DEPARTMENT OF ECONOMICS

Labour values and energy values

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§1. Introduction

At least since the ancient Greek philosophers some thinkers have been attracted by the problematic idea of a single cause of economic value. Today such ideas appear mostly in historical studies on the labour theory of value of Karl Marx. In 1867 Volume 1 of his *Das Kapital* tried to explain the values of commodities by a special common substance, the quantity of abstract labour embodied in them. Marx's labour theory of value was criticised from several angles. Several well-known mainstream economists (for example, Philip Henry Wicksteed, Eugen von Böhm-Bawerk, Knut Wicksell and Vilfredo Pareto) quickly pointed out that labour was not the only common substance in commodities. Another problem was Marx's suboptimal mathematics. In Marx's system the same commodities can appear on the side of the inputs and on the side of the outputs. Today it is well-known that many analytical problems in such circular systems can be solved by means of simultaneous equations of the input-output type. Here the most original contributions were not made by leading economists mentioned above, but by rather unknown outsiders (Wolfgang Mühlpfordt, Vladimir Dmitriev, Georg Charasoff, Maurice Potron), whose pioneering works were neglected for decades. In December 1927, both Piero Sraffa and Wassily Leontief, independently of each other, argued that the reduction of all commodities to labour values was not a unique process; from a formal point of view, it was possible to compute not only labour values, but also wheat values, coal values, etc. Although both Sraffa and Leontief were well-known economic scholars already at a young age, their above insights of 1927 remained rather unnoticed for several decades.

A few decades ago the high impact journal *Science* published some papers by non-economists on the theory of value, advocating the use of *energy values*, which describe the direct and indirect energy content of a commodity. The most conspicuous text of this type in *Science* is the often cited paper *Embodied Energy and Economic Valuation*, by Robert Costanza (1980), claiming that both theory and empiry suggest a close connection between prices and energy values. The works of Costanza and some other influential non-economists are usually discussed outside the networks of orthodox or marxian economics, but the empirical and the theoretical studies on energy values and labour values show several remarkable cases of analogous arguments and imperfections, both trying to support the use of a single substance of economic value.

My paper considers both theoretical and empirical aspects of such discussions on labour values and energy values. I also use a numerical example of a simple three commodity economy, and illustrate the computation of both labour values and energy values in two ways: via a system of simultaneous input-output equations and via subsystems (vertically integrated systems that produce only one net output). The two computation methods are very well-known from the rich literature on labour values. However, my example is useful here, because comparing energy values and labour values forces me to a generalisation of the notion of a subsystem: in order to define this notion unambiguously, I need to specify not only the net output of the subsystem, but also its net input. If the only net input of the subsystem is energy, I use the expression “energy subsystem”. This concept is needed to establish a percentage formula on the deviation between prices and energy values, analogous to my formula for labour values. By generalising Sraffa’s notion of subsystems it is possible to explain why supporters of labour values and energy values both present “good” empirical results for their favourite theory. Both groups should be more aware of the non-uniqueness of their exaggerated claims, and then be more humble when defending their one sided theories and their empiry.

Some studies on theories of value seem to like extraterrestrial points of view, engaging the services of a man “dropped from another planet”, or a “man from the Moon”.[1] For my purposes, I prefer to involve not one, but two such observers. Suppose Lana and Enzo live on a distant planet, where no scarcity exist and everything looks like paradise. For a scholarly mission, they are sent to the Earth, a planet full of economic and other problems. Lana arrives in 1973, Enzo in 2017. Both are eager to study the fundamentals of economics, the queen of the social sciences. They have been told that nearly all economic thinkers use a theory of value as the fundamental building block of their analysis. Therefore Lana and Enzo want to concentrate on this topic. Both have planned to follow the same “scholarly” method to start their research: go to one of the biggest university libraries of the country of arrival, and ask for the most prestigious publications on the theory of value.

§2. Lana discovers labour values

Suppose Lana arrives on the Earth in 1973, in a communist state. About half the Earth’s population then lived in such countries, where every university librarian proclaimed that authoritative economics derives from the works of Karl Marx, and is based upon the labour theory of value. Already in the very first section of *Das Kapital*, in 1867, Marx uses the following argument: the fact that two commodities, for example, one quarter of corn and x cwt. of iron, are exchanged against each other in the market, implies that “there exists in equal quantities something common to both”. Marx then continues:

> The two things must therefore be equal to a third, which in itself is neither the one nor the other. Each of them, so far as it is exchange-value, must therefore be reducible to this third . . . If we then leave out of consideration the use-value of commodities, they have only one common property left, that of being products of labour (Marx 1954 [1867], p. 45).

Then Marx explains the values of commodities by a special common substance, the quantity of abstract labour embodied in them. Marx claims that only one commodity can create more value than its embodied labour time. This special commodity is labour power. Its reproduction requires a certain consumption basket for the worker. Suppose the daily basket embodies 5 hours of labour. If the worker performs 10 hours per day, this means 5 hours more than

[1] See, for example, Kurz and Salvadori (2004) and Gilibert (2006), all inspired by Sraffa.

necessary for the production of his own basket, hence 5 hours of surplus labour time, or surplus value. According to Marx, this surplus labour time explains the existence of profits.[2]

If later Lana is able to get access to every economic publication, including those that do not simply repeat Marx's points of view, she will find both strong criticisms and mathematical refinements of Marx's arguments.

§3. Wicksteed, Böhm-Bawerk, Wicksell and Pareto on the common substance

Many readers of Marx have questioned his arguments about the common substance. The critics included, among others, the following four important mainstream economists who read and criticised some of Marx's works shortly after their publication: Philip Henry Wicksteed, Eugen von Böhm-Bawerk, Knut Wicksell and Vilfredo Pareto.

In Britain Wicksteed at age 40 made his first contribution to economic theory in the socialist journal *To-Day,* in an influential critique of Volume 1 of Marx's *Das Kapital* (Wicksteed 1884). Although he was an important supporter of the marginalist school in economic theory, Wicksteed at the same time showed a lot of sympathy for socialism. His contribution in *To-Day* succeeded in converting George Bernard Shaw from Marx's labour theory of value to Jevons's marginal utility theory. Wicksteed claims that the common something in all exchangeable things is not abstract labour, but abstract utility. According to Wicksteed, Marx had opened the door to this argument by stating that not all labour, but only useful labour counts (Wicksteed 1884; Steedman 1989, Chapter 7).

Böhm-Bawerk made fun of the search for a common substance. Marx had suggested that all types of skilled and unskilled labour should be reduced to abstract labour, and that the latter was the common substance of value in all commodities. Böhm-Bawerk tried to ridicule Marx‘s argument:

> With the very same reasoning one could affirm and argue the proposition that the quantity of material contained in commodities constitutes the principle and measure of exchange value – that commodities exchange in proportion to the *quantity of material incorporated in them*. Ten pounds of material in one kind of commodity exchange against ten pounds of material in another kind of commodity. If the natural objection were raised that this statement was obviously false because ten pounds of gold do not exchange against ten pounds of iron but against 40,000 pounds, or against a still greater number of pounds of coal, we may reply after the manner of Marx that it is the amount of *common average material* that affects the formation of value, that acts as unit of measurement. Skillfully wrought costly material of special quality *counts* only as compound or rather multiplied common material, so that a small quantity of material fashioned with skill is equal to a larger quantity of common material (Böhm-Bawerk 1949 [1896], p. 85).

[2] For more background information, see Faccarello, Gehrke and Kurz (2016)

Wicksell in his *Value, Capital and Rent* (1954 [1893], p. 17) referred to "the 'Hegelian' darkness–and conceit–of Karl Marx", for example with respect to Marx's proof of the labour theory of value by his remarks on the "common substance":

> The so-called proof . . . has . . . scarcely the virtue of being able to be discussed seriously . . . there could generally exist a great number of circumstances besides labour which together could, without being the same for both commodities, constitute the same exchange value. For instance, both have used a certain area of land for the production of raw material as well as for the production of the finished commodity; for both of them a certain quantity of power (coal) was needed to bring them to the market, etc. (Wicksell 1954 [1893], p. 44)

Wicksell never propagates the idea of a land theory of value, or a coal theory of value. It is well-known that he ultimately emphasises the role of marginal utilities in his Marx critique.

Pareto also pointed out the arbitrariness of claims about the common substance, for example, in his book *Les systèmes socialistes*:

> Il plaît à Marx de se placer exclusivement au point de vue du travail; s'il plaisait à un autre auteur de se placer exclusivement au point de vue des services des capitaux fonciers et mobiliers, il mesurerait la valeur d'un objet par la quantité de ces services contenue, *cristallisée*, dans le produit. (Pareto 1965 [1902], vol. 2, p. 343n)

Elsewhere in the same book, Pareto suggested that the classical economists and Marx should have used simultaneous equations, to handle the interdependent nature of their systems (see my §9 below). Pareto and his contemporaries were not aware that the relevant equations for Marx's analysis had been formulated already in 1893 in Königsberg, in a doctoral dissertation that would fall into oblivion till the end of the 20th century.

§ 4. Mühlpfordt formulates Sraffa-like price equations

In his 1893 dissertation in Königsberg, Wolfgang Mühlpfordt (1872-1928) reviewed both some objective theories of value (classical or Marxian) and the subjective value theory of the Austrians. Mühlpfordt (1893, pp. 25-26) formulated the same price equations that were presented by Sraffa (1960, p. 6), for a system of production of *n* commodities by means of the same *n* commodities, where the wage goods for the workers enter the system on the same footing as the other inputs. Two years later, Mühlpfordt's (1895) article on Marx again showed how the simultaneous equations were able to determine the average rate of profit in Marx's system.

Mühlpfordt's used the notation of many modern studies on Leontief-Sraffa systems, i.e., the familiar coefficients with two subscripts, the a_{ij} coefficients, where a_{ij} = physical quantity of commodity j used to produce a unit of commodity i (where i and j vary from 1 to n).[3]

At the very end of his article, Mühlpfordt (1895, p. 99) saw "kein unüberbrückbarer Widerspruch" (no irreconcilable contradiction) between the classical and the Austrian school. He was neither a mathematician nor a marxist, but surely a pioneer of marxian mathematical economics. He spent his career outside the networks of the specialists of value theory; after 1895 he never published on value theory again (he wrote a few articles on other topics, for example, on accountancy and on actual political problems).

