DEPARTMENT OF ACCOUNTANCY AND FINANCE

Portfolio optimization at the frontier: Assessing the diversification benefits of African securities

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Portfolio optimization at the frontier: Assessing the diversification benefits of African securities*

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Abstract

This study investigates the diversification benefits of African securities in comparison with other international investment opportunity sets from the perspective of a US investor. Using data from the most representative S&P Dow Jones traded indices of the US, other developed, emerging and African markets for the period of July 2014 – September 2018, I assess the benefits of diversification over these markets using the traditional and step-down tests of mean-variance spanning, and test the results' robustness by deviating from the normality assumption. My results show that, unlike their peers, African investment opportunity sets offer statistically significant diversification benefits to the benchmark US domestically-diversified minimum-variance and tangency portfolio. More specifically, I find that the "All Africa" set contributes to risk profile of the benchmark set while the "Africa ex-SA" is the only set offering significant improvements to this benchmark's tangency portfolio. These results bring additional evidence to the observation that countries with higher country-risk offer greater potential benefits of global diversification, which justifies to a significant extent the ongoing high appetite of international investors for African securities.

Keywords: International diversification, Mean-Variance Spanning, Frontier markets, Africa
JEL codes: G11, G15, C46

1 Introduction

Since the last decade, African securities have gained an increased focus from international investors who have been particularly responsive to eurobond issues from this continent including from the post-HIPCs (Heavily Indebted Poor Countries). To name a few, after the African eurobond spree that got issues from this countries such as Zambia in 2012, Rwanda in 2013, Cote d'Ivoire in 2015 and Ghana in 2016 receive order books of respectively 15, 8.5, 4 and more than 5 times their book sizes, recent issues by countries such as Kenya, Senegal and Egypt have been oversubscribed for respectively 7, 5 and 4 times their book sizes in 2018. For many observers, this investors' appetite for African assets is mainly driven by the quest for high yields far afield their domestic environments dominated by protracted low growth and sluggish economic recovery, hence their eagerness to partake of the recent records of Africa's economic performance. It is believed that the stimulus packages to address the consequences of the global financial and European

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sovereign debt crises of the mid-2000s and early 2010s in advanced economies have resulted in unprecedented low interest rates that shrank growth possibilities for investors who had no alternative but to explore new opportunities in emerging and frontier markets including Africa.

Despite its plausibility, this general perception trying to portray the enthusiasm of investors for African securities as a desperate –and maybe reckless– move in quest for high yields outside the developed world seems to overlook the benefits of these assets in terms of portfolio diversification that can also justify the interest of international investors in light of the insights from the international portfolio theory (see Moffett et al., 2011, chap. 17). In fact, it is argued that, over time, the increased economic and financial integration between the developed and major emerging markets has compounded the correlations between these two markets' asset returns with, as a consequence, a decrease in the diversification benefits that directed the attention of international investors to new attractive environments (Piljak and Swinkels, 2017). Empirical studies have shown that the current investors' interest in the next generation of emerging markets (the so-called frontier market) has been whetted among others by their low integration with global markets (see, e.g. Berger et al., 2011) that offer significant diversification benefits in terms of both expected return increase and portfolio risk reduction (Li et al., 2003; Jayasuriya and Shambora, 2009; Berger et al., 2013, 2011). Moreover, based on the results of Driessen and Laeven (2007) in the context of emerging markets suggesting that countries with higher country risk offer greater potential benefits of global diversification, it seems justified to consider the high potential diversification benefits of African securities as the main driver of international investors' appetite for these assets.

The revival of economic growth in Africa since the mid-2000s – especially in the Sub-Saharan Africa (SSA) part thanks to a successful implementation of the Heavily Indebted Poor Countries (HIPC) and Multilateral Debt Relief Initiative (MDRI) initiatives[1] – has been backed by an arsenal of structural reforms aiming at the improvement of countries' soft and hard infrastructure to expedite and sustain their development, and create a conducive environment for a private sector-led economic growth. In many countries, these reforms have among others facilitated the development of financial markets to allow domestic and foreign investors to contribute and partake to economic growth. It is even argued that the issue of sovereign eurobonds by SSA countries on international financial markets was, for many of the first time issuers, an opportunity to register on the investors' radar (Bertin, 2016). The local currency bond market has as well registered significant developments allowing the participation of both domestic and international investors (Essers et al., 2016; Dafe et al., 2018). Importantly, financial assets from Africa have been progressively integrated into globally recognized and traded financial indices such as the JP Morgan Emerging Bond Index (EMBI) and MSCI Frontier Markets Index, or constituted specific independently-traded components of these indices (e.g. MSCI Frontier Markets Africa Index. S&P Dow Jones has since recently launched a variety of specific indices for Africa, a move that arguably improves the visibility of African assets within the investment community.

Notwithstanding, the literature on the diversification potential of emerging and frontier markets has timidly covered the African market. So far, the few studies on international diversification beyond the developed and traditional emerging markets have only approached the African market as a constituent of the general frontier market category (see, e.g. Jayasuriya and Shambora, 2009; Driessen and Laeven, 2007; Berger et al., 2011, 2013; Piljak and Swinkels, 2017). Despite the growing interest of international investors in this particular market, little is still know about its individual contribution to the risk-return profile of globally-diversified portfolios, and the relative performance its different investment opportunity sets has not yet been fully investigated. This research zooms into the potential diversification benefits of African securities by considering separately the cases of the whole Africa, Africa excluding South Africa and, when applicable, frontier Africa[2].A comparison is then made between the individual performances of these separated seg-

[1] The Heavily Indebted Poor Countries (HIPC) and Multilateral Debt Relief Initiative (MDRI) initiatives by the International Monetary Fund (IMF) and the World Bank were meant to address the issue of protracted and unsustainable issue of excessive external debt by poor countries, most of them from SSA. Details about these initiatives' motivation, processes and outcomes can be found in Bhattacharya et al. (2004) and IMF (2017).

[2] The term "frontier markets" refers to countries with markets that are smaller and less liquid than those in the more advanced emerging markets (Nellor, 2008). The S&P Africa Frontier BMI index covers Botswana, Cote d'Ivoire, Ghana, Kenya, Mauritius, Namibia, Nigeria and Zambia (see `https://us.spindices.com/indices/equity/sp-africa-frontier-bmi-us-dollar`).

ments of the African market and both the developed and traditional emerging markets. This analysis is kept at the aggregate (continental) level and only covers US dollar-denominated traded indices to allow easy comparability across the asset classes and markets under consideration.

In terms of contribution to the existent literature, this study is to our knowledge the first to specifically focus on Africa and break down the potential diversification benefits of its investable assets per level of their originators' economic and financial market development. Assuming that lower development levels entails high risks, this distinction allows the test of the conclusion of Driessen and Laeven (2007) that the gains from international portfolio diversification appear to be largest for countries with high country risk. Furthermore, in light of the conclusions by Piljak and Swinkels (2017), I test whether the scope of the selected African securities diversification benefits is affected by the prior integration of emerging and/or other developed markets. As a primer to my results, this research shows that 1) unlike the emerging and developed market sets, our selected African investment opportunity sets offer statistically significant diversification benefits to an investor holding the benchmark US diversified portfolio; 2) while the "All Africa" set significantly improve the risk profile of the benchmark portfolio, the "Africa ex-SA Africa" is the only set of the whole investment universe studied to offer statistically significant diversification benefits in terms of expected portfolio return; and 3) the prior integration of investment opportunity sets from emerging or other developed markets does not abate the diversification benefits of the considered African securities.

In the rest of the paper, after a brief summary of the related literature in Section 2, I elaborate on the methodology of the mean-variance spanning tests applied in this analysis in Section 3. Data and empirical results are presented in Section 4 followed by the general conclusions of the paper in Section 5.

2 Literature review

A good number of studies have investigated the diversification potential of emerging and frontier markets including Africa. Though not exhaustive, the following literature has proved closely related to this research. Jayasuriya and Shambora (2009) use daily data of major world equity markets for the period January 2000 to December 2007 and find that US investors would have achieved substantial international diversification benefits in terms of portfolio risk minimization should they have integrated assets from frontier markets and emerging markets of Europe, Latin America, Middle East, Africa and South Asia. Berger et al. (2011) systematically assess the level of integration among global financial markets using daily data on the constituents of the MSCI World Index. Their findings indicate a low level of frontier markets integration with the world market and thereby support the conclusion that these markets offer significant diversification benefits. Beyond the equity markets, Berger et al. (2013) uses exchange-traded funds (ETFs) from the frontier and US markets and find outstanding benefits of diversification over ETFs from frontier markets. A recent study by Piljak and Swinkels (2017) focuses on government bonds using data from the US and selected emerging and frontier markets over the period 2001-2013. Their results indicate a time-varying but zero-averaged correlation between the returns of frontier markets and US government bonds. Importantly, they emphasize the high correlation between US investment grade corporate bonds, US corporate high yield bonds, and US dollar-denominated debt issued by governments of emerging markets, which limits the diversification benefits for a US portfolio containing these asset classes.

Marshall et al. (2015) push further the investigation of frontier markets diversification benefits by measuring and integrating transaction costs associated to trades on these markets. Using high-frequency tick data covering 19 countries included in the MSCI Frontier Markets Index for the period June 2002 – December 2010, they find that transactions costs in frontier markets are almost three times larger than those in the U.S market. They therefore recommend a three month or longer rebalancing period to safeguard the benefits of diversification over these markets.

