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**Optimal pricing of public final and intermediate  
goods in the presence of externalities\***

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## ABSTRACT

The purpose of this paper is to incorporate externalities in the literature on public pricing of both final and intermediate goods. We develop a model that determines welfare-optimal prices which capture the general equilibrium effects on private sector prices and correct for the existence of the externalities. The model does not impose constant returns to scale on private production, and it allows for distributional effects of both the publicly determined prices and private sector profits. Moreover, the externalities are allowed to generate feedbacks in demand, and are assumed to affect both consumers and producers. Optimal pricing rules are derived, carefully interpreted, and the relation of the results with previous models of optimal pricing in the presence of externalities is investigated.

## Introduction

It has long been realized that welfare-optimal prices should systematically deviate from marginal costs when budgetary restrictions are imposed on a public sector characterised by increasing returns to scale (see, e.g., Ramsey (1927), Boiteux (1956), Baumol and Bradford (1970)). An abundant literature has developed in which the traditional Ramsey rule was extended in a variety of ways. First, a number of writers studied the implications of distributional considerations (Feldstein (1972a), Bös (1986)). Second, optimal pricing rules were extended to cover both public final and intermediate goods. Assuming a more general production structure, Yang (1991) confirmed Feldstein's (1972b) result that the Ramsey rules were formally the same for both types of public enterprises' outputs<sup>1</sup>. Recently, both Iwamoto and Konishi (1991), within an optimal taxation framework, and Yang (1993) extended this analysis by explicitly incorporating distributional issues, including the effects of private sector profit distribution. Third, the optimal pricing model had to be extended to deal with externalities. The seminal paper by Sandmo (1975) considered an optimal commodity taxation problem in a general equilibrium setting with a linear production structure and a consumption externality. More recently, Oum and Tretheway (1988) focused on the derivation of public sector Ramsey rules in a partial equilibrium framework with external effects and explicit demand interdependencies. Moreover, Bovenberg and van der Ploeg (1994) constructed a second-best general equilibrium taxation model that incorporates consumption externalities<sup>2</sup>. This model was extended by Mayeres and Proost (1994) to accommodate production externalities and nonidentical households. Unfortunately, the resulting pricing rules are highly complex and difficult to interpret; therefore, a detailed simulation exercise was used to illustrate the implications of their findings.

The purpose of the present paper is to integrate the above literature to study optimal pricing rules for a budget-constrained and externality-generating public sector offering both final and intermediate goods. For purposes of concreteness the model is phrased in terms of a publicly controlled transport sector offering both passenger transportation (a final consumer good) and freight transport (an intermediate good). We develop a model that determines welfare-optimal prices which capture the general equilibrium effects on private sector prices and correct for the existence of the externalities. The model does not impose constant returns to scale on

private production, and it allows for distributional effects of both the publicly determined prices and private sector profits. Moreover, the specification of the externalities, which are assumed to affect both consumers and producers, is quite general. It allows us to accommodate congestion-type externalities (which directly feed back into the demands of consumers and producers) as well as other externalities (such as pollution) which do not have feedback effects<sup>3</sup>.

Structure of the paper is as follows. In a first section we present the model. In Section 2 we derive and interpret the optimal pricing results. Section 3 investigates the relation of our findings to previous models incorporating externalities. Finally, Section 4 concludes.

## 1. Structure of the model

We consider a model consisting of two sectors: a private sector and a publicly controlled transport sector. The latter consists of two modes (e.g., road and rail) which each produce both passenger transport (final good) and freight transport (intermediate good)<sup>4</sup>. To keep the model as simple as possible it is assumed that one transport mode (say, road transport) imposes an externality on both consumers and private producers. The government is interested in determining optimal prices in the transport sector, taking account of the externality, of the impact of public pricing on the private market, and of distributional considerations. Moreover, to remain within the Ramsey tradition it is further assumed that a budgetary restriction is imposed on the transport sector<sup>5</sup>.

### The transport sector

Turning to specifics, let us denote the outputs of the two transport modes by  $z_1$  and  $z_2$ , respectively. Each mode produces both a final consumption good ( $z_i^H$ ,  $i=1,2$ ) and an intermediate good used by the private sector as an input into the production process ( $z_i^F$ ,  $i=1,2$ ).

It is assumed that the first transport mode produces an external effect  $E$ . The latter is simply defined as a weighted sum of the use of the first mode by consumers and the private sector,

i.e.,

$$E = \rho z_1^H + \beta z_1^F$$

where  $\rho$  and  $\beta$  are the relative contributions of the final and intermediate goods to the external effect, respectively<sup>6</sup>. The interpretation is clear. If mode 1 is road transport, then  $z_1^H$  and  $z_1^F$  are the volumes of passenger and freight transport, and the weights reflect different external effects of private cars and trucks.

