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A further augmentation of the Solow model and the empirics of economic growth for OECD countries

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Abstract

This paper extends the human capital augmented Solow growth model by including endogenous accumulation of technological know-how. It is shown that the empirical description of OECD cross-country data in Mankiw, Romer & Weil (1992) improves substantially.

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Keywords: economic growth, augmented neoclassical model, human capital accumulation, accumulation of technological know-how, conditional convergence.

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1. Introduction

Mankiw, Romer and Weil (1992) (MRW hereafter) showed that the augmented Solow model, including accumulation of human as well as physical capital, provides a good description of cross-country data, with the exception of the OECD subsample. The textbook Solow model explains about 60 percent of the cross-country variation in per worker GDP in a comprehensive sample of 98 non-oil producing countries. By including human capital, the augmented Solow model accounts for almost 80 percent of the variation in this sample.

For the OECD subsample, explanatory power of the models is rather poor. The textbook Solow model explains very little of the variation in per capita income levels (less than 6 percent). The performance of the MRW human capital augmented model is somewhat better but still less than 30 percent. Differences in explanatory power between samples largely disappear in specifications that allow for departures from the steady-state. MRW interpret this finding by conjecturing that OECD countries are perhaps further from their steady-state levels than countries in the broader sample.

The difference in explanatory power of the augmented Solow model in a broader sample vs. the OECD sample is possibly due to the similarity of OECD countries and the limited variation in explanatory variables. An alternative explanation offered here is that not all relevant factors of production are included. We therefore suggest a further augmentation of the Solow model by explicitly including the (endogenous) accumulation of technological know-how.

2. A generalization of the augmented Solow model

Following MRW a Cobb-Douglas type production function is assumed. However, m types of capital (e.g. infrastructure, equipment, other physical capital, human capital, ...) are included. In contrast to "new" neo-shumpeterian growth models (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992), technological know-how, in the sense of blueprints for production processes and new products, is considered here as a form of capital, but just as any other input in production. Property rights on know-how are assumed complete so that there is a well functioning market in know-how, so that neoclassical assumptions hold. Contrasting to new growth model, there is no presumption of externalities, spill-overs, imperfect competition or increasing returns from technology. Hence, production Y at time t is given by :

$$Y_t = c \cdot L_t^{(1-\sum_{i=1}^{m}\alpha_i)} \cdot K_{1_t}^{\alpha_1} \cdot \ldots \cdot K_{m_t}^{\alpha_m} \qquad (1)$$

with L (effective) labor, K_i capital of type i (i = 1...m), c and α_i constants. Labor is assumed to grow exogenously at rate n, due to population growth and exogenous growth in labor productivity (e.g. bacause of learning by doing). The model also assumes that a constant fraction s_i of output is invested in each type of capital. Defining k_i as the stock of capital of type i per unit of labor and y as output per unit of labor, the following set of differential equations governs the evolution of the k_is

$$\frac{dk_i}{dt} = s_i \cdot y_t - (n + \delta_i) \cdot k_{i_t} \qquad \forall i = 1 \text{ to } m \tag{2}$$

where δ_i s are the rates of depreciation of each type of capital.

Steady-state values of k_i, i = 1...m (i.e. the solution if all dk_i/dt are zero) are calculated by substituting the production function (1) in the differential equations (2), taking logarithms and solving the resulting linear system. Substituting these steady-state values for k in the production function yields the following steady-state value of per worker income:

$$\ln(y_*) = \frac{c}{1-\Sigma\alpha} + \frac{\alpha_1}{1-\Sigma\alpha}.[\ln(s_1) - \ln(n+\delta_1)] + \ldots + \frac{\alpha_m}{1-\Sigma\alpha}.[\ln(s_m) - \ln(n+\delta_m)] \tag{3}$$

From this generalized model, assuming that countries are in steady-state and allowing for country specific shocks captured by an error term ε_j, different specifications are derived.

The textbook Solow-model (m = 1) has only physical capital in the production function. This yields the following empirical specification:

$$\ln(y_{j*}) = \alpha_0 + \frac{\alpha}{1-\alpha} \cdot \ln(s_j) - \frac{\alpha}{1-\alpha} \cdot \ln(n_j + \delta) + \varepsilon_j \tag{4}$$

As there are no country specific data on depreciation and labor productivity growth, Mankiw et. al. assumed a constant value of 5 percent. The remaining cross-country variation in n_j corresponds to variations in the rate of population growth.

