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Port capacity investment size and timing under uncertainty and congestion

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Abstract

Well-planned port capacity investments are required to accommodate growing maritime trade, especially in developing countries. Such investments prove to be expensive, irreversible and subject to uncertainty. Under these conditions, the application of real options is better suited than the traditionally used NPV approach, given that specific port and market characteristics are taken into account. In this paper, we consider the investment in a new container port. The port is operated by one single actor who also owns the port and absence of competing ports in the neighbourhood is assumed. Two decisions have to be made: the size of the port and when to build it. Our real options model evaluating the option of flexible size and timing of the investment is based on a geometric Brownian motion. This approach has the advantage of analysing the impact of economic growth and uncertainty independently. Additionally, port users differ in aversion to congestion. Our model allows examining this impact on the optimal port investment. This paper shows that ports with more waiting-time averse customers are better off when investment is delayed to install a larger port. This result holds in both privately owned, profit-maximising ports and government-owned, social welfare-maximising ports.

Keywords: port capacity investment; congestion; uncertainty; flexible investment size; flexible timing; real options.

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1 Introduction

Activities performed in ports, such as cargo handling, facilitate international and regional trade, as well as the economic development of a region (Verhoeven, 2015; De Langen *et al.*, 2018). Disposing of an insufficient number of docks, terminals, cranes, pavements and other interrelated elements determining service capacity to handle cargo involves high logistics costs (Novaes *et al.*, 2012). Consequences may involve a slowdown of the growth of a region's GDP, freight delays and increased trade distances (Kauppila *et al.*, 2016).

Port infrastructure capacity investments are irreversible, subject to many sources of uncertainty and involve large sums of money (Vanelslander, 2014). The real options (RO) approach to investment was developed to help determine the flexible, optimal amount of capacity for this type of investment project and the ideal moment to invest.

The port capacity investment decision is complicated by the aversion to waiting-time of the shipping companies using the port's capacity. This waiting time generates delays involving considerable costs (Novaes *et al.*, 2012). Aversion to waiting time differs among users and among goods categories (De Jong, 2000). For example, liners shipping containers with food are more waiting-time averse than liners shipping construction materials. Waiting time in ports generally begins to increase as infrastructure occupancy rates pass 50% and is the result of uncertainty regarding the distribution of ship arrivals at a terminal with a given theoretical capacity (Blauwens *et al.*, 2016). Installing a larger port would lead to lower waiting time and less congestion when handle the same amount of throughput. As a result, travel times would be shorter, since capacity serves as a buffer against waiting time caused by future demand growth. This is of measurable value to shippers. Yet, previous studies have shown that investment in more capacity attracts additional traffic and throughput, offsetting part of this capacity investment benefit (Zhang, 2007). What remains unclear however is how the port authority's investment decision in a new port is influenced by port users' aversion to waiting time.

Port infrastructure capacity investments are irreversible, subject to many sources of uncertainty and involve large sums of money (Vanelslander, 2014). The real options (RO) approach to investment was developed to help determine the flexible, optimal amount of capacity for this type of investment project and the ideal moment to invest. In this paper, we consider the option to invest in a new container port in a growth market under uncertainty. The port is operated by one single actor who also owns the port. In the considered region, we assume only one significant port, leaving port competition beyond the scope of this paper. Two 'once and for all' decisions have to be made: the size of the port and when to build it. We take into account that next to the capacity decision in the first (investment) stage, the port can flexibly adapt its price level in the second (operational) stage to influence throughput and port occupancy. Compared to Balliauw *et al.* (2019), our approach omitting competition allows focusing on the impact of congestion and uncertainty on an individual port's optimal investment decision. Due to preemption in a competitive setting, competing ports are not able to invest in their unrestricted optimal investment strategy. They have to invest sooner. Since this paper considers a single port not experiencing competition, an in depth analysis of the impact of changes in port- and project-related economic characteristics on such a port's unrestricted strategy is required. Additionally, we consider first a privately owned and financed port that maximises profits and second a public port that maximises social welfare. Such investment decisions are relevant in growing developing countries with a lot of uncertainty. Examples include the greenfield container ports in Manaus and Porto Central (both in Brazil) and north of Izmir (in Turkey), as well as port development in African countries, where port competition is limited (Vanelslander, 2014). Another example could be a sole container port on an island, such as the container port of Cagliari in Sardinia.

Our paper compares to a number of other papers, also modelling uncertainty through a geometric Brownian motion (GBM). The GBM is considered to be a more realistic representation of uncertainty than the uniform distribution used by Chen & Liu (2016). The GBM moreover allows analysing the impact of economic growth and uncertainty independently. Meersman (2005) uses an uncertain present value of the port investment project to show that the NPV rule is not correct in case of uncertainty and managerial flexibility. Zheng & Negenborn (2017) apply RO to the investment in a new terminal. They assume total demand to be uncertain, and then use disaggregated origin-destination pairs. In our paper, uncertainty is situated in the intercept of the demand function.

We contribute to the literature by using a real options approach to study how the aversion to waiting of port users affects the optimal investment in new port capacity under demand uncertainty. Our approach to congestion costs allows comparing different projects for different shippers and goods categories, each with their own aversion to waiting. Our RO approach also represents a more accurate continuous time and state model for ports to decide about their optimal investment strategies, compared to models omitting congestion and the costs it involves (Dangl, 1999; Hagspiel *et al.*, 2016; Novaes *et al.*, 2012).

The structure of this paper is as follows. The next section deals with the economic context considered in this paper and explains the methodology used to identify the optimal size and timing of the port capacity investment decision under uncertainty. In Section 3, we elaborate the value calculation of an investment project under uncertainty. Section 4 explains the selection of the parameters used for the numerical application of the model to a new port, since for the rest of the calculations, analytical solutions are not obtainable. In Section 5 we derive the optimal size and timing of the investment decision and analyse the impact of congestion and uncertainty on the decision. A sensitivity analysis in Section 6 shows how the investment decision depends on the various economic parameter values. Section 7 considers the optimal investment decision in a new port that is owned publicly. The final section presents a number of conclusions and potential avenues for future research.

2 Economic setting and methodology

To start, we consider a profit-maximising port with flexible throughput. It is assumed that the port faces at time t the following linear inverse demand function:

$$\rho(t) = X(t) - Bq(t), \tag{1}$$

with $\rho(t)$ the full price, $q(t)$ the throughput, $X(t)$ the value of an exogenous demand shift parameter at time t and B the slope of the inverse demand curve. Throughput is measured in terms of container twenty-foot equivalent units (TEU). It is assumed that the theoretical design capacity of the new port, K, puts an upper limit on the throughput that can be handled. As such it is assumed that $0 \leqslant q(t) \leqslant K$ and as a consequence the capacity utilisation rate $q(t)/K$ is constrained between zero and one. Hence, we disregard exceptional cases where realised throughput $q(t)$ temporarily exceeds K at peak moments (Luo *et al.*, 2010).

Demand uncertainty is introduced in the model by random shifts of the demand function. These are introduced by a random process for $X(t)$ which is assumed to follow a geometric Brownian motion (GBM):

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}Z(t), \tag{2}$$

specified by parameters μ (expected positive drift in demand shift) and σ (variability). Z represents a standard Wiener process. A GBM is often used to model uncertainty, because both drift and variability are included in the random process as independent parameters. The implementation of this approach is however new in the context of port capacity investment decisions under congestion. Chen & Liu (2016) for example use a uniform distribution to model demand uncertainty in a port. Another advantage of the GBM is that with an initial value of $X(t = 0) > 0$, the variable $X(t)$ always remains positive, even if the expected drift is negative (Dixit & Pindyck, 1994). Here, we focus on a base case with a positive drift μ, reflecting economic growth and growing international trade (UNCTAD, 2015). In such a case, demand shifts at the rate of μ, on average, but over certain time intervals, growth may be higher, lower or even negative due to the uncertainty. For the sake of readability, the (t)-dependencies will be omitted in the following equations.