He was deeply disappointed by the lack of recognition that plagued him from the very beginning. His doctoral dissertation deserved better than the insulting jokes in March 1894 in *Die Neue Zeit*, where a hypercritical reviewer failed to understand the dissertation's strong points, and started his sarcastic review as follows:

> Judging from the present treatise, it seems not to be very difficult for the faithful to become a doctor in Königsberg by means of an economic work. What the author has to say about price and income contains absolutely nothing new. (Anonymous 1894)

The rest of the review contained many other impolite remarks, harming the scholarly reputation of young Mühlpfordt.

Mühlpfordt's pioneering approach missed the opportunity to write down explicitly the related (and somewhat simpler) simultaneous equations for the labour values. He started from given labour values λ_1, λ_2, etc. (my notation), and then constructed price equations in which he multiplied λ_1, λ_2, etc. by $x_1 = p_1/\lambda_1$, $x_2 = p_2/\lambda_2$, etc. (the symbols p_1, p_2, were not explicitly visible). These price equations allowed him to determine all prices and the rate of profit; he did not need a system of labour value equations for this purpose.

Note the very early publication date of Mühlpfordt's dissertation. He was only 21 in 1893, and one was still waiting for Marx's own solutions of the transformation of labour values into prices, and of surplus value into profit, in the posthumously published Volume 3 of *Das Kapital* (Marx, 1894). It took more than a century before Mühlpfordt's brilliant originality was discovered by Howard and King (1987), and then generated further comments; see, for example, the bibliographical references given by Quaas (1994).

[3] By the first subscript Mühlpfordt indicates the number of the production process. Production process i, for example, produces good i by means of a_{i1} units of good 1, a_{i2} units of good 2, etc. A similar convention for the subscripts was used by Sraffa in some unpublished texts of the 1940s, and by the young and middle aged Leontief; however, Leontief's later work and most modern input-output textbooks use the reverse ordering for the two subscripts (Parys 2016b, p. 990).

§5. Dmitriev formulates the modern labour value equations

Completely unaware of Mühlpfordt's results, another remarkable case of early input-output analysis was presented by Vladimir Dmitriev (1868-1913), in the first pages of his Russian essay of 1898, reprinted in his 1904 book, after a long search for a publisher in Russia. When Mario Nuti edited an English translation in 1974, he mentioned in his editorial notes (p. 30) that in the West he knew only one original copy of Dmitriev's Russian book, in the rich library of Piero Sraffa. According to the recent catalogue by Giancarlo de Vivo (2014, pp. xxxix, 138), Sraffa acquired a copy of Dmitriev's 1904 book only in the 1960s. Apparently it is not widely known that Leontief too owned the Russian text in the 1960s. There exists no catalogue of Leontief's library, but in a letter of 20 October 1969 to William Jaffé, Leontief writes:

> Under separate cover I am mailing to you – registered mail – a photostat copy of Dmitrieff's book. I have received it a year ago from Russia, and, most likely it is the only copy in this country. So, please return it to me as soon as possible by registered mail (Leontief Papers, HUG 4517.6, General Correspondence 1969-70, G-O, Box 11, folder J).

Dmitriev (1974 [1904]) used modern simultaneous equations of the input-output type to show explicitly how to compute the labour values, but as soon as he studied the price system, he abandoned the simultaneous equations approach, and employed the less general method of finite dated labour series. Samuelson (1975, p. 493) rightly praised Dmitriev for his "remarkable anticipation of Leontief", but also noted that after a few pages "in a sense, he retreats from his brilliant beginning, never quite reattaining the full Sraffa generality". Modern discussions of Marx's value theory construct two systems of simultaneous equations: one system for labour values and one system for prices and the rate of profit. Here Dmitriev and Mühlpfordt each performed 50% of the task: Dmitriev formulated the modern system of labour value equations, but not the related system of prices, whereas Mühlpfordt had explicitly shown us the latter, but not the former.

The 19th century mathematics of Mühlpfordt and Dmitriev used elementary linear equations only, without matrices or sophisticated mathematical theorems, and of course there is no trace of the future Perron-Frobenius algebra. It is well-known (Allisson 2015) that some important Russian authors tried to formulate a synthesis between the objective and the subjective approaches to value theory. The title page of the English edition of Dmitriev (1974) forgets to show that the original Russian edition carried the relevant subtitle: "An attempt at an organic synthesis of the labour theory of value and the theory of marginal utility". Moreover, in some later writings Dmitriev strongly leaned towards the marginal productivity theory (Howard and King 1995, p. 249; Schütte 2003, pp. 186-275). Both his mathematical approach and his non-marxian economic standpoints decreased the official recognition of Dmitriev to nearly zero in Stalin's Soviet Union.

§6. Bortkiewicz misses various opportunities

Ladislaus von Bortkiewicz (1868-1931) knew both the mainstream and the marxist literature unusually well, and supplied important contributions to both currents of economic thought (and to statistics, demography, physics, etc.). It is bizarre that even Bortkiewicz, who normally cited the relevant literature competently and fairly, neglected Mühlpfordt, underused the results of Dmitriev, and underestimated Leontief's dissertation.

Bortkiewicz unexpectedly overlooked the Mühlpfordt (1895) article on Marx in the *Jahrbücher für Nationalökonomie und Statistik*, then one of the leading economic journals. Moreover, this same journal in the same year also published a paper by Bortkiewicz (1895) himself, albeit on statistics, not on value theory. A decade later, the classic papers on Marx's economics by Bortkiewicz (1906-7, 1907) revealed a deep knowledge of the literature, but surprisingly overlooked Mühlpfordt.

Bortkiewicz knew Dmitriev's originality and praised it explicitly in his influential Marx critique (Bortkiewicz 1906-7, Zweiter Artikel, p. 34). Later he even took care of the entry on Dmitriev in the authoritative *Encyclopaedia of the Social Sciences* (Bortkiewicz 1931). In spite of this, Bortkiewicz never used the system of simultaneous input-output equations that was used by Dmitriev for the determination of labour values. In his Zweiter Artikel (p. 34) Bortkiewicz rather laconically mentions about Dmitriev: "Ich habe seine Darstellung nur etwas vereinfacht" (I have only simplified his presentation a bit). That "simplification" deleted Dmitriev's brilliant simultaneous input-output equations.

Later Bortkiewicz also underestimated Wassily Leontief's potential, when in 1928 he wrote a rather critical report on Leontief's doctoral dissertation (Leontief 1991a; Bjerkholt 2016, pp. 32-40). Leontief's other supervisor in Berlin was Werner Sombart, but he lacked the mathematical sophistication necessary to understand Leontief's (and Bortkiewicz's) ideas on value theory. Ironically, Sombart (1905) did not forget to include the article by Mühlpfordt (1895) in his long chronological bibliography of works on Marx. However, the 300 items in Sombart's bibliography were not annotated, and the name Mühlpfordt generated no interest among his contemporaries.

In an ideal world Bortkiewicz, Leontief and Sraffa would have met in September 1931, because all three belonged to the elite group of economists that was invited to the first Meeting of the Econometric Society in Lausanne (Bjerkholt 2015). However, Leontief had to decline the invitation because he had to leave for the U.S. a few weeks earlier, after accepting a stimulating offer from the *National Bureau of Economic Research*, and one year later he was appointed at Harvard. Bortkiewicz was as active as ever at the start of the summer of 1931, but died unexpectedly on 15 July 1931. Sraffa attended the Lausanne conference, he was silent on his own research on value theory, and concentrated on the monetary theory of Henry Lloyd (Sraffa's paper is not extant). Only in the early 1940s did Sraffa learn about Bortkiewicz's German articles on value theory, and he then wrote a large number of unpublished pages on it (Gehrke and Kurz, 2006). Sraffa and Leontief never appeared at the same conference, but in 1950 and later they met a few times in Cambridge, U.K. (Parys 2016b). Many years earlier, in the late 1920s, unaware of each other's work, Leontief and

Sraffa both made an original use of linear equations to reduce all commodities to one common substance, and in addition showed the arbitrary nature of the chosen substance (see my §10 and §11 below).

§7. Charasoff reduces everything to a basket of Urkapital (in the limit)

Unlike other contributors to classical or marxian economics, Georg von Charasoff (1877-1931) did not reduce everything to a *single* substance, but to a specially designed *composite* commodity, a basket of so-called *Urkapital*.

After obtaining a doctoral degree in mathematics in Heidelberg in 1902, Charasoff planned a trilogy on the economic theories of value. First, he presented a short book on Marx (Charasoff 1909), based upon lectures he had organised in the preceding three years. A review by Eckstein (1909) was as negative as possible: "If one wanted to note all the nonsense which is in this book, one would have to transcribe it". Most other reviewers were more polite, but all showed little enthusiasm (for more details see Gehrke 2015, 2016).

The following year Charasoff (1910) published a much stronger and longer work, his highly original *Das System des Marxismus*. For various reasons the book was neglected by most specialists. Charasoff held no university position, lived a rather bohemian life as an independent scholar, had a less than ideal book publisher (who went out of business in 1913) Moreover, Charasoff sometimes behaved in a shocking nonacademic way in everyday life (Gehrke 2015, 2016).

It would have been interesting to observe Bortkiewicz's reaction to Charasoff's works. However, Bortkiewicz never commented upon them. Maybe Bortkiewicz was not stimulated to study Charasoff's second book in 1910, because the first book of 1909 seemed to have no conspicuous content, except a rather pretentious preface, in which Charasoff (1909) explicitly criticised Zuckerkandl and Böhm-Bawerk for their belief that Marx deliberately formulated his doctrine in an unclear dialectical form. Charasoff's preface then also made a similar attack on Bortkiewicz, but in this case without explicitly mentioning a name:

> Recently this belief has been repeated anew by a university professor from Berlin, who asserted that Marx enjoyed playing the role of a Mephisto, who liked to tease and to confuse the learned world by means of ingeniously constructed sophistries. Can a criticism more clearly show its distress and its inability to understand and to appreciate a thinker, when it accuses Marx of disguising his ideas and fooling his fellow men?

