Scheduling Flexibility and the Contribution Maximizing Vehicle Routing Problem with Time Windows

Wout Dullaert

Ufsia-Ruca Faculty of Applied Economics
University of Antwerp,
Prinsstraat 13, 2000 Antwerp, Belgium
wout.dullaert@ua.ac.be

Abstract

In this paper, the concept of scheduling flexibility, flexibility with respect to the moment of service within a day, is introduced for a less-than-truckload carrier. The carrier charges a tariff based on the weight of the shipment and the width of the time window in which it is to be serviced. For a given shipment, customers with a higher scheduling flexibility pay lower freight rates. Carriers can use scheduling flexibility of customers to reduce operating costs by designing more efficient routes. The carrier is thus confronted with a trade-off between possible cost reductions of scheduling flexibility and lower freight rates. The Contribution Maximizing Vehicle Routing Problem with Time Windows (CMVRPTW) can be used to study a number of combined pricing and scheduling issues. A sequential and 2 two-phase insertion heuristics for the CMVRPTW are presented and evaluated. Applications of the model and directions for future research are given.

1 Introduction

Despite the demand for on-the-hour deliveries due to JIT and other zero-inventory philosophies, differences between customers' flexibility with respect to the time of delivery remain. In general, three categories of customers, each having a different degree of scheduling flexibility, can be distinguished. The customers in the first category specify short time periods (e.g. a few hours) for loading or unloading. As they offer very little scheduling flexibility to the carrier, we call them rigid customers. Customers in the second category are characterized by daily flexibility of scheduling. They specify time periods of several hours for receiving service. A carrier can use this type of scheduling flexibility to design more cost efficient routes, which would be infeasible if time windows were

narrower. Both types of scheduling flexibility can be modelled by using time windows. Finally, if customers allow a carrier to choose the day of delivery, they are said to have periodic flexibility of scheduling. This type of scheduling flexibility is related to the Period Vehicle Routing Problem and can be modelled by allowing service on a number of day combinations[1]. If a customer requires two visits each five-day working week, then for example Monday/Wednesday, Tuesday/Thursday and Wednesday/Friday can be the only allowable combinations (Tan and Beasley, 1984, p. 497). Since the last kind of scheduling flexibility is defined over a longer time period, a carrier can design more efficient routes by, for example, assigning customers in the same geographic area to the same day of the week (Ball, 1988, p. 200).

As scheduling flexibility allows a carrier to design more cost efficient routes, he can be interested in persuading rigid customers to become more flexible. Because flexible customers pay lower freight rates than rigid ones, a carrier is confronted with a trade-off between possible cost reductions of scheduling flexibility and lower freight rates. For small Vehicle Routing Problems with Time Windows (VRPTWs), this trade-off can be made manually by experienced dispatchers. If a large number of customers has to be serviced, a more systematic approach is needed to support their decisions. To this purpose the Contribution Maximization Vehicle Routing Problem with Time Windows (CMVRPTW), a generalization of the VRPTW, is introduced in section 2. In section 3 we briefly discuss Solomon's (1987) sequential insertion heuristic I1 for the VRPTW. In section 4 it is adapted to the CMVRPTW for a less-than-truckload carrier with a homogeneous fleet. In the fifth section a set of test problems based on Solomon's (1987) VRPTW problem instances is constructed. Section 6 contains computational results on the effectiveness of a sequential insertion heuristic and 2 two-phase heuristics for a problem set derived from Solomon's (1987) 100-customer VRPTW instances. Conclusions and avenues for future research are formulated in the last section.

2 The Contribution Maximizing Vehicle Routing Problem

The CMVRPTW is a generalization of the VRPTW. Like the VRPTW, it refers to a situation in which capacitated vehicles located at a depot are required to service geographically scattered customers over a limited scheduling period (e.g. a day). Each customer i has a known demand q_i to be serviced (either for pickup or delivery but not both) at time b_i chosen by the carrier. If time windows are hard, b_i is chosen within a time window, starting at the earliest time e_i and ending at the latest time l_i that customer i permits the start of service. In the soft time window case, a vehicle is allowed to arrive too late at a customer but a penalty is incurred. In both cases, a vehicle arriving too early at customer j, has to wait until e_j. If t_{ij} represents the direct travel time from customer i to customer j, and s_i the service time at customer i, then the moment at which

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[1]Note that there are other ways to model periodic flexibility of scheduling. Russell and Igo (1979) specify a required service level (number of deliveries within the period) and prevent the deliveries being too close together in the period by constraining the spacing in days between successive deliveries.

service begins at customer j, b_j, equals $\max\{e_j, b_i + s_i + t_{ij}\}$ and the waiting time w_j is equal to $\max\{0, e_j - (b_i + s_i + t_{ij})\}$. A time window can also be defined for the depot in order to define a 'scheduling horizon' in which each route must start and end (Potvin and Rousseau, 1993).

Although customers in the VRPTW usually have only one time window for receiving service, customers in the CMVRPTW can have several alternative time windows. Assume, without loss of generality, that the freight rate depends only on the size of demand q_i and the width of the delivery time window. For a given q_i, the wider the time window, the lower the price. Each customer is or is assumed to be indifferent between each of his price-time window combinations. The objective is to maximize contribution by servicing each customer within his optimal time window, i.e. the time window that gives the carrier the highest contribution, taking the capacity limits of the vehicles and the time horizon into account.