The MRW model (m = 2) has physical capital (k) and human capital (h) in the production function. Again, all depreciation is assumed equal which yields the MRW specification:

$$\ln(y_j^*) = \alpha_0 + \frac{\alpha_k}{1-(\alpha_k+\alpha_h)}.\ln(s_{k_j}) + \frac{\alpha_h}{1-(\alpha_k+\alpha_h)}.\ln(s_{h_j}) - \frac{\alpha_k+\alpha_h}{1-(\alpha_k+\alpha_h)}.\ln(n_j+\delta) + \varepsilon_j \tag{5}$$

Our model (m=3) has three types of capital viz. physical capital (k), human capital (h) and technological know-how (τ). Assuming all depreciation rates at a constant value δ, this yields the further augmented specification viz.

$$\ln(y_{j^*}) = \alpha_0 + \frac{\alpha_k}{1-(\alpha_k+\alpha_h+\alpha_\tau)}\cdot\ln(s_{k_j}) + \frac{\alpha_h}{1-(\alpha_k+\alpha_h+\alpha_\tau)}\cdot\ln(s_{h_j}) + \frac{\alpha_\tau}{1-(\alpha_k+\alpha_h+\alpha_\tau)}\cdot\ln(s_{\tau_j})$$
$$-\frac{\alpha_k+\alpha_h+\alpha_\tau}{1-(\alpha_k+\alpha_h+\alpha_\tau)}\cdot\ln(n_j+\delta)+\varepsilon_j \qquad (6)$$

The assumption of countries being in steady-state may be relaxed. It may be shown that (see Barro and Sala-i-Martin 1992, 1995)

$$\ln(\frac{y_{jt}}{y_{j0}}) = (1-e^{-\lambda t}).\ln(y_{j^*}) - (1-e^{-\lambda t}).\ln(y_{j0}) \qquad (\lambda>0) \qquad (7)$$

in which the above specifications for the steady-state (y_{j^*}) may be substituted, yielding an estimable specification. The parameter λ indicates the speed of (conditional) convergence toward the steady-state.

3. Empirical results

Basic data sets are from the Barro and Lee data set (Barro & Lee 1994), except for the data on R&D expenditures (OECD 1989). Data used in the regressions are in appendix I. Table I shows the definition and basic data source of the variables used. The sample consists of the 22 OECD countries with populations greater than one million.

Table II reports results for the three models, viz. the textbook Solow model (m=1), the augmented Solow model (m=2) and the extended augmented Solow model (m=3) and assuming observed levels of income correspond to the steady-state.
Similar to MRW the models are estimated unconstrained but also with the constraint that the sum of the coefficients of the logarithm of the investment shares should equal the negative of the coefficient of the logarithm of the sum of population growth, depreciation and exogenous labor productivity growth. We also assumed the same value for depreciation and labor productivity growth viz. 5 percent.

The results of equations m = 1 and m = 2 are very close to results reported in Mankiw et. al. (1992). Differences are due to update in the basic data set and also to the use of a different proxy to measure relative investment in human capital. We approximated the investment share of GDP in human capital by the ratio of direct government expenditure on education to GDP, rather than the percentage of the working-age population in secondary school (SCHOOL) used by MRW. The SCHOOL variable basically reports a kind of an opportunity cost of investment in education wheras the GEETOT variable may be a closer proxy for direct investment in education - at least for the OECD countries in which education is systematically subsidized in most of the cases.

Explanatory performance of the textbook Solow model for the OECD sample is still very poor (adj. $R^2 < 0.06$). Also, the influence of the share of GDP in physical capital is statistically not significant. Furthermore, the implied elasticity of capital investment (viz. about 0.6) is implausibly high.

The augmented Solow model performs much better and explains now almost half of the variation in GDP per working-age population. The new proxy for the investment share in human capital substantially increases both global explanatory power of the regression and the relative precision of the estimated coefficient, compared to MRW results. The present estimates reinforce the MRW results and conclusions.

The model including investment in know-how explains about three quarters of the variation in GDP per capita between OECD countries. Only the share of GDP invested in know-how is significant. The estimated effect on per capita income levels of the share of GDP invested in human capital is no longer statistically significant and its value is substantially smaller than what follows from the augmented Solow model.