Charasoff's reference to a "university professor from Berlin" surely pointed to an innocent use of the word "Mephisto" by Bortkiewicz (1906, p. 4). Furthermore, Charasoff (1909, p. 57) made an explicit criticism against "Herr Prof. L. v. Brotkiewicz" (sic, but name corrected in list of misprints at the end of the book). He unfairly criticised Bortkiewicz for determining the rate of profit independently of the labour theory of value. Most specialist readers of his next book (Charasoff 1910) will see that Charasoff is able to do the same, that is, Charasoff succeeds in determining the rate of profit by means of his original device of Urkapital, without needing labour values. Charasoff's amateurish behaviour as a scholar, his provocative and impolite writing style, and the difficulties in his private life, all contributed to the neglect by most of his contemporaries.

One of the many original insights in Charasoff (1910) is his clever treatment of the circular nature of production. In his analysis we find the same commodities on the side of the inputs and on the side of the outputs. Charasoff (1910: 290) suggests that Marx argues in a circle when he first takes the labour values of the inputs as given and then determines the labour values of the outputs. In a system of production of commodities by means of commodities we need to determine the values of all commodities via a system of simultaneous equations. Charasoff (1910, pp. 290-291) clearly had read Pareto, and quoted one of Pareto's examples of circular production:

> For instance, the cost of production of mineral coal depends on the price of machinery, and the cost of production of machinery depends on the price of this very same coal. And it depends on it even more directly, when one considers the consumption of coal by the machines used in the mine (Pareto 2014 [1906], p. 121, §224)

Charasoff then supports Pareto's claim that most economists do not seem to know how to construct or to handle the underlying system of simultaneous price equations, due to a lack of mathematical expertise. Those who look for a simple unique cause of economic value do not understand the nature of a general equilibrium system (Pareto 2014 [1906], p. 123, §227).

Charasoff invented the device of *Urkapital* ("original capital"), a specially designed basket of different commodities. The basket has the following special property: to produce one unit of output of this basket, we need a certain basket of inputs, where the latter basket is proportional to the former.[4] Charasoff shows that in the limit all commodities can be reduced to the same Urkapital. Suppose we consider two different commodities X and Y. Their input baskets (Charasoff's "capitals of the first order") will correspond to X' and Y', in general these two baskets have different proportions. To produce X' and Y', we need input baskets X'' and Y'' ("capitals of the second order"), again with two different proportions in general. If we go back k rounds we find the "capitals of the k-th order" $X^{(k)}$ and $Y^{(k)}$. The further we go in this series (the higher k), the more the baskets $X^{(k)}$ and $Y^{(k)}$ seem to become proportional to each other.[5] "In the limit" the baskets become exactly proportional to each other, being different quantities of the same composite commodity, namely baskets of Urkapital.

If in the limit X needs three times more Urkapital than Y, then the price of X will be three times higher than that of Y (Charasoff, 1910, Chapter 9 and 10). In this sense, Charasoff considers Urkapital as the common substance in the commodities X and Y that "explains" their relative prices. Strictly speaking, within the usual assumptions of the model (given

[4] From a mathematical point of view, a basket of Urkapital corresponds to an eigenvector of the augmented input matrix: see Egidi and Gilibert (1984), Mori (2011), Parys (2013, 2014). Compare with Sraffa's (1960) standard composite commodity, which corresponds to an eigenvector of the non-augmented input matrix (by the latter I mean the original input matrix, not including consumption by workers).

[5] Charasoff presents an example with three commodities, where the proportions of the Urkapital basket become already visible after four rounds. See the figures in the original tables from Charasoff (1910, p. 114) or their representation in matrices (Parys 2014, p. 998).

inputs and outputs, single product industries, homogeneous labour, circulating capital only, given basket of wage goods for the workers, etc.), Charasoff was the only one whose search for a common substance of value was logically correct. However, remember that his common substance of Urkapital was not an individual commodity, but a basket of commodities, and that Charasoff's construction of it involved the computation of a mathematical limit.

In the preface of his second book (Charasoff 1910, p. XIV), the following footnote appeared (my translation):

> An in-depth critique of the subjective theory of value will be provided in a third book "The Problems of Production and Distribution", which meets its completion

This third German book never appeared, and no drafts of it have been found.[6] Problems in Charasoff's personal life and in the business of his publisher must have prevented the publication. Of course, there are already a few isolated remarks on subjective versus objective theories of value in the second book (Charasoff 1910). Some of Charasoff's remarks are discussed or quoted by Nikolai Bukharin (1927, pp. 56, 127, 179, 184, 188), in *The Economic Theory of the Leisure Class*. Bukharin's book wanted to attack "bourgeois political economy", and paid special attention to the Austrian marginal utility school of Menger, Wieser and above all Böhm-Bawerk.

Sraffa's copy of the 1927 English edition of Bukharin's book contains many annotations. In his unpublished papers (D3/12/3:54) Sraffa makes some positive comments on Bukharin's book. Hence, Sraffa seems to have read Bukharin's book thoroughly. Sraffa surely must have noticed the name Charasoff, especially on pages 127 and 179, where Sraffa places a line in the margin close to or near a Bukharin quote of Charasoff. For example, on p.179 Bukharin provides the full bibliographical reference of Charasoff's 1910 book, and then continues:

> Charasoff's assertion that even certain Marxian studies contain a subjective interpretation of the Marxian theory, is entirely correct; but this is not the place to discuss this question (Bukharin 1927, p. 179)

Sraffa put a straight line in the margin of this passage. However, there is no trace of Charasoff in the catalogue of Sraffa's library (de Vivo 2014), though it contains many German publications on value theory, most of which are much less important than Charasoff.[7] Charasoff's (1910) book introduced some original devices that were related to later Sraffian tools (Sraffa's standard commodity, distinction between basics and non-basics, determination

[6] The preface of Charasoff (1910, p. IX) even seems to suggest the possibility of a fourth book, on actual economic tendencies: "I myself do not attach important scientific value to the few remarks on current tendencies that are scattered in my book, and therefore I would ask the reader to rather completely abstract from it, and to stick exclusively to the positive content of the book, as long as I have not set forth my practical opinions in a work intended specifically for this purpose". ("Ich selbsts lege den in meinem Buche gelegentlich verstreuten Äuβerungen über die Tendenzen der gegenwärtigen Entwicklung keinen allzugroβen wissenschaftlichen Wert bei, und daher möchte ich den Leser bitten, lieber ganz davon zu abstrahieren und sich ausschlieβlich an den positiven Inhalt des Buches zu halten, solange ich meine praktischen Ansichten noch nicht in einem eigens dazu bestimmten Werke dargelegt habe").

[7] Charasoff's 1910 book is now rarely offered for sale. For several years, it was invisible on abebooks.com, but in 2017 an Austrian bookseller offered a slightly stained and uncut copy for 498 euro.

of labour values by dated labour series and subsystems, for example). However, Bukharin did not draw the attention of his readers to these innovating devices. Bukharin quoted rather superficial remarks by Charasoff, and must not have understood the original parts of Charasoff's work. Charasoff finally started getting some recognition only many decades later, when Egidi and Gilibert (1984) drew special attention to many of Charasoff's original results.

§8. Potron's unintended contribution to a solution of some marxian problems

Most studies on Marx (including Mühlpfordt, Dmitriev, Bortkiewicz, Charasoff) concentrate on systems with homogeneous labour. Of course, it is well-known that Marx himself spent several paragraphs on the problem of reducing heterogeneous types of labour to a sort of standard labour, i.e., on the reduction of skilled labour to multiple simple labour. Ironically, one of the most creative technical contributions in this context was implicit in the works of the French Jesuit Maurice Potron (1872-1942), although Potron never studied Marx and surely did not like his ideology.

Potron held a prestigious degree in engineering (Polytechnique), and a doctoral degree in mathematics. He provided an amazing example of amateurish economics and high originality, anticipating many later developments in mathematical economics. Without an effort to study the relevant economic literature, Potron wrote on economics in the periods 1911-1914 and 1935-1942.[8] His economic papers were hyperindividual constructions, pioneering anticipations of input-output economics, but they were published in journals for mathematicians or for Roman Catholic intellectuals, hence outside the networks of economics.

Only at the end of his life did Potron start to look for some visibility among economists. He presented a paper on his mathematical economic system at the International Congress of Mathematicians in Oslo in 1936, in the same session as Ragnar Frisch, but Potron missed the opportunity to make an impression, due to his bad lecturing (he stuttered) and his difficult writing style. He organised six lectures on economics in 1937 in Paris; the final one was attended by François Divisia and René Roy, but again Potron could not convey his original ideas. He became a new member of the Econometric Society in 1938 (in the same year as Paul Samuelson). However, it was too late: soon the Second World War broke out, many young teachers in France had to join the army, and the old Potron was sent to a secondary school in Vannes to replace one of them. Due to his bad health, and lack of fuel in Vannes, Potron died from pneumonia in January 1942. For more than half a century economists neglected his work, until Émeric Lendjel (2000) finally drew attention to Potron's novel economics. A few years later, Kenji Mori (2008) explicitly emphasised that Potron's long 1913 paper *de facto* included remarkable analytical results on the relation between surplus values and profits in marxian systems with heterogeneous labour. Actually, in 1948 the French statistician Michel Huber (1948, p. 138), who knew Potron very well via the *Société*

[8] See, for example, Potron (1911, 1913, 1937). For many details on Potron's biography, the contents of his papers on economics, their recent translation into English, and an annotated Potron bibliography, I refer to Bidard, Erreygers and Parys (2009), Bidard and Erreygers (2010), and Parys (2016a).

de Statistique de Paris, had already referred to Potron's equations and matrices, when studying the direct and indirect wages entering the price of a commodity, which is related to the problem of its direct and indirect labour content. However, Huber did not explicitly make the connection with the classical or marxian literature on labour values.

§9. Simultaneous equations are necessary

Potron's wealthy parents and their family strongly (intellectually and financially) supported Catholic social action, and in this context Maurice Potron became interested in the study of just prices and just wages. Inspired by the encyclical *Rerum Novarum* (Leo XIII, 1891), Potron worked out an original linear model, with many simultaneous equations and inequalities. He pointed out that such a method was needed, because just wages and just prices could not be studied in isolation. To judge whether a wage was just or not, one had to know the prices of the commodities consumed by the workers. To know whether the price of a commodity was just or not, one had to know its cost of production, thus the prices of the physical inputs and especially the wage. Just wages and just prices were interdependent. To handle this interdependence, Potron emphasised that a mathematical approach by means of equations and inequalities was necessary.