The literature on frontier African assets has dominantly focused on the maiden hard-currency sovereign bonds issued by SSA countries on international capital markets (Sy, 2013; Willem te Velde, 2014; Mecagni et al., 2014; Gevorkyan and Kvangraven, 2016; Sy, 2015; Senga et al., 2018). In most cases, these studies have focused on the drivers of African eurobonds' primary and secondary market yields, and disputed the prevalence of global and country-specific factors in influencing these yields' evolution; the perspective of investors seeking diversification benefits has not yet been covered despite the insights from the international portfolio theory. Some other studies have investigated the development of local currency bond markets in these countries, emphasizing the increasing availability of long-term investment opportunities in these markets (Essers et al., 2016; Dafe et al., 2018). To my knowledge, no study has specifically zoomed into the diversification benefits of African assets. An attempt in this avenue has been made by Lagoarde-Segot and Lucey (2007) who limit their investigation on equity markets of the Middle East and North Africa.

The contribution of this research to this literature is twofold. First, it provides a thorough investigation of the potential diversification benefits of dissected segments of the African market, namely the whole of Africa, Africa excluding South Africa and frontier Africa, in comparison with the emerging and other developed markets outside the USA. Second, by covering all the African tradable opportunities including the highly-advertised hard-currency sovereign bonds, it contributes to the ongoing debate on the drivers on the observed enthusiasm of international investors for African securities by bringing in the diversification perspective that has so far been overlooked.

3 Methodology

As in Driessen and Laeven (2007), this paper uses the standard meanvariance framework of Markowitz (1952) assuming the normality of asset returns and a mean-variance utility function for the investor. In this *mean-variance* framework, the investor maximizes the expected utility

$$\max_{w} w'\mu - \frac{\gamma}{2} w'\Omega w \tag{1}$$

with w the vector of optimal portfolio weights of N risky assets, μ the N-vector of expected excess returns over the risk-free asset, Ω the $N \times N$ covariance matrix and γ the risk aversion parameter. Mishra (2015) recalls that the solution to this maximization problem is given by

$$w^* = \frac{1}{\gamma}\Omega^{-1}\mu \tag{2}$$

and that, under the assumption of the sum of the portfolio weights equal to 1 ($w'1_N = 1$), and where a risk-free rate is available and chosen as the zero-beta portfolio and when short-sales are allowed, optimal weights are given by

$$w^* = \frac{\Omega^{-1}\mu}{\mathbf{1}'_N\Omega^{-1}\mu} \tag{3}$$

with $\mathbf{1}_N$ the N-dimensional vector of 1. Given the covariance matrix Ω, the vectors of expected returns μ and and optimal weights w, the expected portfolio return (μ_p) and variance (Ω_p) are obtained respectively by

$$\mu_p = w'\mu \quad \text{and} \quad \Omega_p = w'\Omega w \tag{4}$$

In essence, the Markowitz portfolio theory aims at maximizing the portfolio return for a given level of portfolio risk or, inversely, minimize the portfolio risk for a given portfolio return. However, it has widely been acknowledged that, in practice, expected returns are more difficult to estimate due to the high volatility of returns (Li et al., 2003; Mishra, 2015). For instance, Li et al. (2003) underscore the fact that risk-averse investors with little ability to forecast expected returns might be inclined to assess the diversification benefits in terms of reduction in their portfolio variance. This paper takes into account this perspective by applying a *global minimum variance* portfolio strategy whose asset weights are independent of expected returns of individual assets composing the portfolio.

3.1 Minimum variance portfolio

The minimum variance portfolio strategy is based on the assumption that sample means differ just because of noise (Mishra, 2015). Therefore, given N assets having a covariance matrix Ω, the minimum variance portfolio weights are given by

$$w^* = \frac{\mu\Omega^{-1}}{\mu\mathbf{1}_N'\Omega^{-1}\mathbf{1}_N} = \frac{\Omega^{-1}}{\mathbf{1}_N'\Omega^{-1}\mathbf{1}_N} \tag{5}$$

with all the parameters defined as in (1) and (3).

More insights can be drawn from David Levermore's lecture notes[3] who considers the minimum variance portfolio strategy as a minimization problem of $\sigma = \sqrt{w'\Omega w}$ with $w \in \mathbb{R}^+$ subject to the constraints $\mathbf{1}_N'w = 1$ and $\mu'w = \mu_p$ for a given μ_p. Since $\sigma > 0$, minimizing σ is equivalent to minimizing σ^2, an easy-to-solve quadratic function of w that can be formalized as follows:

$$\begin{cases} \min\limits_{w\in\mathbb{R}^+} & \frac{1}{2}w'\Omega w \\ s.t. & \mathbf{1}_N'w = 1 \quad \text{and} \quad \mu'w = \mu_p \end{cases} \tag{6}$$

hence the following Lagrangian $\mathcal{L}(w, \lambda_1, \lambda_2) = \frac{1}{2}w'\Omega w - \lambda_1(\mathbf{1}_N'w - 1) - \lambda_2(\mu'w - \mu_p)$. By equalizing the first derivatives with respect to w, λ_1, λ_2 to zero, we get

$$\begin{cases} \partial_w \mathcal{L}(w, \lambda_1, \lambda_2) & = \Omega w - \lambda_1\mathbf{1}_N - \lambda_2\mu = 0 \\ \partial_{\lambda_1} \mathcal{L}(w, \lambda_1, \lambda_2) & = -\mathbf{1}_N'w + 1 = 0 \\ \partial_{\lambda_2} \mathcal{L}(w, \lambda_1, \lambda_2) & = -\mu'w + \mu_p = 0 \end{cases} \tag{7}$$

Since Ω is positive definite, we can solve for $w = \lambda_1\Omega^{-1}\mathbf{1}_N + \lambda_2\Omega^{-1}\mu$ and replace w in the other equations to get

$$\begin{cases} \lambda_1\mathbf{1}_N'\Omega^{-1}\mathbf{1}_N + \lambda_2\mathbf{1}_N'\Omega^{-1}\mu = 1 \\ \lambda_1\mu'\Omega^{-1}\mathbf{1}_N + \lambda_2\mu'\Omega^{-1}\mu = \mu_p \end{cases} \tag{8}$$

By setting $a = \mathbf{1}_N'\Omega^{-1}\mathbf{1}_N$, $b = \mathbf{1}_N'\Omega^{-1}\mu$ and $c = \mu'\Omega^{-1}\mu$, the system (8) can be written as

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \mu_p \end{pmatrix}$$

Levermore shows that when $\mathbf{1}_N$ and μ are not co-linear, λ_1 and λ_2 are obtained by

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \frac{1}{ac - b^2} \begin{pmatrix} c & -b \\ -b & a \end{pmatrix} \begin{pmatrix} 1 \\ \mu_p \end{pmatrix} = \frac{1}{ac - b^2} \begin{pmatrix} c - b\mu_p \\ a\mu_p - b \end{pmatrix}$$

Therefore, for each μ_p, we have

$$w(\mu_p) = \frac{c - b\mu_p}{ac - b^2}\Omega^{-1}\mathbf{1}_N + \frac{a\mu_p - b}{ac - b^2}\Omega^{-1}\mu$$

with the associated minimum value of σ^2 given by

$$\begin{aligned} \sigma^2 &= w(\mu_p)'\Omega w(\mu_p) = (\lambda_1\Omega^{-1}\mathbf{1}_N + \lambda_2\Omega^{-1}\mu)'\Omega(\lambda_1\Omega^{-1}\mathbf{1}_N + \lambda_2\Omega^{-1}\mu) \\ &= \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \frac{1}{ac - b^2} \begin{pmatrix} 1 & \mu_p \end{pmatrix} \begin{pmatrix} c & -b \\ -b & a \end{pmatrix} \begin{pmatrix} 1 \\ \mu_p \end{pmatrix} \\ &= \frac{1}{a} + \frac{a}{ac - b^2}\left(\mu_p - \frac{b}{a}\right)^2 \end{aligned} \tag{9}$$

[3] See details on http://www.terpconnect.umd.edu/~lvrmr/2011-2012-S/Classes/MATH420/SLIDES/Risky03.pdf.

Equation (9) corresponds to the hyperbola representing the *frontier portfolios* in the $\sigma\mu_p$-plane, with each point σ, μ_p representing a unique Markowitz portfolio. Replacing a, b and c with meaningful *frontier* parameters $\sigma_{\text{mv}} = \frac{1}{\sqrt{a}}$, $\mu_{\text{mv}} = \frac{b}{a}$ and $\nu_{as} = \sqrt{\frac{ac-b^2}{a}}$, equation (9) can be written as

$$\sigma^2 = \sigma_{\text{mv}}^2 + \left(\frac{\mu_p - \mu_{\text{mv}}}{\nu_{as}}\right)^2$$

After some manipulations, we get the following parts of the hyperbola representing respectively the efficient and inefficient frontiers

$$\mu_p^+ = \mu_{\text{mv}} + \nu_{as}\sqrt{\sigma^2 - \sigma_{\text{mv}}^2} \quad \text{and} \quad \mu_p^- = \mu_{\text{mv}} - \nu_{as}\sqrt{\sigma^2 - \sigma_{\text{mv}}^2} \tag{10}$$

As $\sigma \to \infty$, these frontiers become asymptotic to the lines $\mu_p = \mu_{\text{mv}} + \nu_{as}\sigma$ and $\mu_p = \mu_{\text{mv}} - \nu_{as}\sigma$ respectively.