In order to get a clearer insight to the problem, we adapt the mathematical programming formulation of the VRPTW of Desrosiers et al. (1995, pp. 85-86) to account for contribution maximization and multiple time windows with an associated price for service.

Given a set of customers $N = \{1, \ldots, n\}$ and a set of available vehicles K, indexed by k, each having an origin location (e.g. depot) $o(k)$ and a destination location $d(k)$, the set of all nodes that vehicle k can service, $V^k = N \cup \{o(k), d(k)\}$ can be defined. All arcs are contained in $V^k \times V^k$, and the subset $A^k \subseteq V^k \times V^k$ represents the set of all feasible arcs. All time windows among which customer j is, or is assumed to be indifferent.

For each arc $(i, j) \in A^k$ and each time window w from the set W^j, there is a variable cost c_{ijk}^w of moving vehicle k from customer i to customer j to perform service in his time window w. In the short run this variable cost consists of the cost of the distance travelled $\delta_d d_{ij}$, and the value of time involved in driving from i to j and servicing customer j, $\delta_t (t_{ij} + s_j)$. The kilometer cost δ_d includes fuel consumption, kilometer-related maintenance, mileage allowances of staff, fines and damages and the kilometer-related component of the depreciation cost. In order to value $(t_{ij} + s_j)$, we use the shadow price of time δ_t, i.e. the contribution that can be earned per unit of time. In the short run, capacity is fixed and its costs therefore do not have to be taken into account to maximize profits. In the long run, capacity becomes variable, meaning that a fixed charge c_f of using a vehicle to build a route has to be added to each $c_{o(k),j,k}^w, \forall j \in N, \forall w \in W^j$.

Each customer $j \in N$ has a known demand for service q_j. If all $q_j > 0$, loads have to be delivered to the customers, otherwise loads have to be picked up in one of the alternative time windows $[e_{jw}, l_{jw}]$, $w \in W^j$. If q_j can take positive or negative values, the problem becomes a pick-up and delivery problem with time windows.

The price the carrier charges to service q_j in $[e_{jw}, l_{jw}]$, $p(q_j, w)$ depends on the size of the load and the width of the time window. Given a set of standard delivery sizes $\bar{D}$, indexed from 1 to $\bar{d}$, and a set of standard time window sizes $\bar{W}$, indexed from 1 to $\bar{w}$, a tariff $P = \bar{D} \times \bar{W}$ can be constructed. Service in wider time windows is performed at a lower price. Contribution is defined as $p(q_j, w) - c_{ijk}^w$, $w \in W^j$.

3

$$p(q_j, w) = P_{lk} \text{ if } \begin{cases} q_{l-1} < q_j \leq q_l & l-1, l \in \bar{D} \\ l_{k-1} - e_{k-1} < l_{jw} - e_{jw} \leq l_k - e_k & k-1, k \in \bar{W} \end{cases} \quad (1)$$

The problem of servicing a customer in his contribution maximizing time window is not related to the type of those time windows. The time windows in the CMVRPTW can therefore be either hard or soft. In both cases a vehicle can arrive too soon at a customer and wait for service until one of its time windows $[e_{jw}, l_{jw}]$, $w \subset W^j$ begins. Only in the soft time window case, a vehicle can arrive too late at a customer but a penalty is incurred.

The mathematical programming formulation introduced next involves three types of variables: flow, time and load variables. A binary flow variable X_{ijk}^w, $(i,j) \in A^k$, $k \in K$, $w \in W^j$, equals 1 if arc (i,j) is used by vehicle k to travel from i to j to perform service in its time window w, and 0 otherwise. The start of service at node j in time window w is denoted by the time variable b_{jk}^w, $j \in V^k, k \in K, w \in W^j$. Load variables $Q_{jk}, j \in V^k, k \in K$ are irrespective of w and specify the load of vehicle k just after servicing node j.

$$\max \sum_{w \in W^j} \sum_{k \in K} \sum_{(i,j) \in A^k} \left(p(q_j, w) - c_{ijk}^w \right) X_{ijk}^w - \sum_{k \in K} \sum_{j \in V^k} c_f^k X_{0jk}^w \quad (2)$$

subject to

$$\sum_{w \in W^j} \sum_{k \in K} \sum_{j \in N \cup \{d(k)\}} X_{ijk}^w = 1 \quad \forall i \subset N \quad (3)$$

$$\sum_{w \in W^j} \sum_{k \in K} \sum_{j \in N} X_{o(k),j,k}^w \leq v \quad (4)$$

$$\sum_{w \in W^j} \sum_{j \in N \cup \{d(k)\}} X_{o(k),j,k}^w = 1 \quad \forall k \in K \quad (5)$$

$$\sum_{w \subset W^j} \sum_{i \in N \cup \{o(k)\}} X_{ijk}^w - \sum_{w \in W^i} \sum_{i \in N \cup \{d(k)\}} X_{jik}^w - 0 \quad \forall k \in K, \forall j \in N \quad (6)$$

$$\sum_{w \in W^{d(k)}} \sum_{i \in N \cup \{o(k)\}} X_{i,d(k),k}^w = 1 \quad \forall k \in K \quad (7)$$

$$X_{ijk}^{w_j} \left(b_i + s_i + t_{ij} - b_j^w \right) \leq 0 \quad \forall k \in K, \forall w \in W^j, \forall (i,j) \in A^k \quad (8)$$

$$e_{jw} \leq b_j^w \leq l_{jw} \quad \forall k \in K, \forall w \in W^j, \forall j \in V^k \quad (9)$$

$$X_{ijk}^w \left(Q_{ik} + q_j - Q_{jk} \right) \leq 0 \quad \forall k \in K, \forall w \in W^j, \forall (i,j) \in A^k \quad (10)$$

$$q_j \leq Q_{jk} \leq C^k \quad \forall k \in K, \forall j \in N \cup \{d(k)\} \quad (11)$$

$$Q_{o(k)}^k = q_{o(k)} \quad \forall k \in K \quad (12)$$

4

$$X_{ijk}^w \in \{0,1\} \quad \forall k \in K, \forall w \in W^j, \forall (i,j) \in A^k \qquad (13)$$