Just like Potron, Charasoff held a doctoral degree in mathematics and understood the power of simultaneous equations. Unlike Potron, Charasoff knew mainstream economics, including Pareto's *Manual* (see Charasoff's quoting it, in my §7 above). The *Manual* was not the only place where Pareto called attention to the necessity of using simultaneous equations in a professional way. Pareto raised this point also elsewhere, for example in Chapter XII of his *Les systèmes socialistes* (especially pp. 287-293), where he patiently tried to explain how many old economic theories were deficient in their analysis of interdependent systems. In circular models where A, B, C, etc. depend on A, B, C, etc., it was useless to look for one "single cause" explaining A, B, C, etc. Marx and others were wrong on this point, because they seemed to possess no comparative advantage in handling systems of simultaneous equations.[9]

Sraffa's (1960) book never refers to Pareto, but Sraffa's unpublished papers show how he took notice of Pareto's remarks. In a system where commodities are produced by means of the same commodities, prices have to be determined simultaneously. The same is true for labour values. Kurz and Salvadori (2010, pp. 198, 212) provide several examples from Sraffa's

[9] At the gymnasium in Trier, Marx's graduation exam in 1835 included four questions on mathematics. The seventeen-year-old Marx had problems with two of them, he could answer one other on his own, and then seemed to have copied the answer to the fourth from Edgar von Westphalen, the youngest brother of his future wife Jenny. When Schefold (1998, p. 130) drew attention to this story, he added the following final remark: "It so happens that the fourth question which Marx apparently answered by means of illegal assistance concerned a simple system of linear equations". Schefold refers to Raussen (1990) for more details. Raussen emphasises that the system consisted of three linear equations in three unknowns. The Marx-Engels Gesamtausgabe, MEGA I/1 (1975, p.1204) suggests that the pupils were confronted with relatively advanced mathematics. I agree with Raussen (1990, pp. 212-213) that MEGA is rather misleading in this context. The three linear equations were simply the following: $50\,000 = x + \frac{y}{2}$; $50\,000 = y + \frac{x}{3}$; $50\,000 = z + \frac{x}{4}$ (a few elementary substitutions generate the solution: x = 30 000; y = 40 000; z = 42 500).

unpublished papers, and even discovered an important (strongly abbreviated and rather well-hidden, inconspicuous) reference to Pareto in this context, in Sraffa's 1928 diary.

§10. Sraffa on reducing everything to labour, or to wheat, or to something else

From early 1930 on Sraffa spent a massive amount of time on his edition of Ricardo (Pollitt 1988; Gehrke and Kurz 2002; Naldi 2005). It is probably not widely known that in 1929 Dobb and Sraffa had made plans to edit an English translation of Marx's *Theorien über den Mehrwert*, and asked Kautsky's permission to use his German edition as a starting point (letter from Dobb and Sraffa to Kautsky, 2 December 1929, and letter from Dobb to Kautsky, 15 February 1930, Karl Kautsky Papers D IX 316-317, International Institute of Social History, Amsterdam; the latter also included a recommendation by Sidney Webb).

I suppose Sraffa quickly abandoned the idea of the Marx translation project, in order to concentrate on the academically more prestigious Ricardo edition. By a remarkable coincidence, Sidney Webb's letter recommending Dobb as Marx-editor to Kautsky was written on the same day that the *Royal Economic Society* entrusted the Ricardo edition to Sraffa: 13 February 1930. The rest of the story is well-known: Dobb never translated Marx's *Theorien über den Mehrwert*, but helped Sraffa to edit Ricardo. However, unlike other Cambridge economic colleagues in the 1930s, both Dobb and Sraffa retained a strong interest in Marx's economics.

Sraffa's unpublished notes of the 1940s often contain marxian terminology, much more than in Sraffa's (1960) book. For example, the latter mentioned "proportions of labour to means of production", the former often argued in terms of "organic composition of capital". A striking feature of the unpublished papers is that Sraffa, already in the 1920s, wrote down very sceptical remarks about Marx's labour values (see Sraffa D3/12/9:89):[10]

> It is a purely mystical conception that attributes to human labour a special gift of determining value.

A small card in the archives (Sraffa D3/12/10:71) has the following text:

> Method of "substitution" in equations: corresponds exactly to "historical" method of showing that all values are "due" to labour, or to wheat or to any other thing that enters in the production of every one of them.

Here Sraffa is referring to methods for solving simultaneous equations. From a purely formal point of view, we can define not one, but several common factors. We can reduce all commodities to embodied labour (labour values), but we might as well reduce everything to embodied wheat, or embodied energy, for example (see further on energy values). Kurz

[10] Also quoted by Kurz and Salvadori (2001, p. 168).

(2015, p. 224, n36) emphasises that the above insight from Sraffa's papers dates from as early as December 1927.[11]

§11. Leontief on reducing everything to A or to K

In exactly the same month of December 1927, independently of Sraffa's research in Cambridge, the 22-year-old Leontief submitted the text of his doctoral dissertation in Berlin. Due to bureaucratic problems, Leontief obtained his degree only at the end of 1928. He published his dissertation in the *Archiv für Sozialwissenschaft und Sozialpolitik* (Leontief 1928).[12]

The last algebraic formulas at the end of the dissertation (Leontief 1928, pp. 621-622; Leontief 1991b, pp. 210-211) are relevant for the problem of reducing everything to one common substance. Leontief considers the following system and lay-out:

$$aA^{n-1} + kK^{n-1} \rightarrow A^n$$

$$(1-a)A^{n-1} + (1-k)K^{n-1} \rightarrow K^n$$

Note that the superscripts are not exponents. The notation of the first row simply means the following: in period $n-1$ we put a units of A and k units of K into the production process, and one year later, in period n, we obtain an output of one unit of A. The notation of the second row describes that in period $n-1$ we put $1-a$ units of A and $1-k$ units of K into the production process, and one year later, in period n, we obtain an output of one unit of K. Here a and k, and $1-a$ and $1-k$, are fractions, i.e., coefficients strictly between 0 and 1. Leontief's lay-out with arrows is rather similar to that of Sraffa's (1960) book. I now omit the superscripts and simply write:

$$aA + kK \rightarrow A$$

$$(1-a)A + (1-k)K \rightarrow K$$

The production of 1 unit of A embodies

$aA + kK$, or

$aA + k[(1-a)A + (1-k)K]$, or

$aA + k[(1-a)A + (1-k)\,[(1-a)A + (1-k)K]\,]$ etc.

[11] Hence a few months after Sraffa had moved to England (Naldi 2005, Potier 1991). In a section on "tertium comparationes" Kurz and Salvadori (2010, pp. 202-203) discuss embodied electricity in relation to some Sraffa notes of early 1928.

[12] The very long and well-documented paper by Bjerkholt (2016) contains many fascinating biographical details on Leontief up until the early 1940s. The section on Leontief's dissertation (Bjerkholt 2016, pp. 32-40) uses archival material from Humboldt University in Berlin (formerly Friedrich-Wilhelms-University) to describe Leontief's unusual path to his doctoral degree.

Because 0 < k < 1, the successive coefficients of K become smaller and smaller: first k, then $k(1-k)$, then $k(1-k)^2$, etc. However, it would be wrong to conclude that A has a unique status here. Leontief claims we can reverse the roles of A and K, and substitute A away. With some goodwill, we can see here the same insight as in Sraffa's unpublished files: an anticipation of the modern notion of commodity values. The modern approach chooses an arbitrary commodity L that enters directly or indirectly into all production processes, and computes the commodity L-value of any commodity j, that is the direct and indirect input of commodity L necessary to produce one unit of gross output of commodity j, also referred to as the commodity L-content of commodity j.

Leontief's example above is rather primitive. It presents a no-surplus system. In such a special case the A-value of A is simply one. Leontief should have worked out a better example; it would have been more meaningful if he had computed the A-value of K, or the K-value of A. Or he should have considered a system with a surplus. Then the A-value of A, or the K-value of K, will be smaller than one, which is typical of economies that are able to produce a surplus.[13]

In the late 1920s Leontief resp. Sraffa mentioned the different reductions to a single input only in an old German study resp. personal papers. One of the first published numerical examples in English of computing commodity values was provided in the book *Proportions, Prices and Planning* by András Bródy (1970, pp. 85-86), for a no-surplus system; in this case the labour values were proportional to other commodity values.

§12. Discovering top economic science in the journal *Science*?

Suppose Enzo, after travelling many decades from his distant planet, finally arrives on Earth in 2017, in a big city of a highly developed Western country, where the writings of various competing schools of thought are freely and abundantly available. Its main university employs a large team of bibliometric specialists. They claim to know objective criteria to assess the scholarly prestige of the thousands of journals in their library: the impact factor. Enzo quickly learns from the bibliometric experts that a certain journal in the library seems to be really scientific beyond doubt. It wears the clear and simple name *Science,* its 2016 impact factor was more than 37. Enzo discovers several papers on the theory of value in this journal, advocating the use of *energy values*, which describe the direct and indirect energy content of a commodity, or, in other words, its embodied energy. The most conspicuous text of this type in *Science* is the often cited paper *Embodied Energy and Economic Valuation,* written by Robert

[13] See Gintis and Bowles (1981), and Roemer (1981, 1982). A few years ago *Metroeconomica* published several sophisticated papers on commodity values, in more general frameworks than the simple systems of the pioneers. See Fujimoto and Fujita (2008), Fujimoto and Opocher (2010a, 2010b), Yoshihara and Veneziani (2010), Bellino (2010).

Costanza (1980).[14] I will provide more details on this paper in §16 and §21 below, but first I introduce a numerical example to compare labour values and energy values.

