

DEPARTMENT OF ENGINEERING MANAGEMENT

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Ambulance Routing for Disaster Response with Patient Groups

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Abstract

We consider a routing problem for ambulances in a disaster response scenario, in which a large number of injured people require medical aid at the same time. The ambulances are used to carry medical personnel and patients. We distinguish two groups of patients: slightly injured people who can be assisted directly in the field, and seriously injured people who have to be brought to hospitals. Since ambulances represent a scarce resource in disaster situations, their efficient usage is of the utmost importance. Two mathematical formulations are proposed to obtain route plans that minimize the latest service completion time among the people waiting for help. Since disaster response calls for high-quality solutions within seconds, we also propose a Large Neighborhood Search metaheuristic. This solution approach can be applied at high frequency to cope with the dynamics and uncertainties in a disaster situation. Our experiments show that the metaheuristic produces near optimal solutions for a large number of test instances within very short response time. Hence, it fulfills the criteria for applicability in a disaster situation. Within the experiments, we also analysed the effect of various structural parameters of a problem, like the number of ambulances, hospitals, and the type of patients, on both running time of the heuristic and quality of the solutions.

Keywords: Ambulance routing, Disaster response, Service time, Local search, Large neighborhood search, Metaheuristic

1. Introduction

Recent examples such as Hurricane Katrina in 2005, the Indian Ocean tsunami in 2004, or any of the recent armed conflicts around the globe demonstrate that disasters can have a devastating impact on a society. Regardless of whether their cause is natural (e.g., earthquakes, floods, hurricanes, wildfires) or man-made (e.g., terrorist attacks, war situations), disasters can cause large-scale loss of life as well as damage to a society's infrastructure, housing, and industrial complex. It has been widely recognized (see e.g., [Berkoune et al. \(2012\)](#) and [Holguín-Veras et al. \(2012\)](#)) that the severity of a disaster can be, to a large extent, influenced by the efficacy of the logistics operations during the response phase. Although the disaster *itself* can certainly cause a lot of casualties, a large

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fraction of the victims usually perish because of a lack of medical aid in the immediate aftermath of a disaster. Clearly, the post-disaster situation results in the response actions having to be executed under extremely challenging conditions: limited availability of resources (transportation, supplies, manpower, hospital capacity), damaged transportation and communication infrastructure, as well as uncertain information regarding the number and locations of people in need of medical assistance, see e.g., [Najafi et al. \(2013a,b\)](#) and [Yi et al. \(2010\)](#). Despite these challenges, it is essential that the logistics relief operations are initiated quickly and well planned to be most effective. Hence, there is a strong need for decision support tools that generate solutions to the underlying optimization problems in a few seconds or less ([Berkoune et al., 2012](#)). However, research on transportation problems and vehicle fleet management for disaster response operations is emerging only recently, see [de la Torre et al. \(2012\)](#) and [Pedraza-Martinez and van Wassenhove \(2012\)](#). With this paper, we propose a decision support approach for the routing of ambulances in response to a disaster.

The central task of managing ambulances in a disaster response situation is to provide first aid to slightly injured people and to bring seriously injured people to operating hospitals. Managing the operations of ambulances in the immediate aftermath of a disaster is massively complicated by the dynamics and uncertainty with which the planning conditions (especially the relevant information) change over the course of time. The information required to support the planning of ambulances includes the number and location of people calling for help, the availability of ambulances, the capacity of the nearby hospitals, as well as the accessibility of incident sites due to the damaged infrastructure and the current traffic situation, see [Jotshi et al. \(2009\)](#). Another issue is that, in contrast to the daily operations in the public health care sector, the number of requests for help in a disaster situation strongly exceed the capacity of the available ambulance fleet. Hence, it is of utmost importance to use the ambulances efficiently in such a way that they provide as much medical aid as possible.

The response process that is executed by the responsible organizations in the aftermath of a disaster has to be designed in such a way that it is able to cope with the challenges of a dynamic planning situation for the scarce ambulance resource. The routing of ambulances in such a situation can be treated as a static or a dynamic routing problem. In the static case, a set of emergency requests is collected first and, then, the routing problem is solved for this set of requests. In the dynamic case, the routes of ambulances are updated whenever new help requests arrive, which can reduce the response time. However, this approach requires that communication with the ambulances is possible at all time, which might not be the case in a disaster situation and, furthermore, the rescue teams may perceive this to be disturbing under stressful circumstances. Therefore, in this paper, we consider a three-step response process that aims at solving a static ambulance routing problem, see [Fig. 1](#). The process is executed by a central dispatching unit, which collects requests and manages ambulance operations repeatedly until no further emergency requests are received. The first step is to answer incoming emergency calls and to collect relevant information like the location and the condition of the people being in need of help. The dispatcher collects several requests that are then classified according to their severity in a second process step. The classification reflects the urgency with which a patient should receive help, which is taken into account when routing the ambulances in the third process step. Collecting and classifying a number of requests before actually

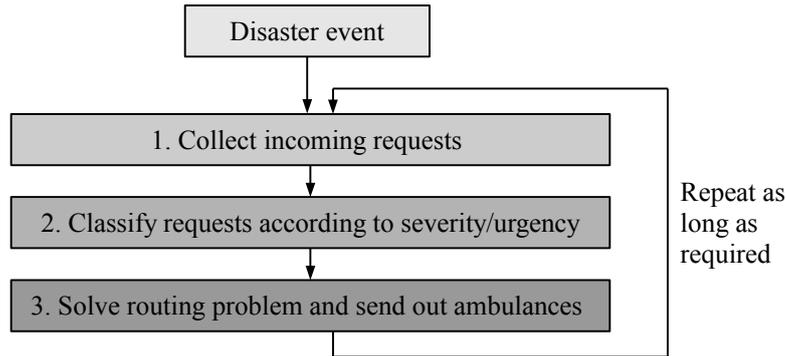


Figure 1: Disaster response process.

sending out the ambulances supports an efficient use of the vehicles, because instead of dispatching ambulances on a first-come first-served basis they can be used to serve the most urgent requests first. Therefore, the first two process steps do not represent a waste of precious time but they collect valuable information to come up with high-quality route plans in the third process step. In fact, the time spent for the first two steps is rather short if numerous requests arrive within short time (as is the case in a disaster event) and if the classification of requests is performed directly while answering an emergency call or automatically from the collected data. Hence, the three-step process can be repeated at high frequency (for example each time a certain number of requests has been collected or a certain time limit has elapsed) such that it causes little delay in the service process. At the same time, up-to-date information regarding the availability of ambulances, infrastructure conditions, etc. can be included in the planning in this way.

The scope of this paper is to investigate the routing problem that occurs in the third step of the response process. The ambulances are used to bring medical personnel to the casualties and to carry injured people to the hospitals. Each ambulance carries medical personnel that can provide first aid to slightly injured people in the field. Seriously injured individuals are accompanied by the medical staff on their way to the hospital where skilled doctors are available. According to this, we distinguish two types of patients:

- *Red code patient*: A person with red code classification is seriously injured and needs to be brought to a hospital by an ambulance.
- *Green code patient*: A person with green code classification is slightly injured and can be helped directly in the field.

There exist more detailed classification schemes for patients (see e.g., [Andersson and Värbrand, 2007](#); [Gennarelli and Wodzin, 2008](#)) but the two types considered here are sufficient to distinguish the fundamental tasks that have to be performed by the ambulances. Moreover this rough classification can be applied relatively easily by the central dispatcher to the incoming emergency requests.

Concerning the routing of ambulances, we assume that each of them can carry one red code patient at a time and that each patient is directly brought to a hospital after having been picked up. The decision to which hospital to bring a patient is part of the routing problem and depends on the capacities of hospitals. Since green code patients can be helped on the field, an ambulance can go directly to the next patient after having served

a green code patient. From this, an ambulance can provide help to multiple people on its route before returning to a hospital. In contrast, if an ambulance has to serve multiple red code patients, it has to visit several hospitals throughout the planning horizon. Therefore, for the purpose of clarity, we refer here to a route as a tour that begins at one hospital, visits one or more patients in a specified sequence, and ends at either the starting hospital or at some other hospital. Hence, an ambulance may perform multiple routes within a solution to the ambulance routing problem.

The optimization problem is then to determine ambulance routes to serve the two groups of patients, red code patients and green code patients, which have been determined in the first two steps of the sketched response process. The objective is to minimize the sum of the latest service completion time among the red code patients and the latest service completion time among the green code patients. The objective strives to reduce the longest waiting time faced by a patient in a group. Although some authors propose to minimize the average waiting time of patients (e.g., [Créput et al. 2011](#)), the objective pursued in our paper ensures that no patient suffers from an excessively long waiting time. This maximizes the probability of survival for the patient that has to wait longest to be served. Furthermore, weights are introduced for the latest completion times of the two patient groups to reflect the higher urgency of red code patients in the planning. In order to assess a solution with regard to these performance measures, not only the routes but also the service start and completion times, i.e., the scheduling of the ambulance operations, have to be determined. With this paper, we provide the first models and algorithms to solve this problem. Since in a disaster situation high-quality ambulance routes have to be determined in short response times, we particularly strive to develop powerful heuristic solution methods.