According to Xiao *et al.* (2012), the full price or gross willingness to pay of customers is the sum of the unit price of services at the port (p) and the cost of congestion the users incur:

$$\rho = p + AX\frac{q}{K^2}. \tag{3}$$

Hence, p can be rewritten as:

$$p = X - Bq - AX\frac{q}{K^2}, \tag{4}$$

with the final term the unit cost of congestion which reduces the customer's gross willingness to pay. Total congestion cost TCC equals $AX(q/K)^2$, an own specification based on queuing theory insights from other authors (De Borger & Van Dender, 2006; Yuen *et al.*, 2008; Xiao *et al.*, 2012; Zhang & Zhang, 2006; Chen & Liu, 2016).

Xiao *et al.* (2012), De Borger & Van Dender (2006), Yuen *et al.* (2008) and Chen & Liu (2016) make delays D linearly dependent on capacity occupancy. However, queuing theory learns that congestion only starts at occupancy rates of about 50%, and increases sharply beyond 75 to 80% (Blauwens *et al.*, 2016; Kauppila *et al.*, 2016). To this end, Zhang & Zhang (2006) use a specification where delay depends on a scaling factor times $q/[K(K-q)]$. This specification is based on estimates from steady-state queuing theory, with the underlying assumption of Poisson distributed arrivals (Lave & DeSalvo, 1968). It satisfies four conditions:

$$\frac{\partial D}{\partial q} > 0, \frac{\partial D}{\partial K} < 0, \frac{\partial^2 D}{\partial q^2} > 0, \frac{\partial^2 D}{\partial q \partial K} < 0. \tag{5}$$

These conditions imply that higher throughput and lower capacity lead to an increase in congestion and that these effects are stronger when congestion is already high.

The condition $\partial^2 D/\partial q^2 > 0$ however does not hold for the linear specification on the one hand. On the other hand, the disadvantage of the specification of Zhang & Zhang (2006) is that delays, and hence costs as well after a multiplication with a monetary scaling factor, are infinitely high at full occupation. This is not the case in reality either. A functional specification that highly resembles the specification of Zhang & Zhang (2006) would be a higher order term of occupancy in the delay function, such as a fourth or fifth order term. This would satisfy all conditions of Zhang & Zhang (2006) in Equation (5), whereas full occupancy would not ex-ante be excluded. However, the exact, analytical optimisations in this paper would become too complicated under such a specification of the delay function, due to the consideration of quantity flexibility. Also a second order dependency would already be more realistic than the first order relationship, also satisfying the required conditions, but would still too much complicate the calculations. Therefore, a proxy for q is required, which can then replace the factor q^1 in q^2. The uncertain reservation price, X, is positively correlated with the optimal throughput at any point in time. This explains why q^2 is replaced by Xq, using X as a proxy for q.

In a next step, delays are translated into costs through the monetary scaling factor A (Xiao *et al.*, 2012). The parameter A depends on the port users and good types involved (Yuen *et al.*, 2008). The monetary scaling factor (A) is an expression of the value of time (De Jong, 2000) and is lower for shipping companies that transport goods which are less urgently needed or less impacted by time (e.g. wood compared to perishables or critical machine components) and for shipping companies with less stringent sailing schemes. In such cases, congestion is less of a problem for the port user. However, different individual liners that will use the new capacity might have different values of time. Therefore, A should be interpreted as the average value of time of the new capacity's users. Moreover, parameter A is fixed over time in this RO model. Therefore, including X in the congestion cost function has the advantage that the excess growth of X compared to the growth of q can account for rising values of time in the future (De Jong, 2000). Moreover, the customer base of a port is relatively stable over time, because container liners became less footloose (Verhoeven, 2015; Heaver *et al.*, 2001). Hence, the assumption of a fixed average scaling factor A per project is reasonable. Nevertheless, the value for A may differ between projects, as the users may have a different aversion to waiting. It is exactly this approach that allows illustrating the effect of different waiting-time aversion of users on the timing and size of the investment in a new port, through the variation of A in the analysis.

The total operational cost function of the port is given by

$$TC_O(q) = cq, \tag{6}$$

where c is the constant marginal operational cost, like was also used by Haralambides (2002). In the literature, no consensus exists about the existence of economies or diseconomies of scale in port operations. For a comprehensive overview, we refer to the literature review of Tovar *et al.* (2007, p. 210). Because of the ambiguity in literature, a simple linear production function is assumed here. The instantaneous port profit, π, can now be calculated as

$$\pi(X, K, q) = p(q) \cdot q - cq - c_h K, \tag{7}$$

as total revenue *minus* operational costs (TC_O) and $c_h K$, the total cost to hold the capacity in place, for example through maintenance. All parameters included in the model are assumed to be positive.

The optimal throughput $q^{opt}(X, K)$ for given X and capacity level K is the result of maximising profit π with respect to throughput q. This leads to the following total congestion cost TCC dependent on given X and K:

$$TCC(X, K) = AX \left(\frac{q^{opt}(X, K)}{K} \right)^2. \tag{8}$$

It is represented by the exponential black curve in Figure 1 for the fixed port capacity level K of 10 million TEU per year.

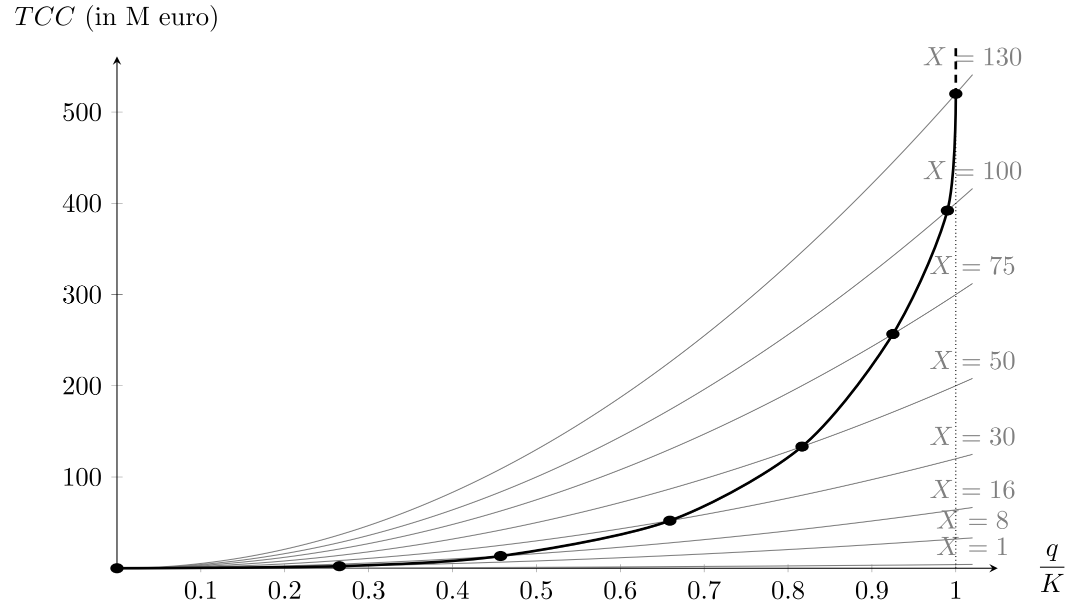

Figure 1: Realised total congestion cost function with installed capacity $K = 10$ million TEU per year. Other parameter values: $A = 4, B = 1, c = 1$ euro per TEU, $c_h = 0.5$ euro per TEU.

