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**PARTON SHOWERS BEYOND THE LEADING ORDER:  
A FACTORIZATION APPROACH \***

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We discuss recent work on methods for incorporating nonleading QCD corrections in parton shower algorithms.

Parton shower Monte Carlo event generators are the main practical tool to describe multi-particle final states in high energy collisions. These event generators couple a leading-order (LO) hard scattering to a showering, treated basically in the leading logarithm approximation. Many nonleading effects are also included in these event generators: for instance, through the exact multi-parton kinematics; through the angular ordering of gluon emission; through optimal choices of the renormalization scale in the running coupling. In some cases there are also treatments (see, e.g., refs.<sup>1,2,3,4</sup>) to approximately include next-to-leading-order (NLO) corrections to the hard scattering. Nevertheless, there is as yet no method for going beyond the leading approximation systematically. This implies that event generators cannot incorporate fully the known NLO (and NNLO) calculations of hard-scattering cross sections.

Recent papers<sup>5,6,7</sup> have started to investigate systematic subtractive methods as a way to tackle this problem. This talk describes some of the work in this direction.

To discuss this, let us schematically represent the cross section in an event generator as

$$\sigma[W] = \sum_{\text{final states } X} W(X) \text{PS} \otimes \hat{H}. \quad (1)$$

Here  $W$  is a weight function that specifies the definition of the particular cross section under consideration. The symbol PS denotes the parton shower and the symbol  $\otimes$  denotes its action on the initial and final partons in the hard scattering, whose cross section is denoted by  $\hat{H}$ .

In a standard Monte Carlo, the hard scattering is taken to the leading order,  $H^{(\text{LO})}$ , and PS denotes showering from the partons in  $H^{(\text{LO})}$ . In an NLO Monte

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Carlo, the cross section involves a structure of the form

$$\text{PS} \otimes \left[ H^{(\text{LO})} + \alpha_s \left( H^{(\text{NLO})} - \text{PS}_I(1) \otimes H^{(\text{LO})} - \text{PS}_F(1) \otimes H^{(\text{LO})} \right) \right]. \quad (2)$$

Here the first term in the square brackets is the LO hard-scattering function, and the second term is the subtracted NLO hard-scattering function.  $H^{(\text{NLO})}$  is the result of computing the partonic cross section from the NLO graphs, while  $\text{PS}_I(1)$  and  $\text{PS}_F(1)$  are the order  $\alpha_s$  approximations to the initial-state and final-state showering. The subtraction terms avoid double counting of events already included by showering from  $H^{(\text{LO})}$ .

Ref.<sup>6</sup> gives a simple example of the structure (2): it treats the photon-gluon fusion process in deep inelastic  $ep$  scattering, using the Monte Carlo algorithm of ref.<sup>8</sup>. This case is special because i) only the subtraction term in  $\text{PS}_I(1)$ , coming from the initial state, is present, and ii) there are no leading-power contributions from soft gluons. Then the geometry of the leading-power regions that contribute to (2) is simple, just consisting of the ultraviolet region and the region collinear to the initial state, with no overlap.

For general NLO processes, leading-power contributions to (2) involve a complicated geometry of possibly overlapping regions, that include soft contributions as well as collinear contributions. A typical case is shown in Fig. 1. Ref.<sup>7</sup> discusses how to extend the method to deal with such cases. A first step in this program is to develop techniques to decompose Feynman graphs into sums of terms over different regions, with the terms arranged so as to correspond to factors in a factorization formula suitable for the event-generator application. Similar problems are encountered in multi-loop calculations based on graph-by-graph methods (see, e.g., ref.<sup>9</sup>). This corresponds to a decomposition for  $H^{(\text{NLO})}$  of the kind

$$H^{(\text{NLO})} = \sum_{\text{regions } R} A_H(R) + \text{nonleading power}, \quad (3)$$

holding uniformly over the whole of the phase space. Each of the pieces in (3) contains counterterms that prevent double counting and provide the suppression for going outside the region in which that particular piece was originally supposed to give a good approximation to the matrix element. This subtractive approach is to be contrasted with approaches based on splitting the phase space in different domains and using different approximations to the matrix element in these different domains (see, e.g., refs.<sup>1,2</sup>).

Semi-analytical<sup>10,11</sup> or fully numerical<sup>12</sup> subtraction methods have been devised to calculate NLO quantities that are infrared safe. These methods are not directly applicable in event generators that simulate the fully exclusive structure of the hadronic final states, since here the quantities being computed are not infrared safe in perturbation theory. In particular, one cannot use a cancellation of soft gluon contributions between real and virtual graphs.

Ref.<sup>13</sup> discusses a strategy to construct a decomposition of the kind (3). This is inspired by the R-operation techniques of renormalization. See ref.<sup>14</sup> for a related

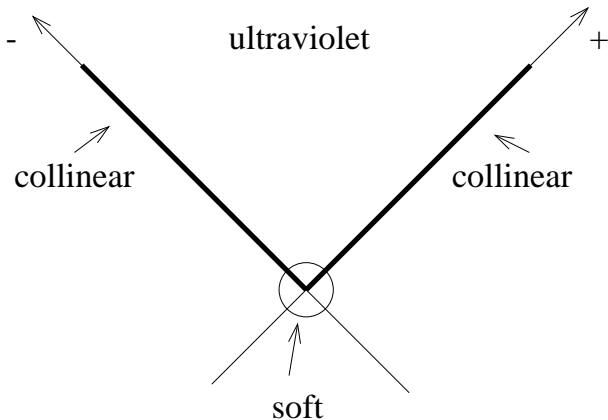


Figure 1: Geometry of leading-power regions for a typical NLO cross section. The axes denote lightcone directions in momentum space.