The paper is organized as follows. In [Section 2](#) we review the relevant literature and relate it to our study. The routing problem is described and modeled in [Section 3](#) together with an illustrating example. In [Section 4](#), we present a Large Neighborhood Search metaheuristic to solve the emergency routing problem. The models and the metaheuristic are computationally tested in [Section 5](#). [Section 6](#) concludes the paper.

2. Literature Review

There exist several streams of research that address locating, dispatching, and routing ambulances and supplies in public health care and in disaster response situations.

Locating ambulances entails finding deployment sites for the vehicles within an (urban) area such that a certain response time is guaranteed to reach the potential emergency sites within this area. Surveys of models and algorithms developed in this field of research are provided by [Brotcorne et al. \(2003\)](#) and [Farahani et al. \(2012\)](#). The approaches typically belong to the class of covering location models, see for example the early work of [Fitzsimmons and Srikar \(1982\)](#). Relocation of ambulances comes into the play when the coverage becomes inadequate due to ambulances that are currently dispatched to incidents. In this case, idle vehicles may have to be relocated to fill gaps in the coverage, which leads to a dynamic ambulance location problem, see [Gendreau et al. \(2001\)](#). Recent research aims at capturing realistic planning situations like traffic-dependent traveling times and congestion phenomena. For example, [Schmid and Doerner \(2010\)](#) consider travel times that vary in the course of a day within an ambulance location problem. From such variations, the coverage achieved throughout a day by a certain deployment of ambulances changes dynamically

which calls for relocations. The authors propose a model and a Variable Neighborhood Search metaheuristic to simultaneously optimize the coverage for various points in time with varying traffic volumes. [Knight et al. \(2012\)](#) present a model to locate ambulances in such a way that the expected survival probability of heterogeneous patients is maximized. The patients differ in the targeted response time and in their medical conditions. An approximation method is proposed to solve this type of ambulance location problem.

Dispatching is the task of assigning incoming emergency requests to ambulances. It is sometimes solved in combination with the ambulance location problem. For example, [Toro-Díaz et al. \(2013\)](#) present an integrated location and dispatching model that captures the impact of queuing patients in congested server systems on the achieved response time and coverage. A Genetic Algorithm is proposed to assign locations and requests to the vehicles. [Andersson and Värbrand \(2007\)](#) dispatch ambulances according to the urgency of requests and the closeness of a vehicle to the site of an incident. The authors combine the dispatching with a relocation of ambulances in order to maintain the coverage of the service area when some of the ambulances are busy serving patients. Also [Schmid \(2012\)](#) combines dispatching and relocation where approximate dynamic programming is used to minimize the expected total response time of requests that occur within the planning horizon. The routing of ambulances is out of scope of these papers, because dispatching is concerned with assigning a single emergency request to a suitable ambulance.

Ambulance routing is considered in some studies as the problem of finding a shortest (fastest) path from one location to another taking into account traffic conditions and the infrastructure damage caused by a disaster, see e.g., [Jotshi et al. \(2009\)](#) and [Goldberg and Listowsky \(1994\)](#). In our paper, ambulance routing is considered as the problem of finding vehicle routes for a set of ambulances to serve a given set of patients. Such problems are often seen as dynamic or real-time vehicle routing problems since emergencies occur in the course of time at unforeseeable locations, see e.g., the surveys of [Ghiani et al. \(2003\)](#) and [Pillac et al. \(2013\)](#). If there is a strong degree of dynamism and stochasticity, the routing problem may be solved by a reactive dispatching policy, cf. [Bertsimas and van Ryzin \(1991\)](#). However, if several requests occur in short time, as is assumed in our study, the problem is to find routes each comprising several patients such that all requests are served. Such a problem is solved in [Créput et al. \(2011\)](#) by means of a multi-agent approach and local search heuristics with the objective of minimizing the average waiting time of patients. [Wex et al. \(2013\)](#) investigate a multiple traveling salesman problem that finds its application in the routing of rescue units that have to serve a given set of incidents. The authors propose several (meta-)heuristics to find routes that minimize the total weighted completion time of the incidents. It is assumed that all patients receive aid in the field such that transportation to hospitals is not part of the problem. Transportation of non-urgent patients among hospitals, from homes to hospitals, or vice versa can be considered as a dial-a-ride problem, which is to relocate patients from their individual origin location to their destination, see e.g., [Parragh \(2011\)](#) and [Parragh et al. \(2012\)](#). A dynamic stochastic version of this problem arises if patients brought to a hospital are discharged the same day with a certain probability such that their return has to be added to the vehicle routes, see [Schilde et al. \(2011\)](#). Since the transport requests are not urgent in these problems, the typical objective is to minimize the travel effort of vehicles or the tardiness of violated time windows. Also, the destination of each patient is prescribed here whereas in disaster response it needs to be decided to which hospital to bring a patient.

Another stream of research is on disaster relief routing for which [de la Torre et al. \(2012\)](#) provide a recent literature survey. Here, the scope is on the distribution of humanitarian aid supplies like water, food, medicine, and survival equipment from distribution centers to demand points like refugee camps with respect to the available transport capacities, see e.g., [Berkoune et al. \(2012\)](#). In this field, various routing problems have been investigated. For example, [Campbell et al. \(2008\)](#) present models and heuristics for traveling salesman and vehicle routing problems that aim at minimizing the latest arrival time or the average arrival time at demand locations as is of relevance in time-critical disaster response actions. [Huang et al. \(2012\)](#) investigate a vehicle routing problem to distribute supplies from a depot with the goal of a fair allocation of scarce supplies if not all demands can be met. [Rath and Gutjahr \(2014\)](#) combine distribution planning with locating supply depots such that a cost measure is minimized and a maximal coverage is achieved. A hierarchical traveling salesman problem where demand locations require supplies with different urgency levels is investigated in [Panchangam et al. \(2013\)](#). A multi-commodity flow problem to serve demand locations in multiple truck trips at minimum time is presented in [Berkoune et al. \(2012\)](#). However, the transportation of patients is out of scope of all these papers.

A few papers propose multi-commodity flow models to combine the distribution of supplies with the transportation of patients. [Yi and Özdamar \(2007\)](#) present such a model with the objective of minimizing the weighted sum of unsatisfied demands and waiting times of injured people. The patients have to be brought to hospitals and to emergency centers that are set up temporarily to cope with the disaster. [Özdamar and Demir \(2012\)](#) provide a similar model that minimizes the total vehicle travel time in order to ensure an efficient utilization of transport capacity and a fast delivery of supplies. [Najafi et al. \(2013a,b\)](#) provide extensions of the model of [Yi and Özdamar \(2007\)](#) to cope with different vehicle types and to support re-planning and robust solutions in dynamic and stochastic planning situations. The approaches investigated in these papers are all based on multi-commodity network flow problems where a detailed routing of vehicles is typically out of scope. Furthermore, although different categories of patients are distinguished, all patients have to be brought to a medical station to be treated. Considering different types of services (first aid in the field for slightly injured people, transportation to hospitals for heavily injured people) is not supported by these models.

From this literature review, we observe that disaster response management is a very active field of research. However, research mainly concentrates on locating and dispatching of ambulances and on the distribution of supplies. Ambulance routing has received only some attention in the literature. In particular, the routing problem investigated in this study, where some patients can be served on the field whereas others need to be brought to hospitals, has not been treated so far.

3. The Ambulance Routing Problem

3.1. Problem Description

The aim of the ambulance routing problem is to find routes for a fleet of ambulances in order to give aid to a set of patients. We formalize this problem using the notation shown in Table 1. Let \mathcal{R} denote the set of red code patients that have to be picked up by ambulances to be brought to the hospitals in set \mathcal{H} . Let \mathcal{G} denote the set of green code patients who can receive aid directly in the field. The set of all patients is denoted

Table 1: Notation used to model the ambulance routing problem.