Assuming a deterministic investment cost $I(K)$ of installing a new port of size K, it is considered to be independent of the investment timing. It is given by the following function:

$$I(K) = FC_I + \gamma_1 K - \gamma_2 K^2 + 0K^3 + \gamma_4 K^4, \tag{9}$$

where FC_I is the fixed investment cost, e.g. resulting from a preliminary feasibility study. The cost of the investment grows with size K. Installing a larger dock involves more dredging activities and installing more quay walls and pavings. This is reflected by a positive first order term. General RO literature (Dangl, 1999; Hagspiel *et al.*, 2016) and port literature (Haralambides, 2002) indicate the existence of economies of scale in the investment size. Investment costs increase less than proportionally with an increase in project size. This is reflected by the negative second order term. Additionally, waterfront land is very costly (Haralambides, 2002). The available land for port infrastructure is limited. A larger investment than the available land would for instance imply expropriating houses, which is very costly. Therefore, a fourth order term is added to the investment cost, reflecting an investment size boundary with a very sharp cost increase. Although a third order term could also be used to model this boundary, the advantage of a fourth order polynomial with the third order term omitted is that the first order derivative of the function is not symmetric about the inflection point. Hence, the proposed mathematical specification does not impose that the economies of scale need to follow the same functional shape as the investment size boundary. These considerations are taken into account in Eq. (9) by selecting appropriate values for the γ's. The γ's should adhere the condition: $|\gamma_1| > |\gamma_2| > |\gamma_4|$.

Additionally, it is important to point out that in practice, the shape and height of the investment cost function depend to a large extent on the characteristics of the project, such as the geographical situation, the production technology to be used, etc. The mathematical specification of the investment cost function needs to be considered as a translation of the reality into a model. In practice, it is necessary for the deciding actor to have a good insight in the different possible sizes of the project and the respective investment costs. The two most important aspects are the correct height of the cost for each size and the first order derivative with respect to size, to express correct cost increases between different project sizes.

Given this information, the decision is to be made when and how much to invest. The former means to decide on the critical value X_T of the demand shift parameter for which it becomes optimal to invest instead of waiting. This implies that investment will take place as soon as X reaches this threshold for the first time from below.[1] If $X(t=0)$ however exceeds X_T, investment takes place right away. At the moment of investing, also the optimal size of the capacity K has to be determined. As long as the demand shift parameter X remains below X_T, it is better to wait and keep the option open. Therefore, this region $X < X_T$ is called the *waiting region*. In this region, for $t < T$ such that $X(t) < X_T$, the value of the option is given by

$$F(X|X < X_T) = \mathrm{e}^{-r\mathrm{d}t}\mathbb{E}\left[F(X) + \mathrm{d}F(X)\right]. \tag{10}$$

At the moment (T) the demand shift parameters X equals X_T for the first time, the demand is sufficiently high to make it optimal to invest. In this *investment region* $X \geqslant X_T$, the value of the option becomes equal to the return of the investment. This return will depend upon the demand at the moment of investment (as determined by X) and the installed capacity K. This capacity will be chosen such that the return of the investment is maximised. This implies that for $t \geqslant T$ where $X(t) \geqslant X_T$, the value of the option is given by

$$F(X|X \geqslant X_T) = \max_K [V(X,K) - I(K)], \tag{11}$$

with V the project value. This project value equals the present value of the annual profits (or losses) resulting from the project after it has been installed.

The model can be summarised as follows. The objective function for the investment maximises the expected future discounted profit stream minus the investment outlay of the project with respect to the timing T, where $X(t=T) \underset{\nearrow}{=} X_T$, and the capacity K (Huberts *et al.*, 2015). If $X(t=0) < X_T$ so that it is not optimal to invest from the beginning, this investment problem objective function is given by

$$\max_{T \geqslant 0, K \geqslant 0} \mathbf{E}\left\{\left[V(X_T,K) - I(K)\right]\mathrm{e}^{-rT}|X(t=0) = X\right\}, \tag{12}$$

[1] Hence, a higher threshold value corresponds to investing later.

with the value of the investment project

$$V = \mathbf{E}\int_0^\infty \max_q\{\pi(T+\tau)\}\mathrm{e}^{-r\tau}\mathrm{d}\tau. \tag{13}$$

The mathematical details and the technical background of the V-calculation and the optimisation of π in a port context with congestion is given in the next section. For these calculations, analytical solutions can be derived. Since this is not the case for the rest of the paper, Section 4 determines numerical values for the model parameters, before calculating numerically the investment decision that maximises the option value F in Section 5.

3 Value of the investment project

The first step of the dynamic programming methodology determines the value V of the new port project, once built at a cost $I(K)$. For a private port, the project value V is the sum of the discounted maximum realised profits (or losses), specified in Eq. (13), in which the discount rate $r > \mu$ in order to guarantee convergence of the model (Dixit & Pindyck, 1994).[2] Once the investment is made and capacity K is determined and becomes fixed, the instantaneous profit under different values for the demand shift parameter X can only be altered and maximised through the pricing mechanism, to result in an optimal throughput quantity q^{opt}. The optimal throughput quantity at each point in time is given by the first order condition for maximising π in Eq. (7) and satisfies the second order condition. It depends on the current value of the demand shift parameter X and the installed capacity K and is given by

$$q^{opt}(X,K) = \begin{cases} 0, & 0 \leqslant X < c : [R_1], \\ \dfrac{(X-c)K^2}{2(XA+BK^2)}, & c \leqslant X < \dfrac{K(2BK+c)}{K-2A} : [R_2], \\ K, & X \geqslant \dfrac{K(2BK+c)}{K-2A} : [R_3]. \end{cases} \tag{14}$$

Through the boundary values for X ensuring $0 \leqslant q^{opt} \leqslant K$ in Eq. (14), three regions can be identified for X: R_1, R_2 and R_3. In R_1, no throughput will be handled, since the cost will always be higher than the return. In R_2, q^{opt} increases in X and in K, which means that throughput increases along with demand and more capacity installed.[3] Since $X > 0$ in a GBM, it can also be seen that R_3 only exists if $K > 2A$. Otherwise, a fully occupied port is always prevented by price increases, as port users are willing to avoid congestion at any expense. In that case, it always holds that $q^{opt} < K$.

Each region identified has a resulting optimal instantaneous profit (or loss):

$$\pi(X,K,q^{opt}(X,K)) = \overline{\pi}(X,K) = \begin{cases} \overline{\pi}_1(X,K) = -c_h K, & X \in R_1, \\ \overline{\pi}_2(X,K) = \dfrac{(X-c)^2K^2}{4(XA+BK^2)} - c_h K, & X \in R_2, \\ \overline{\pi}_3(X,K) = (K-A)\,X - (BK+c+c_h)K, & X \in R_3. \end{cases} \tag{15}$$

Now, Eq. (13) can be rewritten as a dynamic programming problem:

$$V(X,K) = \overline{\pi}(X,K)\mathrm{d}t + \mathrm{e}^{-r\mathrm{d}t}\,\mathbf{E}(V(X,K) + \mathrm{d}V(X,K)). \tag{16}$$

Here, V is a function of the value for X at the moment of investment and the installed capacity K. Using Itô's Lemma and the Bellman equation (Dixit & Pindyck, 1994), we arrive at a differential

[2]Without this convergence, discounted future cash flows would grow infinitely, resulting in eternal postponement of the investment.