3.2 Mean-variance spanning

The notion of mean-variance spanning relates to the efficiency of a benchmark portfolio mean-variance with respect to a given set of test assets positing the impossibility of obtaining either, for a given expected return, a portfolio with a lower variance, or, for a given portfolio variance level, a higher expected return, through a combination of the benchmark and test asset portfolios. Following Huberman and Kandel (1987), the benchmark portfolio of K risky assets is said to span a larger portfolio of $K+N$ risky assets if the minimum-variance frontier of the benchmark K assets is identical to the minimum-variance frontier of the K assets plus an additional test portfolio of N risky assets, thus indicating the impossibility of improving the K asset benchmark portfolio's mean-variance profile through the integration of the N asset test portfolio.

The context and usefulness of the mean-variance spanning analysis is provided in the following summary by Kan and Zhou (2012): "When there exists a risk-free asset and when unlimited lending and borrowing at the risk-free rate is allowed, then investors who care about the mean and variance of their portfolios will only be interested in the tangency portfolio of the risky assets (i.e., the one that maximizes the Sharpe ratio). In that case, the investors are only concerned with whether the tangency portfolio from using K benchmark risky assets is the same as the one from using all $K + N$ risky assets. However, when a risk-free asset does not exist, or when the risk-free lending and borrowing rates are different, then investors will be interested instead in whether the two minimum-variance frontiers are identical. The answer to this question allows us to address two interesting questions in finance. The first question asks whether, conditional on a given set of $K + N$ assets, an investor can maximize his utility by holding just a smaller set of K assets instead of the complete set. This question is closely related to the concept of K-fund separation and has implications for efficient portfolio management. The second question asks whether an investor, conditional on having a portfolio of K assets, can benefit by investing in a new set of N assets. This latter question addresses the benefits of diversification, and is particularly relevant in the context of international portfolio management when the K benchmark assets are domestic assets whereas the N test assets are investments in foreign markets" (see Kan and Zhou, 2012, p. 141).

This paper uses the mean-variance spanning analysis to assess the contribution of African securities to the mean-variance profile of a US domestically-diversified benchmark portfolio. The individual contribution of our selected African securities is evaluated in comparison to that of the portfolios made of securities from developed and emerging economies outside the US market to analyze the rationale behind the recently observed enthusiasm of investors for African securities. This analysis is complemented with the evaluation of the contribution of these African securities against the scenarios where the benchmark portfolio has already integrated one of the developed or emerging markets or both to test the validity of the non-diversification benefits of the kind of Piljak and Swinkels (2017)[4] in the case of African securities.

[4]These authors underscore the high correlation between US investment grade corporate bonds, US corporate high yield bonds, and US dollar-denominated debt issued by governments of emerging markets, which limits the diversification benefits of the latter assets for a US portfolio containing the former two asset classes.

3.3 Tests of mean-variance spanning

The test of mean-variance spanning was first formalized by Huberman and Kandel (HK henceforth) who project the returns of the test portfolio to those of the benchmark portfolio and test the significance of the resulting coefficients. Assuming $R_t = [R'_{1t}, R'_{2t}]$ the raw returns on respectively the K benchmark and N test risky assets at time t, the expected returns on the $K + N$ assets as well as their covariance matrix can be defined as

$$\mu = E\left[R_t\right] \equiv \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad \text{and} \quad \Omega = \text{Var}\left[R_t\right] \equiv \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \tag{11}$$

By projecting R_{2t} on R_{1t}, we have

$$R_{2t} = \alpha + \beta R_{1t} + \varepsilon_t \quad \text{with} \quad E\left[\varepsilon_t\right] = \mathbf{0}_N \quad \text{and} \quad E\left[\varepsilon_t R'_{1t}\right] = \mathbf{0}_{K\text{x}N} \tag{12}$$

where $\mathbf{0}_N$ is an N-vector of zeros and $\mathbf{0}_{K\text{x}N}$ is an $K\text{x}N$ matrix of zeros. Defining $\delta = \mathbf{1}_N - \beta\mathbf{1}_K$ with $\delta = \mathbf{1}_N$ and $\delta = \mathbf{1}_K$ N and K-vectors of ones respectively, HK check the necessary and sufficient conditions for spanning by testing the following joint hypothesis of

$$H_0: \quad \alpha = \mathbf{0}_N \quad \text{and} \quad \delta = \mathbf{0}_N \tag{13}$$

When the hypothesis (13) holds, the N test portfolio is said to be spanned (i.e. dominated) by the K benchmark portfolio as it is possible to find a portfolio of K benchmark assets that has the same mean but a lower variance than the N test portfolio.

3.3.1 Traditional tests

HK use a likelihood ratio (LR) test to test the hypothesis (13). This test compares the likelihood functions under the null and the alternative hypothesis. Recalling that equation (12) can be written in the following matrix form

$$Y = XB + E$$

with Y a $T\text{x}N$ matrix of R_{2t}, X a $T\text{x}(K+1)$ matrix of $[1, R'_{1t}]$ rows, $B = [\alpha, \beta]'$, and E a $T\text{x}N$ matrix of ε'_t. It is assumed that $T \geqslant N + K + 1$ and $X'X$ is singular. In order to obtain the exact distributions of the test statistics, it is further assumed that, conditional on R_{1t}, the error terms ε_t are independent and identically distributed as multivariate normal with mean zero and variance Σ. The maximum likelihood estimators of B and Σ are given by

$$\hat{B} \equiv [\hat{\alpha}, \hat{\beta}]' = (X'X)'(X'Y) \quad \text{and} \quad \hat{\Sigma} = \frac{1}{T}(Y - X\hat{B})'(Y - X\hat{B})$$

Assuming $\tilde{\Sigma}$ the constrained maximum likelihood estimator of Σ and $U = |\hat{\Sigma}|/|\tilde{\Sigma}|$, the LR test of hypothesis (13) is given by

$$LR = -T\ln(U) \sim \chi^2_{2N} \tag{14}$$

Kan and Zhou (2012) contend that, numerically, the performance of the constrained estimation is not needed in order to obtain the LR test statistic. In fact, the null hypothesis (13) can be written as $H_0 : \Theta = 0_{2\text{x}N}$ with $\Theta = [\alpha, \delta]'$. The maximum likelihood estimator of Θ is given by $\hat{\Theta} \equiv [\hat{\alpha}, \hat{\beta}]' = A\hat{B} + C$ with

$$A = \begin{bmatrix} 1 & 0'_K \\ 0 & -1'_K \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0'_N \\ 1'_N \end{bmatrix}$$

The authors further define

$$\hat{G} = TA(X'X)^{-1}A' = \begin{bmatrix} 1+\hat{\mu}_1'\hat{V}_{11}^{-1}\hat{\mu}_1 & \hat{\mu}_1'\hat{V}_{11}^{-1}1_K \\ \hat{\mu}_1'\hat{V}_{11}^{-1}1_K & 1_K'\hat{V}_{11}^{-1}1_K \end{bmatrix}$$

with $\hat{\mu}_1' = \frac{1}{T}\sum_{t=1}^{T} R_{1t}$ and $\hat{V}_{11}^{-1} = \frac{1}{T}\sum_{t=1}^{T}(R_{1t}-\hat{\mu}_1)(R_{1t}-\hat{\mu}_1)'$ and contend that

$$\tilde{\Sigma} - \hat{\Sigma} = \hat{\Theta}'\hat{G}^{-1}\hat{\Theta}$$

so that $1/U$ can be obtained from the unconstrained estimate alone as

$$\begin{aligned} \frac{1}{U} &= \frac{\tilde{\Sigma}}{\hat{\Sigma}} = |\hat{\Sigma}^{-1}\tilde{\Sigma}| = |\hat{\Sigma}^{-1}(\hat{\Sigma}+\hat{\Theta}'\hat{G}^{-1}\hat{\Theta})| \\ &= |I_N + \hat{\Sigma}^{-1}\hat{\Theta}'\hat{G}^{-1}\hat{\Theta}| = |I_2 + \hat{H}\hat{G}^{-1}| \end{aligned} \tag{15}$$

with

$$\hat{H} = \hat{\Theta}\hat{\Sigma}^{-1}\hat{\Theta}' = \begin{bmatrix} \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} & \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\delta} \\ \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\delta} & \hat{\delta}'\hat{\Sigma}^{-1}\hat{\delta} \end{bmatrix}$$

Denoting λ_1 and λ_2 as the two eigenvalues of $\hat{H}\hat{G}^{-1}$ with $\lambda_1 \geqslant \lambda_2 \geqslant 0$, we have $1/U = (1+\lambda_1)(1+\lambda_2)$ such that the LR test can be written as

$$LR = T\sum_{i=1}^{2}\ln(1+\lambda_i) \tag{16}$$

The authors indicate that, besides the LR test, the hypothesis (13) can be tested using the Wald test (W) and the Lagrange multiplier test (LM) test given by

$$W = T(\lambda_1+\lambda_2) \sim \chi^2_{2N} \quad \text{and} \quad LM = T\sum_{i=1}^{2}\frac{\lambda_i}{1+\lambda_i} \sim \chi^2_{2N} \tag{17}$$

However, following the observation by Gibbons et al. (1989) and others, Kan and Zhou (2012) warn that, although LR, W and LM all have an asymptotic χ^2_{2N} distribution, we have $W \geqslant LR \geqslant LM$ in finite samples inducing conflicting results with LM favoring acceptance and W favoring rejection (see also Sentana, 2009). Therefore, they recommend the following F-test

$$\begin{cases} \text{If} \quad N \geqslant 2: \left(\frac{1}{U^{\frac{1}{2}}}-1\right)\left(\frac{T-K-N}{N}\right) \sim F_{2N,2(T-K-N)} \\ \text{If} \quad N = 1: \left(\frac{1}{U}-1\right)\left(\frac{T-K-1}{2}\right) \sim F_{2,T-K-1} \end{cases} \tag{18}$$

3.3.2 Step-down test

One of the criticisms of the traditional spanning tests is that they jointly test the two components of the spanning hypothesis (13), i.e. $\alpha = 0_N$ and $\delta = 0_N$. Kan and Zhou (2012) argue that, this practice does not take into account the economic significance of the departure from the spanning hypothesis as, though statistically significant, a small difference in the global minimum-variance portfolios is not necessarily as economically important as the statistically hard-to-detect potential big difference in the tangency portfolios. This weakness call on some caution in the interpretation of the tests' results as a low p-value does not always imply an economically significant difference between the two frontiers, the same as a high p-value does not always imply a null contribution of the test assets to the mean-variance profile of the benchmark assets.