Sets:	
\mathcal{R}	set of red code patients
\mathcal{G}	set of green code patients
\mathcal{P}	set of all patients, $\mathcal{P} = \mathcal{R} \cup \mathcal{G}$
\mathcal{H}	set of hospitals
\mathcal{K}_h	set of ambulances that are initially located at hospital $h \in \mathcal{H}$
\mathcal{K}	set of all ambulances, $\mathcal{K} = \cup_{h \in \mathcal{H}} \mathcal{K}_h$
\mathcal{A}	set of arcs in a problem, $\mathcal{A} = \{\mathcal{P} \times \mathcal{P}\} \cup \{\mathcal{H} \times \mathcal{P}\} \cup \{\mathcal{P} \times \mathcal{H}\}$
Parameters:	
f_h^k	binary parameter, 1 iff ambulance k is initially located at hospital h (i.e. $k \in \mathcal{K}_h$)
t_{ij}	travel time from i to j with $(i, j) \in \mathcal{A}$
d_i	service time of patient $i \in \mathcal{P}$
d_h	transfer time to drop off a red code patient at hospital $h \in \mathcal{H}$
c_h	capacity of hospital $h \in \mathcal{H}$
w_R	priority given to red code patients
w_G	priority given to green code patients
Decision variables:	
x_{ij}^k	binary, 1 iff ambulance k serves patient i directly before patient j (3-Index Model)
x_{ij}	binary, 1 iff any ambulance serves patient i directly before patient j (2-Index Model)
u_{ih}	binary, 1 iff red code patient i is brought to hospital h
b_i	visiting time of patient $i \in \mathcal{P}$, $b_i \geq 0$
e_R	latest service completion time among all red code patients
e_G	latest service completion time among all green code patients

by $\mathcal{P} = \mathcal{R} \cup \mathcal{G}$. The fleet of ambulances available to give aid to patients is denoted by \mathcal{K} . Each ambulance is initially located at a hospital. We denote by $\mathcal{K}_h \subseteq \mathcal{K}$ the subset of ambulances that are initially located at hospital $h \in \mathcal{H}$. A corresponding binary parameter f_h^k indicates whether ambulance k is initially located at hospital h ($f_h^k = 1$) or not ($f_h^k = 0$). Furthermore, we denote by $\mathcal{A} = \{\mathcal{P} \times \mathcal{P}\} \cup \{\mathcal{H} \times \mathcal{P}\} \cup \{\mathcal{P} \times \mathcal{H}\}$ the set of arcs that are of relevance for the routing problem where t_{ij} is the travel time needed by an ambulance to traverse arc $(i, j) \in \mathcal{A}$. A service time d_i is associated to each patient $i \in \mathcal{P}$. For red code patients, d_i denotes the time required to prepare the patient for transportation to a hospital. For green code patients, d_i denotes the time needed to give first aid to the patient in the field. For the ease of notation, we also define a transfer time d_h for each hospital $h \in \mathcal{H}$, which represents the time required to drop off a red code patient at this hospital. Finally, c_h denotes the capacity of hospital $h \in \mathcal{H}$ in terms of the maximum number of red code patients that can be brought to this location. We assume that the total capacity of all hospitals is sufficiently large to host all red code patients, i.e., $\sum_{h \in \mathcal{H}} c_h \geq |\mathcal{R}|$. We also assume that each ambulance can carry at most one red code patient at a time and that an ambulance has to go directly to a hospital with residual capacity after taking up a red code patient. In contrast, having served a green code patient in the field, an ambulance can go directly to the next patient (green code or red code) on its route. We assume that each ambulance finishes its last route at any hospital.

In order to evaluate the quality of a solution, we define the *service completion time* of a red code patient as the point in time when the patient is dropped off at the assigned hospital. The service completion time of a green code patient is given by the completion of the first aid. The objective of the ambulance routing problem is to minimize a weighted linear combination of the latest service completion time e_R among all red code patients

and the latest service completion time e_G among all green code patients. The latest service completion time among the patients of a group is considered here because it minimizes the worst case waiting time. Furthermore, e_R and e_G are weighted by parameters w_R and w_G , respectively, to express the relative importance that a decision maker probably assigns to the patient groups. In particular, if red code patients shall be served with utmost priority, a setting $w_R \gg w_G$ will ensure that these patients are served early in the routing. We next provide an illustrative example. Afterwards, two mathematical models of the problem are presented respectively in Sections 3.3 and 3.4.

3.2. Illustrative Example

We illustrate the problem on a small artificial example. This instance contains three red code patients $\mathcal{R} = \{r_1, r_2, r_3\}$ and seven green code patients $\mathcal{G} = \{g_1, g_2, \dots, g_7\}$. Two hospitals $\mathcal{H} = \{h_1, h_2\}$ are available to take up the red code patients, with respective capacities $c_{h_1} = c_{h_2} = 3$. Two ambulances a_1 and a_2 are initially located at hospital h_1 , i.e., $\mathcal{K}_{h_1} = \{a_1, a_2\}$. A third ambulance a_3 is initially located at h_2 , i.e., $\mathcal{K}_{h_2} = \{a_3\}$. Figure 2 illustrates the locations of all patients and hospitals. We assume here that the travel times t_{ij} of arcs $(i, j) \in \mathcal{A}$ correspond to the Euclidean distance between locations i and j . The service times d_i are set to 30 time units for red code patients $i \in \mathcal{R}$ and to 10 time units for green code patients $i \in \mathcal{G}$. We assume that dropping off a patient at a hospital $h \in \mathcal{H}$ can be done in no time, i.e., $d_h = 0$.

Figure 2 shows a potential route plan for the three ambulances. In this solution, each ambulance performs two routes. Ambulance a_1 starts its first route at hospital h_1 , picks up red code patient r_1 and brings this patient to hospital h_1 . On its second route, ambulance a_1 serves three green code patients g_1, g_2 , and g_4 . Ambulance a_2 first picks up red code patient r_2 and brings it to hospital h_2 . Afterwards, it serves patient g_3 before returning to hospital h_1 . Ambulance a_3 starts at hospital h_2 and combines the service of two green code

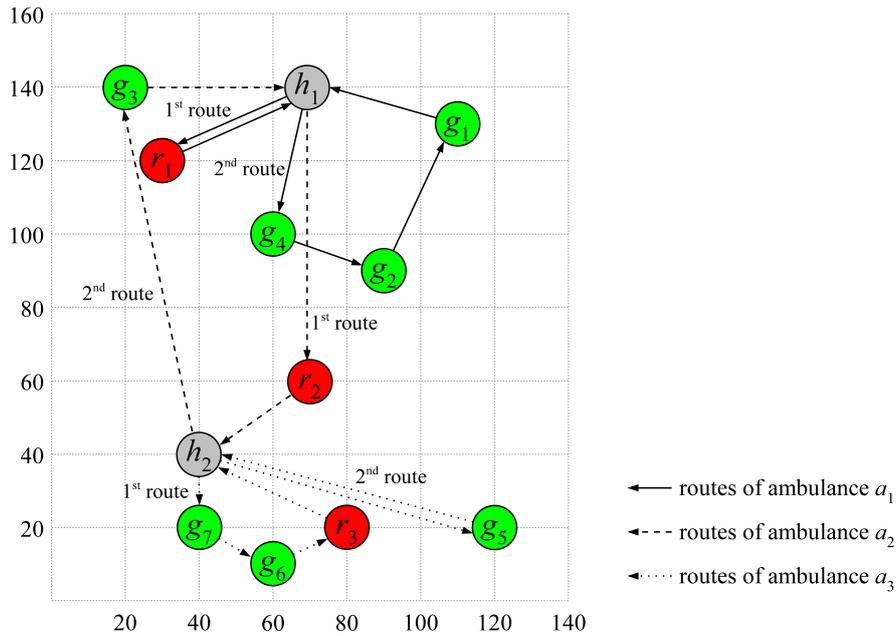


Figure 2: Example solution.

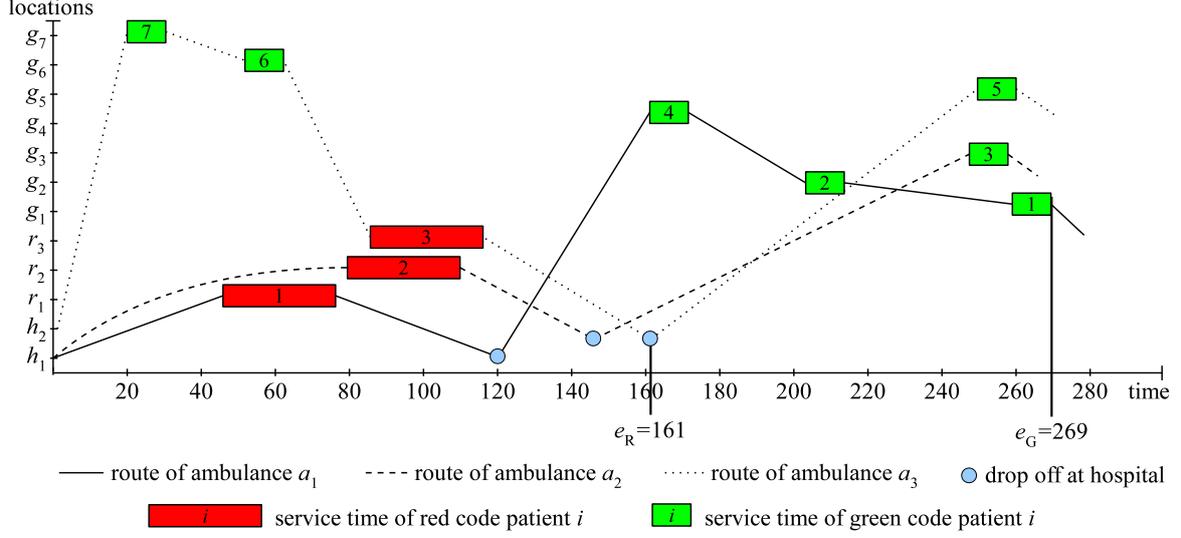


Figure 3: Time-space representation of the example solution.

patients with the service of red code patient r_3 . After having brought r_3 to hospital h_2 , patient g_5 is served in a second route. In order to determine the quality of this solution, the time-space diagram in Fig. 3 shows the positions of all ambulances over the course of time. It can be seen that the latest drop off of a red code patient at a hospital takes place at time $e_R = 161$. The latest completion time of serving a green code patient is $e_G = 269$. Note that a route can start and end at different hospitals, which enables solutions of high quality where patients are served as quickly as possible. In our example, this is the case for the routes of ambulance a_2 .

