THE MALMQUIST PRODUCTIVITY INDEX AND PLANT CAPACITY UTILISATION

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Abstract

This note proposes a new decomposition of the Malmquist productivity index to account for changes in plant capacity utilisation. Using a primal, non-parametric specification of technology, the Malmquist index is decomposed into technical efficiency change, variations in plant capacity utilisation, and frontier shifts. It provides an alternative to the available methods to incorporate capacity utilisation changes in measures of productivity change. The latter are based upon parametric (and, in many cases, dual) technology specifications; moreover, they typically do not allow for technical inefficiency.

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I. Introduction

The Malmquist productivity index, defined by Caves, Christensen and Diewert (1982) as a ratio of distance functions, has gained some popularity in applied work (see, e.g., Berg, Forsund and Jansen (1992)). Recently, Färe et al. (1995) developed a straightforward computational procedure to calculate the Malmquist index relative to non-parametric frontier technologies by exploiting the inverse relationship between output distance functions and output-oriented technical efficiency measures[1]. Furthermore, they relaxed the implicit hypothesis of technical efficiency maintained in Caves, Christensen and Diewert (1982) and showed that the Malmquist productivity index can be decomposed into technical efficiency changes and technology shifts.

One potentially important issue ignored in applications of the Malmquist productivity index is that changes in technical efficiency may be partially due to changes in the utilisation of production capacity. The purpose of this paper is therefore to suggest a further decomposition of the Malmquist productivity index to separate technical efficiency changes from variations in capacity utilisation. More specifically, we integrate Johansen's (1968) measure of plant capacity utilisation (also see Färe, Grosskopf and Kokkelenberg (1989)) into the Malmquist index. This allows us to decompose productivity changes into frontier shifts, variations in technical efficiency, and variations in capacity utilisation.

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[1] Given the difficulties involved in computing distance functions at the time, Caves, Christensen and Diewert (1982) approximated the Malmquist productivity index by a Törnqvist index. The latter is computable from price and quantity data only, and does not require a precise knowledge of technology. Unfortunately, the conditions under which it provides a good approximation to the Malmquist index are quite stringent. The Färe et al. (1995) methodology makes recourse to the Törnquist unnecessary.

Integrating variations in plant capacity utilisation in the development of the Malmquist productivity index is a welcome addition to the literature for several reasons. First, although for parametric specifications of technology various productivity decompositions have been suggested to incorporate measures of capacity utilisation (see, among others, Hulten (1986) and Morrison (1985, 1993)), such decompositions are as yet unavailable for non-parametric technology representations. Second, unlike the available parametric approaches the decomposition suggested in this paper incorporates the possibility of technical inefficiency. Third, since the Malmquist has so far been defined for primal technologies only, we use a primal concept of capacity, unlike previous studies based on dual representations of technology (e.g., (Morrison (1985) and Squires (1987)). A primal approach is a useful alternative that may be especially relevant in situations where prices are unreliable or unavailable.

Structure of this note is as follows. In Section 2 we first review the Malmquist productivity index and the Johansen (1968) measure of plant capacity utilisation. We then show how to decompose Malmquist productivity changes into frontier shifts, variations in technical efficiency, and variations in capacity utilisation. Section 3 concludes.

II. Malmquist Productivity Indexes and Capacity Utilisation

Malmquist Productivity Indexes

Assume that for periods t = 1,...,T one observes m inputs ($x^t \in \Re_+^m$) producing n outputs ($y^t \in \Re_+^n$). In each period t, the production technology is defined by the set of feasible input/output vectors: $S^t = \{(x^t,y^t)|\ x^t \text{ can produce } y^t\}$. The output set $P^t(x^t)$ denotes all output vectors y^t that can be produced from the input vector x^t, i.e., $P^t(x^t) = \{y^t | (x^t, y^t) \in S^t\}$. The output distance function is defined as:

$$D_o^t(x^t, y^t) = \min\{\theta | (y^t/\theta) \in P^t(x^t)\}. \tag{1}$$

It treats the inputs as given and expands, in a proportional way, the output vector until $y^t/D_o^t(x^t,y^t)$ belongs to the isoquant of the output set (Isoq $P^t(x^t)$). For future reference it is important to note that output distance functions are inversely related to the radial technical efficiency measures in the outputs (Färe, Grosskopf and Lovell (1994)). Denoting radial output efficiency by $DF_o^t(x^t, y^t)$ we have