The step-down test proposed by Kan and Zhou (2012) is a sequential test that first tests the hypothesis $\alpha = 0_N$, and then $\delta = 0_N$ conditional on the constraint $\alpha = 0_N$. The first hypothesis is tested using

$$F_1 = \left(\frac{T-K-N}{N}\right)\left(\frac{|\bar{\Sigma}|}{|\hat{\Sigma}|}-1\right) = \left(\frac{T-K-N}{N}\right)\left(\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}\right) \tag{19}$$

where $\hat{a} = \hat{\mu}'\Omega^{-1}\hat{\mu}$, $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} R_t$, $\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}(R_t - \hat{\mu})(R_t - \hat{\mu})'$, and $\hat{a_1} = \hat{\mu_1}'\Omega_{11}^{-1}\hat{\mu_1}$ related to the K benchmark assets, and where $\hat{\Sigma}$ is the unconstrained estimates of Σ and $\bar{\Sigma}$ its estimate with the only constraint $\alpha = 0_N$. Under the null hypothesis, F_1 has a central F-distribution with N and $T-K-N$ degrees of freedom.

The second hypothesis is tested using

$$\begin{aligned} F_2 &= \left(\frac{T-K-N+1}{N}\right)\left(\frac{|\tilde{\Sigma}|}{|\bar{\Sigma}|} - 1\right) \\ &= \left(\frac{T-K-N+1}{N}\right)\left[\left(\frac{\hat{c}+\hat{d}}{\hat{c}_1+\hat{d}_1}\right)\left(\frac{1+\hat{a}_1}{1+\hat{a}}\right) - 1\right] \end{aligned} \tag{20}$$

where $\hat{c} = 1'_{N+K}\Omega^{-1}1_{N+K}$, $\hat{d} = \hat{a}\hat{c} - \hat{b}^2$ with $\hat{b} = \hat{\mu}'\Omega^{-1}1'_{N+K}$, defined as previously $\hat{c}_1$,$\hat{d}_1$ related to the K benchmark assets, and where $\tilde{\Sigma}$ is the constrained estimate of Σ with both constraints $\alpha = 0_N$ and $\delta = 0_N$. The authors show that, under the null hypothesis, F_2 is independent of F_1 and has a central F-distribution with N and $T-K-N+1$ degrees of freedom.

Unlike the traditional tests, the step-down test has the advantage of unveiling the cause of the rejection. In fact, a rejection due to the first test means that the two tangency portfolios are statistically very different while the one due to the second test indicates a significant statistically difference between the two global minimum-variance portfolios.

3.3.3 Tests under non-normality

The traditional spanning tests are based on the assumption that the error term ε_t is normally, and independently and identically distributed. Kan and Zhou (2012) explore two cases cases of non-normality where 1°) ε_t is non-normal but still independently and identically distributed conditional on R_{1t}, and 2°) ε_t exhibits conditional heteroskedasticity, i.e. its variance is time-varying and can be expressed as a function of R_{1t}. The authors contend that, for the first case, the conditional homoskedasticity, the traditional tests are still asymptotically χ^2_{2N} distributed under the null hypothesis but their finite sample distributions will not be the same as the ones presented in paragraph 3.3.1, though without major harm to their ability to provide a very good approximation for the small sample distribution of the non-normality case.

However, in the case of conditional heteroskedasticity, the traditional test statistics are no longer χ^2_{2N} distributed under the null hypothesis, which affects their suitability. As an alternative, Kan and Zhou (2012) propose a generalized method of moments (GMM) approach based on Ferson et al. (1993). Assume $x_t = [1, R'_{1t}]'$ and $\varepsilon_t = R'_{2t} - B'x_t$, the GMM moment conditions for the estimation of B are

$$E[g_t] = E[x_t \otimes \varepsilon_t] = 0_{(K+1)N}$$

Assuming stationary R_t with finite fourth moments, the sample moments are given by

$$\bar{g}_T(B) = \frac{1}{T}\sum_{t=1}^{T} x_t \otimes (R'_{2t} - B'x_t)$$

and the GMM estimate of B is obtained by minimizing $\bar{g}_T(B)'S_T^{-1}\bar{g}_T(B)$ with S_T a consistent estimate of $S_0 = E[g_t g'_t]$, assuming the absence of serial correlation of g_t. The GMM version of the Wald test is then written as

$$W_a = T\mathrm{vec}\left(\hat{\Theta}'\right)'\left[(A_T \otimes I_N) S_T (A'_T \otimes I_N)\right]^{-1}\mathrm{vec}\left(\hat{\Theta}'\right) \sim \chi^2_{2N} \tag{21}$$

where

$$A_T = \begin{bmatrix} 1+\hat{a}_1 & -\hat{\mu}_1\hat{\Sigma}_{11}^{-1} \\ \hat{b}_1 & -1'_K\hat{\Sigma}_{11}^{-1} \end{bmatrix}$$

Kan and Zhou (2012) illustrate the case of conditional heteroskedasticity by assuming R_t independently and identically distributed as a non-degenerate multivariate *elliptical distribution* with finite fourth moments. Given the kurtosis parameter

$$\kappa = \frac{E\left[\left((R_t-\mu)'\,\Sigma^{-1}\,(R_t-\mu)\right)^2\right]}{(N+K)(N+K+2)} - 1$$

they argue that the GMM Wald test of spanning is then given by

$$W_a^e = T\,\mathrm{tr}\left(\hat{H}\hat{G}_a^{-1}\right) \sim \chi^2_{2N} \tag{22}$$

with $\hat{H}$ defined as in (15) and

$$\hat{G}_a = \begin{bmatrix} 1+(1+\hat{\kappa})\hat{a}_1 & (1+\hat{\kappa})\hat{b}_1 \\ (1+\hat{\kappa})\hat{b}_1 & (1+\hat{\kappa})\hat{c}_1 \end{bmatrix}$$

where $\hat{\kappa}$ is a consistent estimate of the kurtosis parameter κ.

4 Data and empirical results

4.1 Data

This paper uses weekly data of stock and bond indices for the US, other developed, emerging and African markets downloaded from the S&P indices website[5] for the period of July 2014 – September 2018. The starting date is justified by the availability of certain African indices, notably the S&P Africa Hard Currency Sovereign Bond Index that has been launched in November 2014 with values starting in June 2014. I specifically use the S&P500, S&P500 Corporate Bond and S&P US Treasury Bond indices for US market, the S&P Developed Ex-US Broad Market Index (BMI), S&P International Corporate Bond, and S&P Global Developed Sovereign ex-US Bond indices for other developed markets. The emerging market is represented by the S&P Emerging ex-Africa and S&P Global Emerging Sovereign Inflation-Linked Bond indices for emerging markets. As concerns Africa, I distinguish the following three investment opportunity sets: the "All Africa" set made of the S&P All Africa stock index, the S&P Africa Sovereign Bond index and the S&P Africa Hard Currency Sovereign Bond index; the "Africa ex-SA" set excluding South-Africa and made of the S&P All Africa ex-South Africa stock index and the S&P Africa Sovereign ex-South Africa Bond index; and the "Frontier Africa" set where we combine the S&P Africa Frontier BMI stock index and the S&P Africa Sovereign ex-South Africa Bond index. I consider the US dollar-denominated version of these indices to ensure comparability and overcome the currency-related risk deemed the single greatest source of volatility in local-currency emerging market debt (Zamora, 2016). Also, these indices are considered at their total return values when applicable to take into account all the possible financial influxes generated by investments in these assets.

The summary statistics and correlations are presented in Table 1 and 2. The indices average and median returns as well as their volatility (standard deviation) are presented in percentage annualized approximations computed from weekly data. As expected, these statistics indicate higher volatility levels for stocks compared to bonds across all the considered markets, with the "All Africa" stock index exhibiting the highest volatility level followed by its "Frontier Africa" counterpart. As far as the correlation are concerned, three main observations can be drawn from these figures. One, there seems to be significantly high correlations within the equity asset class across all the considered markets, indicating a higher integration level of equity markets compared to the other considered asset classes. Two, the US government bond index displays negligible correlation levels with the rest of of the considered asset classes and markets; it seems to be only correlated

[5] Data and details on the indices can be found on https://us.spindices.com/index-finder/

with the US market and, to a certain extent, with only government bonds from other developed markets. Last, the "Africa ex-SA government bonds" and "Frontier Africa stocks" sets appear to be the least correlated with other markets and asset classes, particularly with the US market. Worth mentioning is also the rejection of the return normality assumption by the Jarke-Bera test for almost all the considered indices, except the US corporate bond and developed government bond indices.