3.3. Mathematical Formulation: A 3-Index Model

Using the notation introduced in this section, we propose a mathematical formulation of the routing problem. The model uses 3-indexed binary decision variables x_{ij}^k , which take value 1 if ambulance k serves patient i directly before patient j and 0 otherwise. Binary variables u_{ih} take value 1 if red code patient $i \in \mathcal{R}$ is brought to hospital h and 0 otherwise. The visiting time of patient i , i.e., the arrival time of the ambulance that gives aid to this patient is represented by a continuous variable $b_i \geq 0$. The ambulance routing problem is modeled by equations (1)–(13).

$$\min w_R \cdot e_R + w_G \cdot e_G \quad (1)$$

s.t.

$$\sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{hj}^k = f_h^k \quad \forall h \in \mathcal{H}; k \in \mathcal{K} \quad (2)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{ji}^k = 1 \quad \forall i \in \mathcal{P} \quad (3)$$

$$\sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{ji}^k = \sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{ij}^k \quad \forall i \in \mathcal{P}; k \in \mathcal{K} \quad (4)$$

$$\sum_{h \in \mathcal{H}} u_{ih} = 1 \quad \forall i \in \mathcal{R} \quad (5)$$

$$\sum_{i \in \mathcal{R}} u_{ih} \leq c_h \quad \forall h \in \mathcal{H} \quad (6)$$

$$b_i + d_i + t_{ij} \leq b_j + \left(1 - \sum_{k \in \mathcal{K}} x_{ij}^k\right) \cdot M \quad \forall i \in \mathcal{G} \cup \mathcal{H}; j \in \mathcal{P} \quad (7)$$

$$b_i + d_i + t_{ih} + d_h + t_{hj} \leq b_j + \left(2 - \sum_{k \in \mathcal{K}} x_{ij}^k - u_{ih}\right) \cdot M \quad \forall i \in \mathcal{R}; j \in \mathcal{P}; h \in \mathcal{H} \quad (8)$$

$$e_G \geq b_i + d_i \quad \forall i \in \mathcal{G} \quad (9)$$

$$e_R \geq b_i + d_i + t_{ih} + d_h - (1 - u_{ih}) \cdot M \quad \forall i \in \mathcal{R}; h \in \mathcal{H} \quad (10)$$

$$b_i \geq 0 \quad \forall i \in \mathcal{P} \cup \mathcal{H} \quad (11)$$

$$u_{ih} \in \{0, 1\} \quad \forall i \in \mathcal{R}; h \in \mathcal{H} \quad (12)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}; k \in \mathcal{K} \quad (13)$$

The objective function (1) aims to minimize the weighted sum of the latest service completion time among all red code patients and the latest service completion time among all green code patients. Constraints (2) ensure that each ambulance originates from the hospital where it is initially located. According to Constraints (3), each patient is visited exactly once by one of the ambulances. Constraints (4) enforce that an ambulance visiting a patient also has to leave that patient's location. Consequently, ambulances finish their routes in one of the hospitals. Constraints (5) and (6) enforce that each red code patient is assigned to exactly one hospital and that the capacity of each hospital is respected. Constraints (7) and (8) propagate the arrival times of ambulances at the patients locations. According to (7), the arrival time b_j of an ambulance at a patient j is determined by the arrival time b_i at the location i (a green code patient or a hospital) visited immediately prior to patient j , the service time d_i at location i , and the travel time t_{ij} to go from i to j . If, however, i represents a red code patient, the detour to bring i to its assigned hospital needs to be included into the calculation of the arrival time at j . This is ensured by (8). Here, if an ambulance serves a red code patient i immediately prior to patient j (i.e., $\sum_{k \in \mathcal{K}} x_{ij}^k = 1$) and if i is assigned to hospital h (i.e., $u_{ih} = 1$) then (8) ensures that the arrival time b_j at patient j is at least as large as $b_i + d_i + t_{ih} + d_h + t_{hj}$, which also includes the time to go to hospital h and drop off patient i before proceeding to patient j . This approach allows to include multiple intermediate returns of an ambulance to a hospital into a solution. Constraints (9) determine the latest service completion time e_G among all first aid services provided to green code patients. Constraints (10) determine the latest service completion time e_R of all red code patients. Note that the service of a red code patient is completed at the time when the patient is dropped off at the assigned hospital. Constraints (11)-(13) define the domains of the decision variables.

Considering the special case of the routing problem with a single ambulance, a single hospital, no red code patients $\mathcal{R} = \emptyset$, and neglected service times $d_i = 0$ of green code patients $i \in \mathcal{G}$, the problem reduces to the traveling salesman problem which is known to be NP-hard (Karp, 1972). Therefore, the problem studied in this paper is also NP-hard.

3.4. A 2-Index Model

Model (1)–(13) uses 3-indexed variables x_{ij}^k for the routing of ambulances $k \in \mathcal{K}$. However, since all ambulances are identical except for their initial locations, we can reformulate

the model by dropping index k . Hence, the size of the model can be reduced with an expected positive impact on the computation time needed by a MIP solver to find an optimal solution to the routing problem. It should be noticed that the 2-Index Model cannot be used in case of non homogeneous ambulances. In that situation the mathematical formulation proposed in the 3-Index Model should be adopted. In order to reformulate the model, we introduce the binary variable $x_{ij} \in \{0, 1\}$, which takes value 1 if any ambulance travels directly from the patient (or hospital) at location i to the patient at location j . All further notation is as before.

$$\min \quad w_R \cdot e_R + w_G \cdot e_G \quad (14)$$

s.t.

$$\sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{hj} \leq |\mathcal{K}_h| \quad \forall h \in \mathcal{H} \quad (15)$$

$$\sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{ji} = \sum_{j \in \mathcal{P} \cup \mathcal{H}} x_{ij} = 1 \quad \forall i \in \mathcal{P} \quad (16)$$

$$\sum_{h \in \mathcal{H}} u_{ih} = 1 \quad \forall i \in \mathcal{R} \quad (17)$$

$$\sum_{i \in \mathcal{R}} u_{ih} \leq c_h \quad \forall h \in \mathcal{H} \quad (18)$$

$$b_i + d_i + t_{ij} \leq b_j + (1 - x_{ij}) \cdot M \quad \forall i \in \mathcal{G} \cup \mathcal{H}; \forall j \in \mathcal{P} \quad (19)$$

$$b_i + d_i + t_{ih} + d_h + t_{hj} \leq b_j + (2 - x_{ij} - u_{ih}) \cdot M \quad \forall i \in \mathcal{R}; j \in \mathcal{P}; h \in \mathcal{H} \quad (20)$$

$$e_G \geq b_i + d_i \quad \forall i \in \mathcal{G} \quad (21)$$

$$e_R \geq b_i + d_i + t_{ih} + d_h - (1 - u_{ih}) \cdot M \quad \forall i \in \mathcal{R}; h \in \mathcal{H} \quad (22)$$

$$b_i \geq 0 \quad \forall i \in \mathcal{P} \cup \mathcal{H} \quad (23)$$

$$u_{ih} \in \{0, 1\} \quad \forall i \in \mathcal{R}; h \in \mathcal{H} \quad (24)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A} \quad (25)$$

The objective function in (14) is identical to the objective function (1) of the 3-index model. Constraints (15) ensure that at most $|\mathcal{K}_h|$ ambulances start from hospital h . Constraints (16) conserve the flow of ambulances at patient locations. Constraints (19) and (20) determine the arrival times at the patients based on the new routing variables x_{ij} . Constraints (25) define these binary variables. Constraints (17)–(18) and (21)–(24) are taken from the 3-index model.

4. Solution Approach

The circumstances in which the ambulance routing problem described in this paper is solved require a fast and robust solution approach. To be useful in disaster situations, the ambulance routing problem must be solved within seconds in order to respond properly to the emergency requests and to replan the routing if updated information becomes available. In addition, the quality of the solutions is an important aspect determining the waiting times for the patients, which should obviously be as short as possible. These reasons warrant the development of a (meta)heuristic solution approach, which is usually faster than an exact approach, and is expected to produce solutions of near-optimal quality.

We propose here a Large Neighborhood Search (*LNS*) metaheuristic to solve the ambulance routing problem. The iterative nature of the *LNS* metaheuristic and the presence of diversification mechanisms allow the procedure to escape from local optima such that various parts of the solution space can be explored in a limited amount of computing time. The operating principle of the *LNS* metaheuristic is based on three stages:

- *Initial stage*: An initial solution for the ambulance routing problem is generated by one of two randomly selected heuristic approaches, which are described in Section 4.1.
- *Intensification stage*: The current solution is improved by a large scale neighborhood search (so-called *Variable Neighborhood Descent (VND)* heuristic), which uses nine different local search operators. The *VND* heuristic is described in Section 4.2.
- *Diversification stage*: To reach unexplored areas of the solution space, the current solution is first partially destroyed by selecting randomly one of three destroy operators described in Section 4.3. Afterwards it is reconstructed by applying a repair operator. The modified solution then becomes the input of the intensification stage.

