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Composite vortices in two-component Bose-Einstein condensates

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Abstract. Motivated by the recent observation of composite vortices in two-gap superconductors, we investigate the occurrence and stability of composite vortices in a two-component Bose condensate. As a function of the inter-component interaction strength, the vortices in the two components of the Bose mixture can overlap and form a composite vortex object. However, when two such composite vortices approach each other, their mutual interaction may cause the composite object to dissociate in its component vortices. We derive the phase diagram for the stability of composite vortices as a function of the interaction strengths of the Bose condensates, based on a variational calculation. The effective forces between the component vortices of the composite object are also derived within this approach, and allow for an interpretation of the phase diagram.

1. Introduction

The discovery of superconductors with multiple superconducting gaps provides new prospects for the study of vortices. Magnesium diboride is an example of a two-gap superconductor, which has been named “type 1.5 superconductor” [1] since one gap corresponds to much more strongly bound pairs than the other gap. In magnesium diboride, vortices have been observed to form stripe patterns, which do not come about due to pinning centers, defects or edge effects [2]. This seems to indicate that the interactions between the vortices may have a dipolar component, due to the *composite* nature of vortices in these materials. The precise interpretation of the observed patterns in these materials with two superconducting components is still under discussion.

Motivated by this debate on composite vortices, we turn our attention to the simpler system of a two-component atomic superfluid. Recent advances in ultracold atom physics have allowed for an unprecedented level of control over the experimental parameters of ultracold atomic gases. The number of particles, temperature, confinement geometry, dimensionality and interaction strength can be tuned accurately over a broad range of values. Due to this experimental flexibility, ultracold atoms are also known as a quantum simulator for many-body systems and theories [3].

In this contribution, we consider a mixture of two Bose gases, with tunable interactions, and with vortices in both components. These two components reflect the presence of the two types of Cooper pairs present in two-gap superconductors. We consider the regime where a vortex in this system is actually not a single vortex, but consists of two vortices: one in each component. This we will call a composite vortex. Interactions between composite vortices are more complicated than interactions between single vortices and have not yet been fully understood. In this

contribution, we first investigate the influence of the inter-component interaction on the core size of the component vortices. Next the interaction between two composite vortices is studied as a function of the distance between them. Finally, we will study the stability of two interacting composite vortices against dissociation (separation of both vortices in a composite vortex).

2. Gross-Pitaevskii description of the two-component condensate

A mixture of two Bose condensed atomic gases can be described by two order parameters Ψ_1 and Ψ_2 . These reduce to the individual order parameters of each component of the mixture, in the absence of interactions between atoms of different components. The number of atoms of each component can be experimentally tuned, and to describe this we introduce chemical potentials μ_1 and μ_2 for components 1 and 2, respectively. The Gross-Pitaevskii energy functional for a mixture of two bosonic gases in the regime of constant chemical potentials is given by

$$\begin{aligned} E[\Psi_1, \Psi_2] &= \mathcal{E}[\Psi_1, \Psi_2] - \mu_1 N[\Psi_1] - \mu_2 N[\Psi_2] \\ &= \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m} |\nabla_{\mathbf{r}} \Psi_1(\mathbf{r})|^2 + (V_1(\mathbf{r}) - \mu_1) |\Psi_1(\mathbf{r})|^2 \right\} \\ &+ \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m} |\nabla_{\mathbf{r}} \Psi_2(\mathbf{r})|^2 + (V_2(\mathbf{r}) - \mu_2) |\Psi_2(\mathbf{r})|^2 \right\} \\ &+ \int d\mathbf{r} \left\{ \frac{1}{2} g_{11} |\Psi_1(\mathbf{r})|^4 + g_{12} |\Psi_1(\mathbf{r})|^2 |\Psi_2(\mathbf{r})|^2 + \frac{1}{2} g_{22} |\Psi_2(\mathbf{r})|^4 \right\}. \end{aligned} \quad (1)$$

Here, m is the mass of the atoms, taken to be the same for both components. Typically, different hyperfine spin states can be trapped simultaneously in a optical trap, and brought into the Bose-Einstein condensed regime. In this way, a mixture of Bose condensates has been realized experimentally[4, 5]. The trapping potentials for components 1 and 2 are indicated by $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$, respectively. The interatomic interactions, in the ultracold regime where the de Broglie wavelength is much larger than the range of the interatomic potential, is well characterized by a contact pseudopotential with given strength g_{ij} for the interactions between an atom of component i with an atom of component j . The strength of the contact potentials is related to the s-wave scattering lengths a_{ij} through $g_{ij} = 4\pi\hbar^2 a_{ij}/m$.