[3]As expected and required, $\partial q^{opt}/\partial X > 0$ in R_2. A higher X leads to a higher potential demand. Economic theory states that, if available capacity allows, optimal realised throughput q^{opt} increases when demand shifts upwards and the supply curve has a positive inclination.

equation for project value as a function of the investment threshold and capacity installed $V(X,K)$, which must be solved in each region R_j:

$$\frac{\sigma^2}{2}X^2\frac{\partial^2 V}{\partial X^2}(X,K)+\mu X\frac{\partial V}{\partial X}(X,K)-rV(X,K)+\overline{\pi}(X,K)=0. \tag{17}$$

The solution for $j=1,2,3$ is given by

$$V(X,K)|_{X\in R_j}=V_j(X,K)=G_{j,1}(K)X^{\beta_1}+G_{j,2}(K)X^{\beta_2}+\overline{V}_j(X,K), \tag{18}$$

where $\overline{V}_j(X,K)$ is a particular solution of the differential equation (17) with $\overline{\pi}=\overline{\pi}_j$. The roots β_1 and β_2 satisfy the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r=0$ (see Dixit & Pindyck (1994)), and are equal to

$$\beta_1=\frac{\frac{\sigma^2}{2}-\mu+\sqrt{\left(\frac{\sigma^2}{2}-\mu\right)^2+2r\sigma^2}}{\sigma^2}>1,\qquad \beta_2=\frac{\frac{\sigma^2}{2}-\mu-\sqrt{\left(\frac{\sigma^2}{2}-\mu\right)^2+2r\sigma^2}}{\sigma^2}<0. \tag{19}$$

Particular solutions are calculated in Appendix A, which contains more details on the calculation of the solution to the differential equation (17).

We imposed six[4] boundary conditions (see Dixit & Pindyck (1994) and Dangl (1999)) in order to calculate the $G_{j,i}$:

$$\begin{cases}
V(0,K)=\mathbf{E}\int_0^{\infty}-c_hK\mathrm{e}^{-rt}\mathrm{d}t=\frac{-c_hK}{r},\\
\lim_{X\to+\infty}(V(X,K)-\overline{V}_3(X,K))=0,\\
\lim_{X\overset{<}{\to}c}V(X,K)=\lim_{X\overset{>}{\to}c}V(X,K),\\
\lim_{X\overset{<}{\to}c}\frac{\partial V}{\partial X}(X,K)=\lim_{X\overset{>}{\to}c}\frac{\partial V}{\partial X}(X,K),\\
\lim_{X\overset{<}{\to}\frac{K(2BK+c)}{K-2A}}V(X,K)=\lim_{X\overset{>}{\to}\frac{K(2BK+c)}{K-2A}}V(X,K),\\
\lim_{X\overset{<}{\to}\frac{K(2BK+c)}{K-2A}}\frac{\partial V}{\partial X}(X,K)=\lim_{X\overset{>}{\to}\frac{K(2BK+c)}{K-2A}}\frac{\partial V}{\partial X}(X,K).
\end{cases} \tag{20}$$

The first condition implies that $G_{1,2}=0$ and the second that $G_{3,1}=0$. This leaves the final four (value matching and smooth pasting) conditions to define the values for $G_{1,1}$, $G_{2,1}$, $G_{2,2}$ and $G_{3,2}$ (see Hagspiel *et al.* (2016)). $G_{1,1}$ represents the option to start production, $G_{3,2}$ the option of flexible throughput levels below full capacity and $G_{2,1}$ and $G_{2,2}$ are two correction factors in R_2. The $G_{j,i}$ are dependent only on K, since X is evaluated in c and $\frac{K(2BK+c)}{K-2A}$.

The derivation of the marginal value of the project with respect to capacity ($\partial V(X,K)/\partial K=v(X,K)$), as also calculated by Dangl (1999), shows that this marginal value is higher when there is more demand uncertainty. This is because more port capacity is required to serve the potentially higher upward shifts in demand (see Eq. (2)), contributing to a higher option value under higher uncertainty.

4 Numerical example: parameter determination

In order to illustrate the working of the model and calculate numerical solutions in the remainder of this paper, we apply our theoretical model to a new container port of about 8 to 14 million TEU, to be installed in a developing region, such as the South West of Africa or South America. The parameter values of our example were calibrated using real available data (Port of Antwerp, 2016; Zuidgeest, 2009).

[4]If R_3 does not exist, the final two conditions can be omitted, and $G_{2,1}$ and $G_{3,2}$ both equal zero.

Various construction technologies available give rise to an investment cost of between one and three billion euros. In the example, throughput (q) and capacity (K) are expressed in million TEU per year, while price p and costs $c(=1)$ are in euros per TEU and $c_h = 0.5$ euros per TEU per year (Meersman & Van de Voorde, 2014; Vanelslander, 2014). B is normalised to 1. This yields realistic elasticities of demand. Profit is calculated in million euros. The investment cost $I(K)$ is also expressed in million euros and adheres the conditions derived in Section 2. The values for drift ($\mu = 0.015$) and drift variability ($\sigma = 0.1$ to 0.2) can be validated empirically in the port context. We regressed an exponential growth model on annual container throughput of significant existing ports in Antwerp and Rotterdam between 2010 and 2015 (Vlaamse Havencommissie, 2016), resulting in a growth rate of between 1.5% and 2% [$p < 0.05$]. The root of the squared error of the regression led to a standard deviation of 15% to 17%, justifying the 10% to 20% interval for σ. Both parameters are however varied in the analysis. In transport infrastructure studies, a discount rate in the range of 4% (Blauwens, 1988) to 8% (Centraal Planbureau, 2001) is often used. We selected the commonly used $r = 0.06$ in the base case. As the choice of the discount rate is often subject to much discussion, we vary this value as well in the sensitivity analysis in Section 6.

One of the most difficult aspects was to determine suitable values for the monetary scaling factor A. Ultimately, we selected several different values for parameter A from a range which produces realistic port investment strategies and occupancy rates. It should be kept in mind that our objective is to show the impact of differing customer aversions to delays on the investment decision, rather than estimating the project's exact A. An overview of the selected parameters is given in Table 1.

Table 1: Overview of the model and the selected parameters.

Variables			
	p	=	price
	q	=	throughput
	K	=	capacity
Inverse demand function: $p = X - Bq - AX\frac{q}{K^2}$			
	$B(=1)$	=	slope
	$A(\in [4;5])$	=	monetary scaling factor of congestion cost
Demand shift parameter X**:** $\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}Z(t)$			
	t(=annual)	=	time horizon
	Z	=	standard Wiener process
	$\mu(= 0.015)$	=	drift of Z
	$\sigma(\in [0.1; 0.2])$	=	drift variability of Z
Total cost $TC = cq + c_h K$			
	$c(= 1)$	=	constant marginal operational cost
	$c_h(= 0.5)$	=	cost to hold one unit of capital in place
Investment cost $I = FC_I + \gamma_1 K - \gamma_2 K^2 + 0K^3 + \gamma_4 K^4$			
	$FC_I(= 80)$	=	fixed investment cost
	$\gamma_1(= 180)$	=	first order coefficient
	$\gamma_2(= 19)$	=	coefficient reflecting economies of scale of investment cost
	$\gamma_4(= 0.12)$	=	coefficient posing boundary to maximum size of the project

5 Investment decision: optimal size and timing

In this section, the numerical values for the parameters and the calculated value of a new port project at the time of investment are used as inputs. Based on this, we determine the optimal capacity and timing threshold in order to maximise the expected discounted profit stream *mi-*

nus the investment cost and identify the ideal point in the trade-off between gaining additional information and foregoing income by delaying investment.

The option value included in the option to postpone the port capacity investment can be written as

$$F(X) = \max\{\mathrm{e}^{-r\mathrm{d}t}\,\mathbf{E}\,[F(X) + \mathrm{d}F(X)], \max_{K}[V(X,K) - I(K)]\}, \tag{21}$$

where the outer maximisation indicates whether the investment should be made or not yet (Dangl, 1999).