Two parameters I and L determine the behavior of the *LNS* procedure. I is the total number of iterations performed by the algorithm. L is a limit on the number of iterations without improvement. If L iterations have been performed without finding a new best solution, *LNS* restarts the search process by generating a new initial solution. This generation process is randomized to produce a solution that likely has not been investigated earlier in the search process. The *LNS* method is outlined in further detail in Algorithm 1. After the initialization phase, the method starts its first iteration ($i = 0$) by constructing an initial solution, see lines 9 to 17. Here, one of the two available heuristics for the generation of new solutions is picked randomly. Furthermore, a parameter α (described in Section 4.1) is determined and given to these procedures to guide the randomized solution construction process. In later iterations, when a solution already exists, the search process is diversified to escape from local optima by randomly selecting a destroy operator and by repairing the resulting solution, see lines 19 to 27. The search is intensified by applying the *VND*-local search method to the current solution in line 28. In line 29, it is checked whether a new best solution is found, which is then stored. In this case, counter l for the number of iterations without improvement is reset, otherwise incremented (line 33). If the counter reaches the limit L , the *LNS* method generates a new initial solution, see again line 9. After a total of I iterations, the *LNS* metaheuristic terminates by returning the best solution found. The components of the *LNS* metaheuristic are described in detail in the following sections.

4.1. Initial Stage

The *LNS* procedure embeds two different heuristics to generate initial feasible solutions to the ambulance routing problem. The first heuristic is referred to as *Insertion heuristic*. It starts by building a single route for all green code patients and then inserts the red code patients one after the other. The second heuristic is called the *Constructive heuristic*. It builds routes simultaneously for green and red code patients. Both procedures are designed to respect the different services required by green code patients (which can be served in the field where, afterwards, the ambulance can go directly to the next patient) and red code patients (which have to be picked up and brought to a hospital with free capacity).

Algorithm 1: *LNS* Metaheuristic

```
1 Initialize metaheuristic parameters  $I$  and  $L$ ;  
2 Let  $s^*$  be the best solution found so far and  $f(s^*)$  be its objective function value;  
3 Let  $s$  be the current solution and  $f(s)$  be its objective function value;  
4  $s^*, s \leftarrow \emptyset, f(s^*), f(s) \leftarrow \infty$ ;  
5 Let  $i$  be the iteration counter;  
6 Let  $l$  be the counter for iterations without improvement;  
7  $i \leftarrow 0, l \leftarrow 0$ ;  
8 while ( $i < I$ ) do  
9   if ( $i = 0 \vee l = L$ ) then // Initial stage  
10      $random \leftarrow RandInt([0, 1])$ ;  
11      $\alpha \leftarrow RandInt([2, 5])$ ;  
12     switch ( $random$ ) do  
13       case ( $random = 0$ )  
14          $s \leftarrow InsertionHeuristic(\alpha)$ ;  
15       case ( $random = 1$ )  
16          $s \leftarrow ConstructiveHeuristic(\alpha)$ ;  
17      $l \leftarrow 0$ ;  
18   else // Diversification of existing solution  
19      $random \leftarrow RandInt([0, 2])$ ;  
20     switch ( $random$ ) do  
21       case ( $random = 0$ )  
22          $s \leftarrow Rem_2(s)$ ;  
23       case ( $random = 1$ )  
24          $s \leftarrow Rem_{rand}(s)$ ;  
25       case ( $random = 2$ )  
26          $s \leftarrow Rem_{all}(s)$ ;  
27      $s \leftarrow Repair(s)$ ;  
28      $s \leftarrow VND(s)$  // Intensification  
29     if ( $f(s) < f(s^*)$ ) then // New best solution?  
30        $s^* \leftarrow s$ ;  
31        $l \leftarrow 0$ ;  
32     else  
33        $l++$ ;  
34      $i++$ ;  
35 return  $s^*$ ; // Return best solution
```

Furthermore, both methods contain random components to deliver different solutions as is needed to exploit the restart capability of the *LNS* metaheuristic.

The *Insertion heuristic* is outlined in Algorithm 2. It initially produces a single giant route that connects all green code patients $i \in \mathcal{G}$, see line 2. For this purpose, we solve a traveling salesman problem (TSP) using the well known heuristic of Lin and Kernighan (1973). We have chosen the Lin-Kernighan heuristic as it is considered to be one of the most effective methods for the TSP, and has found the best-known solutions to a large number of benchmark problems. The method used in our paper is the modified Lin-Kernighan heuristic proposed in Helsgaun (1998, 2000, 2006), which is a highly efficient implementation from a computational point of view. Once the giant route has been generated for

Algorithm 2: Insertion heuristic

```
1 Let  $a$  be a randomly selected ambulance and let  $h$  be the hospital where  $a$  is located;  
2 Let  $r$  be a TSP route that starts and ends at  $h$  visiting all patients in  $\mathcal{G}$ ;  
3 Assign route  $r$  to ambulance  $a$ ;  
4 while (not all patients have been visited) do  
5   Let  $j$  be a randomly selected and so far unvisited red code patient;  
6   Let  $C$  be a candidate list of  $\alpha$  least-cost positions to insert  $j$  into current routes;  
7   Randomly select an insertion position  $i \in C$ ;  
8   Split the corresponding route right after position  $i$ ;  
9   Append  $j$  to the first sub-route and close this route by appending the nearest hospital  
   with free capacity;  
10  Let  $\hat{a}$  be the ambulance that becomes available earliest in the current solution;  
11  Let  $\hat{h}$  be the current location of  $\hat{a}$ ;  
12  Let the second sub-route start at  $\hat{h}$  and assign this route to  $\hat{a}$ ;  
13 return solution  $s$ ;
```

the green code patients, the red code patients are inserted using a variant of the insertion heuristic proposed by Solomon (1987). Since red code patients have to be brought to hospitals, including such a patient in a route actually means to split this route into two new ones. More precisely, the insertion procedure iterates through the red code patients and includes one patient $j \in \mathcal{R}$ into the route plan per iteration. For this purpose it determines a candidate list C of α feasible and least-cost insertion positions to include j into one of the current routes, see line 6. Then, the procedure randomly selects an insertion position $i \in C$ and splits the corresponding route into two sub-routes, see lines 7 and 8. The first sub-route contains the patients up to and including i . The second sub-route contains patient $i + 1$ and the following ones. Then, red code patient j is appended to the first sub-route and this route is closed by selecting the hospital that takes up this patient, see line 9. For the second sub-route, a new ambulance and, thus, starting location is determined in lines 10 to 12. The procedure ends if all patients have been added to the solution.

The *Constructive heuristic* is outlined in Algorithm 3. It builds a solution step-by-step by adding one patient at a time to a route plan until all patients \mathcal{P} are served by the ambulances. The procedure starts by creating a new route for one of the ambulances in lines 2 to 4. Then, using a greedy randomized selection process, a patient i is randomly selected from a restricted candidate list C^P of α unserved patients and added to the route, see lines 5 to 7. If the selected patient i is a red code patient, the route is closed by selecting a hospital from a candidate list C^H of α closest hospitals with free capacity, see lines 8 to 11. Otherwise, if the patient i is a green code patient, the route is potentially extendible by adding further patients, which is investigated in lines 12 to 34. For this purpose, an unserved patient j that is nearby patient i is randomly selected in lines 14-15. If patient j is a red code patient, j is added to the route, a hospital is selected, the route is closed, and the procedure restarts a new iteration, see lines 16 to 21. Otherwise, if j is a green code patient, it is checked in lines 23 to 26 whether there exists an alternative ambulance \hat{a} that can reach j at an earlier point in time than the currently considered ambulance a . If this is not the case, j is appended to the current route r of ambulance a , see lines 27 to 29. If, however, ambulance \hat{a} can reach j earlier than ambulance a , route r of ambulance