$$DF_o^t(x^t,y^t) = 1/D_o^t(x^t,y^t). \tag{2}$$

An output-based Malmquist productivity index with base period t was defined by Caves, Christensen and Diewert (1982) as the ratio of two output distance functions:

$$M_o^t(x^t,y^t, x^{t+1},y^{t+1}) = D_o^t(x^{t+1}, y^{t+1}) / D_o^t(x^t, y^t), \tag{3}$$

where $D_o^t(x^t, y^t)$ and $D_o^t(x^{t+1}, y^{t+1})$ are output distance functions relating observations in period t and t+1, respectively, to a period t technology. Of course, a Malmquist productivity index in the outputs with base period t+1 can similarly be defined as

$$M_o^{t+1}(x^t,y^t, x^{t+1},y^{t+1}) = D_o^{t+1}(x^{t+1}, y^{t+1}) / D_o^{t+1}(x^t, y^t). \tag{4}$$

Therefore, to avoid an arbitrary choice of base period Färe et al. (1995) proposed to define the output-oriented Malmquist productivity index as a geometric mean of (3) and (4):

$$\begin{aligned} M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) &= \sqrt{M_o^t(x^t, y^t, x^{t+1}, y^{t+1}).M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1})} \\ &= \sqrt{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}.\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)}}. \end{aligned} \tag{5}$$

The base period of this productivity index changes over time. It can be conceptualised as an index computed in a two year window sliding over the observations in time. Moreover, the Malmquist index (5) can be decomposed into two mutually exclusive components:

$$M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \sqrt{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \cdot \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)}}. \qquad (6)$$

The first component measures the change in technical efficiency over time while the second is related to the shifts of the frontier of the production technology, i.e., it captures technical change. If $M_o^{t,t+1}(x^t,y^t, x^{t+1},y^{t+1})$ is larger (smaller) than unity, this indicates an improvement (deterioration) in productivity.[2] A similar interpretation applies to the separate components.

Capacity Utilisation

To maintain consistency with the primal approach underlying the Malmquist productivity index this paper focuses on a primal definition of capacity. In particular, we follow the seminal contribution by Johansen (1968) who defines plant capacity as the maximal amount that can be produced per unit of time with existing plant and equipment without restrictions on the availability of variable production factors. It is clear that this capacity notion is a technical (engineering) concept that, unlike economic (cost) capacity notions, is not based on optimising behaviour.[3] Of course, from an economic point of view this can be considered a disadvantage. However, this should be traded-off against various more attractive features of Johansen's concept. For example, Johansen (1968) shows that, almost independent of the production model

[2] The technical efficiency change component has been further decomposed into variations in technical efficiency, scale efficiency and congestion (Färe, Grosskopf and Lovell (1994)). The important issue of identifying scale effects in the technical change component has led to a discussion from which no consensus has yet emerged (see, e.g., Balk (1998) and Färe, Grosskopf and Roos (1998)).

[3] One capacity notion considers the output produced at short run minimum average total cost, given existing plant and factor prices (e.g., Morrison (1985)). Another definition looks at the output for which short and long run average total costs curves are tangent (e.g., Segerson and Squires (1990)). Both coincide under constant returns to scale.

one is assuming (fixed proportions, neo-classical production, or putty-clay models), his capacity concept meets many of the criteria typically required (existence, attainability, aggregation, consistency, etc.). He also argues that most estimation methods, such as engineering and survey-based methods, explicitly or implicitly have this specific plant capacity concept in mind.[4] Finally, avoiding any strong hypotheses on the optimising behaviour of the scrutinised organisations may be a clear advantage in cases where the hypotheses are unlikely to hold (e.g., in the public sector). Rather than embarking on a full discussion of the relative merits of different capacity notions we simply note that the relations between several economic and technical capacity measures are well known (Nelson (1989)).

A method for calculating Johansen's plant capacity utilisation measure using non-parametric, deterministic specifications of technology has been introduced in Färe, Grosskopf and Kokkelenberg (1989) and Färe, Grosskopf and Valdmanis (1989). Partitioning the input vector $x^t = (x_f^t, x_v^t)$ into fixed (x_f^t) and variable (x_v^t) inputs at period t, plant capacity utilisation ($\mathrm{PCU}^t(x^t, x_f^t, y^t)$) at time t is defined as follows:

$$\mathrm{PCU}_o^t(x^t, x_f^t, y^t) = \frac{\mathrm{DF}_o^t(x^t, y^t)}{\mathrm{DF}_o^t(x_f^t, y^t)}. \tag{7}$$

where $\mathrm{DF}_o{}^t(x^t, y^t)$ and $\mathrm{DF}_o{}^t(x_f^t, y^t)$ are output efficiency measures relative to technologies including, respectively excluding, the variable inputs. By definition $\mathrm{DF}_o{}^t(x_f^t, y^t) \geq \mathrm{DF}_o{}^t(x^t, y) \geq 1$, hence $\mathrm{PCU}^t(x^t, x_f^t, y^t) \leq 1$. Any deviation from unity is interpreted as a proportional decrease in actual outputs compared to outputs at full plant capacity. Importantly, note that the plant capacity utilisation index (7) is obtained by first removing any existing technical inefficiency: indeed, it is