First, to obtain the new port's optimal capacity K^* for a given level of X at which investment takes place, we examine a now or never investment decision, which is the inner maximisation of Eq. (21). This is equivalent to obtaining the value $K = K^*(X)$ such that

$$v(X,K) - \frac{\mathrm{d}I(K)}{\mathrm{d}K} = 0. \tag{22}$$

It allows ports to know their optimal design capacity K^*, should they wish to invest at an exogenously determined moment.

In addition to inner maximisation, outer maximisation must also be solved. The option to delay investment with flexible throughput from Eq. (21) satisfies a second order differential equation (Dangl, 1999):

$$\frac{\sigma^2}{2}X^2\frac{\partial^2 F}{\partial X^2}(X,K) + \mu X\frac{\partial F}{\partial X}(X,K) - rF(X,K) = 0. \tag{23}$$

The solution is given by

$$F(X) = H_1X^{\beta_1} + H_2X^{\beta_2}. \tag{24}$$

The conditions required to find H_1, H_2 and the solution for state variable X (i.e. $X_T^*(K)$) are:

$$\begin{cases} F(0) = 0 \rightarrow H_2 = 0 \\ H_1X_T^{*\beta_1} = V(X_T^*, K(X_T^*)) - I(K(X_T^*)) \\ \beta_1H_1X^{\beta_1-1} = \dfrac{\partial}{\partial X}(V(X,K(X)) - I(K(X)))\Big|_{X=X_T^*} \end{cases}, \tag{25}$$

in which the last two conditions, known as value matching and smooth pasting, guarantee a continuous option value function (Dixit & Pindyck, 1994). The solution of this system is the optimal investment threshold X_T^* as a function of K, which can be useful for determining the optimal investment timing of a new port with a predetermined capacity.

In the next step, combining the optimal timing and size functions in the system

$$\begin{cases} X_T^{**} = X_T^*(K^{**}), \\ K^{**} = K^*(X_T^{**}), \end{cases} \tag{26}$$

yields the optimal investment strategy (X_T^{**}, K^{**}) wherein both the timing and size of the investment are optimal. Using this solution, we can plot the option value $(F(X))$ and value of the project $(V(X,K^{**}) - I(K^{**}))$ as a function of X, with K fixed at K^{**}. Figure 2 gives an example for $A = 5$ and $\sigma = 0.1$. In the example, an optimum can only be found in R_2, where capacity is not fully utilised. The port will invest before $X \in R_3$, and select capacity K^{**} so that $X_T^{**} \in R_2$.

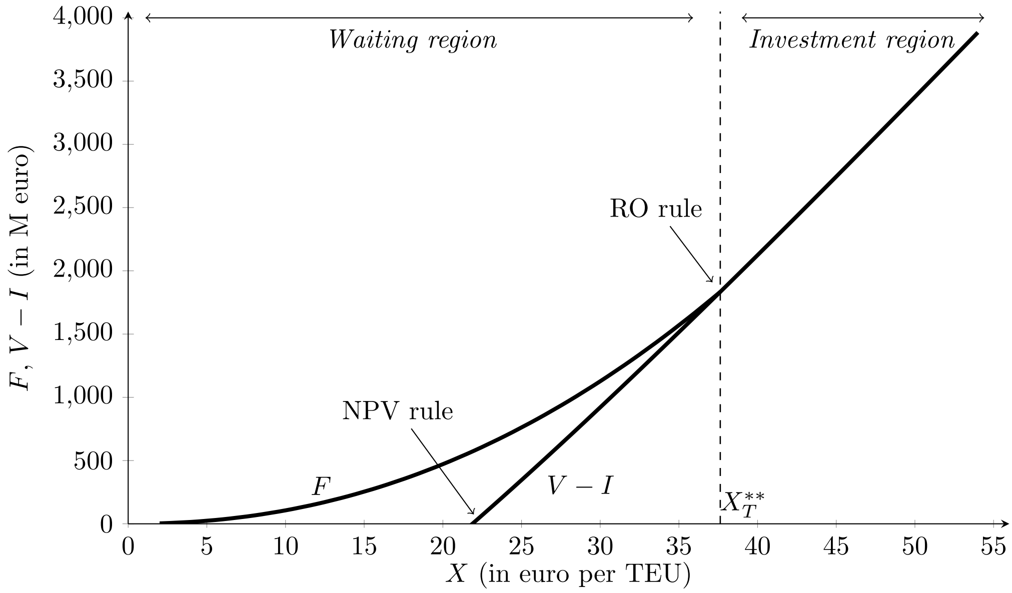

Figure 2: Comparison between the option value $F(X)$ and the value of the project minus the investment cost $V(X, 11.17) - I(11.17)$ for $A = 5$ and $\sigma = 0.1$ in a private port. The optimal investment threshold is $X_T^{**} = 37.63 \in R_2$, whereas the optimal capacity is $K^{**} = 11.17$.
Parameter values: $B = 1, c = 1, c_h = 0.5, \mu = 0.015, r = 0.06, FC_I = 80, \gamma_1 = 180, \gamma_2 = 19, \gamma_4 = 0.12$.

Figure 2 shows that in ports too, the general real options theory holds. The option to postpone investment bears a value and leads to postponement of investment until $V \geqslant I + F$, as opposed to the traditional NPV rule $V > I$ (Dixit & Pindyck, 1994).[5] In the example, the new port will dispose of capacity K^{**}(= 11.17 million TEU) and will be installed at threshold X_T^{**}(= 37.63 euros per TEU), leading to an optimal throughput at the moment of investment of 7.3 million TEU. The initial occupancy rate will therefore be 65%, which is below full occupancy. The initial price per TEU after investment, according to Eq. (4), would be 19.31 euros, which is necessary for obtaining a discounted project value of 3.35 billion euros at the time of investment and compensating for the project's investment costs of 1.59 billion euros.

In order to further analyse the impact of congestion costs and uncertainty on the investment decision, we calculate a number of other investment pairs (X_T^{**}, K^{**}) for altered values of A and σ, as shown in Table 2.

Table 2: Optimal private port investment decision (X_T^{**}, K^{**}) for different values of A and σ.

		Uncertainty		
		$\sigma = 0.10$	$\sigma = 0.15$	$\sigma = 0.20$
Congestion	$A = 0$	(18.61, 8.15)	(22.82, 9.20)	(24.40, 10.31)
monetary	$A = 4$	(33.67, 10.83)	(44.74, 12.21)	(63.27, 13.98)
scaling	$A = 4.5$	(35.96, 11.02)	(48.44, 12.55)	(70.60, 14.53)
factor	$A = 5$	(37.63, 11.17)	(52.32, 12.88)	(78.97, 15.10)

Other parameter values: $B = 1, c = 1, c_h = 0.5, \mu = 0.015, r = 0.06, FC_I = 80, \gamma_1 = 180, \gamma_2 = 19, \gamma_4 = 0.12$.
Source: Authors' calculations.

The first result from the table is consistent with a well-known RO finding (Dangl, 1999; Huisman & Kort, 2015):

[5]The RO rule leads to postponing the investment to $X_T = 37.63$, as opposed to $X_T = 22.49$ under NPV.

Result 1. *More demand uncertainty leads to later but more extensive investments in port capacity.*

When uncertainty is lower (or zero), the option value of waiting will be lower. Consequently, the investment in less port capacity will take place at a lower threshold value. Less capacity is required to handle smaller upward deviations from the expected market demand for throughput, leading to congestion. Hence, when developing countries can facilitate demand uncertainty reductions, they can benefit sooner from the new port, installed at optimal conditions.

Another important question to be answered is how the port investment decision is influenced by congestion costs. If congestion costs were not considered ($A = 0$), the new port would be too small and it would be installed at a threshold X_T^{**} that is too low. Table 2 demonstrates this major result.