Table III shows results, relaxing the assumption that observed levels of per capita income correspond to steady-state values. These results are at variance with MRW. Our estimates of the textbook Solow model are far more better than those reported in MRW: explanatory power is higher (MRW find an adjusted R^2 of 0.62), the coefficient of s_k is much larger (compare with MRW-value of 0.392) and more precisely estimated. Estimates of the human capital augmented model also differ substantially. In our estimates the human capital augmented model performs worse explaining the data than the textbook Solow model. The further augmented model adds in explanatory power explaining almost 3/4 of the variation in per capita income levels. At least in the unrestricted regression, the influence of investment in know-how is statistically meaningful (10 percent significance level).

The estimates also support the hypothesis of conditional convergence between countries. The implied value of convergence (λ) in the textbook Solow model and the human capital augmented model is 1.7 to 2 percent, and up to 2.6 percent in the further augmented model.

4. Conclusions

This paper extends the MRW model by generalizing the augmented Solow growth model. Apart from physical capital and human capital, we specifically include endogenous accumulation of technological know-how. For the sample of OECD countries, and assuming observed levels of per capita income correspond to steady-state levels, our empirical results reinforce MRW findings for the textbook Solow model and the human capital augmented model. Our fully augmented model (i.e. including physical capital, human capital and know-how) explains 3/4 of the variation in per capita income levels between OECD countries. However, in relaxing the assumption that economies are close to the steady-state and explicitly allowing for conditional convergence, our results are at variance with MRW. We find that the influence of human capital investment is far less important.

Table I. Definition of variables

Variable	definition
$\ln Y_t$ $\ln Y_0$	Natural log of real GDP per working-age population (i.e. age 15 to 65) in 1985 and 1960 (1985 international prices). Basic data source: Summers-Heston v.5.5
s_k	Average of annual ratios of real domestic investment to real GDP (1960-1985). Basic data source: Summers-Heston v.5.5
s_h	Average of annual ratios of total nominal government expenditure on education to nominal GDP (1960-1985). Basic data source: UNESCO
s_τ	Average of annual ratios gross domestic expenditure on research and development to nominal GDP (of available observations during 1975-1985). Basic data source: OECD (1989)
n	Annual population growth 1960-1985 (computed as ln(pop85/pop60)/25). Basic data source: Summers-Heston v.5.5

Table II. Least squares estimation results. Dependent variable: $\ln Y_t$

Independent variable	Textbook Solow model m = 1	Augmented Solow model m = 2	Extended Solow model m = 3
Unrestricted			
$\ln(s_k)$	+0.652 (0.574)	+0.592 (0.419)	+0.395 (0.305)
$\ln(s_h)$	-	+0.702 (0.167)°°	+0.145 (0.179)
$\ln(s_\tau)$	-	-	+0.379 (0.090)°°
$\ln(n+.05)$	-0.614 (0.826)	-0.739 (0.603)	+0.037 (0.472)
constant	+8.769 (2.538)°°	+10.504 (1.896)°°	+12.433 (1.441)°°
adj.R^2	0.007	0.472	0.756
s.e.e.	0.378	0.275	0.198
Restricted			
$\ln(s_k)-\ln(n+.05)$	+0.639 (0.425)	+0.405 (0.319)	+0.095 (0.257)
$\ln(s_h)-\ln(n+.05)$	-	+0.684 (0.162)°°	+0.151 (0.187)
$\ln(s_\tau)-\ln(n+.05)$	-	-	+0.355 (0.093)°°
constant	+8.681 (0.670)°°	+9.229 (0.511)°°	+10.169 (0.462)°°
adj.R^2	0.057	0.486	0.700
s.e.e.	0.368	0.272	0.208

	Textbook Solow model	Augmented Solow model	Extended Solow model
Implied Shares of:			
Effective Labor	0.61	0.48	0.63
Total capital	0.39	0.52	0.37
of which:			
% physical capital	100	37.20	15.80
% human capital		62.80	25.10
% know how			59.10

note: standard errors in parenthesis. °: significant at 10%.°°: significant at 5% or better.