It helps to rewrite the Gross-Pitaevskii functional in a dimensionless form. For this purpose, we write

$$\Psi_1(\mathbf{r}) = \rho_1 f_1(\mathbf{r}) e^{i\theta_1(\mathbf{r})}, \quad (2)$$

$$\Psi_2(\mathbf{r}) = \rho_2 f_2(\mathbf{r}) e^{i\theta_2(\mathbf{r})}, \quad (3)$$

with real functions $f_j(\mathbf{r})$ for the amplitudes and $\theta_j(\mathbf{r})$ for the phase fields. The constants ρ_1^2 and ρ_2^2 represent the homogeneous bulk particle density of the components, far away from any perturbation like a vortex core. Hence, we require $f_j(r \rightarrow \infty) = 1$. These bulk densities are naturally related to the chemical potentials. Extremizing $E_{bulk} = E[\rho_1, \rho_2]$ one straightforwardly finds

$$\rho_1^2 = \frac{\mu_1 g_{22} - \mu_2 g_{12}}{g_{11} g_{22} - g_{12}^2}, \quad (4)$$

$$\rho_2^2 = \frac{\mu_2 g_{11} - \mu_1 g_{12}}{g_{11} g_{22} - g_{12}^2}. \quad (5)$$

The mixture is stable (with respect to unmixing and component separation) when the extremum in E_{bulk} is a minimum, requiring

$$g_{11} g_{22} > g_{12}^2, \quad (6)$$

and $g_{11} > 0$ or $g_{22} > 0$. In order to obtain a dimensionless form for the Gross-Pitaevskii potential, we must choose length and energy scales. We will relate those to the first component, and set

$$\tilde{\xi}_1 = \sqrt{\frac{\hbar^2}{2mg_{11}\rho_1^2}} \quad (7)$$

as length scale. If inter-component interactions are absent, this represents the coherence length of the first component condensate. The energy scale that we use is $\epsilon_0 = \hbar^2/(2m\tilde{\xi}_1^2)$. With these conventions we obtain for the energy $\Delta E = (E[\Psi_1, \Psi_2] - E_{bulk})/\epsilon_0$ to be extremized

$$\begin{aligned} \Delta E[f_1, \theta_1, f_2, \theta_2] = \int d\mathbf{r} \left\{ (\nabla_{\mathbf{r}} f_1)^2 + \delta (\nabla_{\mathbf{r}} f_2)^2 + f_1^2 (\nabla_{\mathbf{r}} \theta_1)^2 + \delta f_2^2 (\nabla_{\mathbf{r}} \theta_2)^2 \right. \\ \left. + \frac{1}{2}(1 - f_1^2)^2 + \gamma \delta (1 - f_1^2)(1 - f_2^2) + \frac{1}{2}\beta \delta^2 (1 - f_2^2)^2 \right\}, \end{aligned} \quad (8)$$

with

$$\beta = \frac{g_{22}}{g_{11}}, \quad \gamma = \frac{g_{12}}{g_{11}}, \quad \delta = \frac{\rho_2^2}{\rho_1^2}. \quad (9)$$

For experiments with mixtures of ^{87}Rb condensates in the $F = 2, m_F = +1$ and $F = 1, m_F = -1$ states[5], the experimental values of the constants are $\beta = 1.062(12)$ for the ratio of intraspecies scattering lengths[6] and $\gamma = 1.030(10)$ for the dimensionless interspecies interaction strength[7]. Feshbach resonances are used experimentally to tune the interaction strengths[8]. In the current context these are magnetically tunable resonances between scattering states of the atoms in the open scattering channel, and a bound (molecular) state in a closed scattering channel. Typically, it is difficult to independently fix β and γ to an arbitrary value, as the changes in both β and γ will be coupled when sweeping the magnetic field through the Feshbach resonance. Nevertheless, at least one of these parameters can be made large (increasing the s-wave scattering length up to 10^4 Bohr radii) through the use of the resonance.