Result 2. *Ports with customers that are on average more waiting-time averse, should invest in more capacity, but later.*

This observation is the result of three effects with conflicting outcomes. First, an increase in the monetary scaling factor of congestion, A, leads to a lower profit. This makes the investment less attractive, resulting in a lower capacity K^*. Second, the port may also wish to wait longer before investing to increase the project's value, resulting in a higher value for threshold X_T^*. Because of the positive inclination of the $K^*(X)$-function, this results at the same time in an increase in the optimal capacity K^*. The final effect is a direct increase in capacity K^*, since congestion now poses a greater problem for port users and lower occupancy rates q/K are required. The final two effects dominate, since capacity has a higher marginal value when it is not fully occupied. Moreover, increasing the size of the project implies taking more advantage of the investment size scale economies. Developing countries that not want to delay new port investment are advised to attract less waiting-time averse customers.

6 Influence of the economic and operational context on the optimal investment strategy for the port

In order to test the sensitivity of the results with respect to changes in other parameters included in the model, Table 3 shows how the optimal capacity (K^*) and timing (X_T^*) for the new port, displayed in Figure 2, alter individually following a limited increase in a single parameter and what the new optimum (X_T^{**}, K^{**}) is.

Table 3: Reaction of the optimal $X_T^*(K)$, $K^*(X)$ and (X_T^{**}, K^{**}) pair to a 10% increase in the other parameters.

Parameter	**Timing** $X_T^*(K = 11.17)$	**Size** $K^*(X = 37.63)$	**Decision** (X_T^{**}, K^{**})
Base case	*37.63*	*11.17*	*(37.63, 11.17)*
c	37.79 (+)	11.16 (−)	(37.87 (+), 11.19 (+))
c_h	37.77 (+)	11.16 (−)	(37.85 (+), 11.19 (+))
FC_I	37.75 (+)	11.17 (=)	(37.86 (+), 11.20 (+))
γ_1	40.59 (+)	11.04 (−)	(42.39 (+), 11.62 (+))
γ_2	41.11 (+)	10.88 (−)	(42.29 (+), 11.45 (+))
γ_4	40.38 (+)	10.74 (−)	(39.52 (+), 10.97 (−))
r	39.74 (+)	10.74 (−)	(38.43 (+), 10.83 (−))
μ	37.37 (−)	11.34 (+)	(38.50 (+), 11.45 (+))
B	38.70 (+)	11.01 (−)	(38.51 (+), 11.12 (−))

Base case parameter values:
$A = 5, B = 1, c = 1, c_h = 0.5, \mu = 0.015, \sigma = 0.1, r = 0.06, FC_I = 80, \gamma_1 = 180, \gamma_2 = 19, \gamma_4 = 0.12.$

Source: Authors' calculations.

This table shows some important results. First of all, the direct impact of the increase on the optimal level of capacity $K^*(X)$ at a given moment is as expected for all parameters: higher operating, capital and investment costs lead to investing in a smaller port, since they reduce the project's attractiveness. One exception is the fixed investment cost parameter FC_I. Since it is a constant in the investment cost function $I(K)$, it has no influence on the optimal capacity K. A steeper demand curve caused by a higher value for B also leads to a lower investment size K, as the market is less profitable and the port can then alter its price more freely without attracting or losing many customers. If B is lower, the port stands to gain by reducing the price slightly and attracting additional customers. For this to work, however, sufficient capacity is required. Investment size is also lower for a higher discount rate, since future revenues will be of lower value, especially compared to the immediate investment cost. In contrast, a higher expected average economic growth (μ) leads to investment in a larger port.

Moreover, we see that a parameter change which has a positive direct impact on optimal capacity $K^*(X)$ has a negative direct impact on optimal timing threshold X_T^*, and vice versa. This is plausible, because the response to a positive economic effect might be either to increase the size of a project at a given moment (to realise more of the feasible profit, growing with the economy), or to implement a project sooner (X_T^* can be lower to obtain the same profitability). Nonetheless, there is one exception. Although an increase in FC_I has no impact on $K^*(X)$, it does have a positive impact on $X_T^*(K)$. If the fixed expenditure for making an investment increases, a port may wish to postpone investment, so that higher revenues can compensate for this increased fixed cost.

As opposed to a negative direct effect, the indirect effect of an increase in capacity (K^*) on the optimal timing is positive, because $K^*(X)$ and $X_T^*(K)$ are both increasing functions. Hence, if size and timing are both considered jointly, the net effect on the (X_T^{**}, K^{**}) equilibrium is ambiguous. One of the variables always moves in the direction of the individual direct impact. The evolution of the other variable depends on the relative sizes of the indirect and direct effects.

Table 3 contains a number of results concerning the sensitivity of the model, based on our numerical simulations. After numerically simulating with a lot of different parameter settings, the reaction of the new optimum to a change of one parameter in Results 1, 2 and 3 turns out to be independent of the other parameter settings. The changes of the new optimum in the other results however do depend on the specific parameter settings.

Result 3. *Higher port costs result in later investments in more capacity.*

Result 4. *If the discount rate r increases, smaller and/or later port capacity investment will be made. The impact on the new optimum depends on the specific parameter values.*

Result 5. *If the growth rate increases, it is much more attractive to invest in a larger port and/or at an earlier timing. The impact on the new optimum depends on the specific parameter values.*

Result 6. *A steeper demand curve leads to smaller and/or later investment in port capacity. The impact on the new optimum depends on the specific parameter values.*

To further explore Result 5, we calculated the impact of a negative growth rate by examining a case with a negative growth rate ($\mu = -0.005$), finding that the optimal investment decision would then equal (44.30, 10.32). The port would be much smaller than in the case of positive drift, since the project would be less attractive due to decreasing profits. In such a situation, less capacity would be needed. Additionally, in order to compensate for future decreases in demand, the project would only be implemented at a much higher threshold, which is less likely to be reached.

The discount rate is the subject of much discussion in the literature, so we opted to vary it between the usual 4% and 8%. Recently however, interest rates have been falling, so we calculated the investment decision at 3% too. Starting from the base case in Table 3, the following results have been obtained:

- $\boldsymbol{r = 0.03}$: (87.81, 21.88)
- $\boldsymbol{r = 0.04}$: (43.96, 14.27)

Table 4: Optimal private port investment decision (X_T^{**}, K^{**}) for different values of r and σ.

		Uncertainty		
		$\boldsymbol{\sigma = 0.09}$	$\boldsymbol{\sigma = 0.10}$	$\boldsymbol{\sigma = 0.11}$
Dis-	$\boldsymbol{r = 0.05}$	(35.13, 11.73)	(37.72, 12.10)	(40.75, 12.51)
count	$\boldsymbol{r = 0.06}$	(35.67, 10.90)	(37.63, 11.17)	(39.87, 11.46)
rate	$\boldsymbol{r = 0.07}$	(37.46, 10.45)	(39.14, 10.67)	(41.01, 10.90)

Other parameter values: $A = 5, B = 1, c = 1, c_h = 0.5, \mu = 0.015, FC_I = 80, \gamma_1 = 180, \gamma_2 = 19, \gamma_4 = 0.12$.

- $\boldsymbol{r = 0.06}$: (37.63, 11.17)
- $\boldsymbol{r = 0.08}$: (41.24, 10.36)

A higher discount rate reduces the value of future profits and the value of the project, making the port investment less attractive. This leads to smaller investment sizes. At low discount and growth rates, the investment decision is also very sensitive to the growth rate. A growth rate of 1% in combination with a discount rate of 3% results in an investment of 13.64 million TEU at threshold $X_T^{**} = 34.26$, which is much more similar to the base case decision. Secondly, at historically low interest and hence discount rates, we can expect these rates to rise again in the future. Such volatility has a significant negative impact on investment size (Cassimon *et al.*, 2002) and moderates the increase in investment triggered by low discount rates. Such discount rate volatility was not included in our real options model and should be considered in future real options research.