	N	Mean	Median	St. dev	Skew	Kurtosis	Jarke-Bera
US stocks	217	12.89	17.68	10.36	-0.47	2.74	79.02***
US corp. bonds	217	2.93	4.08	3.30	-0.24	0.32	3.22
US gov. bonds	217	1.09	2.37	2.52	-0.41	0.76	11.66***
Developed stocks	217	4.66	20.87	13.22	-0.46	0.84	14.59***
Developed corp. bonds	217	-0.84	1.13	7.60	-0.46	0.71	12.73***
Developed gov. bonds	217	-0.92	-3.00	7.26	0.02	0.38	1.56
Emerging stocks	217	3.03	28.42	15.49	-0.56	1.13	23.94***
Emerging gov. bonds	217	-3.69	4.54	14.52	-0.45	0.77	13.21***
All Africa stocks	217	-0.42	9.52	20.59	-0.41	0.63	10.14***
Africa LC gov. bonds	217	-1.88	-1.45	11.28	-1.50	6.92	527.07***
Africa HC gov. bonds	217	3.98	7.24	6.76	-0.40	1.20	19.71***
Africa ex SA stocks	217	-10.37	-9.18	14.26	-0.48	0.30	9.30**
Africa ex SA LC gov. bonds	217	-3.13	6.51	10.47	-6.68	60.47	35246***
Frontier Africa stocks	217	-9.40	-3.49	17.01	-0.72	5.95	347.85***

Note: Mean, median and standard deviation are presented in percentage annualized approximations computed from weekly data. As in Li et al. (2003), assets' weekly average returns have been multiplied by 52 and their corresponding standard deviations by $\sqrt{52}$ to get their annual approximate values. ***,** and * indicate the statistically significance at 1%, 5% and 10% respectively.

Table 1: Summary statistics and normality test

4.2 Empirical results

The results of the evaluation of the diversification benefits of the different considered investment opportunity sets are presented in this section. While keeping the perspective of the US investor, I investigate five scenarios starting with my case of interest, i.e. the diversification over the selected investments possibilities in Africa. Then, for the sake of comparison, this scenario is analyzed alongside the scenario where the investor diversifies over the emerging and developed markets outside his domestic US market. The three other case scenarios investigate African securities diversification benefits when the benchmark US portfolio is already diversified over the emerging or other developed markets, or both. These results are first visualized using efficient frontiers corresponding to each of these scenarios followed formal mean-variance spanning tests to assess the statistical significance of the potential benefits of the US benchmark portfolio diversification over the considered investment opportunity sets.

4.2.1 Efficient frontiers

In order to set the ground, I plot on Figure 1 the investment universe selected for this study in a $\sigma\mu$-plane to visualize the individual risk-return profile of the considered asset class indices, and how their combination produce better profiles thanks to diversification. On the individual basis, the graph shows that US government bonds are the least volatile of all the considered assets while the "All Africa" stock index lies on the other extreme of the spectrum with more than 20% annual volatility. However, an interesting observation is that, when grouped per opportunity set, these indices yield improved *minimum-variance* levels compared to the sum of their individual risk levels exactly inline with the portfolio theory predictions. The most stunning case is that of the combination of the "All Africa" stock, local-currency and hard-currency indices that produce the second-best minimum-variance level after the US combination and ahead of both their counterparts from emerging and developed markets.

		A	B	C	D	E	F	G	H	I	J	K	L	M	N
A	US stocks	1.00***													
B	US corp. bonds	-0.04	1.00***												
C	US gov. bonds	-0.23***	0.82***	1.00***											
D	Developed stocks	0.62***	0.07	-0.11	1.00***										
E	Developed corp. bonds	0.15*	0.24***	0.20**	0.40***	1.00***									
F	Developed gov. bonds	-0.06	0.37***	0.43***	0.15*	0.67***	1.00***								
G	Emerging stocks	0.53***	0.18**	0	0.83***	0.35***	0.20**	1.00***							
H	Emerging gov. bonds	0.25***	0.26***	0.16*	0.34***	0.38***	0.27***	0.52***	1.00***						
I	All Africa stocks	0.47***	0.19**	0.06	0.60***	0.42***	0.33***	0.65***	0.46***	1.00***					
J	Africa LC gov. bonds	0.19**	0.22**	0.17*	0.40***	0.38***	0.40***	0.46***	0.41***	0.52***	1.00***				
K	Africa HC gov. bonds	0.33***	0.28***	0.14*	0.58***	0.37***	0.35***	0.59***	0.45***	0.66***	0.56***	1.00***			
L	Africa ex SA stocks	0.34***	0.14*	0	0.47***	0.30***	0.22**	0.55***	0.39***	0.65***	0.36***	0.44***	1.00***		
M	Africa ex SA LC gov. bonds	0.07	0.14*	0.11	0.09	0.11	0.11	0.15*	0.14*	0.1	0.69***	0.13	0.24***	1.00***	
N	Frontier Africa stocks	0.11	0.04	-0.07	0.23***	0.08	0.04	0.30***	0.09	0.22**	0.14*	0.26***	0.57***	0.16*	1.00***

Note: Pairwise Pearson correlations of the assets' weekly returns. ***,** and * indicate the statistically significance at 1%, 5% and 10% respectively.

Table 2: Pairwise Pearson correlations

It can also be observed on this figure that, though the USA opportunity set has the least volatility of all, it still lies at a distance from the *global minimum-variance* point of the *efficient frontier* produced by the combination of all the considered investment opportunity sets, which can arguably be considered as an indication of potential international diversification benefits for a US investor holding a benchmark domestically-diversified portfolio.

The insights on the diversification benefits of the selected African opportunity sets are provided on Figure 2. To start with, the benchmark scenario corresponding to US diversified portfolio is presented on Figure 2a. The related efficient frontier sets the limits in terms of both minimum variance and expected return the investor can envisage by choosing efficient combinations of the considered assets on this market. The scenario corresponding to the diversification over the African market is presented on Figure 2b. This figure indicates the possibility of substantial expansion of the efficient frontier once the considered African investment opportunity sets are integrated to the benchmark portfolio. It can be observed that, while the "Africa ex-SA" and "Frontier Africa" sets seem not to affect the minimum-variance level of the benchmark combination, the "All Africa" set clearly induces a up-leftwards move of the benchmark efficient frontier indicating the possibility of improvement of the minimum-variance level thanks to the diversification over this investment opportunity set. Worth mentioning is also the apparent improvement in the tangency portfolio that appears to be induced by the integration of the "Africa ex-SA" set. These diversification benefits indication will be subjected to rigorous significance tests in the following paragraphs.

On Figure 2c, the potential diversification benefits of the "All Africa" set is compared to those of the "Developed" and "Emerging" sets. The figure clearly displays the distant position of the global minimum-variance point of the "USA + All Africa" efficient frontier compared to the considered alternatives, suggesting that the "All Africa"' set outperforms these alternatives when it comes to improving the risk profile of the benchmark portfolio. This outstanding performance of African sets appears to be robust to prior integration of the other investment opportunities from emerging and developed markets outside the USA as shown on Figure 2d, Figure 2e and Figure 2f.