Constraint 3 ensures that each customer j is serviced exactly once in one of its allowable time windows by one single vehicle. Side-constraint 4 can be used to set the number of vehicles smaller or equal to v. All vehicles must leave their origin location $o(k)$, and travel to one of the $j \in N$ customers or its destination location $d(k)$ (5). After performing service, each vehicle must leave the customer (6) and finally has to arrive at its destination location $d(k)$ (7). Feasibility of the time schedule is guaranteed by constraints 8 and 9. Constraints 10 to 12 ensure feasible loads and the binary conditions on the flow variables are defined in 13. A linearization of constraints 8 and 10 can be found in Desrosiers et al. (1995, p. 86).

Because the Vehicle Routing Problem (VRP) is NP-hard, the VRPTW and the CMVRPTW are NP-hard by restriction. Although Kohl et al. (1999) are able to solve a number of Solomon's (1987) problem instances to optimality, (meta)heuristics often remain preferable for real-life applications. Initial heuristics generate a feasible solution for the VRPTW which can then be improved by means of an improvement heuristic ((Or, 1976), (Solomon et al., 1988)) or a metaheuristic [2] in short time spans. Moreover, metaheuristics are more capable in coping with difficult objective functions and/or constraints than exact algorithms (Reeves and Beasley, 1993, p. 11). Several authors have pointed out the importance of the quality of initial heuristics on the performance of metaheuristics. Liu and Shen (1999) conclude from the results reported by (Garcia et al., 1994), (Thompson and Psaraftis, 1993), and (Potvin and Rousseau, 1995) that algorithms which concentrate only on improving a poor initial solution do not perform very well within a limited computation time. (Louis et al., 1999) report on the impact of good initialization on solution quality and computational speed for genetic algorithms. Van Breedam (2001) demonstrates the dependence of descent heuristics and tabu search on the quality of the initial solution.

Two-phase heuristics are able to generate good initial solutions for the VRPTW. By using a sequential route-building heuristic in the first phase to determine the number of seed routes, they share the sequential heuristics' focus on utilization of scarce resources. By applying a parallel heuristic in the second phase, well-separated routes are designed.

Given Solomon's (1987) sequential insertion heuristic's track record as an initial heuristic for the VRPTW, we use it to construct a sequential insertion heuristic and 2 two-phase heuristics in section 4.

3 Solomon's (1987) sequential insertion heuristic I1

After starting the current route with an initialization criterion, a sequential insertion heuristic uses the insertion criterion $c_1(i,u,j)$ to calculate for each unrouted stop u the best place and associated cost for insertion between two

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[2]Best heuristic solutions on several of the original Solomon's (1987) instances are currently obtained by (Tan et al., 2000), (Homberger and Gehring, 1999), (Chiang and Russell, 1997), (Thangiah et al., 1996), (Potvin and Bengio, 1996), (Rochat and Taillard, 1995), (Thangiah et al., 1994), (Desrochers et al., 1992).

adjacent customers i and j in the current partial route $(i_0, i_1, ..., i_m)$ in which i_0 and i_m represent $o(k)$ and $d(k)$ (e.g. the depot). The cheapest insertion cost and the associated insertion place is determined for each unrouted customer u as

$$c_1(i, u, j) = \min_p [c_1(i_{p-1}, u, i_p)], \quad p = 1, ..., m \qquad (14)$$

in which

$$c_1(i, u, j) = \alpha_1 c_{11}(i, u, j) + \alpha_2 c_{12}(i, u, j) \text{ with} \qquad (15)$$
$$\alpha_1, \alpha_2 \geq 0 \text{ and } \alpha_1 + \alpha_2 = 1$$
$$c_{11}(i, u, j) = d_{iu} + d_{uj} - \mu d_{ij}, \mu \geq 0$$
$$c_{12}(i, u, j) = b_{u,j} - b_j$$

In a second step, the customer that is best according to the selection criterion $c_2(i, u, j)$ is selected. The selected customer u^* is then inserted in the route between i and j.

$$c_2(i, u^*, j) = \max_u [c_2(i, u, j)] \quad u \text{ unrouted and feasible} \qquad (16)$$

$$c_2(i, u, j) = \lambda d_{0u} - c_1(i, u, j), \lambda \geq 0 \qquad (17)$$

If no remaining unrouted customer has a feasible insertion place, a new route is initialized and identified as the current route.