Another macroeconomic phenomenon that requires attention is the relation between interest rates and uncertainty. When uncertainty rises, households increase precautionary savings, which in turn has a negative effect on interest rates (Hartzmark, 2016). The impact of these combined effects can be seen in Table 4. As discussed before, both an increase in uncertainty and a decrease in the interest rate, used as the discount factor, lead to an increase in the size of the port investment. As a result, the outcome of the combined effect is a considerable increase of the project size. Although increased uncertainty leads to investment postponement, the impact of the discount rate on timing was already shown to be ambiguous and dependent on the height of the discount rate. Therefore, the net impact of the combined effects on timing depends on the height of the interest rate as well.

7 The influence of public port ownership

Because a large number of ports is partly or fully owned publicly, we extend in this section our analysis to a public port. A public port does not maximise profit in Eq. (13), but social welfare:

$$SW(q, X, K) = \pi(q, X, K) + \lambda q + CS(q) = \pi(q, X, K) + \lambda q + Bq^2/2, \tag{27}$$

with λ the local spillover benefits per unit q as a result of throughput handled in the port and CS the consumer surplus, calculated as $Bq^2/2$ (Xiao *et al.*, 2012). According to Coppens *et al.* (2007) and Benacchio & Musso (2001), spillover effects within the port perimeter are estimated between 20 and 60 percent of the cost c to process one TEU, depending on the level of aggregation of the local government's jurisdiction. Here λ is set to 0.4, to account for a port's spillover effects in the entire country.

The optimal investment strategy of a public port is displayed in Table 5 for different levels of uncertainty and costs of congestion. Moreover, the results are compared to the strategies displayed in Table 2 for a private port.

The numbers in Table 5 illustrate that Results 1-2 also hold for a public port. Increased uncertainty and aversion to waiting leads to delayed investment in a larger port. Also the other results and findings in this paper do not differ qualitatively between public and private ports. However, Table 5 allows deriving an additional result:

Table 5: Optimal public port investment decision (X_T^{**}, K^{**}) for different values of A and σ, compared to the private port decision.

Case	Public (X_T^{**}, K^{**})	Private (X_T^{**}, K^{**})
$A = 4, \sigma = 0.1$	**(27.20, 10.97)**	(33.67, 10.83)
$A = 5, \sigma = 0.1$	**(31.40, 11.39)**	(37.63, 11.17)
$A = 4, \sigma = 0.2$	**(46.10, 13.26)**	(63.27, 13.98)
$A = 5, \sigma = 0.2$	**(58.65, 14.35)**	(78.97, 15.10)

Other parameter values:
$A = 5, B = 1, c = 1, c_h = 0.5, \mu = 0.015, \sigma = 0.1, FC_I = 80, \gamma_1 = 180, \gamma_2 = 19, \gamma_4 = 0.12, \lambda = 0.4.$
Source: Authors' calculations.

Result 7. *Public ports invest sooner than private ports, due to more project benefits being considered in the operational objective function.*

This result confirms the finding of Asteris *et al.* (2012). As we already indicated, investment anticipation has an indirect effect of smaller investment. In the first two examples however, public port investment does not only take place earlier, but also involves a larger project, which allows benefiting more from the economies of scale in the investment cost. This finding is exceptional in real options analyses. Consequently, if governments of developing countries want to dispose of a port sooner, they have a clear incentive to participate in the investment, especially if sufficient space is available. However, if the government wants to force a private port to invest in the optimal strategy of a public port, a subsidy can be given, based on the amount of throughput. Based on Eq. (27), this subsidy S would have to equal

$$S(q) = \lambda q + Bq^2/2. \tag{28}$$

Next to full public ownership, ports can also be owned by both public and private actors jointly. In such a case, Xiao *et al.* (2012) define Π, the operational objective function of the port, as the weighted sum of the individual owners' objectives. The shares of ownership, s_G for the public owner and $1 - s_G$ for the private owner, are used as the weights:

$$\begin{aligned} \Pi(X, K, q) &= (1 - s_G) \cdot \pi(X, K, q) + s_G \cdot SW(X, K, q) \\ &= \pi(X, K, q) + s_G \cdot \lambda q + s_G \cdot Bq^2/2. \end{aligned} \tag{29}$$

Where the case of $s_G = 0$ yields a private port and $s_G = 1$ a public port, the case of $s_G = 50\%$ allows studying a port where the public and private actor own the same number of shares. In case $A = 5$ and $\sigma = 0.1$, the optimal investment of this port would equal (34.74, 11.27). As can be intuitively expected, both the timing and size are between those of the private and the public port from Table 5. So, increased public ownership, leading to a higher relative focus on social welfare rather than profit alone, will lead to an anticipation of the investment timing and an increased size of a new port.

8 Conclusions and future research

Developing countries are confronted with growing but uncertain maritime trade flows. In order to accommodate the demand for container transport and avoid delays and a slowdown of growth, new port investments are required. In this paper, we used a real options model to derive a decision rule for determining the optimal size and timing of the investment in a new port, taking into account the uncertainty, irreversibility and lumpiness of such investments. The paper is innovative in the sense that it applies the theory of real options to the investment in a new port, including both congestion and a continuous stochastic process with independently settable parameters. A variable monetary scaling factor allowed us to differentiate the impact of waiting according to the goods categories and port users involved.

The main finding of this paper is that considering congestion costs in the analysis under uncertainty alters the investment decision considerably. If congestion costs are not considered, a port may make an early investment that is not large enough. As the example in the paper shows, ports should invest in more capacity but at a later time if the port customers are more waiting-time averse. The results also indicate that increased net economic growth, which is often encountered in developing countries, leads to larger port investments becoming much more attractive. However, uncertainty is also higher in developing countries. This leads to the advice for port authorities to postpone investment but also to invest in more capacity. This investment postponement will allow them to gather more market information to justify the investment. As a result, developing countries willing to advance the investment in a new port are advised to take measures to reduce the uncertainty. Moreover, government ownership of the new port will anticipate the optimal timing as well.

In future research, it would be worthwhile to include other sources of variability in the real options analysis, such as operational and investment cost or interest rate uncertainty.

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A Solution to the differential equation (17)

In this appendix, we show how the solution to the differential equation (17) can be obtained. We need to solve the following equation:

$$\frac{\sigma^2}{2}X^2\frac{\partial^2 V}{\partial X^2}(X,K)+\mu X\frac{\partial V}{\partial X}(X,K)-rV(X,K)=-\overline{\pi}(X,K) \tag{30}$$

where $\overline{\pi}(X,K)$ is either a constant or rational function:

$$\pi(X,K,q^{opt}(X,K))=\overline{\pi}(X,K)=\begin{cases}\overline{\pi}_1(X,K)=-c_hK, & X\in R_1,\\ \overline{\pi}_2(X,K)=\dfrac{(X-c)^2K^2}{4(XA+BK^2)}-c_hK, & X\in R_2,\\ \overline{\pi}_3(X,K)=(K-A)\,X-(BK+c+c_h)K, & X\in R_3.\end{cases} \tag{31}$$

First we consider the homogeneous equation:

$$\frac{\sigma^2}{2}X^2\frac{\partial^2 V}{\partial X^2}(X,K)+\mu X\frac{\partial V}{\partial X}(X,K)-rV(X,K)=0. \tag{32}$$