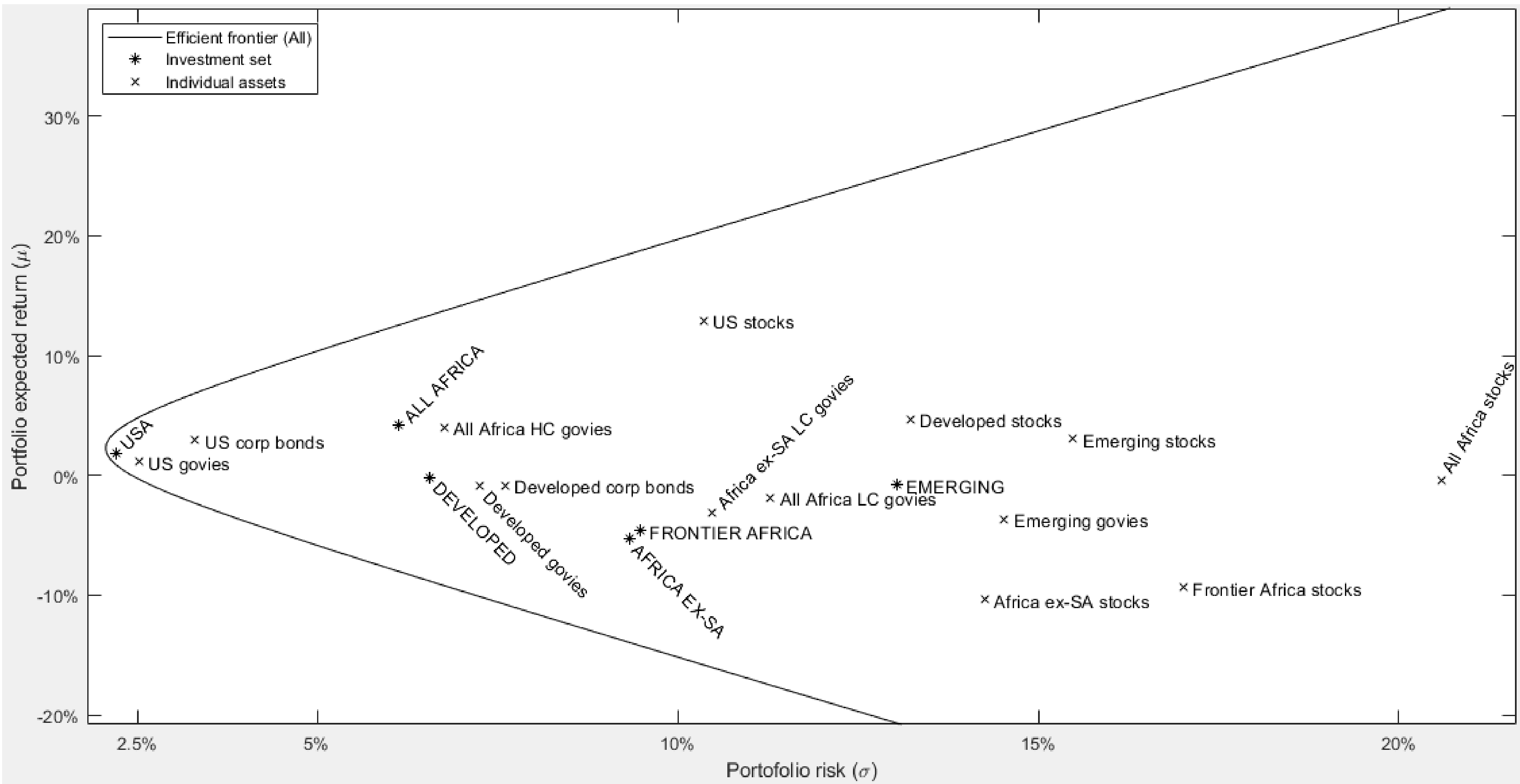
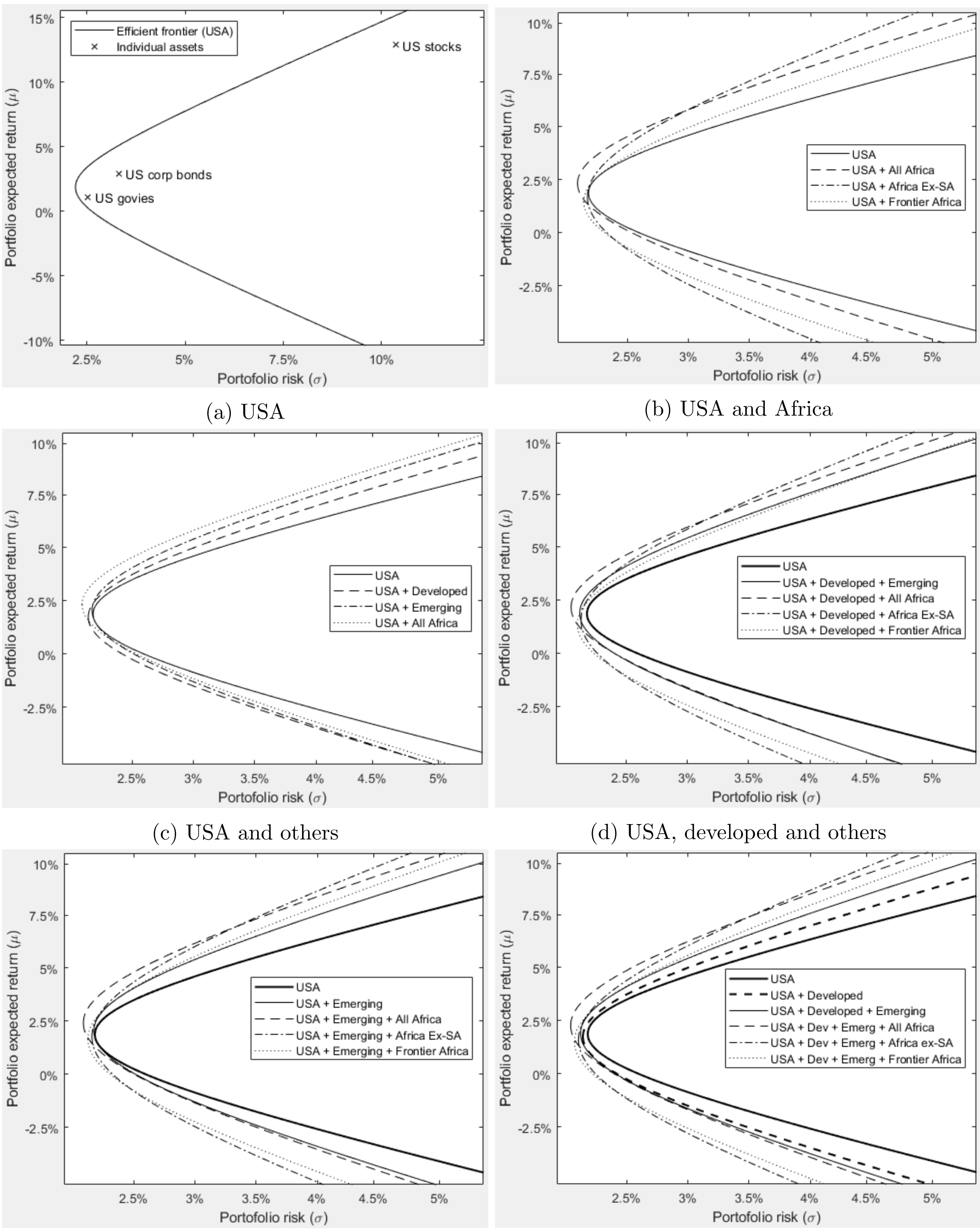


Figure 1: Selected investment universe



(a) USA

(b) USA and Africa

(c) USA and others

(d) USA, developed and others

(e) USA, emerging and others

(f) USA, developed, emerging and others

Figure 2: Efficient frontiers

The international diversification benefits of African securities is further illustrated in Table 3. This table presents the optimal results of the investment scenarios investigated on Figure 2 using two strategies, namely the minimum variance strategy that aims at minimizing the portfolio risk and the mean-variance optimization strategy that maximizes the Sharpe ratio, i.e. the portfolio's expected return per unit of risk. To start with the minimum variance strategy, these results show that, across all the investigated scenarios, the lowest portfolio volatility is obtained by the integration of the "All Africa" set. For instance, it can be observed that the integration of assets from emerging and other developed markets reduces the benchmark portfolio annualized volatility from 2.20% to 2.19% and 2.16% respectively. However, these opportunity sets are outperformed by the "All Africa" set whose integration lowers the benchmark's volatility to this scenario's minimum of 2.11% and highest Sharpe ratio of 1.10.

	Minimum variance			Mean-variance		
	μ_p	σ_p	SR	μ_p	σ_p	SR
USA	1.86	2.20	0.85	6.36	4.08	1.56
USA + Developed	1.74	2.16	0.80	8.04	4.66	1.72
USA + Emerging	2.04	2.19	0.93	8.11	4.38	1.85
USA + All Africa	**2.32**	**2.11**	**1.10**	7.27	3.75	1.94
USA + Africa ex-SA	1.68	2.19	0.77	**12.83**	**6.07**	**2.11**
USA + Frontier Africa	1.45	2.16	0.67	10.23	5.75	1.78
USA + Developed + Emerging	1.89	2.14	0.88	8.51	4.56	1.87
USA + Developed + All Africa	**2.16**	**2.07**	**1.04**	7.93	3.97	2.00
USA + Developed + Africa ex-SA	1.56	2.15	0.72	**13.57**	**6.37**	**2.13**
USA + Developed + Frontier Africa	1.38	2.12	0.65	11.56	6.16	1.88
USA + Emerging + All Africa	**2.40**	**2.11**	**1.14**	7.91	3.83	2.06
USA + Emerging + Africa ex-SA	1.77	2.18	0.81	**12.64**	**5.84**	**2.17**
USA + Emerging + Frontier Africa	1.65	2.14	0.77	10.91	5.53	1.97
USA + Developed + Emerging + All Africa	**2.26**	**2.06**	**1.10**	8.24	3.95	2.09
USA + Developed + Emerging + Africa ex-SA	1.60	2.13	0.75	**13.38**	**6.15**	**2.17**
USA + Developed + Emerging + Frontier Africa	1.50	2.09	0.72	11.62	5.84	1.99

Note: Portfolio expected return (μ_p) and volatility (σ_p) expressed in percentage annualized approximations as in Table 1. SR stands for the Sharpe ratio indicating the return per unit of risk. The mean-variance strategy is presented to illustrate the contribution of the Africa ex-SA set to the benchmark's tangency (return per risk) portfolio. The figures in **bold** correspond to optimal results per investment strategy and diversification scenario.

Table 3: Optimal portfolio results

As far as the mean-variance portfolio strategy is concerned, these results show that, despite its minimum volatility across the investigated scenarios, the "All Africa" set is no longer the most appealing of all the considered investment opportunity sets as it fails to produce the highest portfolio expected return per unit of risk (Sharpe ratio). The results show that this role is better performed by the "Africa ex-SA" set whose integration helps attain the highest Sharpe ratio for each of the investigated scenarios. In short, these results underscore the dominance of the "All Africa" and "Africa ex-SA" sets' diversification benefits in terms of minimum variance and tangency portfolios respectively once diversified over by an investor holding the US domestically-diversified portfolio. Similar to the observations on Figure 2, this performance is not affected by the prior (or simultaneous) integration of investment opportunity sets from emerging or other developed markets (or both) unlike the observation by Piljak and Swinkels (2017).