The insertion criterion $c_1(i, u, j)$ of insertion heuristic I1 is time-space based in the sense that the best insertion place is the one that minimizes a weighted average of the additional distance and time needed to include a customer in the current partial route. Solomon (1987) equals the additional time needed, $c_{12}(i, u, j)$ to the difference between the new time at which service begins at customer j after inserting u, $b_{u,j}$, and the original start of service at j, b_j. The weighting factors α_i, μ and λ are used to guide the heuristic to different (local) optima. The selection criterion $c_2(i, u, j)$ is a generalization of the Clarke and Wright (1964) savings algorithm as it tries to maximize the benefit derived from inserting a customer in the current partial route rather than on a new, direct route.

4 Insertion heuristics for the CMVRPTW

As the CMVRPTW is an extended version of the VRPTW, elements of successful VRPTW heuristics can be used to construct heuristics able to maximize a carrier's contribution over its customers' set of alternative price-time window combinations. Routes are initialized with the unrouted customer that is farthest from the depot or with the one with the earliest deadline. The initialization customer is scheduled within its most narrow (and thus most expensive) time window $[e_{u1}, l_{u1}]$ at price $p(q_j, w_{j1})$.

After starting the current route with the initialization criterion, the insertion criterion $co_1(i, u, j)$ determines the highest contribution margin for each unrouted customer u under each of his price-time window combinations and their

6

associated insertion places between two adjacent customers i and j in the current partial route $(i_0, i_1, ..., i_m)$. The insertion distance $c_{11}(i, u, j)$, independent of w, is multiplied by the kilometer cost δ_d. The insertion time $c_{12}(i, u(w), j)$ depends on the choice of w and is valued at the opportunity cost of time δ_t. The new service time at j, given that customer u in his time window w is inserted in the route, is denoted by $b_{u(w),j}$. If we would steer the heuristic only by the kilometer cost δ_d and the opportunity cost of time δ_t, we would obtain only one solution based on the actual cost data. Since it may well be possible to obtain a better solution based on factors different from the original δ_d and δ_t, they are multiplied by two weighting factors $\alpha_1, \alpha_2 \geq 0$. Contrary to Solomon (1987) α_1 and α_2 do not have to sum up to 1. If $\alpha_1 = \alpha_2 = 1$, the original cost levels are used to guide the insertion heuristic.

$$co_1^*(i, u(w), j) = \max_p \left[co_1(i_{p-1}, u(w), i_p) \right], p = 1, ..., m \text{ and } \forall w \in W^u \quad (18)$$

$$co_1(i, u, j) = p(q_u, w_u) - [\alpha_1 \delta_d c_{11}(i, u, j) + \alpha_2 \delta_t c_{12}(i, u(w), j)],$$
$$c_{11}(i, u, j) = d_{iu} + d_{uj} - \mu d_{ij}, \mu \geq 0 \quad (19)$$
$$c_{12}(i, u(w), j) = b_{u(w),j} - b_j$$

Selection criterion $co_2(i, u, j)$ then chooses the customer with the largest difference in contribution between insertion in a new, direct route in its most narrow time window (w_{u1}) at the highest price $p(q_u, w_{u1})$, and insertion in the current route in the chosen time window. Since the contribution of inserting a customer in a new or an existing route can be either positive or negative, the maximum difference between $co_1(i, u, j)$ and $[p(q_u, w_{u1}) - \lambda(\alpha_1 \delta_d d_{ou} + alpha_2 \delta_t t_{ou}) - \delta t s_u - c_f]$ is taken.

$$co_2^*(i, u^*, j) = \max_u \left[co_2(i, u, j) \right] \quad u \text{ unrouted and feasible} \quad (20)$$

$$co_2(i, u, j) = co_1(i, u, j) - [p(q_u, w_{u1}) - \lambda(\alpha_1 \delta_d d_{ou} + \alpha_2 \delta_t t_{ou}) - \delta t s_u - c_f], \lambda \geq 0 \quad (21)$$

If no remaining unrouted customer has a feasible insertion place, a new route is initialized and made the current route.

For a two-phase heuristic the initialization customers of the sequential phase are used to build seed routes for the parallel insertion phase. In the insertion criterion $co_1(i, u(w), j)$ is then used to compute, for each unrouted customer u, a feasible insertion place with the highest associated contribution over all n_r seed routes and all time windows $w \in W^u$.

$$co_1^*(i_r, u(w), j_r) = \max_{p=1,...,m} \left[co_{1_r}(i_{r_{p-1}}, u(w), i_{r_p}) \right],$$
$$r = 1, \ldots, n_r \text{ and } w \in W^u \quad (22)$$

$$co_{1_r}(i_r, u(w), j_r) = p(q_u, w_u) - [\alpha_1 \delta_d co_{11_r}(i, u, j) + \alpha_2 \delta_t co_{12_r}(i, u(w), j)],$$
$$co_{11}(i, u, j) = d_{i_r, u} + d_{u, j_r} - \mu d_{i_r, j}, \mu \geq 0 \quad (23)$$
$$c_{12}(i, u(w), j) = b_{u(w), j_r} - b_{j_r}$$

Customers are then selected by means of the traditional savings criterion

$$co_2\left(i, u\left(w\right), j\right) = \max_{u \in W^u} \left\{ co_1\left(i_r, u, j_r\right) - \left[p\left(q_u, w_{u1}\right) - \lambda\left(\alpha_1 \delta_d d_{ou} + \alpha_2 \delta_t t_{ou}\right) - \delta t s_u - c_f\right] \right\}$$