### The private sector

Implicitly, the private sector consists of a large number of competitive firms whose production functions contain labor, the quantities of the intermediate inputs, and the external effect  $E$  as arguments. However, to save on notation, and without loss of generality, we aggregate the private sector and assume it produces output  $y$  using public inputs  $z_i^F$  ( $i=1,2$ ) and labor  $L_y$  according to a production plan  $(y, z_1^F, z_2^F, L_y, E) \in Y$ , where  $Y$  is the set of feasible production plans. In order to avoid the problems associated with possible externality-induced nonconvexities, it is assumed that the production set is convex for given externality parameters (Baumol and Bradford (1972)). Moreover, it is assumed that the sector maximizes profit given feasible production plans  $Y$

$$\text{Max} \quad qy - p_1^F z_1^F - p_2^F z_2^F - L_y$$

$$y, z_1^F, z_2^F, L_y$$

$$\text{s.t. } (y, z_1^F, z_2^F, L_y, E) \in Y$$

Solution of this problem defines the output supply function

$$y(q, p_1^F, p_2^F, E),$$

the system of input demand functions

$$z_i^F(q, p_1^F, p_2^F, E) \quad (i=1,2),$$

$$L_y(q, p_1^F, p_2^F, E)$$

and the corresponding profit function

$$\pi(q, p_1^F, p_2^F, E),$$

where  $q$  is output price, and the  $p_i^j$  are the respective input prices.

### The behavior of households

Households are assumed to maximize utility subject to a budget restriction. They derive income from two sources in this model. First, labor income is generated by working at the numeraire wage; second, they share in private sector profits. The constrained optimisation problem can be summarized as follows

$$\begin{aligned} & \text{Max} && u^h(x^h, z_1^h, z_2^h, L^h, E) \\ & && x^h, z_1^h, z_2^h, L^h \\ & \text{s.t.} && q x^h + p_1^H z_1^h + p_2^H z_2^h = L^h + \delta^h \pi \end{aligned}$$

where  $x^h$  is household  $h$ 's consumption of the private good,  $z_i^h$  is consumption of transport mode  $i$  by  $h$ , and  $L^h$  is family  $h$ 's labor supply. The prices of  $z_1^H$  and  $z_2^H$  are denoted by  $p_1^H$ , and  $p_2^H$ , respectively. Finally,  $\delta^h$  is  $h$ 's share in private sector profit  $\pi$ .

The solution of the consumer's problem yields the system of demand functions

$$\begin{aligned} & z_1^h(q, p_1^H, p_2^H, \delta^h \pi, E) \\ & z_2^h(q, p_1^H, p_2^H, \delta^h \pi, E) \\ & x^h(q, p_1^H, p_2^H, \delta^h \pi, E), \end{aligned}$$

the labor supply function

$$L^h(q, p_1^H, p_2^H, \delta^h \pi, E),$$

and the corresponding indirect utility function

$$v^h(q, p_1^H, p_2^H, \delta^h \pi, E).$$

### The nature of the externality

The externality considered in this paper is allowed to be of a fairly general nature. To see this, consider for example the effect of an increase in the price of the first mode to consumers,  $p_1^H$ . This directly affects the externality level through its impact on the demand for  $z_1^H$ . However, the increase in  $E$  itself results in a further indirect ('feedback') effect, because at higher externality levels demand for both  $z_1^H$  and  $z_1^F$  is affected. The total impact of an increase in the price of the first mode to passengers on  $E$  is easily analyzed as follows.

Totally differentiating  $E = \rho z_1^H + \beta z_1^F$ , using the specifications of the demand functions

$$z_1^H = \sum_h z_1^h (q, p_1^H, p_2^H, \delta^h \pi, E)$$

$$z_1^F = z_1^F (q, p_1^F, p_2^F, E)$$

and taking account of the impact of E on profit through the profit function  $\pi (q, p_1^F, p_2^F, E)$ , yields

$$\frac{dE}{dp_1^H} = \left\{ \rho \sum_h \left[ \frac{\partial z_1^h}{\partial p_1^H} + \frac{\partial z_1^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^H} + \frac{\partial z_1^h}{\partial E} \frac{dE}{dp_1^H} \right] + \beta \frac{\partial z_1^F}{\partial E} \frac{dE}{dp_1^H} \right\}$$

This can be rearranged to yield

$$\frac{dE}{dp_1^H} = \frac{\left\{ \rho \sum_h \frac{\partial z_1^h}{\partial p_1^H} \right\}}{1 - \eta}$$

$$\text{where } \eta = \left\{ \rho \sum_h \left[ \frac{\partial z_1^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial z_1^h}{\partial E} \right] + \beta \frac{\partial z_1^F}{\partial E} \right\}.$$

The interpretation of this result is straightforward. The numerator measures the direct impact of the price increase on the externality level. The denominator, however, corrects the direct effect for the feedback effect: the increase in the externality level E feeds back into the demand functions, which itself again affects the externality level. An obvious example of an externality with these general characteristics is congestion: total road transport generates the overall congestion level, which itself feeds back into all consumer demand functions for both publicly and privately provided goods, and into the production function and factor demands in the private sector.