Algorithm 3: Constructive heuristic

```
1 while (not all patients have been visited) do
2   Let  $a$  be the ambulance that becomes available earliest in the current solution;
3   Let  $h$  denote the hospital where ambulance  $a$  is currently located;
4   Define a new route  $r$  for ambulance  $a$  with starting point  $h$ ;
5   Let  $C^P$  be a candidate list of  $\alpha$  unserved patients that are located closest to  $h$ ;
6   Let  $i$  be a patient randomly selected from  $C^P$ ;
7   Append  $i$  to route  $r$ ;
8   if ( $i \in \mathcal{R}$ ) then // Red code patient requires closing the route
9     Let  $C^H$  be a candidate list of  $\alpha$  available hospitals that are closest to  $i$ ;
10    Let  $h$  be a hospital randomly selected from  $C^H$ ;
11    Close route  $r$  by appending hospital  $h$ ;
12  else // Route  $r$  is potentially extendible
13    while (not all the patients have been visited) do
14      Let  $C^P$  be a candidate list of  $\alpha$  unserved patients that are located closest to  $i$ ;
15      Let  $j$  be a patient randomly selected from  $C^P$ ;
16      if ( $j \in \mathcal{R}$ ) then // Red code patient requires closing the route
17        Append  $j$  to route  $r$ ;
18        Let  $C^H$  be a candidate list of  $\alpha$  available hospitals that are closest to  $j$ ;
19        Let  $h$  be a hospital randomly selected from  $C^H$ ;
20        Close route  $r$  by appending hospital  $h$ ;
21        break; // Restart from line 1
22      else // Route  $r$  can be extended
23        Let  $\hat{a} \neq a$  be the ambulance that becomes idle earliest in current solution;
24        Let  $T_{\hat{a}}$  denote the time at which  $\hat{a}$  becomes idle;
25        Let  $\hat{h}$  be the hospital where ambulance  $\hat{a}$  is currently located;
26        Let  $b_i$  be the time at which patient  $i$  is visited by ambulance  $a$ ;
27        if ( $b_i + d_i + t_{ij} \leq T_{\hat{a}} + t_{\hat{h}j}$ ) then // Extend route  $r$  of ambulance  $a$ 
28          Append  $j$  to route  $r$ ;
29           $i \leftarrow j$ ;
30        else // Close route  $r$  of ambulance  $a$ 
31          Let  $C^H$  be a list of  $\alpha$  hospitals that are closest to  $i$ ;
32          Let  $h$  be a hospital randomly selected from  $C^H$ ;
33          Close route  $r$  by appending hospital  $h$ ;
34          break; // Restart from line 1
35 return solution  $s$ ;
```

a is closed and the algorithm starts a new iteration, see lines 30 to 34. Patient j is then added to a new route in one of the following iterations of the *Constructive heuristic*. The procedure ends if all patients have been added to the solution.

4.2. Intensification Stage

During the intensification stage of the *LNS* metaheuristic, solutions are improved by means of local search. For this purpose, we have adopted several of the most common local search operators for vehicle routing problems (Bräysy and Gendreau, 2005). In total, we use nine local search operators that are listed in Table 2 and described afterwards. The four *Intra Route Operators* search for improvements within a route:

Table 2: Local search operators used in the VND heuristic.

N_λ	Intra Route Operators	N_λ	Inter Route Operators
N_1	Internal Patients Relocate	N_5	External Patients Relocate
N_2	Internal Patients Swap	N_6	External Patients Swap
N_3	Internal Patients 2-Opt	N_7	External Patients 2-Opt
N_4	Single Hospital Change	N_8	Hospitals Swap
		N_9	Route reassignment

- *Internal Patients Relocate*: A green code patient contained in a route is relocated to another position within this route. If the route contains a red code patient, the green code patient is not allowed to be placed behind the red code patient, because the latter needs to be brought to a hospital directly. Therefore, red code patients appear only at the end of a route and, hence, relocating them using the local search is not an option.
- *Internal Patients Swap*: The positions of two green code patients that both belong to the same route are exchanged.
- *Internal Patients 2-Opt*: Two edges $(i, i+1)$ and $(j, j+1)$ contained in one route are replaced by edges (i, j) and $(i+1, j+1)$. Such a neighborhood move also reverses the order in which the patients in between $i+1$ and j are served. Again, moves that would relocate a red code patient are forbidden.
- *Single Hospital Change*: The hospital at which the route ends is replaced by another hospital. If the route involves a red code patient, the new destination hospital has to have at least one free capacity unit.

The following five *Inter Route Operators* address the routings of two different ambulances:

- *External Patients Relocate*: A patient i is removed from an ambulance route and inserted into another route. Let j denote the patient in the new route behind which patient i is inserted. The neighborhood move replaces edges $(i-1, i)$, $(i, i+1)$ and $(j, j+1)$ by edges $(i-1, i+1)$, (j, i) and $(i, j+1)$. Note that the operation is only feasible if j is not a red code patient.
- *External Patients Swap*: Two patients i and j that are served in different routes are exchanged. The neighborhood move replaces edges $(i-1, i)$, $(i, i+1)$, $(j-1, j)$ and $(j, j+1)$ by edges $(i-1, j)$, $(j, i+1)$, $(j-1, i)$ and $(i, j+1)$.
- *External Patients 2-Opt*: This operator, also known in literature as Crossover operator, considers two patients i and j belonging to different routes. Both routes are split right after patients i and j and the detached sub-routes are exchanged. In other words, all patients that followed patient i on its original route now follow patient j and vice versa. The move is performed by replacing edges $(i, i+1)$ and $(j, j+1)$ by edges $(i, j+1)$ and $(j, i+1)$.
- *Hospitals Swap*: The destination hospitals at which two different routes end are exchanged. For example, if route r_1 originally ends at hospital h_1 whereas route r_2 ends at hospital h_2 , the swap move effects that r_1 now ends at h_2 and r_2 now ends

at h_1 . The swap is only feasible if it does not violate the hospital capacities. For example, if route r_1 includes a red code patient and r_2 does not, the new destination hospital h_2 of route r_1 must have a free capacity unit.

- *Route reassignment*: This operator removes a route from its ambulance and assigns it to another ambulance.

The local search operators are combined in a *Variable Neighborhood Descent (VND)* heuristic that is outlined in Algorithm 4. The *VND* heuristic improves the current solution by exploring the nine neighborhoods one after the other. Each neighborhood is examined by a first-improvement descent strategy, accepting only feasible moves that lead to an improvement of the current solution. Although the order in which the neighborhoods are investigated may have an impact on the quality of the obtained solutions, we did not observe such an effect in some preliminary experiments. Therefore, *VND* explores the neighborhoods in the order shown in Table 2. If, during the search process, a solution is found that is better than the best solution known so far, the best solution is updated and *VND* intensifies the search by restarting from the first neighborhood. The procedure terminates if the current solution cannot be further improved by any of the local search operators and, thus, a common local optimum has been reached for all the neighborhoods.

Note that the quality of a solution can only be improved by modifying routes of those ambulances that serve the green code patient and the red code patient with the latest service completion times e_G and e_R . We refer to the vehicles serving these patients as the *critical green ambulance* and the *critical red ambulance*. In order to speed up the *VND* heuristic, we restrict the local search to the routes of the two critical ambulances. Since each of these ambulances may have assigned multiple routes in a solution, all nine neighborhoods can yield improvements even if the search is restricted to the two vehicles.

4.3. Diversification Stage

In order to escape from the local optima that are reached in the *intensification stage*, a diversification mechanism is used to reach unexplored areas of the solution space. The diversification strategy consists of a *destroy step* that eliminates some of the routes contained in the best solution found so far. Afterwards, a *repair step* is performed to assign the now unserved patients to non-destroyed routes or to build new routes for them. Then,

Algorithm 4: VND heuristic

```

1 Let  $s$  be the current solution and  $f(s)$  be its objective function value;
2 Let  $s^*$  be the best solution found so far and  $f(s^*)$  be its objective function value;
3  $\lambda \leftarrow 1$ ; // Start with first neighborhood
4 repeat
5    $s \leftarrow N_\lambda(s)$ ; // Find best solution within neighborhood
6   if ( $f(s) < f(s^*)$ ) then // New best solution?
7      $s^* \leftarrow s$ ;
8      $\lambda \leftarrow 1$ ; // Restart with first neighborhood
9   else
10     $\lambda++$ ; // Continue with next neighborhood
11 until ( $\lambda = 9$ );
12 return  $s^*$ ;

```

LNS again applies the *intensification stage* to improve the obtained solution and so on. For the *destroy step*, three different operators have been implemented:

- *Remove two routes (Rem_2)*: This operator destroys two routes of the current solution. The first route to be destroyed is the one that contains the green code patient with the latest service completion time, i.e., the patient that determines e_G . The second route is the one that contains the red code patient who determines value e_R . If both these patients are served in the same route, only this single route is destroyed.
- *Remove a random number of routes (Rem_{rand})*: Let the *critical green ambulance* (*critical red ambulance*) be the vehicle that serves the green (red) patient that determines the objective value e_G (e_R). Each of these vehicles can perform more than one route in the current solution to bring various red code patients to hospitals and to serve a number of green code patients. Therefore two random numbers $rand_G$ and $rand_R$ are uniformly generated in the ranges $[1, N_G]$ (for the critical green ambulance) and $[1, N_R]$ (for the critical red ambulance) respectively, where N_G represents the maximum number of routes contained in the critical green ambulance, while N_R is the maximum number of routes assigned to the critical red ambulance. Then the destroy operator removes $rand_G$ and $rand_R$ of these routes from the green and red critical ambulances.
- *Remove all routes (Rem_{all})*: All the routes of both critical ambulances are destroyed. If the critical green ambulance coincides with the critical red ambulance, only the routes performed by this single ambulance are removed.

Each time the *diversification stage* is applied, one of the aforementioned destroy operators is randomly selected. The destroyed solution is then repaired using the greedy randomized selection mechanism proposed for the *Constructive heuristic* of Section 4.1.

5. Computational Study

In this section, we describe the computational experiments that were executed to evaluate the performance of the proposed models and heuristics. Since the ambulance routing problem described in this paper has not been studied before, no benchmark instances are available in the literature. We therefore generate a large set of test instances in Section 5.1, which are made available to other researchers upon request.