(24)

or the generalized regret criterion. Following Potvin and Rousseau (1993), a generalized regret selection criterion can be constructed for the CMVRPTW as follows. First, a sequential insertion heuristic analogous to (23) is used to calculate for each unrouted customer u, the best insertion place in each of the n_r seed routes. If a customer can be inserted in a route, his regret measure for that route is equal to the insertion contribution for that route r,

$$co_{1_r}\left(i_r, u\left(w\right), j_r\right) = p\left(q_u, w_u\right) - \left[\alpha_1 \delta_d co_{11_r}\left(i, u, j\right) + \alpha_2 \delta_t co_{12_r}\left(i, u\left(w\right), j\right)\right].$$

(25)

If a customer cannot be inserted in a route, his regret measure for that route is set equal to an arbitrarily large negative amount L. Since Potvin and Rousseau (1993) consider a (cost) minimization problem in which L is a large positive amount, their generalized regret measure for each customer u is defined as the summation over the differences of the regret measures of all seed routes but the best, $(r \neq r')$, and the best r' (i.e. having the lowest generalized insertion costs). In order to remain valid for a (contribution) maximization problem, the generalized regret measure $co_2\left(i, u, j\right)$ must be defined as the summation over the differences between the best regret measure (i.e. having the highest contribution) over all seed routes and the regret measure of all other seed routes but the best, $(r \neq r')$.

$$co_2^*(i, u\left(w\right), j) = \max_u \left[co_2\left(i, u\left(w\right), j\right)\right]$$

(26)

$$co_2\left(i, u\left(w\right), j\right) = \sum_{r \neq r'} \left[co_{1r'}^*\left(i_{r'}, u\left(w\right), j_{r'}\right) - co_{1r}^*\left(i_r, u\left(w\right), j_r\right)\right] \quad \text{where}$$

(27)

$$co_{1r'}\left(i_{r'}, u\left(w\right), j_{r'}\right) = \max_{r-1,\ldots,n_r} \left[co_{1r}^*\left(i_r, u\left(w\right), j_r\right)\right]$$

(28)

Since the selection criterion, $co_2\left(i, u\left(w\right), j\right)$, selects the customer with the largest generalized regret, it will automatically select a customer with the smallest number of feasible insertion routes. Since the contribution from inserting u in feasible routes is also used to calculate the regret measure for each of the routes, it can be used to discriminate between two customers with the same number of feasible routes. Notice that the seed route reduction procedure suggested by Potvin and Rousseau (1993) was not implemented for the generalized regret based two-phase heuristic to allow comparison of both selection criteria.

5   Development of problem sets

The Solomon problem instances for the VRPTW consisting of randomly generated customer coordinates (set R), clustered customers (set C) or both (the so-called semi-clustered sets RC), were adapted for the CMVRPTW with one

8

Table 1: Tariff

k/d	0	1	2	3	4
0	70	80	90	100	110
1	75	85	95	105	115
2	80	90	100	110	120
3	85	95	105	115	125
4	90	100	110	120	130
5	95	105	115	125	135
6	100	110	120	130	140
7	105	115	125	135	145
8	110	120	130	140	150
9	115	125	135	145	155

depot and one to possibly ten alternative price - time window combinations per customer. The data on customer coordinates, their demand and service time was not modified and the earliest and latest time at which service must start, was used to model the customer's original time window.

The carrier's price structure is modelled as a 2-dimensional tariff. Appendix 1 describes its construction and the generation of the CMVRPTW problem sets. The tariff in Table 1 consists of 10 rows, and 5 columns. The 10 rows correspond to 10 standard time window sizes, ranging from 100% to 10% of the scheduling horizon. The columns represent 5 standard demand sizes, ranging from 10 to 50 units. The multidimensional tariff is linear in the time window size k and the weight class d. Servicing a load in the smallest standard demand class, $d = 0$, in the most flexible time window, $k = 0$, costs $20 + 50$ Euros. For a given weight class d, the freight rate increases with $5k$ Euros. If a customer's demand for service falls within the kth standard time window, and the dth standard demand class, the freight rate equals $20 + 5k + 50 + 10d$. Relative time window sizes were preferred to absolute ones, to take the differences in scheduling horizon into account (e.g. 230 units of time in R101 and 1236 units in C101).

Given that the profit or surplus lost by using an n segment multipart tariff instead of an optimal nonlinear tariff is approximately proportional to $1/n^2$ (Wilson, 1993, pp. 190-193), our tariff may seem to be too complicated. The large number of standard time windows and demand sizes is, however, necessary to safeguard information vital to the analysis. Although it is indeed possible to construct an efficient, less complicated tariff for a single problem instance, applying the same tariff to all problem instances under study could be inappropriate in some cases. If, for example, we would only consider four standard time windows covering the entire scheduling horizon, all customers from C105 and the majority of customers from C106, C108 and RC105 would be priced according to the first standard time window.

After determining each customer's current standard time window, the number of time window relaxations is calculated by modelling the customer's inclination to accept a relaxation. Time window relaxations are modelled around the original time window's center and are rounded to the nearest integer. Only time windows that fall within the scheduling horizon can be accepted. Each customer is assumed to be indifferent between its original price-time window combination and the next relaxation with probability 0.5. An even wider time

Table 2: Hour and time coefficients in Euros for 1999 (Blauwens et al., 2001)

Carrying capacity	hour coefficient	kilometer coefficient
delivery van 0.5 t	16.03	0.10
lorry 5 t	17.14	0.15
lorry 8 t	18.06	0.17
lorry 20 t	20.88	0.21
truck and trailer 28 t	21.75	0.24

window is accepted with probability $(0.5)^2$, and an n larger one with probability $(0.5)^n$. If a customer prefers its original time window, he is serviced within the bounds of his original time window. Should he accept one or more relaxations, his original time window is replaced by the corresponding standard time window.