3. Composite vortex

The first question to be answered is how much the vortex cores, seen as holes in the density of the atomic clouds, are renormalized by the coupling. For this, we take as variational trial functions

$$f_j(\mathbf{r}) = \tanh \left[r / \left(\sqrt{2}\xi_j \right) \right], \quad (10)$$

$$\theta_j(\mathbf{r}) = \phi, \quad (11)$$

where cylindrical coordinates $\mathbf{r} = \{r, \phi, z\}$ are used. Firstly, note that both single-component vortices are placed on the z -axis, i.e. on top of each other in the otherwise homogeneous situation. If there are no inter-component interactions, there is (in the homogeneous case) no need to place the phase singularity in the first component on top of the phase singularity in the second component. Only if the inter-component interaction is attractive, $\gamma < 0$, does this lower the energy. Indeed, each component will have a hole in the atom density at the vortex core, and since atoms of different components attract, the lowest energy configuration will occur when the holes in the density overlap. Secondly, note that we have introduced ξ_j as a typical size of the vortex core in component j : these are now adjustable variational parameters and they can differ

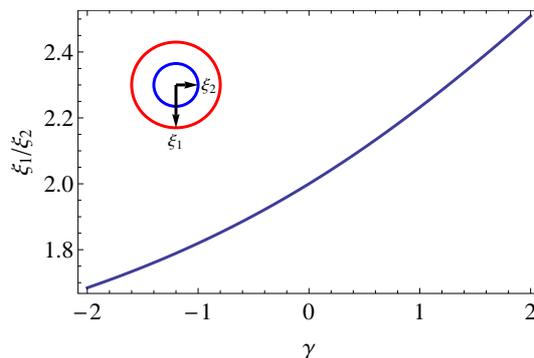


Figure 1. The ratio between the core sizes in component 1 and 2, as obtained from the variational calculation, is shown as a function of the inter-component interaction strength. The composite vortex considered in this calculation is such that the component vortices have coinciding centers.

from the unrenormalized healing lengths $\tilde{\xi}_j$ introduced before. The variational energy becomes

$$\begin{aligned} \Delta E(\xi_1, \xi_2) = & \ln(1/\xi_1) + \delta \ln(1/\xi_2) + \frac{\ln(16) - 1}{3} (\xi_1^2 + \beta \delta \xi_2^2) \\ & + \delta \gamma \int_0^\infty r \operatorname{sech}^2\left(\frac{r}{\sqrt{2}\xi_1}\right) \operatorname{sech}^2\left(\frac{r}{\sqrt{2}\xi_2}\right) dr. \end{aligned} \quad (12)$$

In order to get a grasp of the typical behavior to be expected for vortices in two-component condensates, we study as a concrete example the case with $\delta = 1$ (the same bulk particle density for both components), and $\beta = 4$. In the absence of inter-component interactions ($\gamma = 0$), the renormalised healing lengths are equal to the unrenormalised healing lengths, $\xi_j = \tilde{\xi}_j$. In that case, the unrenormalised healing lengths are by definition (7) related as $\tilde{\xi}_2 = \tilde{\xi}_1/\sqrt{\delta\beta}$ and the vortex core of component 2 is half the size of the vortex core in component 1. To get to a nontrivial example, we need the inter-component coupling. We allow the coupling to vary between $\gamma = -2, \dots, 2$, satisfying the stability criterion (6) which can be rewritten as $\gamma^2 < \beta$. The result for these parameter values is shown in Figure 1. We find that the effect of the interaction-induced renormalization on the ratio ξ_2/ξ_1 is relatively small, of the order of 20% in the full range of possible γ values. In comparison, $\xi_2/\tilde{\xi}_2$ ranges from 0.68 to 1.23, and $\xi_1/\tilde{\xi}_1$ ranges from 0.87 to 1.12 (where in both cases the size of the core is largest for $\gamma = -2$). So, the individual core sizes get influenced more strongly than the ratio between core sizes, but all in all, the estimates of the core sizes based on the non-interacting component analysis is roughly right.