The computational experiments are divided into three parts. In the first experiment, described in Section 5.2, we compare the two optimization models regarding their potential to produce optimal solutions for the ambulance routing problem. In Section 5.3, we assess the performance of the *LNS* metaheuristic. In the third experiment, presented in Section 5.4, we perform sensitivity tests to analyse the relationship between the structure of a problem instance and its solution.

5.1. Test Instances and *LNS* Parameter Setting

In order to test both the mathematical models and the metaheuristic algorithm presented in this paper, a large set of test instances has been generated to capture various planning situations. We have produced instances with a varied number of red code and green code patients, hospitals, hospital capacities, and ambulances. These parameters are varied as follows:

Table 3: Variation of weights evaluated in the experiments.

Weights		Relative importance of	# of
w_G	w_R	red code patients	instances
1	1	50%	324
1	2	67%	324
1	5	83%	324
1	10	91%	324
Total			1296

- Total number of patients:
low ($|\mathcal{P}| = 10$), *medium* ($|\mathcal{P}| = 25$), *high* ($|\mathcal{P}| = 50$)
- Percentage of red patients:
low ($|\mathcal{R}| = 25\% \cdot |\mathcal{P}|$), *medium* ($|\mathcal{R}| = 50\% \cdot |\mathcal{P}|$), *high* ($|\mathcal{R}| = 75\% \cdot |\mathcal{P}|$)
- Number of hospitals: $|\mathcal{H}| = 1, 2, 3$ or 4
- Hospital capacity:
low ($\sum_{h \in \mathcal{H}} c_h = 1 \times |\mathcal{R}|$), *medium* ($\sum_{h \in \mathcal{H}} c_h = 1.5 \times |\mathcal{R}|$), *high* ($\sum_{h \in \mathcal{H}} c_h = 2 \times |\mathcal{R}|$)
- Number of ambulances:
low ($|\mathcal{K}| = 0.05 \times |\mathcal{P}|$), *medium* ($|\mathcal{K}| = 0.25 \times |\mathcal{P}|$), *high* ($|\mathcal{K}| = 0.5 \times |\mathcal{P}|$)

One test instance was produced for each combination of the above parameters, which yields a total of 324 instances. For each instance, the locations of hospitals and patients was randomly drawn in an area of size 200×200 , with travel times t_{ij} corresponding to the Euclidean distance. The service duration d_i of red code patients $i \in \mathcal{R}$ was randomly drawn from the interval $[2, 15]$. The service duration d_i of green code patients $i \in \mathcal{G}$ was selected in the interval $[5, 35]$. Dropping off a red code patient at a hospital is done in no time, i.e., $d_h = 0 \forall h \in \mathcal{H}$. The available $|\mathcal{K}|$ ambulances and the hospital capacity are shared randomly among the hospitals contained in an instance.

In order to test the impact of the priority assigned to red and green code patients, we further associated the 324 instances to different combinations of weights w_G and w_R . The four combinations shown in Table 3 were considered. Associating these parameters to each of the test instances yielded a total of 1296 instances for the experiments.

To perform the experiments, the *LNS* metaheuristic was coded in Java. The metaheuristic requires two parameter to be set: the number of iterations I to perform and the number of non-improving iterations L after which the heuristic generates another initial solution. It is clear that a higher value I makes finding better solutions more likely at the expense of a longer computation time. In preliminary experiments, we observed that the heuristic converges quickly, and that $I = 40$ offers a good compromise between runtime and solution quality. Moreover, we set $L = I/3$ as this delivered good results in the pretests. Finally, since the metaheuristic involves various elements of randomness, finding better solutions may also be achieved by repeating the solution process a number of times. For this purpose, we repeat the heuristic 20 times when solving an instance. In the remainder of this section the reported *cpu*-times are the total for all runs. All experiments have been performed on an Intel core i7-2760QM 2.40 GHz processor with 4 GB RAM.

5.2. Evaluation of the Optimization Models

To compare the two models presented in Section 3, we solve them using the MIP-solver CPLEX 12.4, see Ilog (2013). More precisely, we apply CPLEX to solve both models for each of the 324 test instances with weights w_G and w_R both set to 1. For each instance, a maximum computation time of one hour was defined as a stopping condition whenever the optimal solution is not obtained. We report in Table 4 aggregated results for each model and for the sets of 108 instances with a low ($|\mathcal{P}| = 10$), a medium ($|\mathcal{P}| = 25$), and a high ($|\mathcal{P}| = 50$) number of patients. The table shows the number of integer feasible solutions ($\#feas$) found for an instance set, the number of optimal solutions ($\#opt$), the average lower bound (LB), the average objective function value (obj), and the average computation time (cpu) required by CPLEX. Column imp provides the percentage gap between the average objective function values of the solutions found by the 2-index model and the 3-index model.

From the results, we see that the 3-index model consistently delivers feasible solutions only for the small sized instances, whereas the 2-index model delivers feasible solutions for all instances. The 2-index model also yields more optimal solutions, stronger lower bounds, and feasible solutions with a lower objective function value. Hence, for medium and large instances, the 3-index model is clearly outperformed by the 2-index model, where the latter achieves average improvements of up to 42.1%. The results of all 324 test instances are aggregated in Table 5, where also the average optimality gap and the change (Δ) of each performance measure between the 3- and the 2-index model is reported. It becomes clear that the 2-index model is superior with respect to all the key performance measures. However, even the computation time required by this model clearly exceeds what is considered applicable in a disaster response process. For this reason, the model’s results can be used for an assessment of heuristics but it appears inappropriate to apply the model itself to solve the ambulance routing problem in practice.

5.3. Evaluation of the LNS Metaheuristic

For the second experiment, all 1296 instances are solved using the *LNS* metaheuristic. The heuristic is evaluated by comparing its results to those obtained by the 2-index model. Table 6 reports key performance measures for both approaches and each subset of 324 test instances with differing weights w_G and w_R . We see that the 2-index model shows almost

Table 4: Computational results achieved by the two proposed models.

# of patients	# of instances	3-Index Model					2-Index Model					
		$\#feas$ [-]	$\#opt$ [-]	LB [-]	obj [-]	cpu [sec.]	$\#feas$ [-]	$\#opt$ [-]	LB [-]	obj [-]	cpu [sec.]	imp [%]
<i>low</i>	108	108	79	634	881	1702	108	82	657	881	1152	0.0
<i>medium</i>	108	105	0	243	1639	3600	108	23	386	1070	2930	34.7
<i>high</i>	108	71	0	179	2430	3600	108	0	252	1408	3600	42.1

Table 5: Aggregated results achieved by the two models.

	3-Index Model	2-Index Model	Δ
$\#feas$	284	324	+14%
$\#opt$	79	105	+33%
<i>avg. opt. gap</i>	4.57%	3.66%	-20%
<i>avg. cpu</i> [sec.]	2967	2561	-14%

Table 6: Comparison of results delivered by the 2-index model and the *LNS* metaheuristic.

Weights		# of instances	2-Index Model			<i>LNS</i>		
w_G	w_R		#opt	gap	cpu [sec.]	#opt	gap	cpu [sec.]
1	1	324	105	4.33%	2560	91	4.12%	0.2
1	2	324	105	2.62%	2601	93	2.58%	0.2
1	5	324	103	1.20%	2607	92	1.18%	0.2
1	10	324	108	0.62%	2565	98	0.61%	0.3
Average:			105	2.19%	2583	94	2.12%	0.2

the same number of optimal solutions and the same *cpu*-times for all combinations of w_G and w_R . The *gap* even decreases the higher the priority of red code patients gets, which is a promising results because in practice $w_R > w_G$ is likely to be used to reflect the higher urgency of seriously injured people. The *LNS* metaheuristic produces less optimal solutions for each of the considered instance sets but the optimality *gap* (measured by using the best textitLNS-solutions out of the 20 runs for each instance) of the heuristic solutions is even slightly lower than the *gap* achieved by the 2-index model. Actually, the metaheuristic is capable of delivering high quality solutions with a *gap* clearly below 1% in case that red code patients receive high priority. Furthermore, the average computation time of this method is just about 0.2 seconds per instance, which is several orders of magnitude faster than the MIP-solver. These computation times show that the *LNS* method is applicable in a dynamic disaster response process, as it produces solutions quasi-instantaneously. In particular, it achieves excellent solution quality if high priority is given to red code patients.

Since the metaheuristic involves several elements of randomness, we also determine the contribution of these techniques to the generation of high quality solutions. In particular, the metaheuristic randomly decides whether to use the *Insertion heuristic* or the *Constructive heuristic* to generate initial solutions. Moreover, in the diversification stage, the destroy operator used is randomly selected. Finally, the *Insertion heuristic* and the *Constructive heuristic* both use greedy randomized mechanisms to select patients, hospitals, and insertion positions from restricted candidate lists of size α . The value α is a random integer in the interval $[2, 5]$, which is drawn by the *LNS* metaheuristic each time a new initial solution is generated. In order to evaluate the contribution of the different randomization techniques, the following methodology was used. For each of the final solutions of the 1296 instances, the heuristic that was used to generate the corresponding initial solution, the applied destroy operator, and the value of α were tallied. Table 7 illustrates the relative frequencies observed for the different settings of these parameters while we solved the 1296 instances 20 times each. It can be seen that the *Insertion heuristic*, the *Rem_{all}* destroy operator, and a value of $\alpha = 2$ most often result in the best solution. Nevertheless, the

Table 7: Contribution of *LNS*-components to finding best solutions.