Being the starting and ending point of each route, the depot is "serviced" free of charge in its unique time window $[e_0, l_0]$. Travel times are taken equal to the corresponding Euclidean distances.

To determine the fixed cost of having a vehicle to construct a route c_f^k, the time and kilometer coefficients δ_t and δ_d for the adapted Solomon problem instances, we use real-life cost data to obtain an acceptable cost structure for testing our heuristics.

Since a pick-up or delivery problem is modelled, average sample data on the cost structure of a lorry with a loading capacity of 20 tons is used. Because the lorry's fixed costs are expressed per hour (20.88 EUR), they are multiplied with the maximum statutory driving time (9 hours) to obtain the daily cost of owning the lorry (187.92 EUR). The hour coefficient from Blauwens et al. (2001) is divided by 60 to approximate the time coefficient δ_t, expressing the opportunity cost of time. Indeed, in the long run the average opportunity cost of time equals the average cost of owning a vehicle. In the short run, the opportunity cost will depend on the carrier's potential customers of that moment, making it higher during peak periods than during off-peak periods. The distance coefficient is set equal to the kilometer coefficient δ_d.

6 Computational results

The three insertion heuristics performance are evaluated for short-run contribution and long-run profit maximization. In the literature, VRPTW heuristics are traditionally evaluated on the best solution they obtain over the set of parameters considered. Solomon (1987) uses two initialization criteria: the farthest unrouted customer and the customer with the earliest deadline, and four $(\mu, \lambda, \alpha_1, \alpha_2)$ settings: $(1, 1, 1, 0)$, $(1, 2, 1, 0)$, $(1, 1, 0, 1)$, and $(1, 2, 0, 1)$. The best of these eight runs of each problem instance is used to calculate the average solution quality for each problem set. Solution quality is measured by a lexicographic preference ordering on the number of vehicles, minimum schedule time, minimum distance and sometimes also minimum waiting time.

Since α_2 is either 1 or 0 in Solomon's original parameter set, 50 percent of all runs per instance ignores the time related component of inserting a customer in a route. Therefore $\alpha = (0.5, 0.5)$ is added to the list of possible weighting

Table 3: Long run CMVRPTW

		SI	GS	%Δ	GR	%Δ
R SET	contribution	7006.15	6881.67	-1.78	6831.13	-2.50
	costs	3900.10	4040.00	3.59	4085.54	4.75
	routes	13.92	14.75	5.99	14.75	5.99
	time	2630.40	2676.46	1.75	2731.17	3.83
	distance	1539.57	1403.20	-8.86	1513.63	-1.69
C SET	contribution	5666.78	5646.54	-0.36	5490.60	-3.11
	costs	5750.45	5771.23	0.36	5950.51	3.48
	routes	10.44	10.67	2.13	11.00	5.32
	time	10146.08	10163.35	0.17	10444.73	2.94
	distance	1070.38	957.95	-10.50	1035.93	-3.22
RC SET	contribution	7138.19	6966.10	-2.41	6832.39	-4.28
	costs	3938.68	4130.15	4.86	4265.11	8.29
	routes	13.63	14.75	8.26	15.13	11.01
	time	2767.35	2769.26	0.07	2897.40	4.70
	distance	1730.14	1644.29	-4.96	1727.20	-0.17

factors, giving equal weighting to the distance and time related component of an insertion. Because the objective is to maximize contribution in the short-run or profit in the long-run, the traditional lexicographic preference ordering on cost components cannot not be used. Therefore of each set of 12 runs, the one with the highest contribution is chosen. All calculations are coded in C++ using double floating-point precision.

In Table 3 the sequential insertion heuristic (SI) dominates both two-phase heuristics. The differences in contribution are the highest when the sequential insertion heuristic is compared to the generalized regret-based two-phase heuristic (GR). The differences in percentages reported in Table 3 are the combined result of each heuristic's ability to design efficient routes and to relax the "right" customers' service time window. To isolate both effects, Table 4 reports on each heuristics' average solution quality for problem instances in which customers are all serviced at 100 Euros in their original time windows. The relative difference in performance of both two-phase heuristics with respect to the sequential one, is smaller in Table 3 than in Table 4. This indicates that both two-phase heuristics decide wiser on time window relaxations but that the higher revenue is partially offset by higher routing costs. Despite the fact that they are able to weigh up the advantage of relaxing a customer's service time window in several seed routes, it just fails to compensate the sequential insertion's focus on high capacity utilization. If all freight rates $p(q_j, w)$ in the tariff would be cheaper with respect to the cost of operating a vehicle, the relative importance of minimizing the number of vehicles would increase. Under these circumstances the sequential insertion heuristic would even more clearly outperform both two-phase heuristics.

In the short run, capacity costs are fixed and become irrelevant in the CMVRPTW. Both two-phase heuristics' tendency to have more routes than the sequential insertion heuristic is then no longer penalized. In all cases considered, the generalized savings criterion (GS) dominates the sequential insertion heuristic (SI), which in turn is superior to the generalized regret criterion (GR).

