visual servoing task with moving target was presented. The performances of the fractional-order PI controller are

analyzed and compared with integer-order PI controller. The simulation results indicate that the fractional-order controller outperform the classical integer-order controllers. Due to the flexibility of the fractional controller introduced by the extra parameter, the system can fulfill more specification but its implementations require additional effort in comparison with classical PI controller. However, given the improvements in performance the trade-off in having a somewhat higher computational effort is justified.

REFERENCES

- [1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2006.
- [2] F. Chaumette and S. Hutchinson, "Visual servo control, part i: Basic approaches," *IEEE Robotics and Automation Magazine*, vol. 13, no. 4, pp. 82–90, 2006.
- [3] T. Sim, G. Hong, and K. Lim, "Multirate predictor control scheme for visual servo control," *IEE Proceedings - Control Theory and Applications*, vol. 149, no. 2, pp. 117–124, 2002.
- [4] C. Perez-Vidal, L. Garcia, N. Garcia, and E. Cervera, "Visual control of robots with delayed images," *Advanced Robotics*, no. 23, pp. 725–745, 2009.
- [5] C. Copot, C. Lazar, and A. Burlacu, "Predictive control of nonlinear visual servoing systems using image moments," *IET Control Theory and Applications*, vol. 6, no. 10, pp. 1486–1496, 2012.
- [6] C. Copot, A. Burlacu, C. Ionescu, C. Lazar, and R. De Keyser, "A fractional order control strategy for visual servoing systems," *Mechatronics*, vol. 23, no. 7, pp. 848–855, 2013.
- [7] M. Llama, R. Kelly, and V. Santibanez, "Stable computed-torque control of robot manipulators via fuzzy self-tuning," *IEEE Trans. Syst. Man Cybern.*, vol. 30, pp. 143–150, 2000.
- [8] O. Barambones and V. Etxebarria, "Robust neural control for robotic manipulator," *Automatica*, vol. 38, pp. 235–242, 2002.
- [9] E. Cruz and A. Morris, "Fuzzy-ga-based trajectory planner for robot manipulators sharing a common workspace," *IEEE Transaction on Robotics*, vol. 22, pp. 613–624, 2006.
- [10] G. Kronreif, "Mechatronic assistance for surgical applications," *Acta Polytechnica Hungarica*, vol. 13, no. 1, pp. 31–42, 2016.
- [11] A. Oustaloup, *La derivation non entiere*. Hermes, 1995.
- [12] E. Dulf, D. Timis, and C. Muresan, "Robust fractional order controllers for distributed systems," *Acta Polytechnica Hungarica*, vol. 14, no. 1, pp. 163–176, 2017.
- [13] J. A. Gangloff and M. F. de Mathelin, "High speed visual servoing of a 6 dof manipulator using multivariable predictive control," *Advanced Robotics*, vol. 17, no. 10, pp. 993–1021, 2003.
- [14] H. Fujimoto, "Visual servoing of 6 dof manipulator by multirate control with depth identification," in *Proc. of 42 nd IEEE Conference on Decision and Control*, Hawaii, 2003, pp. 5408–5413.
- [15] F. Chaumette and S. Hutchinson, "Visual servo control, part II: Advanced approaches," *IEEE Robotics and Automation Magazine*, vol. 14, no. 1, pp. 109–117, 2007.
- [16] F. Chaumette, S. Hutchinson, and P. Corke, *Handbook of Robotics*, 2nd ed. Springer, 2016, ch. Visual servoing, pp. 841–866.
- [17] C. Monje, Y. Chen, B. Vinagre, D. Xue, and V. Feliu, *Fractional-order Systems and Controls. Fundamentals and Applications*. Springer, 2010.
- [18] Y. Chen and C. Ionescu, "Applied fractional calculus in modelling, analysis and design of control systems," *International Journal of Control (Taylor and Francis Group)*, vol. 90, no. 6, pp. 1155–1156, 2017.
- [19] Y. Luo and Y. Chen, *Fractional order motion controls*. John Wiley & Sons, Ltd, Chichester, UK, 2013.
- [20] R. Caponetto, G. Dongola, L. Fortuna, and I. Petras, *Fractional Order Systems: Modeling and Control Applications*. World Scientific Publishing, 2010.
- [21] K. Oldham and J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*. Dover, New York, NY, USA, 2006.
- [22] E. Cervera, "Visual servoing toolbox," <http://vstoolbox.sourceforge.net/>, Castellon, 2003, Jaume I University.