Initial solution		Destroy step		α	
Method	Frequency	Operator	Frequency	Value	Frequency
<i>Insertion heuristic</i>	55%	<i>Rem₂</i>	34%	2	40%
<i>Constructive heuristic</i>	45%	<i>Rem_{rand}</i>	23%	3	32%
		<i>Rem_{all}</i>	43%	4	18%
				5	10%

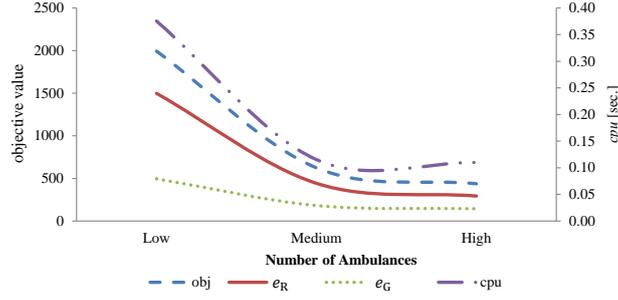


Figure 4: Impact of the number of ambulances.

other components are involved in the generation of a sizeable number of best solutions.

5.4. Problem Structure and Solution Quality

In the third experiment, we analyse the relationship between the structure of an instance and the best solution found by the *LNS* metaheuristic. We first investigate the impact of the number of ambulances. For this purpose, we distinguish subsets of instances with a low, a medium, and a high number $|\mathcal{K}|$ of ambulances as defined in Section 5.1. The objective weights w_R and w_G are both set to 1. Figure 4 shows the following performance measures averaged over all instances belonging to a subset: objective function value *obj*, latest service completion times e_R and e_G of red code and green code patients, and computation time *cpu*. Note that e_R is larger than e_G in these results, because the service of red code patients ends at their delivery to a hospital whereas the service of green code patients ends directly after having been treated in the field. As expected, a larger number of ambulances results in a better service (lower objective values). We see that the latest completion times of both red code patients and green code patients benefit from a medium number of ambulances. However, the marginal contribution of additional vehicles decreases such that the objective values do not decline further if a high number of ambulances is available. These computations show that the presented approach can be useful in determining the fleet size required in a disaster situation.

Figure 5 analyses the impact of the number of hospitals (left figure) and the impact of the total hospital capacity (right figure). Regarding the number of hospitals, we distinguish instances with $|\mathcal{H}| = 1, 2, 3,$ and 4 hospitals. The total hospital capacity $\sum_{h \in \mathcal{H}} c_h$ is varied from low to medium and to high in relation to the number of red code patients as described in Section 5.1. The considered test instances comprise all the combinations of objective weights w_R and w_G from Table 3. According to Figure 5 (left), a larger number

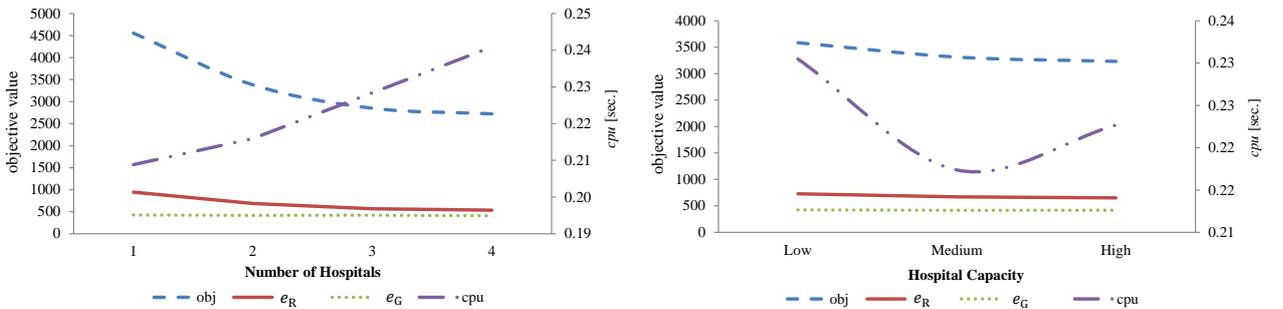


Figure 5: Impact of the number of hospitals (left) and the hospital capacity (right).

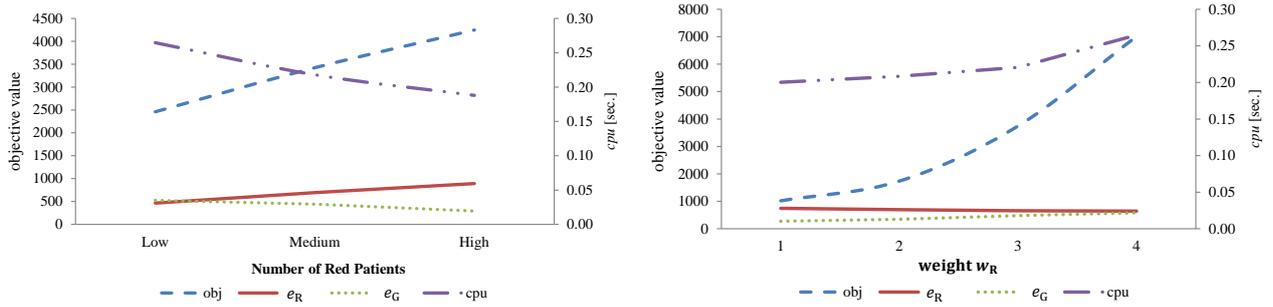


Figure 6: Impact of the number of red code patients (left) and the objective weight w_R (right).

of hospitals clearly helps serving red code patients by reducing the trip duration to transport them to a hospital of free capacity. In fact, the larger the number of hospitals is, the lower the value e_R becomes. This experiment shows that the method can also be used to determine whether patients would benefit from setting up temporary hospitals like medical camps. Although the average computation time grows with a larger number of hospitals, it stays clearly below one second even for the largest number of hospitals. The capacity of hospitals seems to have only a minor effect on the obtained solutions, see Figure 5 (right). Although a higher capacity means that there is a higher chance for red code patients to find free capacity at a nearby hospital, we do not observe a significant reduction in the latest service completion time here. The explanation is that the patients and hospitals are widely spread over the whole area in these instances such that most red code patients find a hospital in their surrounding even if the overall capacity is low.

Finally, we investigate the impact of having a low, a medium, or a high percentage of red code patients, see Figure 6(left), and of considering red code patients equally important as green code patients ($w_R = w_G = 1$) or more important ($w_R = 2, 5, 10$), see Figure 6(right). As expected, if the percentage of red code patients increases, the latest service completion time among these patients increases because the ambulances have to bring more patients to the hospitals. At the same time, the latest service completion time e_G of green code patients reduces, because fewer such patients need assistance. Interestingly, the computation time decreases although a higher percentage of red code patients means more decisions to assign patients to hospitals. However, an increase in the percentage of red code patients also means a reduction in the number of green code patients, which in turn reduces the number of decisions to group and sequence green code patients on a same ambulance route. The relative importance of red code patients in a solution is controlled by parameter w_R . In Figure 6 (right) we see that a larger value w_R indeed reduces the latest service completion time e_R of red code patients at the cost of the latest service completion time e_G of green code patients. Also the average objective function value increases with a larger value of w_R , but this can be attributed to the fact that the objective function sums up the two weighted service completion times. This experiment confirms that the metaheuristic effectively considers the weights w_R and w_G to produce ambulance routes that reflect the different priorities of red code patients and green code patients.

6. Conclusions

In this paper an ambulance routing problem for disaster response is investigated, where patients require different types of services. We have proposed two mathematical models.

Computational tests show that a 2-index formulation outperforms a 3-index formulation. However, although problem instances will be of rather small size because the routing problem is solved at high frequency in disaster response, the exact solution of the optimization model takes an unacceptably long time. Therefore, a Large Neighborhood Search metaheuristic has been proposed to solve the ambulance routing problem in sub-second computing, with the aim to assist all patients as fast as possible. Experiments on a very large set of test instances show that the heuristic delivers solutions of excellent quality. Several further experiments demonstrate that the proposed planning approach can be used to support decisions regarding the fleet size of ambulances and the number and capacities of hospitals. Furthermore, the metaheuristic can be controlled to produce routes that take into account the different urgency of slightly and seriously injured patients.

Future research may aim at incorporating further aspects such as different types of ambulances, time windows, or constraints on the route length, e.g., before refuelling is needed. Moreover, the ambulance routing problem may be adapted to the public health care sector. In fact the two classes of patients considered in this paper may also be used to model patients requiring services at their homes (like, for example, physiotherapy) and patients that have to be transported to certain health care facilities to receive hospital treatment.

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