11

Table 4: Long run VRPTW

		SI	GS	%Δ	GR	%Δ
R. SET	contribution	6211.22	6113.61	-1.57	5944.47	-4.29
	costs	3788.78	3886.40	2.58	4055.53	7.04
	routes	13.58	14.08	3.68	14.67	7.98
	time	2548.46	2634.20	3.36	2707.95	6.26
	distance	1455.58	1346.47	-7.50	1487.50	2.19
C SET	contribution	4383.00	4381.97	-0.02	4186.42	-4.48
	costs	5617.00	5618.03	0.02	5813.58	3.50
	routes	10.00	10.00	0.00	10.78	7.78
	time	10063.32	10110.25	0.47	10212.63	1.48
	distance	982.36	918.61	-6.49	975.92	-0.66
RC SET	contribution	6201.75	6030.96	-2.75	5876.62	-5.24
	costs	3798.25	3969.04	4.50	4123.38	8.56
	routes	13.25	14.00	5.66	14.50	9.43
	time	2652.94	2735.48	3.11	2850.25	7.44
	distance	1604.53	1609.22	0.29	1694.38	5.60

Table 5: Short run CMVRPTW

		SI	GS	%Δ	GR	%Δ
R. SET	contribution	9634.95	9665.93	0.32	9629.73	-0.05
	costs	1274.22	1257.40	-1.32	1291.10	1.33
	routes	14.33	15.00	4.65	15.33	6.98
	time	2614.04	2654.08	1.53	2714.70	3.85
	distance	1518.89	1390.77	-8.43	1443.28	-4.98
C SET	contribution	7629.50	7667.13	0.49	7577.49	-0.68
	costs	3787.73	3757.87	-0.79	3863.62	2.00
	routes	10.44	11.00	5.32	11.33	8.51
	time	10146.08	10136.79	-0.09	10364.78	2.16
	distance	1070.38	959.46	-10.36	1069.49	-0.08
RC SET	contribution	9726.19	9743.99	0.18	9685.54	-0.42
	costs	1354.43	1352.89	-0.11	1414.46	4.43
	routes	14.25	15.13	6.14	15.38	7.89
	time	2731.15	2765.30	1.25	2898.67	6.13
	distance	1683.31	1627.34	-3.32	1690.50	0.43

Table 6: Short run VRPTW

		SI	GS	%Δ	GR	%Δ
R SET	contribution	8773.35	8767.54	-0.07	8719.76	-0.61
	costs	1226.65	1232.46	0.47	1280.24	4.37
	routes	13.75	14.58	6.06	15.08	9.70
	time	2533.73	2610.98	3.05	2673.91	5.53
	distance	1437.13	1349.34	-6.11	1457.15	1.39
C SET	contribution	6273.47	6293.24	0.32	6244.82	-0.46
	costs	3726.53	3706.76	-0.53	3755.18	0.77
	routes	10.11	10.56	4.40	11.11	9.89
	time	10047.50	10045.14	-0.02	10132.65	0.85
	distance	958.32	879.38	-8.24	954.22	-0.43
RC SET	contribution	8695.53	8689.89	-0.06	8626.39	-0.80
	costs	1304.47	1310.11	0.43	1373.61	5.30
	routes	13.50	14.38	6.48	14.75	9.26
	time	2644.28	2693.60	1.87	2813.62	6.40
	distance	1601.10	1553.05	-3.00	1643.62	2.66

Table 7: Non-parametric Friedman test ($n = 1044$)

	test statistic	p-value
short run CMVRPTW	3.5253	0.1716
long run CMVRPTW	24.4831	0.0001

Following Golden and Stewart (1985) we use the nonparametric Friedman test to check whether all three heuristics have the same expected value over the entire test set ($n = 1044$). For the short run CMVRPTW, the performance of the sequential insertion heuristic and the 2 two-phase heuristics does not statistically differ (test Friedman's test statistic = 3.5253, p-value = 0.1716).

For the long run CMVRPTW, Friedman's test statistic (5.9051, p-value = 0.0001) strongly rejects the null hypothesis that all heuristics perform equally well. The Wilcoxon signed rank statistic for comparing heuristics SI and GS equals 116750 and the one-sided p-value equals 0.0439. At $\alpha = 0.05$ we accept the null hypothesis of equal heuristic performance. Because the p-value for the Wilcoxon test on GR and SI is extremely small (p-value = 0.0001), we can reject the null hypothesis with much conviction. Because of the equal performance of SI and GS for the short run CMVRPTW, it should come as no surprise that the null hypothesis of equal performance of GR and SI is also rejected (p-value = 0.0006).

In conclusion, SI and GS perform equally well and outperform GR at a confidence level $\alpha = 0.05$ for the long run CMVRPTW. This means that the additional computational effort of using a two-phase heuristic does not pay off on average over the entire test set ($n = 1044$). Also if the best run for each problem instance is used, no higher contribution can be attained by using a two-phase heuristic.

For the long run, the use of the generalized savings selection criterion in the sequential insertion heuristic (SI) and the generalized savings two-phase heuristic (GS) dominates the generalized regret criterion. Regardless of whether

13

Table 8: Wilcoxon signed ranks test (using the normal approximation)

	test statistic	p-value
GS vs. SI	116750	0.0439
GR vs. SI	108480	0.0001
GR vs. GS	112678	0.0006

customers can be serviced in a single time window (see Tables 4 and 6) or in multiple time windows (see Tables 3 and 5), the generalized regret criterion always underperforms. This contrasts with the interesting results that Potvin and Rousseau (1993) obtained for the generalized regret criterion combined with a seed route reduction procedure. Without seed route reduction[3], the generalized regret selection criterion seems to be inferior to the generalized savings criterion, both for the CMVRPTW as for the VRPTW.