§13. A simple corn-energy-labour economy

To simplify my exposition, I often use the following numerical example of an economic system where a unit of corn means one kilo of corn, a unit of energy means one kilo of coal, and a unit of labour means one hour of labour time. It is an example of ex post accounting: at the end of the year, after it is known which quantities of inputs and outputs have been used and produced (including knowledge of the workers' consumption and the labour hours they performed), an accountant expresses all quantities per kilo of corn, per kilo of coal and per hour of labour. I make the traditional simplifying assumption that all inputs are completely used up (no fixed capital):

0.4 units of corn + 0.5 units of energy + 0.05 units of labour → 1 unit of corn

0.1 units of corn + 0.4 units of energy + 0.25 units of labour → 1 unit of energy

0.2 units of corn + 0.5 units of energy + 0.15 units of labour → 1 unit of labour

These data can be compatible with various economic theories. An orthodox marxist could interpret the last row above as the outcome of tumultuous struggles about the length of the working day, such that in the past year every individual worker ultimately performed 10 hours of labour per day, and every day consumed 2 kilos of corn, burned 5 kilos of coal for heating at home, and needed 1.5 hours of labour services at home.[15]

§14. Lana computes labour values

If Lana has access to modern Western books that discuss the simple mathematics of labour values, she will use the following method. Labour has a special status in the above system, is separated from the rest, and is considered the ultimate source of value. Lana first concentrates on the production conditions in the two non-labour sectors, i.e., corn and energy. She uses the following lay-out:

0.4 units of corn + 0.5 units of energy + *0.05 units of labour* → 1 unit of corn

0.1 units of corn + 0.4 units of energy + *0.25 units of labour* → 1 unit of energy

[14] Later Costanza et al. (1997) published a paper on the value of the world's ecosystem services and natural capital, not in *Science*, but in the equally prestigious journal *Nature*. In April 2018 Google Scholar mentioned nearly twenty thousand citations to this paper.

[15] In my example all input coefficients are positive. If one prefers the more traditional assumption of zero units of labour input in the consumption basket of the workers, the methods for determining labour values and other values in the next paragraphs remain the same.

The quantities in italics are the *direct labour inputs*; the other inputs are considered to be indirect labour (labour embodied in corn or energy inputs). Let λ_c and λ_e be the labour values for a unit of corn and a unit of energy. They are computed via simultaneous equations:

$$0.4\ \lambda_c + 0.5\,\lambda_e + 0.05 = \lambda_c$$

$$0.1\ \lambda_c + 0.4\,\lambda_e + 0.25 = \lambda_e$$

The first equation shows how the labour value λ_c of one unit of corn is computed by the sum of the labour embodied in its corn and energy inputs (0.4 λ_c + 0.5 λ_e) plus the 0.05 direct labour added in the production process. I repeat that I assume that all inputs are completely used up in production, and thus 100% of the labour embodied in the inputs is transferred to the output.

The two simultaneous equations above give $\lambda_c = 0.5$ and $\lambda_e = 0.5$

Note that the example gives equal labour values for a unit of corn and energy, even though the direct labour inputs of their production processes are very different (0.05 versus 0.25).

I use the following notation:

Matrix of direct non-labour inputs $= \boldsymbol{A} = \begin{bmatrix} 0.4 & 0.5 \\ 0.1 & 0.4 \end{bmatrix}$;

Vector of direct labour inputs $= \boldsymbol{\ell} = \begin{bmatrix} \ell_c \\ \ell_e \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.25 \end{bmatrix}$

Vector of labour values of corn and energy $= \boldsymbol{\lambda} = \begin{bmatrix} \lambda_c \\ \lambda_e \end{bmatrix} = = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

In general the system of labour values is $\boldsymbol{A}\,\boldsymbol{\lambda} + \boldsymbol{\ell} = \boldsymbol{\lambda}$

How much labour time is necessary to “reproduce” labour power? A working day (10 hours) involves the consumption of 2 corn, 5 coal and 1.5 labour, hence $2\lambda_c$ +$5\lambda_e$ +1.5 = 1+2.5+1.5 = 5 hours of necessary labour. Hence every working day consists of 5 hours of necessary labour and 5 hours of surplus labour. Marx’s rate of surplus value is 100 % here.

§ 15. Labour subsystems

I define *the labour subsystem of corn* as a sort of vertically integrated system for the production of corn, with labour as the only net input and corn as the only net output.

Assume constant returns to scale, and consider the system of §14 above. Multiply the quantities for the corn process by $\frac{120}{62}$ and those of the energy process by $\frac{100}{62}$. This gives:

$$\frac{48}{62} \text{ units of corn} + \frac{60}{62} \text{ units of energy} + \frac{6}{62} \textit{ units of labour} \rightarrow \frac{120}{62} \text{ units of corn}$$

$$\frac{10}{62} \text{ units of corn} + \frac{40}{62} \text{ units of energy} + \frac{25}{62} \textit{ units of labour} \rightarrow \frac{100}{62} \text{ units of energy}$$

The gross output of energy = $\frac{100}{62}$ equals its total input = $\frac{60}{62} + \frac{40}{62}$.

The only positive net output is corn = $\frac{120}{62} - \frac{48}{62} - \frac{10}{62} = \frac{62}{62}$ = 1 unit of corn

The only net input is labour = $\frac{6}{62} + \frac{25}{62} = \frac{31}{62}$ = 0.5 units of labour.

We can project the net input into the net output, and claim that the 0.5 units of labour are now embodied (incorporated, crystallised, congealed) in 1 unit of corn. This is just an alternative method to compute the labour value of corn, λ_c = 0.5. This type of subsystem is well known from Sraffa (1960, Appendix A). A very detailed empirical example of such a vertically integrated system was already computed by Leontief (1953), see my §19 below.

I called the above system *the labour subsystem of corn*. In a similar way I can construct *the labour subsystem of energy*. The latter will generate the labour value of energy.

§ 16. Enzo computes energy values

If Enzo uses the journal *Science* and applies the ideas of Costanza (1980) to the simple corn-energy-labour economy of my §13, he would consider energy as the ultimate source of value, and he gives the energy input a special status, separated from the two other inputs (corn and labour). He treats corn and labour, the non-energy sectors, as the two endogenous sectors, and therefore prefers the following lay-out:

0.4 units of corn + 0.05 units of labour + *0.5 units of energy* → 1 unit of corn

0.2 units of corn + 0.15 units of labour + *0.5 units of energy* → 1 unit of labour

Both corn and labour require a direct input of 0.5 units of energy. However, it would be wrong to conclude that their energy values are the same.[16] To the 0.5 units of direct energy input, we have to add the indirect energy input, i.e., the energy embodied in the corn and labour inputs. Hence the energy values for corn and labour, denoted ε_c and ε_ℓ, are computed via simultaneous equations:

[16] A slight change in the data of my example can produce the following situation: direct energy input for commodity c is smaller than for commodity ℓ, but energy value for c is higher than for ℓ. The input-output equations allow us to take into account all indirect inputs. Many engineers (not Costanza) study methods of energy saving, neglect some indirect inputs, and then reach questionable conclusions on policies for energy saving. Baumol and Wolff (1981) explain how errors happen when some engineers replace full input-output analysis by more short sighted computations.

$$0.4\ \varepsilon_c + 0.05\ \varepsilon_\ell + 0.5 = \varepsilon_c$$

$$0.2\ \varepsilon_c + 0.15\ \varepsilon_\ell + 0.5 = \varepsilon_\ell$$

My example gives $\varepsilon_c = 0.9$ and $\varepsilon_\ell = 0.8$.

In this context, Enzo uses the following notation:

Matrix of direct non-energy inputs $= \boldsymbol{B} = \begin{bmatrix} 0.4 & 0.05 \\ 0.2 & 0.15 \end{bmatrix}$;

Vector of direct energy inputs $= \boldsymbol{e} = \begin{bmatrix} e_c \\ e_\ell \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Vector of energy values $= \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_c \\ \varepsilon_\ell \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.8 \end{bmatrix}$

In general the system of energy values is $\boldsymbol{B}\,\boldsymbol{\varepsilon} + \boldsymbol{e} = \boldsymbol{\varepsilon}$ **.**

At the end of §14 we asked how much labour was directly and indirectly necessary to produce the consumption basket of labour. Similarly, we can now ask how much energy (coal) is necessary to produce one unit of energy (one kilo of coal). We know (see §13 above) that 1 unit of energy required 0.1 corn, 0.25 labour and 0.4 energy itself, hence a total energy value of $0.1\ \varepsilon_c + 0.25\ \varepsilon_\ell + 0.4 = 0.09 + 0.2 + 0.4 = 0.69$. Hence in the production of 1 unit of energy (coal) we use up an energy value = 0.69 and thus we create 1- 0.69 = 0.31 surplus energy value here. Just like Lana found a positive rate of surplus *labour* value, Enzo finds a positive rate of surplus *energy* value (0.31/0.69 = 0.45 or 45 %). In my example it takes less than a kilo of direct and indirect coal to produce 1 kilo of coal. Such a condition is also mentioned in Sraffa's unpublished drafts of 22 September 1944: "If condition 'one ton of coal requires dir. + indir. less than 1 ton of coal' is satisfied . . . " (Sraffa Papers D3/12/39:41).

§17. Energy subsystems

I define *the energy subsystem of corn* as a sort of vertically integrated system for the production of corn, with energy as the only net input and corn as the only net output**.**

Consider again the system with exogenous energy of §16 above. Multiply the quantities consumed in the corn process by 1.7 and those consumed by labour by 0.1. This gives:

0.68 units of corn + 0.085 units of labour + *0.85 units of energy* → 1.7 units of corn

0.02 units of corn + 0.015 units of labour + *0.05 units of energy* → 0.1 units of labour

Both on the left and the right we find a total of 0.1 labour.

The only net input = 0.85 + 0.05 = 0.9 energy.

The only net output is 1.7 – 0.68 – 0.02 = 1 corn.

Again, we can project the net input into the net output, and claim that the 0.9 units of energy have been embodied into 1 unit of corn. This is an alternative method to compute ε_c = 0.9, the energy value of corn.

I called the above system *the energy subsystem of corn*. In a similar way I can construct *the energy subsystem of labour*. The latter will generate the energy value of labour. [17]

§ 18. Subsystems with one net input IN and one net output OUT

I can generalise the above arguments to systems with an arbitrarily large number of commodities. Suppose IN is an arbitrary basic input, and consider any other commodity OUT. Then we can construct a subsystem with commodity IN as the only net input and one unit of commodity OUT as the only net output. Such an IN-subsystem of OUT will show how much IN is embodied in OUT, or in other words it shows the IN-value of OUT.