4.2.2 Mean-variance spanning tests

The results of the traditional mean-variance spanning tests are presented in Table 4. First of all, as predicted by Kan and Zhou (2012), Sentana (2009) and Gibbons et al. (1989) in case of finite samples, these results consistently feature the inequality $W \geqslant LR \geqslant LM$ across all the investment sets under scrutiny. However, despite this inequality, there appears to be a consistency across the three test statistics with regards to the rejection of the spanning hypothesis, which increases the confidence to be attributed to the conclusions of the test.

According to these results, the spanning hypothesis over the "All Africa" set is rejected at 1% indicating highly significant benefits of diversification over this investment set. This hypothesis is also rejected, though at 5%, for the "Africa ex-SA" and "Frontier Africa" sets, which equally support their potential improvement of the benchmark risk-return profile once diversified over. Nonetheless, these three tests have all failed to reject the null hypothesis of spanning in the cases of "Developed" and "Emerging" sets indicating the impossibility of statistically significant diversification benefits from the diversification over these sets.

	W	p-value	LR	p-value	LM	p-value
All Africa	22.6398	0.0016	21.6374	0.0017	20.6961	0.0019
Africa Ex-SA	10.5773	0.0379	10.3697	0.0379	10.1680	0.0379
Frontier Africa	12.1766	0.0201	11.8508	0.0205	11.5366	0.0210
Developed	10.5967	0.1178	10.4206	0.1173	10.2489	0.1167
Emerging	5.1498	0.2879	5.0900	0.2889	5.0311	0.2899
Developed + Emerging	16.2140	0.1187	15.8466	0.1179	15.4916	0.1171
Developed + All Africa	33.1065	0.0022	31.2768	0.0025	29.5918	0.0028
Developed + Africa Ex-SA	19.4231	0.0495	18.9228	0.0486	18.4412	0.0477
Developed + Frontier Africa	22.3721	0.0209	21.4896	0.0218	20.6569	0.0229
Emerging + All Africa	25.3427	0.0084	24.1500	0.0091	23.0358	0.0099
Emerging + Africa Ex-SA	14.2583	0.0932	13.9992	0.0915	13.7469	0.0898
Emerging + Frontier Africa	18.3462	0.0265	17.8976	0.0258	17.4650	0.0251
Developed + Emerging + All Africa	36.6596	0.0060	34.4535	0.0068	32.4375	0.0078
Developed + Emerging + Africa Ex-SA	25.8340	0.0445	25.0009	0.0434	24.2062	0.0424
Developed + Emerging + Frontier Africa	30.3568	0.0136	28.9619	0.0141	27.6608	0.0147

Note: W, LR and LM stand respectively for the Wald, Likelihood Ratio and Langrange Multiplier tests.

Table 4: Traditional mean-variance spanning tests

Furthermore, the combination of the "Developed" and "Emerging" sets does not help reject the conclusion of these assets' mean-variance spanning conclusion, thus underplaying their diversification benefits for the considered US benchmark portfolio. On the other hand, these results show that African sets still bear significant diversification benefits even in the scenario where the benchmark portfolio is already diversified over the "Developed" set. It is however important to stress that the spanning hypothesis for the "Africa ex-SA" set is only rejected at 10% once combined with the "Emerging" set suggesting a shrink in this set's benefits significance in the case of prior integration of the "Emerging" set into the benchmark portfolio. This threat to the significance of the diversification benefits of the African investment opportunity sets vanishes when the spanning test is applied to the scenario where the benchmark portfolio is already diversified over both the "Developed" and "Emerging" sets.

In substance, these results validate the insights from the efficient frontiers discussed above that it seems unlikely to improve the risk-return profile of the benchmark US diversified portfolio by diversification over assets from emerging and/or other developed markets; Only the diversification over investment opportunity sets from Africa are likely to induce diversification benefits to this benchmark portfolio. All the same, it is worth underscoring that this conclusion does not mean that all the studied African investment sets are equivalent with respect to the significance of diversification benefits to the benchmark portfolio. In fact, besides rejecting these assets' mean-variance hypothesis at different levels, these tests provide no indication on whether the rejection of the spanning hypothesis is attributed to the significance of the considered African assets' diversification benefits in terms of tangency or minimum-variance portfolio. This question is answered by the step-down test.

	F	p-value	F_1	p-value	F_2	p-value
All Africa	3.5954	0.0017	1.7223	0.1635	5.5434	0.0011
Africa Ex-SA	2.5632	0.0379	3.9793	0.0201	1.1708	0.3121
Frontier Africa	2.9343	0.0205	1.4341	0.2406	4.4765	0.0125
Developed	1.7092	0.1172	0.6989	0.5537	2.7468	0.0439
Emerging	1.2505	0.2889	1.9399	0.1463	0.5681	0.5674
Developed + Emerging	1.5544	0.1178	0.8124	0.5420	2.3205	0.0445
Developed + All Africa	2.5905	0.0025	0.9905	0.4327	4.2829	0.0004
Developed + Africa Ex-SA	1.8628	0.0485	1.6276	0.1540	2.1093	0.0656
Developed + Frontier Africa	2.1218	0.0218	0.8410	0.5220	3.4576	0.0050
Emerging + All Africa	2.3919	0.0091	1.4099	0.2219	3.4125	0.0055
Emerging + Africa Ex-SA	1.7211	0.0915	2.1874	0.0715	1.2647	0.2850
Emerging + Frontier Africa	2.2103	0.0258	1.4116	0.2312	3.0352	0.0184
Developed + Emerging + All Africa	2.1275	0.0068	0.9156	0.5045	3.4110	0.0011
Developed + Emerging + Africa Ex-SA	1.7535	0.0433	1.2530	0.2754	2.2731	0.0299
Developed + Emerging + Frontier Africa	2.0407	0.0141	0.8316	0.5622	3.3139	0.0023

Note: The F-test corresponds the traditional GRS spanning test where F_1 and F_2 relate to the step-down test presented in equations (19) and (20).

Table 5: Step-down mean-variance spanning test

The results of the GRS F-test of mean-variance spanning as well as the step-down test are presented in Table 5. In the first place, the results of the GRS test corroborate the conclusions of the traditional spanning tests performed above as they also reject the null hypothesis of spanning for only the "All Africa", "Africa ex-SA" and "Frontier Africa" sets but not the "Developed" and "Emerging" sets. On their side, the step-down test goes deep into this analysis by investigating the cause of this rejection and therefore answer the question of which of the tangency or the minimum-variance contribution the investigated asset can be accounted for.

With respect to the objective of this study, the step-down test provides valuable additional information regarding the potential value addition of these individual sets to the optimization of the benchmark portfolio. On one hand, it shows that, though deemed overall significant in terms of diversification, the considered African sets do not affect in the same way the tangency and minimum-variance benchmark portfolios. The null hypothesis of the spanning test is only rejected by the F_2 and not the F_1 tests for the "All Africa" and "Frontier Africa" sets, indicating that the diversification over these sets affect only the minimum-variance and not the tangency portfolio. The same applies to even the "Developed" set that was deemed not significant by the traditional and GRS tests. On the other hand, while the "Emerging" set is the only one without sufficient evidence against the spanning hypothesis across all the tests, the "Africa ex-SA" set appears to be the only one to experience the rejection of the F_1 but not the F_2 step-down spanning test. Combined with the results of the traditional and GRS tests, this observation indicates that the "Africa ex-SA" set affects rather the tangency portfolio and not the minimum-variance one unlike the other African and the "Developed" sets. It is the only one of the whole considered investment universe to bear this type of diversification benefits if we believe the results of the step-down test, results that confirm the insights of the efficient frontiers discussed above.

4.2.3 Robustness check

The tests of mean-variance spanning performed above using return samples of the US benchmark assets and the test assets from African, emerging and other developed markets have converged to the statistically significance of the benefits of diversification over the investment opportunity sets from Africa, the relative significance of those from the developed and non-significance at all for the investment opportunity sets from emerging markets. However, the normality assumption underlying the above-performed tests lowers the confidence to be attributed to these conclusions from the policy or practitioner point of view given that, in practice, it is widely recognized that asset returns exhibit patterns that deviate from the normal distribution. This robustness check in performed in order to proof-check the validity of these conclusions when the normality assumption is relaxed. I therefore apply, as an alternative, the two GMM versions of the Wald test presented in equations (21) and (22).

	W	p-value	W_a^e	p-value	W_a	p-value
All Africa	22.6398	0.0009	13.7530	0.0325	32.8520	0.0000
Africa Ex-SA	10.5773	0.0317	7.3834	0.1170	11.0184	0.0264
Frontier Africa	12.1766	0.0161	4.2812	0.3693	11.2619	0.0238
Developed	10.5967	0.1017	8.3466	0.2138	14.8342	0.0216
Emerging	5.1498	0.2723	4.5001	0.3425	4.4144	0.3528
Developed + Emerging	16.2140	0.0937	12.6648	0.2430	20.2091	0.0273
Developed + All Africa	33.1065	0.0009	21.9357	0.0382	61.2872	0.0000
Developed + Africa Ex-SA	19.4231	0.0352	10.3731	0.4084	31.1562	0.0006
Developed + Frontier Africa	22.3721	0.0133	8.2639	0.6031	36.6161	0.0001
Emerging + All Africa	25.3427	0.0047	16.4284	0.0880	35.2697	0.0001
Emerging + Africa Ex-SA	14.2583	0.0753	8.9988	0.3424	14.7666	0.0638
Emerging + Frontier Africa	18.3462	0.0188	7.4833	0.4855	18.6251	0.0170
Developed + Emerging + All Africa	36.6596	0.0023	24.9808	0.0702	65.9381	0.0000
Developed + Emerging + Africa Ex-SA	25.8340	0.0272	12.9182	0.5330	35.4137	0.0013
Developed + Emerging + Frontier Africa	30.3568	0.0068	11.7601	0.6256	41.1654	0.0002

Note: W corresponds to the traditional Wald spanning test while W_a and W_a^e correspond respectively to the general Wald-GMM and the specific case of Wald-GMM under the assumption of conditional heteroskedasticity with R_t independently and identically distributed as a non-degenerate multivariate *elliptical distribution*.