Note that the second order derivative has a coefficient that is quadratic in X, and likewise for the other terms. Hence, if V is given by a power function, all terms in the differential equation will prove to be the same power functions. Thus, the two independent solutions will be given by power functions. Plugging $V=X^\beta$ into the equation, we see that it can be simplified to the quadratic equation

$$\frac{\sigma^2}{2}\beta(\beta-1)+\mu\beta-r=0, \tag{33}$$

which has the two distinct real roots β_1 and β_2 given in Eq. (19). Note that the condition $r>\mu$ means that $\beta_1>1$ and $\beta_2<0$. Hence, the solution to the homogeneous equation in the j-th region ($j=1,2$) is given by

$$V_{h,j}(X,K)=G_{1,j}(K)X^{\beta_1}+G_{2,j}(K)X^{\beta_2}. \tag{34}$$

In region R_1, the right-hand side of the differential equation (30) is a constant. A constant particular solution is then clear:

$$\overline{V}_1(X,K)=-\frac{c_hK}{r}. \tag{35}$$

In region R_2, the right-hand side is more complicated:

$$\begin{aligned}\overline{\pi}_2(X,K) &= \frac{(X-c)^2K^2}{4(AX+BK^2)}-c_hK\\ &= \frac{K^2}{4A}X-\left[\frac{K^2}{4A}\left(2c+\frac{BK^2}{A}\right)+c_hK\right]+\frac{K^2(c+\frac{BK^2}{A})^2}{4A}\frac{1}{X+\frac{BK^2}{A}}.\end{aligned} \tag{36}$$

This right-hand side consists of a polynomial part (of degree one) and a rational part. Finding a particular solution for the polynomial part is straightforward: if we enter $V=aX+b$ into the differential equation, we obtain

$$\overline{V}_{2,\text{pol}}(X,K)=\frac{K^2}{4A(r-\mu)}X-\frac{1}{r}\left[\frac{K^2}{4A}\left(2c+\frac{BK^2}{A}\right)+c_hK\right]. \tag{37}$$

For the final part of the solution it is easier to consider the monic form of the differential equation, i.e. we divide (36) by $\sigma^2X^2/2$. In order to find a particular solution for the remaining rational (in X) part,

$$\overline{\pi}_{2,\text{rat,mon}}(X,K)=\frac{K^2\left(c+\frac{BK^2}{A}\right)^2}{2A\sigma^2}\frac{1}{X^2\left(X+\frac{BK^2}{A}\right)}, \tag{38}$$

we use the method known as variation of parameters or constants. That is, we propose a solution in the same form as the solution to the homogeneous equation, but now with coefficients that also depend on X:

$$\overline{V}_{2,\mathrm{rat}}(X,K) = C_1(X,K)X^{\beta_1} + C_2(X,K)X^{\beta_2}. \tag{39}$$

As a result of this method, the coefficients are given by

$$C_1(X,K) = \int \frac{1}{W} X^{\beta_2} \overline{\pi}_{2,\mathrm{rat}}(X,K)\,\mathrm{d}X, \tag{40}$$

$$C_2(X,K) = -\int \frac{1}{W} X^{\beta_1} \overline{\pi}_{2,\mathrm{rat}}(X,K)\,\mathrm{d}X, \tag{41}$$

where W is the Wronskian, i.e. the determinant of the two independent solutions to the homogeneous equation and their derivatives:

$$W = \begin{vmatrix} X^{\beta_1} & X^{\beta_2} \\ \beta_1 X^{\beta_1 - 1} & \beta_2 X^{\beta_2 - 1} \end{vmatrix} = (\beta_2 - \beta_1) X^{\beta_1 + \beta_2 - 1}. \tag{42}$$

Besides for integer values of β_1 and β_2, the integrals in equations (40)-(41) cannot be evaluated using elementary functions alone. Instead, their solution can be written in terms of hypergeometric functions (Abramowitz & Stegun, 1972). We obtain the solution

$$\begin{aligned} \overline{V}_{2,\mathrm{rat}}(X,K) &= -\frac{\left(c + \frac{B}{A}K^2\right)^2}{2\sigma^2 B(\beta_2 - \beta_1)} \\ &\quad \times \left[\frac{1}{\beta_1} {}_2\mathrm{F}_1\left(1, -\beta_1; 1-\beta_1; -\frac{AX}{BK^2}\right) - \frac{1}{\beta_2} {}_2\mathrm{F}_1\left(1, -\beta_2; 1-\beta_2; -\frac{AX}{BK^2}\right)\right], \end{aligned} \tag{43}$$

valid for $X < BK^2/A$, or its analytic continuation for $X \geq BK^2/A$. The sum of the particular solutions for the polynomial and rational parts, found in (37) and (43) respectively, produces a particular solution for the differential equation (30) in the region R_2. If we add the solution of the homogeneous equation to this particular solution, we will still obtain a particular solution. Consequently, we can add a linear combination of X^{β_1} and X^{β_2} to our particular solution. The reason for this stems from the analysis of the asymptotic behaviour of our solution. With a large X, the first hypergeometric function in (43) will increase as a constant times X^{β_1}. However the next term in the asymptotic expansion at infinity will merely be linear, even if $\beta_1 > 2$. This can be shown by writing down the series expansion of the hypergeometric functions at infinity. Thus, we subtract this term in X^{β_1}, calculated as

$$\frac{\Gamma(\beta_1)\Gamma(1-\beta_1)}{(BK^2)^{\beta_1}} A^{\beta_1} X^{\beta_1},$$

in order to obtain the particular solution omitting speculative bubbles resulting from the term in X^{β_1} (see Dixit & Pindyck (1994), Chapter 6). For a behaviour near zero omitting speculative bubbles, no further steps are needed. Since the hypergeometric series is a power series around zero, it converges in the neighbourhood of zero and no divergent terms (i.e. terms in X^{β_2}) are present. Hence, no correction in X^{β_2} is required.

Finally, in region R_3, the right-hand side is polynomial in X:

$$\overline{\pi}_3(X,K) = (K - A)\,X - (BK + c + c_h)K. \tag{44}$$

Again, by plugging $V = aX + b$ into the differential equation, we obtain

$$\overline{V}_3(X,K) = \frac{K - A}{r - \mu} X - \frac{BK^2 + cK + c_h K}{r}. \tag{45}$$

All of these considerations lead to the following particular solution in each region R_j:

$$\overline{V}_j(X,K) = \begin{cases} -\dfrac{c_h K}{r}, & j=1; (X \in R_1), \\ \dfrac{K^2}{4A(r-\mu)}X - \dfrac{1}{r}\left[\dfrac{K^2}{4A}\left(2c + \dfrac{K^2}{A}\right) + c_h K\right] & \\ \quad -\dfrac{\left(c+\frac{K^2}{A}\right)^2}{2\sigma^2 B(\beta_2-\beta_1)}\left[\dfrac{1}{\beta_1}{}_2\mathrm{F}_1\left(1,-\beta_1;1-\beta_1;-\dfrac{AX}{BK^2}\right)\right. & \\ \quad -\dfrac{1}{\beta_2}{}_2\mathrm{F}_1\left(1,-\beta_2;1-\beta_2;-\dfrac{AX}{BK^2}\right) & \\ \quad \left.-\dfrac{\Gamma(\beta_1)\Gamma(1-\beta_1)(AX)^{\beta_1}}{(BK^2)^{\beta_1}}\right], & j=2; (X \in R_2), \\ \dfrac{K-A}{r-\mu}X - \dfrac{BK^2+cK+c_hK}{r}, & j=3; (X \in R_3). \end{cases} \tag{46}$$

This needs to be plugged into the solution for V in each region R_j:

$$V_j(X,K) = G_{j,1}(K)X^{\beta_1} + G_{j,2}(K)X^{\beta_2} + \overline{V}_j(X,K). \tag{47}$$