§19. Leontief's early empirical examples of labour values and subsystems

In the 1920s the young Leontief in Russia read a very large amount of texts from classical economists and Marx, but Leontief never emphasised the importance of labour values for his own economic analysis. However, at Harvard he implicitly computed them to study problems of employment, using the expression "total employment coefficients" (Leontief 1944, p. 312).

At the end of his career Leontief was rather often interviewed. In an interview in French, edited by Bernard Rosier, Leontief discussed the theory of value of the classical economists and Marx; here Leontief expressed his scepticism as follows:

> Je pense toujours que c'est un peu métaphysique. C'est pourquoi j'ai toujours des difficultés avec les marxistes. Quelle est la valeur, qu'est-ce que la plus-value? Vous pouvez interpréter les choses de la manière que vous voulez. Cependant on peut calculer le quotient de travail quand on a un tableau, mais aussi celui de l'acier, ou maintenant celui de l'énergie. Tout cela est identique *du point de vue du calcul* (Leontief, in Rosier 1986, p. 89)

Here Leontief clearly points to possible computations of labour values or steel values or energy values, and at the same time he does not believe that such single input theories provide a useful theory of value. Ironically, Leontief (1944), in his paper *Output, Employment,*

[17] One can construct many other cases where labour values and energy values generate similar concepts and propositions. Suppose the same rate of profits rules in the production of all commodities (see also footnote 23). Then relative prices and labour values are equal if and only if equal organic composition of capital holds. A simple criterion to check this uses the direct input coefficients of the matrix A and the vector ℓ only (as defined in §14): it is necessary and sufficient that all ratios $A_j\ell/\ell_j$ are equal, where A_j denotes the j th row of the matrix A (Parys 1977). Similarly, relative prices and energy values are equal if all ratios $B_j e/e_j$ are equal, where B_j denotes the j th row of the matrix B, and where B and e are defined as in §16.

Consumption, and Investment, was perhaps the first who computed real world labour values by means of input-output methods. His paper wanted to solve questions such as “How many new jobs will be created by the consumers’ demand for an additional one million of passenger cars?” (Leontief 1944, p. 290). His Table IV (Leontief 1944, p. 312) contains “the direct and the total employment coefficients” for different industries (thousands of persons per one million dollars worth of final demand).

In my notation the total coefficients correspond to the ratios $\frac{\lambda_1}{p_1}$, $\frac{\lambda_2}{p_2}$, $\frac{\lambda_3}{p_3}$, etc. In Leontief (1944) the labour time is not expressed in hours, but in thousands of man-years; the physical units for commodities are not kilos or litres, for example, but are all chosen such that one unit of every commodity has a price of one million dollar. Leontief’s table was thus tailor-made for empirical studies on the deviation between prices and labour values, but Leontief was not interested in this question, and supporters of labour values for decades missed the opportunity to look at such input-output data.

Unlike Sraffa, Leontief spent a large percentage of his time and resources on empirical research. After many years of extensive statistical research at Harvard, both the statistical data and the analytical procedures were readily available to Leontief in the early fifties for his seminal work on international trade. Here Leontief (1953) used a large 200-industry input-output table of the American economy for the year 1947, sometimes consolidated into 50 sectors.

Leontief’s main goal in his classic 1953 paper was the study of the labour intensity of American imports versus exports, and he concluded that the U.S. exports were more labour intensive than its imports. This “unexpected result”, the Leontief Paradox, captured wide attention and generated numerous debates among specialists, and standard paragraphs in many textbooks on international economics.

Some tables of Leontief (1953) implicitly contained comprehensive empirical examples of the concept of a subsystem, seven years before Sraffa’s theoretical remarks on such systems were finally published. Parts II and III of Leontief (1953) concentrate on the production of automobiles, and contain his important Table 1 that presents a vertically integrated system for the production of automobiles: its only net output consists of automobiles; it is a detailed description of a subsystem of automobiles (commodity 145).[18] It produces a net output of one

[18] Many reprints of this Leontief (1953) paper omit the original opening quotation, which Leontief took from a “French Cookbook by Tante Marie”, and which warned its readers about the lack of stylistic ambitions of the publication. Leontief’s opening quotation suggested he wanted to follow Tante Marie, who did not hesitate to repeat the same word several times in a sentence, in order to be absolutely clear. However, in case of Leontief’s use of “motor vehicles”, the readers of Leontief’s paper might be confused, because the same expression has two meanings: in the consolidated 50-industry table, “motor vehicles” is number 26, and it turns out to be an aggregation of three industries of the more detailed 200-industry table, in which it is divided into Industry 145, also called “motor vehicles”, Industry 146, called “truck trailers”, and Industry 147, called “ automobile trailers”. Leontief’s example of a vertically integrated system actually studies Industry 145, hence, not including truck trailers and automobile trailers. Again forgetting Tante Marie, Leontief’s informal descriptions of his results do not only use the term “motor vehicles” for Industry 145, but elsewhere he also uses “automobiles” or “finished cars” to designate the same Industry 145. In the following I will simply refer to “automobiles” and call them commodity 145. Similarly, I treat the output of Industry 109, called “nuts, bolts and screw machine products”, as commodity 109 and simply refer to “nuts”.

unit (one million dollars worth) of automobiles and its total direct labour requirement is approximately 200 man-years (the exact figure is 201.476). The Table 2 of Leontief (1953) offers a summary of the figures for all other subsystems. Take the nuts, commodity 109, for example. The total direct labour requirement of its subsystem is 216.333 man-years (again per unit, i.e., per million dollars worth). My nonrandom example of automobiles and nuts leads to a 7.38 % deviation of relative prices from relative labour values:

$$\frac{p_{145}/\lambda_{145}}{p_{109}/\lambda_{109}} = \frac{1/201.476}{1/216.333} = \frac{216.333}{201.476} = 1.0738$$

If I take commodity 145 as the base (switch numerator and denominator), I obtain:

$$\frac{p_{109}/\lambda_{109}}{p_{145}/\lambda_{145}} = \frac{1/216.333}{1/201.476} = \frac{201.476}{216.333} = 0.9313$$

hence a 6.87 % deviation. Rounding leads to a 7% deviation in both computations, a real world example of a 93 % labour theory of value, at least for this example of automobiles and nuts. Of course, my choice was not random: in other examples large deviations exist. Moreover the percentage deviation depends on the choice of numerator and denominator (Consider a secondary school example with the numbers 8 and 5. We find 8/5 = 1.6, hence a 60 % deviation; 5/8 = 0.625, hence a 37.5 % deviation).

§20. Deviations of prices from labour values and energy values

If there are n commodities, I propose a simple solution to tackle this arbitrariness: take all possible pairwise deviations $\frac{p_i/\lambda_i}{p_j/\lambda_j}$ for all possible variations of *i* and *j*, and then compute their mean and the median. By variations of 2 indices out of n, we mean all possible pairs of 2 different indices, where the order of the selected indices matters. A different ordering is considered as a different variation. In total there are n (n-1) possible variations of 2 out of n.

The mean and the median of all percentage deviations are simple, transparent measures of the deviation of prices from labour values. These measures are not influenced by the choice of a numeraire or by the choice of units of measurement for prices and labour values. The mean is more sensitive to a few extreme observations. If one wants to avoid this, the median of all pairwise percentage deviations seems a more interesting measure.[19]

[19] For a comprehensive discussion of many other measures of deviation between prices and labour values, see the recent books by Mariolis and Tsoulfidis (2016), and by Shaikh (2016, Chapter 9). Both books contain many bibliographical references about recent empirical studies on the labour theory of value. They do not discuss the literature on energy values, but implicitly they pay attention to many subtle problems of empirical measurement that are overlooked in the energy literature.

In the early 1990s I used such measures in various computations on four Belgian input-output tables. The deviations were much smaller for labour values (direct and indirect labour content) than for direct labour, and much smaller for the subset of 20 manufacturing sectors than for the whole economy. The median of all percentage deviations for the 1980 table, manufacturing sectors only, was 6.17 % for labour values versus 16.06 % for energy values. Note that labour performed better than energy values or other values. Actually, energy values did not do better than transport values; the latter uses the rather freak idea that production is nothing more than changing the position of some items, and therefore computes the total embodied transport content.[20]

Many experiments showed that aggregation can cancel out many extreme differences. A disaggregated study would use more than just a few dozen sectors. A perfect study for a modern Western economy surely needs to consider more than a dozen million different commodities. Remember the planning problems of the old Soviet Union, where consumers enjoyed less variety than in the West:

> In the USSR at this time there are *12 million identifiably different products* (disaggregated down to specific types of ball-bearings, designs of cloth, size of brown shoes, and so on) (Nove 1983, pp. 33-34).

Do also note that the real world prices differ from the "prices of production" analysed in many theoretical studies on Marx. The latter prices assume a uniform rate of profit r in all sectors. In addition, suppose now that such prices and labour values are expressed in comparable units: take an arbitrarily chosen single or composite commodity h and choose units for labour values and prices so that:

$$\lambda_h = p_h = 1$$

Then for an arbitrary commodity *j*, Parys (1982, p. 1210) established the following "percentage formula":

$$\frac{p_j}{\lambda_j} = 1 + r\,(k_j - k_h)$$

where r is the uniform rate of profit, k_j is the capital-labour ratio of the labour subsystem of commodity j, and k_h is the capital-labour ratio of the labour subsystem of the numeraire commodity h. To obtain this simple formula, it is necessary to measure capital in prices, not in labour values, and to consider the capital-labour ratio of a subsystem, not that of an individual industry.[21]

[20] I presented the unpublished results to the Department of Labour Economics, University of Antwerp, on 11 January 1993, in Dutch, in a Workshop entitled "Relatieve prijzen, arbeidskosten en energiekosten: waardetheorie en Belgische input-output empirie". I am very indebted to Koen Hendrickx for the computations.

[21] See the numerical example in Parys (1982, p. 1211). It shows that Marx's approach in terms of organic compositions of individual industries and measurement in terms of labour values can sometimes be strongly misleading. Sraffa, already in the 1940s, suggested that the ranking of the organic composition of capital of two subsystems could be the reverse of that of the two original industries (Sraffa Papers, D3/12/36/60).