Table III. Least squares estimation results. Dependent variable: $\ln(Y_t / Y_0)$

Independent variable	Textbook Solow model $m=1$	Augmented Solow model $m=2$	Extended Solow model $m=3$
Unrestricted			
$\ln Y_0$	-0.343	-0.390	-0.495
	(0.056)°°	(0.079)°°	(0.093)°°
$\ln(s_k)$	+0.650	+0.642	+0.560
	(0.202)°°	(0.204)	(0.195)°°
$\ln(s_h)$	-	+0.097	-0.001
		(0.113)	(0.118)
$\ln(s_\tau)$	-	-	+0.130
			(0.067)°
$\ln(n+.05)$	-0.576	-0.596	-0.359
	(0.291)°	(0.294)°	(0.302)
constant	+2.967	+3.627	+5.412
	(1.022)°°	(1.283)°°	(1.535)°°
adj.R^2	0.705	0.700	0.738
s.e.e.	0.133	0.134	0.125
Restricted			
$\ln Y_0$	-0.343	-0.386	-0.473
	(0.055)°°	(0.077)°°	(0.089)°°
$\ln(s_k)- \ln(n+.05)$	+0.624	+0.595	+0.470
	(0.149)°°	(0.155)°°	(0.165)°°
$\ln(s_h)- \ln(n+.05)$	-	+0.089	-0.007
		(0.108)	(0.117)
$\ln(s_\tau)- \ln(n+.05)$	-	-	+0.114
			(0.067)
constant	+2.798	+3.259	+4.398
	(0.546)°°	(0.784)	(1.002)
adj.R^2	0.720	0.715	0.742
s.e.e.	0.129	0.131	0.121
implied λ	1.7%	2.0%	2.6%

note: standard errors in parenthesis. °: significant at 10%.°°: significant at 5% or better.

Appendix I. Data set

	Y_{85}	Y_{60}	s_k	s_h	s_τ	n
Canada	23060	12361	0.2542	0.0682	0.0125	0.0197
USA	25014	16364	0.2397	0.0594	0.0255	0.0154
Japan	17669	4648	0.3658	0.0477	0.0240	0.0124
Austria	16646	7827	0.2828	0.0471	0.0110	0.0036
Belgium	16876	8609	0.2645	0.0541	0.0140	0.0045
Denmark	19406	10515	0.2915	0.0627	0.0110	0.0058
Finland	17776	8630	0.3852	0.0576	0.0120	0.0076
France	18546	9650	0.2972	0.0437	0.0205	0.0099
Germany	17969	9819	0.3095	0.0403	0.0245	0.0050
Greece	9492	3164	0.2885	0.0204	0.0020	0.0070
Ireland	12054	5454	0.2877	0.0505	0.0080	0.0105
Italy	16055	7086	0.3139	0.0376	0.0095	0.0064
Netherland	16937	10008	0.2789	0.0695	0.0205	0.0138
Norway	22107	8977	0.3494	0.0626	0.0145	0.0068
Portugal	7925	2965	0.2608	0.0267	0.0035	0.0060
Spain	11876	4916	0.2817	0.0190	0.0045	0.0090
Sweden	20826	11364	0.2636	0.0710	0.0225	0.0031
Switzerland	22428	14532	0.3142	0.0450	0.0230	0.0084
Turkey	5150	2884	0.2323	0.0355	0.0020	0.0271
UK	17034	10004	0.2067	0.0525	0.0225	0.0033
Australia	20617	12824	0.3128	0.0473	0.0105	0.0200
New Zealand	17319	13569	0.2680	0.0467	0.0095	0.0170

REFERENCES

AGHION, P. (1992), and HOWITT, P. "A Model of Growth Through Creative Destruction", *Econometrica*, 60(2), p. 323-351.

BARRO, R. (1992), and SALA-I-MARTIN, X., "Convergence", *Journal of Political Economy*, 100(2), p. 223-251.

BARRO, R. (1994), and LEE, J, "Data Set for a Panel of 138 Countries", Harvard University.

BARRO, R. (1995), and SALA-I-MARTIN, X, "Economic Growth", MacGraw and Hill, Forthcoming.

GROSSMAN, G. (1990), and HELPMAN, E., "Trade, Innovation and Economic Growth", *American Economic Review*, 80, p. 797-815.

GROSSMAN, G. (1991), and HELPMAN, E., "Quality Ladders in the Theory of Growth", *Review of Economic Studies*, 58, p. 43-61.

MANKIW, N. (1992), ROMER, D. and WEIL, D., "A Contribution to the Empirics of Economic Growth", *Quarterly Journal of Economics*, 107(2), p. 407-437.

OECD, (1989), Science and Technology Indicators (report) 3, technical annex, p. 95 -132.

ROMER, P. (1990), "Endogenous Technological Change", *Journal of Political Economy*, 98(5), p. S71-S102.

SUMMERS, R. (1991), and HESTON, A., "The Penn World Table (Mark 5): an Expanded Set of International Comparisons", *Quarterly Journal of Economics*, CVI(2), p. 327-368.

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