4. Interactions between composite vortices

Now we consider two composite vortices, approaching each other. We fix the distance a between the vortex cores in the first component. This means that for the first component, we have

$$f_1(\mathbf{r}) = \tanh\left[\frac{r}{\sqrt{2}\xi_1}\right] \tanh\left[\frac{\sqrt{r^2 + a^2 - 2ra \cos \phi}}{\sqrt{2}\xi_1}\right], \quad (13)$$

$$\theta_1(\mathbf{r}) = \phi + \arctan\left(\frac{r \sin \phi}{r \cos \phi - a}\right). \quad (14)$$

The amplitude of the order parameter of the first component has two holes drilled in it, one at the origin and one at a distance a away on the x -axis. The phase field is the sum of the angle around the origin, and the angle around the center of the second vortex. In the second component, the positions of the vortex cores are adapted variationally, and we have

$$f_2(\mathbf{r}) = \tanh \left[\frac{|\mathbf{r} - \mathbf{r}_1|}{\sqrt{2}\xi_2} \right] \tanh \left[\frac{|\mathbf{r} - \mathbf{r}_2|}{\sqrt{2}\xi_2} \right], \quad (15)$$

where \mathbf{r}_1 and \mathbf{r}_2 are the positions of the vortices in component 2. The phase in component 2 will likewise be a sum of the angle around the first vortex plus the angle around the second vortex. When these expressions are introduced in the energy functional, a rather cumbersome expression appears. Insight can be gained by realizing that the variational energy functional is dominated by three contributions: firstly there is a repulsion between the vortices in the first component; secondly there is the analogous repulsion between vortices in the second component; and thirdly there is an attraction between vortices from component 1 and component 2. These contributions, for our case with $\beta = 4$, $\gamma = -2$ are shown in Figure 2.

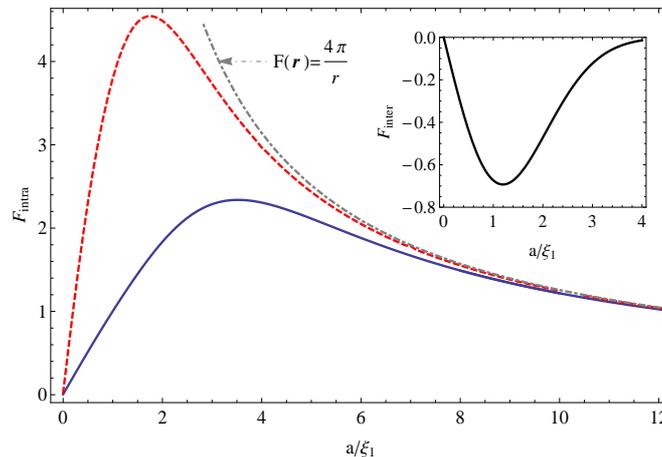


Figure 2. The individual contributions to the force between the vortices at $\beta = 4$, $\gamma = -2$. Full blue: the repulsion between component 1 vortices. Dashed red: the repulsion between component 2 vortices. Inset: inter-component attraction.

This figure depicts the (negative) gradients in the energy, i.e. the forces, as a function of distance. When the vortices within a same component are far apart, they repel each other with an inverse distance force $F(\mathbf{r}) = 4\pi/r$ that corresponds to the Lorentz force felt by a vortex due to the superfluid velocity field of the other vortex. At shorter distances, the cores start to overlap and the force is reduced. The repulsive force becomes maximal at a distance near 2ξ where ξ is representative for the vortex core radius. The renormalization of the vortex core size (the healing length) at short intervortex distance is taken into account, but does not change these results significantly. With only the intra-component vortex-vortex interaction, there is a repulsive force at all length scales. The inter-component vortex-vortex interaction, on the other hand, is attractive, and proportional to γ , the interatomic interaction strength between atoms of the two components. This force is due not to the kinetic energy of superfluid velocity fields, but it is due to the overlap of the component order parameters. Hence, it is short range as the vortices of different components “feel” each other only when the cores are overlapping. The attraction is maximal when the vortex cores (of sizes represented by ξ_1, ξ_2) are just touching, at an intervortex distance of the order of $\xi_1 + \xi_2$. Once the “smaller” vortex core is captured

inside the “larger” vortex core, the force diminishes again: the “small vortex core” is free to rattle around in the well provided by the “large” vortex core. For the parameters used in Figure 2 (with a modest γ), the attraction provided by the inter-component force is relatively weak in comparison to the repulsion; however it scales with γ .