Table 6: Alternative mean-variance spanning tests

The results of the alternative tests are presented in Table 6 alongside those of the traditional Wald-test for sake of comparison. These results provide contradicting indications regarding the rejection of the spanning hypothesis with the W_a validating the conclusions of the traditional Wald test and the W_a^e failing to reject them all except for the "All Africa" set. So, according to the W_a test, all the investigated sets but the emerging one bear significant diversification benefits while, following the W_a^e test, only the "All Africa" set can be accounted for statistically significant diversification benefits for the benchmark US diversified portfolio. These confirmed diversification benefits of the "All Africa" seem to vanish when this set is combined with the emerging set, according to the W_a^e test.

Several reasons can explain the observed conflict between the W_a^e and W_a tests. One, while the W_a test results are valid for all distributions, the W_a^e is a specific case where returns are assumed to follow a multivariate elliptical distribution and its results are only valid when this assumption holds. Since this assumption has not been tested, it may be possible that these conflicting results emanate from the failure of the W_a^e test due to a misspecification of the underlying returns distribution. Two, Kan and Zhou (2012) demonstrate that these two tests tend to suffer from the sample characteristics with W_a tending to be inflated such that

$W_a^e \leqslant W_a$ in small samples. They stress that, in addition to the compliance to the underlying distribution, the W_a^e test requires a sufficiently large N and preferably a small T to produce reliable results while the W_a test is rather precise with a small N. With $T = 217$ and $N \leqslant 8$ in this case, it seems reasonable to lean towards the validity of the W_a test which is deemed more precise in such cases. In this perspective, the validation of the tradition test results by the W_a test can be seen as a proof of these results' robustness to conditional heteroskedasticity in the returns. Either way, a more conservative opinion would tend to also consider the W_a^e results and conclude for the overwhelming rejection of the spanning hypothesis for the sole "All Africa" set across all the test specifications, thus indicating the unequivocal benefits of diversification over this investment opportunity set for a US investor holding the considered benchmark US diversified portfolio.

In summary, three main conclusions can be drawn from the results of this investigation of international diversification benefits of selected African investment opportunity sets from the perspective of a US investor in comparison to their counterparts from emerging and other developed markets. Firstly, there are substantial evidence in favor of the benefits of international diversification over the selected African investment opportunity sets for an investor holding a US domestically-diversified portfolio. Secondly, there are strong indications of the possibility of improving both the risk and return profiles of the US benchmark portfolio by an adequate selection of African investment opportunity sets. In fact, the results of the step-down test have shown that the integration of the "All Africa" or "Frontier Africa" sets can improve the minimum-variance of the benchmark portfolio while the "Africa ex-SA" set can potentially improve the benchmark tangency portfolio. Finally, the prior diversification of the benchmark over the international set does not seem to abate the diversification potential of these African sets; some caution is however recommended when the benchmark is already diversified over the emerging set.

Overall, these results are in one way or the other related to the general conclusions of the studies on diversification benefits of emerging and frontier markets (see Berger et al., 2011, 2013; Driessen and Laeven, 2007; Jayasuriya and Shambora, 2009; McDowell, 2017). For instance, the significantly high performance of selected asset classes from Africa supports the main conclusion by Driessen and Laeven (2007) that the gains from international portfolio diversification appear to be largest investments in developing countries, particularly those with high country risk. However, they qualify in the case of Africa the conclusion by Piljak and Swinkels (2017) about the limited diversification benefits of US dollar-denominated debt issued by governments of frontier markets due to their substantially high correlation with the US market; the inclusion of the African hard currency government bond index in the "All Africa" set does not seem to have affected this set's outstanding performance in comparison to the other available investment opportunity sets. Finally, exception made for the "Africa ex-SA" set, these results substantiate the observation by McDowell (2017) that the international diversification benefits the benchmark US investor more in terms of reduction in the portfolio risk rather than the increase in portfolio expected return.

Nevertheless, the overwhelming evidence of the international diversification benefits of African assets should not overshadow the transaction cost headwinds attached to financial trades in frontier markets. Marshall et al. (2015) underscore that these costs can be considerably high and inhibit the materialization of diversification benefits, particularly for case of monthly portfolio rebalancing. Investors should therefore consider these authors' recommendation for a quarterly or longer portfolio rebalancing period to safeguard the diversification gains from this market. In the same token, the incentives to invest in these rewarding opportunities in Africa should be appreciated with due consideration to the risks they entail not only to the investor and but also to the borrowing countries. So far, the enthusiasm of investors seems to have been directed towards the hard-currency denominated assets which, in most cases, expose borrowing countries to exchange rate risks that might be particularly severe for commodity-dependent countries such as those of Africa. Ideally, investors would consider participating in domestic local currency-denominated markets and find a way to diversify the related risks. Some valuable insights in this direction can be borrowed from the analysis by Zamora (2016) in the case of emerging markets (EM henceforth) local-currency sovereign bonds. The authors breaks down these assets' total returns into three components, namely the duration (including the return of local currency bonds after hedging out the currency risk), the carry (capturing the differential between money market rates in EM and those in US dollars) and currency (capturing the changes in spot exchange rates in EM currencies versus the US dollar) components and spots the different risks associated to each of these components. Although he acknowledges the currency-related risks as the major source of EM total return volatility, he also highlights a significantly positive and less-volatile contribution of the carry component to these returns, thus indicating some offsetting possibilities between currency-risks and carry-gain exposures that, if well strategized, could help investors generate significant gains from their investments in such assets. Moreover, the availability of currency risk hedging possibilities for emerging and frontier markets by institutions such as the Currency Exchange Fund (TCX)[6] should be harnessed by investors interested in diversification opportunities in Africa.

5 Conclusion

This study has investigated the diversification potential of African securities from the perspective of a US investor. Building on the observed enthusiasm of international investors for sovereign eurobonds from Africa, I strove to bring the portfolio optimization perspective into the debate about this investors' enthusiasm by analyzing the potential contribution of African securities to a benchmark US diversified portfolio in comparison to that of their peers from the emerging and developed markets outside the USA. A mean-variance spanning approach has been used and a battery of tests has been applied to a sample of weekly returns on seleted asset classes from these markets for the period July 2014 – September 2018 to assess the statistical significance of the potential contribution of African securities to the risk-return profile of the benchmark portfolio.

Three main points summarize the findings of this study. First, unlike their counterparts from emerging and other developed markets, the selected African investment opportunity sets offer statistically significant diversification benefits to the benchmark US diversified portfolio made of the S&P500, S&P500 Corporate Bond and S&P US Treasury Bond indices. In fact, all the deployed mean-variance spanning tests have failed to reject the null hypothesis of the benchmark portfolio spanning the selected emerging and developed markets' asset classes, indicating the impossibility for these assets to impact the benchmark portfolio's risk-return profile once diversified over; only the considered African assets have proved to bear significant diversification benefits to this benchmark portfolio. Second, the diversification benefits of the "All Africa" and "frontier Africa" sets have bee proved only significant in terms of variance minimization but not in terms of increased expected portfolio return. An exactly opposite observation has been made for the "Africa ex-SA" that is indicated to have no significant impact in terms of risk but rather a significantly positive contribution to the return profile of the benchmark portfolio, making it the only one of our whole considered investment universe to bear significant diversification benefits in terms of tangency portfolio. Third, the prior integration of investment opportunity sets from the emerging or developed world does not abate the diversification benefits of these African securities.

Against this backdrop, it seems reasonable to conclude for the possibility of an investor holding a US domestically-diversified portfolio to reach substantial international diversification benefits by investing in selected the African securities. This observation constitutes to a certian extent a justification for the observed appetite of international investors for African securities as, beyond the influence of global and debtor's economic conditions, investors may rightfully get interested in these securities for international diversification purposes. The ongoing strides in domestic capital markets across Africa, the availability of investable indices as well as the existence of hedging possibilities covering most of the African currencies should incentivize international investors to fully tap the diversification opportunities of the African market beyond the sole hard currency government bonds advertised and traded on major developed financial markets.

This research has used weekly returns covering a four year period, a reasonably short period to assumes the stability of the coefficients of the return-generating model underlying the different mean-variance spanning tests. Based on the observation by Huberman and Kandel (1987), it is possible that this coefficients' stability

[6]The Currency Exchange Fund (TCX) was founded in 2007 by a group of development finance institutions (DFIs), specialized microfinance investment vehicles (MIVs) and donors to offer solutions to manage currency risks in emerging and frontier markets. It offers hedges for currencies of most of Africa, Asia and Latin America for maturities up to 30 years. More details can be found on https://www.tcxfund.com/

assumption would no longer hold once a longer period is considered. As more data become available, it would be interesting to test the robustness of my conclusions to changes in the sample size and/or data frequency. Besides, some studies have underscored the burden of high transaction costs associated to trades in frontier markets, especially when investors consider shorter portfolio rebalancing periods. The investigation of how these results are affected by the consideration of transaction costs constitutes, in my view, an interesting avenue for future research in this domain.

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