Subsystems contain various individual industries and therefore the capital-labour ratios of subsystems are often less different than those of individual industries. Moreover the expression $(k_j - k_h)$ is multiplied by r, usually a small number (rates of profit are usually much lower than 100 %, thus r is much lower than 1). This is one of the reasons why the percentage deviation $r\,(k_j - k_h)$ can be rather small in some empirical studies, especially when the numéraire h is close to a sort of average commodity. Moreover, the empirical exercises employ input-output tables, which contain only a small number of heavily aggregated industries instead of the millions of different individual commodities of the real world, and they usually aggregate outputs and inputs by simply using prices, including the reduction of heterogeneous labour and heterogeneous energy to homogeneous labour and energy.[22] This sort of aggregation also cancels out many extreme differences and reduces the reported deviations of prices from labour values or energy values.

Consider now special cases where the uniform rate of profit r also rules in the price equation for labour.[23] If we then choose units for prices and energy values so that $\varepsilon_h = p_h = 1$, some word processing generates a similar expression for the deviation between prices and energy values:

$$\frac{p_j}{\varepsilon_j} = 1 + r\,(E_j - E_h)$$

where the symbol E_j refers to the energy subsystem of commodity j, and it denotes the ratio between non-energy inputs (aggregated at prices) and the energy input in this subsystem; similarly E_h refers to the energy subsystem of the numeraire commodity h.

§21. Costanza's regressions

Costanza's work on energy values never establishes such subsystems or percentage formulas. His main *Science* paper (Costanza 1980) concentrated on econometric regressions. In the first page of this paper, Costanza (1980, p. 1219) invoked the prestige of a Nobel laureate, not in economics, but in chemistry, namely Frederick Soddy (1877-1956), who tried to unite physical and social sciences into one framework, emphasising the unique role of energy:

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[22] There is a large marxist literature on the reduction problem of heterogeneous labour to homogeneous labour, and there are many energy papers on the reduction of various energy inputs to one homogeneous physical unit. Despite these sophisticated theories, the empirical studies on labour values or energy values usually have to choose simple aggregation methods, by means of the corresponding wages or energy costs.

[23] Denote the price of a unit of labour by p_ℓ, denote the price of the corresponding consumption basket by p_b, and then define the symbol r_ℓ such that $p_b\,(1 + r_\ell) = p_\ell$. The rate r_ℓ can be equal to r, the uniform rate of profit in all the other price equations, in the following two special cases. First, when a pure slave system rules, where slave labourers are "produced" at the profit rate r just like any other product. Second, consider a capitalist system where workers are "free", earn a wage, and have a positive savings rate s. Then by definition $s = (p_\ell - p_b)/p_\ell$ and thus $s = r_\ell/(1 + r_\ell)$. The rate r_ℓ then equals the uniform rate of profit r, if the savings rate $s = r/(1 + r)$. For example, if $r = 0.10$, then $r = r_\ell$ if the savings rate $s = 1/11$.

> If we have available energy, we may maintain life and produce every material requisite necessary. That is why the flow of energy should be the primary concern of economics (Soddy 1933, p. 56).

Soddy's switch from chemistry to economics was not well-received by mainstream economists.[24] Some energy studies supported the energy theory of value with the same enthusiasm that many marxists used in their defense of the labour theory of value. At a rather late stage, the energy values supporters turned their eyes to input-output methods, and publication in *Science* generated a lot of prestige and funding. After the ambitious paper by Costanza (1980), some later input-output computations also tried to support the energy theory, for example in work by Cleveland, Costanza, Hall and Kaufmann (1984) and by Costanza and Herendeen (1984). In the authoritative *Encyclopedia of Energy* Costanza (2004) mentions a sort of 93% argument for the energy values: they are not 100 % exact, but a very good empirical approximation. Elsewhere Costanza (1997: xviii) had complained that his results "have been more or less completely ignored by both conventional economists and environmentalists" (see also Costanza 2016, p. 51). [25]

Costanza (1980) employed American input-output tables, for an ambitious empirical exercise on the relation between prices and energy values. He presented the results of various regressions, for example the following result:

$$\text{DOUT} = 0.5203 + 0.00115\ \text{BTU} \qquad (R^2 = 0.9455), \quad \text{where}$$

- DOUT of sector i = price of total yearly output of sector i, in 10^9 dollars, and

- BTU of sector i = energy value of total yearly output of sector i, in 10^{12} British Thermal Units (BTU = British Thermal Unit = the amount of heat needed to increase the temperature of one pound of water by one degree Fahrenheit).

The very high determination coefficient R^2 was employed by Costanza to suggest a strong relation between prices and energy values, and thus a strong case for the energy theory of value. Ironically, in the same period, Wolff (1979) in the *American Economic Review* had reported equally high determination coefficients, when regressing prices on labour values, using similar American input-output tables. Wolff (1979, p. 335) suggested that "this indicates that empirically relative prices were close to relative labour values for the U.S. economy". Most authors of *Science* do not read intensely the *American Economic Review*, and vice versa.

[24]There is a surprisingly large group of non-economists who produced heterodox economic theories based upon a special role for energy. See Berndt (1985), Martinez-Alier (1987) and Mirowski (1988) for some historical details on the economic ideas of such adventurous outsiders.

[25] However, Nicholas Georgescu-Roegen wanted to attack the fundamentals of Costanza's approach, but *Science* rejected his criticism; see the long letter of 20 May 1991, from Georgescu-Roegen to James Berry, published by Bonaiuti (2011, pp. 236-240); see also Georgescu-Roegen (1986). Note that *Science* published a short comment by Huettner (1982), who concentrated on problems of double counting, but Costanza (1982) could counter it. More fundamental criticisms never reached the pages of *Science*.

It is risky to overestimate the importance of high values of R^2. Consider my following example:

commodity *i*	commodity 1	commodity 2	commodity 3
total price p_i	102	106	112
total energy value ε_i	1	3	6

Even without any knowledge of econometrics, it is easy to check that in my example a perfect linear relation holds:

$$p = 100 + 2\ \varepsilon$$

In such cases an econometric regression of total price on total energy value will yield $R^2 = 1$. Do we meet a 100 % perfect energy theory of value in this case? No! On the one hand, relative prices are 106/102, 112/102, 112/106, thus all close to 1. On the other hand, the corresponding relative energy values are extremely different: 3/1, 6/1, 6/3. Instead of the energy theory of value, the explanation might be quite different. For example, imagine a criminal dictator controls every firm in the economy, and chooses unit prices according to the linear formula above, including a "tax" of 100 transferred to himself or his corrupt family. To explain relative prices here a theory of corruption would contribute more insight than a theory of energy values.

§22. Problems of imperfect communication

Usually energeticists and marxists belonged to two separated intellectual networks, with different conferences, different journals, etc. Of course, a minority of scholars studied both marxian and energy accounting approaches. In point of fact, Marx himself tried to read widely in the natural sciences. In the final years of Marx's life, he and Engels paid some attention to novel ideas on the relation between energy and labour, presented by the Ukrainian socialist Sergei Podolinsky (1850-1891).

After publishing a Russian version of his thoughts, Podolinsky tried to spread his ideas in the West. He sent a letter to Marx on 30 March 1880, including a paper (in French) "to which your work *Das Kapital* has given me the first stimulus".[26] He received a reply from Marx (not extant), thanked him (letter of 8 April), and then published a revised version on 20 June in *La Revue Socialiste* (Podolinsky 1880). A somewhat longer Italian text appeared the next year in *La Plebe* (Podolinsky 1881), and a slightly different German version two years later in *Die*

[26] Marx-Engels Papers D 3701-3702 (D VI 240-241): letters from Podolinsky, Archives of the International Institute of Social History, Amsterdam.

Neue Zeit (Podolinsky 1883).[27] At that moment Marx had just passed away, and Podolinsky had stopped his creative activities, due to an incurable mental breakdown. In recent years Podolinsky's work received increasing attention in studies on ecology and marxism, and this led to English translations of the above Italian and German articles (Podolinsky, 2004, 2008).

The existence of different versions might be slightly confusing. For example, the French and the Italian version refer to Marx, but not to Quesnay. The German version mentions Quesnay, but not Marx, etc. Engels read the Italian version, which contains the following passage on the opening page:

> According to the theory of production formulated by Marx and accepted by socialists, human labour, expressed in the language of physics, accumulates in its products a greater quantity of energy than that which was expended in the production of the labour power of the workers. Why and how is this accumulation brought about? (Podolinsky 2004 [1881])

Some modern readers might be tempted to answer Podolinsky's question by means of simultaneous equations and energy surplus values, but Podolinsky himself concentrated on the general distribution of energy in the universe, the role of energy on the earth, the special properties of human labour, etc., without offering equations or well-rounded conclusions. In December 1882, Engels wrote two letters to Marx about Podolinsky's ideas. Engels did not believe that a sort of energy accounting could be a useful addition or improvement of Marx's work:

> The energy value, according to the production costs, of a hammer, a screw, a sewing needle, is an impossible quantity. In my opinion, it is completely impossible to express economic relations in physical magnitudes. (Engels to Marx, letter of 19 December 1882).[28]

The extensive study by Martinez-Alier (1987) and other 20th century writings might have suggested that Marx and Engels handled Podolinsky's work rather hastily, but some more recent literature challenged this interpretation. Foster and Burkett (2017, Chapter 2) mention that there exists around 1,800 words long extracts from Podolinsky's texts in Marx's notebooks, and that such yet unpublished material will be included in a future volume IV/27 of the Marx-Engels Gesamtausgabe (MEGA).

The majority of contemporary economists raise eyebrows when confronted with strong results claimed by adherents of labour values or energy values. Sometimes it is clear that these adherents believe that their approach possesses a simple "definitional" obviousness. Compare

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[27] Another German text, entitled "Der Sozialismus und die Einheit der physischen Kräfte" appeared in Budapest in the *Arbeiter-Wochen-Chronik* (Sozial-ökonomisches Volksblatt. Zentralorgan der Hungarischen Allgemeinen Arbeiterpartei) in three installments in 1881 (no. 32, 33, 37), but the German version in Kautsky's *Die Neue Zeit* (Podolinsky 1883) was more visible in Western Europe.