approach. Given the list of the leading regions, determined by standard power counting arguments<sup>15</sup>, we proceed from “smaller” to “larger” regions. See Fig. 2. For each region  $R$ , we remove the contribution from smaller regions, and construct an approximation to the matrix element valid in  $R$ , up to power suppressed corrections. Then we subtract any divergences that appear in this expression coming from larger regions. To ensure that the splitting between the terms is defined gauge-invariantly, at each step we demand that the counterterms be constructed from matrix elements of Wilson line operators,

$$V(n) = \mathcal{P} \exp \left( ig \int_0^{+\infty} dy n \cdot A(y n) \right), \quad (4)$$

with suitable directions  $n$  for the lines. Evolution equations in  $n$  enable one to connect the results corresponding to different directions. Taking  $n$  along lightlike directions gives back the eikonals of the leading infrared approximation, while taking  $n$  off the light cone provides effective cut-off parameters — but in such a way that even a formalism involving off-shell partons (see, e.g., ref.<sup>4</sup>) could be treated with gauge invariance. It has been shown how to apply this procedure to virtual loops<sup>13</sup> and real emission corrections<sup>7</sup>.

Ref.<sup>7</sup> uses this method to construct a decomposition (3) for one-gluon emission graphs in deep inelastic scattering. The term corresponding to the ultraviolet region gives the subtracted hard-scattering function to be used in Eq. (2). The collinear terms correspond to the evolution kernels to be used in the showering. The soft term would correspond to a new element in the Monte Carlo, but ref.<sup>7</sup> shows that this term can be eliminated by a suitable choice of the directions  $n$  for the Wilson lines. This is a result analogous to one in ref.<sup>10</sup>. Whether or not this result generalizes to all orders remains to be investigated and is very important to the construction of

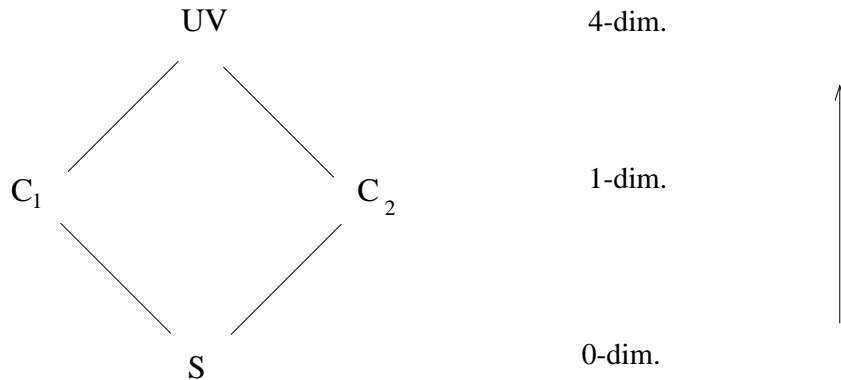


Figure 2: The soft (S), collinear ( $C_1$ ,  $C_2$ ), ultraviolet (UV) regions, corresponding to the model geometry of Fig. 1.

NLO parton showers.

The decomposition of ref.<sup>7</sup> entails a specific definition for the collinear factors, which will not necessarily coincide with the definitions used in any current event generator. The issue of deriving a showering algorithm that corresponds to the subtracted collinear terms is the subject of current work. A correct answer to this question will encompass soft-gluon coherence and angular ordering. It will involve evolution equations with respect to the direction of the Wilson line, Eq. (4), in terms of which the collinear subtractions are defined.

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### References

1. M.H. Seymour, *Comput. Phys. Commun.* **90**, 95 (1995).
2. G. Corcella and M.H. Seymour, *Phys. Lett. B* **442**, 417 (1998).
3. J. André and T. Sjöstrand, *Phys. Rev. D* **57**, 5767 (1998).
4. S. Mrenna, e-print hep-ph/9902471.
5. C. Friberg and T. Sjöstrand, e-print hep-ph/9906316, in *Proceedings of the DESY Workshop “Monte Carlo Generators for HERA Physics”*, eds. A.T. Doyle, G. Grindhammer, G. Ingelman and H. Jung (Hamburg, 1999), p.181.
6. J.C. Collins, *JHEP* 0005:004 (2000).
7. J.C. Collins and F. Hautmann, e-print hep-ph/0009286.
8. M. Bengtsson and T. Sjöstrand, *Z. Phys. C* **37**, 465 (1988).
9. T. Binoth and G. Heinrich, *Nucl. Phys.* **B585**, 741 (2000).
10. S. Catani and M.H. Seymour, *Nucl. Phys.* **B485**, 291 (1997).
11. S. Frixione, Z. Kunszt and A. Signer, *Nucl. Phys.* **B467**, 399 (1996).
12. D.E. Soper, *Phys. Rev. Lett.* **81**, 2638 (1998).

13. J.C. Collins and F. Hautmann, Phys. Lett. B **472**, 129 (2000).
14. F.V. Tkachov, e-print hep-ph/9703423; Int. J. Mod. Phys. **A8**, 2047 (1993).
15. S.B. Libby and G. Sterman, Phys. Rev. D **18**, 3252 (1978); *ibid.* **18**, 4737 (1978).