[4] Christiano (1981) presents an overview of both data-based and survey methods for estimating capacity utilisation. He acknowledges (p. 171) that many survey respondents have an engineering concept in mind.

computed as a ratio of efficiency measures. Elimination of inefficiencies implies that it is not downward biased, in contrast to most traditional capacity utilisation measures.

Malmquist Productivity Indexes and Plant Capacity Utilisation

In order to isolate changes in capacity utilisation in the definition of productivity increases we suggest the following decomposition of the Malmquist productivity index so as to explicitly separate variations in plant capacity utilisation, changes in efficiency, and pure technical change. We start by noting that technical efficiency can be written as

$$\begin{aligned} \mathrm{DF}_{\mathrm{o}}^{\mathrm{t}}(x^t, y^t) &= \mathrm{DF}_{\mathrm{o}}^{\mathrm{t}}(x_f^t, y^t).\frac{\mathrm{DF}_{\mathrm{o}}^{\mathrm{t}}(x^t, y^t)}{\mathrm{DF}_{\mathrm{o}}^{\mathrm{t}}(x_f^t, y^t)} \\ &= \mathrm{DF}_{\mathrm{o}}^{\mathrm{t}}(x_f^t, y^t).\mathrm{PCU}_{\mathrm{o}}^{\mathrm{t}}(x^t, x_f^t, y^t). \end{aligned} \tag{8}$$

The last equality is based on (7) above (see also Färe, Grosskopf and Kokkelenberg (1989)). In other words, technical efficiency equals the product of technical efficiency relative to a full capacity (short-run) technology and plant capacity utilisation.

Using (8) and the relation between the radial efficiency measure and the output distance functions in (2), we can decompose the technical efficiency change component of the Malmquist productivity index $\mathrm{M}_{\mathrm{o}}^{\mathrm{t,t+1}}(x^t,y^t,x^{t+1},y^{t+1})$. Indeed, incorporating (2) and (8) in (6), we obtain:

$$\begin{aligned} \mathrm{M}_{\mathrm{o}}^{\mathrm{t,t+1}}(x^t, y^t, x^{t+1}, y^{t+1}) = &\frac{\mathrm{D}_{\mathrm{o}}^{\mathrm{t+1}}(x_f^{t+1}, y^{t+1})}{\mathrm{D}_{\mathrm{o}}^{\mathrm{t}}(x_f^t, y^t)}.\frac{\mathrm{PCU}^{\mathrm{t}}(x^t, x_f^t, y^t)}{\mathrm{PCU}^{\mathrm{t+1}}(x^{t+1}, x_f^{t+1}, y^{t+1})}. \\ &\sqrt{\frac{\mathrm{D}_{\mathrm{o}}^{\mathrm{t}}(x^{t+1}, y^{t+1})}{\mathrm{D}_{\mathrm{o}}^{\mathrm{t+1}}(x^{t+1}, y^{t+1})}.\frac{\mathrm{D}_{\mathrm{o}}^{\mathrm{t}}(x^t, y^t)}{\mathrm{D}_{\mathrm{o}}^{\mathrm{t+1}}(x^t, y^t)}}. \end{aligned} \tag{9}$$

Expression (9) shows that productivity changes are the combined result of three separate phenomena. The first component measures the change in technical efficiency assuming a constant degree of capacity utilisation. Specifically, it evaluates the change in technical efficiency relative to a full capacity output technology between periods t and t+1. The second component captures the change in the degree of plant capacity utilisation between t and t+1, holding technical efficiency levels constant. The third component is the same as in (6) and reflects pure technical change. When any of the components is larger (smaller) than unity, this indicates an improvement (deterioration) in the corresponding component, except for the component indicating changes in plant capacity utilisation. For the latter a number smaller (larger) than unity indicates an improvement (deterioration). In other words, this decomposition of the Malmquist productivity index provides a straightforward procedure to relate productivity growth to the dynamics of capacity utilisation.