[28] Der Energiewert, den Produktionskosten nach, eines Hammers, einer Schraube, einer Nähnadel ist eine unmögliche Gröβe. Ökonomische Verhältnisse in physikalischen Maβen ausdrücken zu wollen, ist meiner Ansicht nach rein unmöglich (Marx and Engels 1973, p. 134).

for example Hunt and Costanza. Hunt defended Marx's labour theory of value in the *Cambridge Journal of Economics*:

> The natural environment must be transformed if it is to support human life. Production is this transformation. And production has only one universally necessary social or human ingredient: labour. [. . .] The sun is as essential to human productive activity as is the crust of the earth. But no one speaks of the sun being a factor of production on the same footing as labour (Hunt 1983, pp. 337-338).

Hunt was surely not familiar with the Costanza paper on energy values in *Science* a few years earlier. Here Costanza attaches great relevance to the obvious fact that in the universe the sun and its energy can exist without human beings, but not vice versa:

> The question might be asked whether the same thing we have done with energy could not be done with any of the other currently defined primary factors and thus support capital, labor, or government service theories of value. The answer is that on paper this could be done. We must look to physical reality to determine which factors are net inputs and which are internal transactions. No one would seriously suggest that labor creates sunlight (Costanza 1980, p. 1224).

§23. Afterthoughts

Often the studies on labour values and energy values neglect each other, and also neglect the criticism from more sophisticated economists. Many empirical studies on labour values concede that the labour theory has logical deficiencies, but is a good empirical approximation. Ironically, this approach does not defend Marx, who claimed that labour values were absolutely necessary to explain prices and the rate of profit, but it supports Stigler's Ricardo. Stigler (1958) coined the expression "Ricardo's 93% labour theory": he claims that Ricardo did not hold an analytical labour theory of value, but an empirical one: labour values often give a good approximation in practice.

Most contemporary economists would like to use such a 93% approach in a rather humble way, to provide transparent short answers to a restricted set of interesting questions. Suppose a layman asks an economist: "Why is it that today a Broadway theatre ticket is much dearer than a simple watch or radio? Wasn't it quite the reverse ninety years ago?" In this special case the economist might draw inspiration from the classic paper by Baumol (1967), on unbalanced productivity growth in different industries. Using the strong assumption that prices correspond to labour costs only, Baumol provided insightful explanations of why modern societies are confronted with ever rising prices of performing arts, health care and education, compared to the prices of basic watches, radios or many other industrial products.[29]

[29] Baumol (1967, p. 416) mentioned the absence of productivity growth possibilities in certain live performances: "A half hour horn quintet calls for the expenditure of 2.5 man hours in its performance, and any attempt to increase productivity here is likely to be viewed with concern by critics and audience alike".

Of course, this does not imply that Baumol's textbooks propagate the labour theory of value as a general modern theory of value.

I suppose that especially the supporters of energy values do not realize that the critique by Steedman (1977) and others with respect to labour values also holds for energy values. Both labour values and energy values are ex post accounting magnitudes. In the real world an extremely complex mechanism, involving actions of numerous decision makers (consumers, producers, politicians, etc.), all operating under many complicated influences and constraints, leads to a certain combination of inputs and a certain gross output. Ex post, and making some simplified assumptions, accountants like Lana and Enzo above can then compute labour values or energy values afterwards. These values are derived quantities; they are not theoretically necessary for explaining prices or profit rates.

§24. Conclusion

At least since the ancient Greek philosophers some thinkers have discussed the idea of a single cause of economic value. My paper concentrated on some developments after 1867, when Marx in the opening pages of *Das Kapital* singled out labour as the ultimate determinant of value, the common substance in all commodities, the source of the surplus, etc.

Mainstream economists like Wicksteed, Böhm-Bawerk, Wicksell, and Pareto casted doubts on Marx's arguments. Pareto also questioned the amateurish logic of many non-mathematical economists, and recommended the use of simultaneous equations for a professional treatment of interdependent systems. Today such simultaneous equations are the standard tool for determining labour values and discussing their relation with prices. In the 1890s and later, Pareto, Bortkiewicz, and many others neglected the pioneering price equations of Mühlpfordt (1895), despite their publication in the *Jahrbücher für Nationalökonomie und Statistik*, which was internationally regarded as one of the top journals of economics in that period. Unaware of Mühlpfordt, Dmitriev formulated the complementary input-output equations for labour values. Bortkiewicz read Dmitriev's Russian essay, and "simplified it a bit", thereby missing the opportunity to use Dmitriev's pioneering input-output coefficients. In 1928 Bortkiewicz also underestimated the potential of his doctoral student Leontief, who received a Nobel Prize in 1973 for his input-output analysis. Working outside the intellectual networks of all the above economists, and therefore neglected by them, Charasoff (1910) reduced all commodities "in the limit" to a special composite commodity (Urkapital), and anticipated several analytical tools used in Sraffa's (1960) book. Sraffa intensively studied Bukharin's book on economic theory, which contained several references to Charasoff, but Bukharin concentrated on rather superficial Charasoff references, and did not draw attention (and probably did not understand) the original devices created by Charasoff. Sraffa's impressive library does not contain a copy of Charasoff.

Both Leontief and Sraffa constructed the fundamentals of their theories on their own. In 1927, at an early stage of their career, unaware of each other, both realised that the reduction of all commodities to labour values was not a unique process. From a formal point of view, it was possible to compute wheat values, coal values, etc. Leontief never published this result in English, until his German dissertation was partially translated in 1991. Sraffa's remarks on this problem remained hidden in his unpublished papers until after his death. In the 1940s Leontief's statistical computations on direct and total employment coefficients yielded important information for debates on the deviation of prices from labour values, but it took rather long before Marx scholars took notice of such empirical input-output material. Moreover, it took nearly a century before it was detected that the Roman Catholic system of Potron (1913) had implicitly solved some marxian problems about the relation between surplus values and profits in a system with heterogeneous labour.

The energy crisis of the early 1970s generated an increasing interest in energy problems, and led some influential non-economists to advocate an energy theory of value. This culminated in the prestigious paper on energy values that Costanza (1980) ultimately published in *Science*, claiming that input-output computations revealed a close connection between relative prices and relative energy values.

I presented a generalisation of Sraffian subsystems for energy values, and a formula for the percentage deviation of prices from energy values (formally similar to my formula for labour values), and I discuss why both adherents of labour values and energy values often claimed "good" empirical support for their favourite theory of value. The histories of labour values and energy values present unusual combinations of originality, amateurism, and several cases of imperfect communication, though Marx and Engels themselves paid critical attention to the papers on energy by Podolinsky.

Mainstream economists of the 21st century will regret that both adherents of labour values and energy values often miss the opportunity to study more sophisticated modern approaches to value theory. Moreover, many studies on energy values and labour values underestimate the similarity of each other's arguments. Not only the formal input-output analysis often looks analogous, but even the methodological arguments can look rather similar, for example, when both currents of thought, often unaware of each other, claim that their favourite input (labour resp. energy) obviously is the unique ultimate input. Finally, note that both currents of thought also discuss much wider issues than the pure theory of value, for example when presenting broad historical studies. In this context Marx and Engels (1848) made the well-known statement that the history of all hitherto existing societies is the history of class struggles. Here Marx and Engels seem to be rather modest, limiting themselves to human societies on the Earth, whereas the scope of some energy specialists is infinitely much wider, as they emphasise the dominating role of energy for the history of all human and nonhuman systems in the Universe.[30]

[30] See the statement by Cleveland (2009, p. xv) on energy: "its central role in human affairs dates to the time of the first humans, its role in shaping the physical and biological nature of the Earth dates to the planet's birth, and the story of Big Bang itself is told in terms of energy and matter."

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Abstract

In the history of economic thought various thinkers have been attracted by the problematic idea of a single cause of economic value. Today such ideas appear mostly in studies on the labour theory of value of Karl Marx. In 1867 his "Das Kapital" tried to explain the values of all commodities by a special common substance, the quantity of abstract labour embodied in them, i.e., the direct and indirect labour time necessary to produce them. Several well-known mainstream economists (for example, Wicksteed, Böhm-Bawerk, Wicksell and Pareto) quickly pointed out that labour was not the only common substance in commodities. Another problem was Marx's deficient mathematics. In Marx's system the same commodities can appear on the side of the inputs and on the side of the outputs. Today it is well-known that many analytical problems in such circular systems can be solved by means of simultaneous equations of the input-output type. Here the most original contributions were not made by leading economists mentioned above, but by rather unknown outsiders (Mühlpfordt, Dmitriev, Charasoff, Potron), whose pioneering works were neglected for decades. In December 1927, both Sraffa and Leontief, independently of each other, argued that the reduction of all commodities to labour values was not a unique process; from a formal point of view, it was possible to compute not only labour values, but also wheat values, coal values, etc. For various reasons, their insights of 1927 remained rather unnoticed for several decades.

A few decades ago the high impact journal Science published some papers advocating the use of energy values, corresponding to the direct and indirect quantity of energy necessary to produce a commodity. The most conspicuous text of this type is the often cited paper "Embodied Energy and Economic Valuation" by Robert Costanza (published in Science in 1980), claiming that both theory and empiry suggest a close connection between prices and energy values. The energy theory of value is usually discussed outside the networks of orthodox or marxian economics, but the empirical and theoretical studies on energy values and labour values show several remarkable cases of analogous arguments and imperfections, both trying to support the use of a single substance of economic value.

My paper considers both theoretical and empirical aspects of such discussions on labour values and energy values. I also use a simple numerical example, to illustrate the computation of both labour values and energy values via two methods: via a system of simultaneous input-output equations and via subsystems (vertically integrated systems that produce only one net output). The two computation methods are well-known from the rich literature on labour values, but comparing energy values and labour values forces me to a generalisation of the notion of a subsystem: in order to define this notion unambiguously, I need to specify not only the net output of the subsystem, but also its net input. If the only net input of the subsystem is energy, I use the expression "energy subsystem". This concept is needed to establish a percentage formula for the deviation of prices from energy values, analogous to my formula for the deviation of prices from labour values. By generalising Sraffa's notion of subsystems it is possible to explain why supporters of labour values and energy values both present misleadingly "good" empirical results for their favourite theory. Both groups should be more aware of the non-uniqueness of their exaggerated claims for their one sided theories of value.

Keywords: *Marx, Sraffa, Leontief, Costanza, energy values, labour values, input-output analysis*

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