What does this mean for the interaction between composite vortices? Two composite vortices at a large distance from each other will repel each other with a force inversely proportional to their distance; and the force for the constituent component 1 and component 2 vortices are of the same size. So, the composite vortices move apart, but retain their character as composite objects (i.e. the component 2 vortex stays bound to the component 1 vortex). We can now envisage the following experiment: pin the component 1 vortices, and slowly move them closer together while allowing the component 2 vortices to adapt their position so as to minimize the energy. The inter-component attraction will favour keeping component 2 and component 1 vortices together in a composite object. The intra-component repulsion will try to push the component 2 vortices away from each other, regardless of the position of component 1 vortices. At some distance, the intra-component repulsion will overcome the inter-component attraction and the component 2 vortices will unbind from their component 1 vortex. The composite vortices, when moved closer than a critical distance, can decompose. We calculated the phase diagram for this process using the variational calculation outlined in Sec. II and the results are summarized in Figure 3.

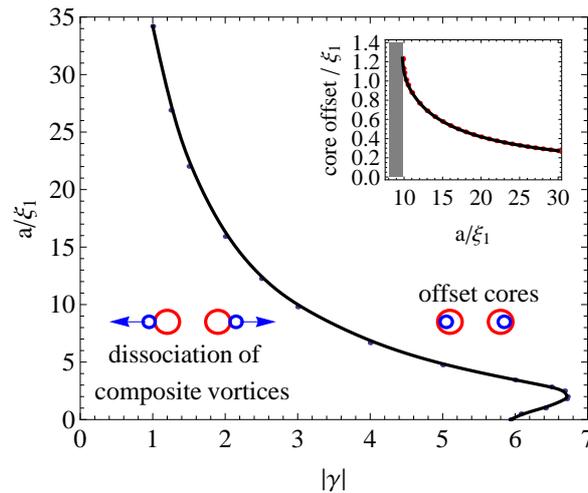


Figure 3. Phase diagram at $\beta = 4$ of the composite vortex interaction as function of the distance a between the cores in component 1 and ratio $|\gamma|$ of the attractive inter-component interaction strength to the interaction strength in component 1. Inset: Core offset of the component 2 vortices relative to the center of the component 1 vortices as function of distance a .

Figure 3 shows which phase is realized, for a set of distances a separating the component 1 vortices and for a set of inter-component interaction strengths $|\gamma|$, assumed to correspond to an attractive interaction¹. Left of the phase separation line, the composite vortices decompose as the component 2 vortices unbind from their component 1 partner and move apart. Right

¹ We show results also for the regime $|\gamma| > \sqrt{\beta}$, which according to our criterion (6) leads to unmixing. However, this criterion was derived in essence in the grand canonical ensemble (working with fixed chemical potentials rather than with fixed numbers of atoms during the unmixing transition), whereas for a closed system, strong attractive interspecies interactions need not lead to unmixing.

of the curve, the composite vortices stay composite in the sense that the component 2 vortex remains bound to the component 1 vortex, even though it will be offset from the center of the component 1 vortex. This offset is due to the fact that the “small” component 2 vortex can move around in the “large” component 1 core without this costing too much energy. The offset gradually increases as the composite vortices are forced closer together, as shown in the inset of Figure 3. When one wants to be able to bring the composite vortices closer together (a smaller), a stronger attraction γ is needed to glue the component 2 vortex to its component 1 partner. But this seems not to be the case at the smallest distances: note the bending over of the phase separation line in Figure 3. What happens there is that, as the vortex cores of the component 2 vortices start to overlap, the repulsion between the component 2 vortices weakens, whereas the attraction keeping them bound to the component 1 cores only weakens at an even shorter scale, as can be seen in Figure 2. The result is twofold: for certain inter-component interaction strengths there is reentrant behavior, and there exists a critical inter-component interaction strength that results in component vortices remaining stable with respect to dissociation at all distance scales. For the situation in Figure 3 this value is approximately $\gamma = -6.72$.

5. Conclusion

In this article, the interaction between two composite vortices in a two-component Bose-Einstein condensate was discussed. First it was shown that the coupling between the two components implies a change in both the size of the vortex cores and their ratio, with the latter experiencing the smallest amount of modification. This renormalization was taken into account in the subsequent calculations, but was found to have no essential qualitative influence. Next, the force between two composite vortices was analysed by considering the three contributions separately. We found two strong, long range repulsive contributions (corresponding to the intra-component vortex interaction) and one weak, short range attractive contribution for the inter-component interaction, due to the overlap of the component order parameters. This results finally in a phase diagram with two distinct areas: a first where the composite vortices dissociate and a second where they remain stable despite their offset cores. It was found that for a sufficiently strong inter-component interaction, the system will be stable with respect to dissociation at all distance scales, meaning that the composite vortices will remain intact.

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