7 Conclusions and suggestions for future research

Both less-than-truckload and full truckload customers, that are flexible with respect to the moment of receiving service, often pay lower prices. Some customer can be indifferent between receiving service in several price time window combinations at different prices. The carrier's associated routing and scheduling problem can be formulated as a Contribution Maximizing Vehicle Routing Problem with Time Windows (CMVRPTW). Because the CMVRPTW is NP-hard, we developed three insertion-based heuristics.

Solomon's (1987) problem instances for the VRPTW were used to construct a test set for the CMVRPTW. The CMVRPTW data set can be obtained from the author on request.

Solution quality was formally evaluated by the non-parametric Friedman and Wilcoxon tests. For the short run CMVRPTW, all three heuristics performed equally well. For the long run CMVRPTW, statistical testing showed the sequential insertion heuristic and the generalized savings two-phase heuristic to be superior to the generalized regret two-phase heuristic. The first two-phase heuristic used the traditional generalized savings selection criterion, the second was based on Potvin and Rousseau's (1993) generalized regret selection criterion. Apart from the selection criterion used, the two-phase heuristics were identical, meaning that the seed route reduction procedure suggested Potvin and Rousseau (1993) was not implemented for the generalized regret based two-phase heuristic. Despite the fact that Potvin and Rousseau (1993) obtained interesting results for the VRPTW, the generalized regret selection criterion was always dominated by the generalized savings criterion in our calculations. This indicates that the generalized regret criterion should not be used without invoking a seed route reduction procedure. For the short run CMVRPTW, best results were obtained for the generalized savings-based two-phase heuristic.

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[3] Route reduction procedures have also been proposed in local search based on Ejection Chains (EC) (Glover, 1991; Glover, 1992). For the VRPTW, EC remove (eject) and insert customers from/in routes until a customer can be inserted in a neighboring route without the need to remove any customer. EC have been applied to the VRPTW in Caseau et al. (1999), Rousseau et al. (2000), and Bräysy (2001).

The CMVRPTW and the heuristics suggested in this paper can be used to study a number of combined pricing and scheduling issues. At the operational level of trucking operations it can help dispatchers to design more profitable routes. The model identifies which customers should be contacted for time window relaxations. At the same time, the model is able to determine the maximum discount for a time window relaxation. This maximum discount is equal to the contribution that is gained by relaxing a customer's time window. At the strategic level of trucking operations the CMVRPTW can be used to investigate the list of customers. A set of problem instances can be used to simulate trucking activity over a period of time. The model can then be used to identify those customers who regularly generate low contributions because of high routing costs. At the same time, the contribution that is lost when a customer only accepts service in its original time window can be determined. By recording the contribution that could be realized by servicing a customer in wider time windows, a carrier can estimate the cost of a customer's lack of scheduling flexibility.

In this paper, the carrier's price structure is considered to be exogenous. In future research, the price structure will be endogenized. Research on designing a multidimensional freight tariff on both the weight of the shipment and the scheduling flexibility is currently underway.

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Algorithm 1 Generation of CMVRPTW data

Calculate standard price - time window combinations
$schedulingHorizon = l_0 - e_0$
$maxDemand = \max_{q_i} \{$all customers $i \in$ sets R, C and RC$\}$
for standardTW $k = 1$ to K **do**
  for standardDemands $d = 1$ to D **do**
    $standardTW[k] = standardTW[k-1] + schedulingHorizon/(K+1)$
    $standardD[d] = standardD[d-1] + maxDemand/(D+1)$
    $price[k][d] = pTWMax - k \cdot (pTWMax - pTWMin)/(K+1) + pDMax$
    $- d \cdot (pDMax - pDMin)/(D+1)$
  end for
end for

for all customers i **do**
  Determine current standard time window
  $TWSize = l_i - e_i$
  $TWCentre = e_i + (l_i - e_i)/2$
  $currentTW = currentD = 0$
  for standardTW $k = 0$ to K **do**
    for standardDemands $d = 0$ to D **do**
      if $TWSize \leq standardTW[k]$ **and** $q_i \leq standardD[d]$ **then**
        $currentTW = k$
        $currentD = d$
        break
      end if
    end for
  end for

  Determine the number of time window relaxations
  for $k = currentTW + 1$ to K **do**
    $criticalValue = 0.5^{(k-currentTW)}$
    if $TWCentre - 0.5 \cdot standardTW[k] \geq e_0$
    and $TWCentre + 0.5 \cdot standardTW[k] \leq l_0$
    and $randomNumber \leq criticalValue$ **then**
      $numberOfTWRelaxations = k - currentTW$
    else
      break
    end if
  end for

  Generate price - time window combinations
  for $k = 0$ to numberOfTWRelaxations **do**
    if numberOfTWRelaxations $!= 0$ **then**
      $price = price[currentTW+k][currentDemand]$
      $e_i = TWCentre - 0.5 \cdot standardTW[currentTW+k]$
      $l_i = TWCentre + 0.5 \cdot standardTW[currentTW+k]$
    else
      $price = price[currentTW][currentDemand]$
      $e_i = e_i$
      $l_i = l_i$
    end if
  end for
end for