The suggested decomposition of the Malmquist index is easily calculated for non-parametric technologies. Since (9) only requires data on inputs and outputs (no price information is necessary) and given the importance of correctly evaluating changes in capacity utilisation, this index may have relevant empirical applicability. At the micro level for example, firm panel data could be used to separate variations in technical efficiency from variations in capacity utilisation. At the macroeconomic level, given the relevance of capacity utilisation as a leading indicator for inflation and business cycle movements (Corrado and Mattey (1997)), an easily computable procedure for separating efficiency changes and variations in capacity utilisation may be important for the correct interpretation of both macro-economic times series and cross-national comparisons of macro data. In this respect, this index provides an interesting alternative to macro-economic estimation methods based on surveys (Christiano (1981)).

III. Conclusion

In this note we have integrated the measurement of plant capacity into the definition of the Malmquist productivity index. The latter was decomposed into technical efficiency changes, variations in capacity utilisation, and technical change. This is important since what might otherwise appear as inefficiency in a standard application of the Malmquist index may to some extent be explained by variations in capacity utilisation.

We end by noting an important avenue of research. It would be highly desirable to develop discrete time dual technical change indexes (see Balk (1998) for a recent proposal). These could then be similarly extended by means of economic capacity notions to capture changes in capacity utilisation.

References

Balk, B. (1998), *Industrial Price, Quantity, and Productivity Indices: The Micro-Economic Theory and an Application*, Kluwer, Boston.

Berg, S.A., Førsund, F.R. and Jansen, E.S. (1992), Malmquist Indices of Productivity Growth during the Deregulation of Norvegian Banking, 1980-89, *Scandinavian Journal of Economics 94*, s211-s228.

Caves, D., Christensen, L., and Diewert, W.E. (1982), The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity, *Econometrica 50*, 1393-1414.

Christiano, L. (1981), A Survey of Measures of Capacity Utilization, *IMF Staff Papers 28*, 144-198.

Corrado, C., and Mattey, J. (1997), Capacity Utilization, *Journal of Economic Perspectives 11*, 151-167.

Färe, R., Grosskopf, S., and Kokkelenberg, E. (1989), Measuring Plant Capacity, Utilization and Technical Change: A Nonparametric Approach, *International Economic Review 30*, 655-666.

Färe, R., Grosskopf, S. and Lovell, C.A.K. (1994), *Production Frontiers*, Cambridge University Press, Cambridge.

Färe, R. et al. (1995), Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach, in A. Charnes et al. (eds) *Data Envelopment Analysis: Theory, Methodology and Applications*, Kluwer, Boston, 253-272.

Färe, R., Grosskopf, S. and Roos, P. (1998), Malmquist Productivity Indices: A Survey of Theory and Practice, in R. Färe, S. Grosskopf, R. Russell (eds) *Index Numbers: Essays in Honour of Sten Malmquist*, Kluwer, Boston, 127-190.

Färe, R., Grosskopf, S., and Valdmanis, V. (1989), Capacity, Competition and Efficiency in Hospitals: A Nonparametric Approach, *Journal of Productivity Analysis 1*, 123-138.

Hulten, C. (1986), Productivity Change, Capacity Utilization, and the Sources of Productivity Growth, *Journal of Econometrics 33*, 31-50.

Johansen, L. (1968), Production Functions and the Concept of Capacity, Namur, Recherches Récentes sur la Fonction de Production (Collection "Economie Mathématique et Econometrie", n° 2) [also in F.R. Førsund (ed.) (1987) *Collected Works of Leif Johansen*, Vol. 1, Amsterdam, North Holland, 359-382].

Morrison, C. (1985), Primal and Dual Capacity Utilization: An Application to Productivity Measurement in the U.S. Automobile Industry, *Journal of Business and Economic Statistics 3*, 312-324.

Morrison, C. (1993), *A Microeconomic Approach to the Measurement of Economic Performance: Productivity Growth, Capacity Utilization, and Related Performance Indicators*, Springer Verlag, New York.

Nelson, R. (1989), On the Measurement of Capacity Utilization, *Journal of Industrial Economics 37*, 273-286.

Segerson, K., and Squires, D. (1990), On the Measurement of Economic Capacity Utilization for Multi-Product Industries, *Journal of Econometrics 44*, 347-361.

Squires, D. (1987), Long-Run Profit Functions for Multiproduct Firms, *American Journal of Agricultural Economics 69*, 558-569.