Note that in the case of congestion the feedback effect is negative, so that the denominator of the above expression is larger than one.

Two remarks are in order. First, it is clear that not all externalities generate feedback effects. These externalities can however easily be considered to be special cases of the general specification analyzed here. For example, externalities such as air pollution do enter consumers' utility functions, but probably do not affect demand<sup>7</sup>. As a consequence the denominator is equal to one and only the direct effect matters. Second, analogous expressions can easily be derived for exogenous increases in  $p_2^H$ , in the prices of the intermediate goods  $p_1^F$ , and in private sector prices  $q$ . One obtains by a similar procedure

$$\frac{dE}{dp_2^H} = \frac{\rho \sum_h \frac{\partial z_1^H}{\partial p_2^H}}{1 - \eta}$$

$$\frac{dE}{dp_1^F} = \frac{\left\{ \rho \sum_h \frac{\partial z_1^h}{\partial I^h} \frac{\partial I^h}{\partial p_1^F} + \beta \frac{\partial z_1^F}{\partial p_1^F} \right\}}{1 - \eta}$$

$$\frac{dE}{dp_2^F} = \frac{\left\{ \rho \sum_h \frac{\partial z_1^h}{\partial I^h} \frac{\partial I^h}{\partial p_2^F} + \beta \frac{\partial z_1^F}{\partial p_2^F} \right\}}{1 - \eta}$$

and

$$\frac{dE}{dq} = \frac{\left\{ \rho \sum_h \left[ \frac{\partial z_1^h}{\partial q} + \frac{\partial z_1^h}{\partial I^h} \frac{\partial I^h}{\partial q} \right] + \beta \frac{\partial z_1^F}{\partial q} \right\}}{1 - \eta}$$



### Private sector market equilibrium

Finally, market equilibrium in the private sector requires

$$\sum_h x^h(q, p_1^H, p_2^H, \delta^h \pi, E) = y(q, p_1^F, p_2^F, E)$$

### The planning problem

The planner chooses all prices so as to maximise welfare, subject to two constraints, viz. the market equilibrium restriction, and a budgetary restriction imposed on the transport sector.

Total transport sector profit is given by

$$\text{Profit } \pi^* = p_1^H z_1^H + p_1^F z_1^F + p_2^H z_2^H + p_2^F z_2^F - G(z_1^H, z_1^F, z_2^H, z_2^F)$$

where  $G(\cdot)$  is the cost function of the transport sector<sup>8</sup>. Using the specifications of the demand functions resulting from utility maximizing behavior by consumers we can more succinctly write the sector's profits as

$$\pi^*(q, p_1^H, p_2^H, p_1^F, p_2^F, E)$$

With the above notation, the planner is assumed to look for the solution to the following problem

$$\begin{aligned} \text{Max} \quad & W \left\{ v^1(q, p_1^H, p_2^H, \delta^1 \pi, E), \dots, v^n(q, p_1^H, p_2^H, \delta^n \pi, E) \right\} \\ & q, p_1^H, p_2^H, p_1^F, p_2^F \\ \text{s.t.} \quad & \pi^*(q, p_1^H, p_2^H, p_1^F, p_2^F, E) = 0 \quad (\lambda) \\ & \sum_h x^h(q, p_1^H, p_2^H, \delta^h \pi, E) = y(q, p_1^F, p_2^F, E) \quad (\mu) \end{aligned}$$

## **2. Solution of the model: optimal pricing results**

There are different ways to summarize the optimal pricing rules that result from the first-order conditions of the welfare optimisation problem<sup>9</sup>. A convenient way from our

perspective is the following

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^H} \right] = \alpha_1 z_1^H + \gamma \frac{dq}{dp_1^H} - \theta \left[ \frac{dE}{dp_1^H} + \frac{dE}{dq} \frac{dq}{dp_1^H} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_2^H} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_2^H} \right] = \alpha_2 z_2^H + \gamma \frac{dq}{dp_2^H} - \theta \left[ \frac{dE}{dp_2^H} + \frac{dE}{dq} \frac{dq}{dp_2^H} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^F} \right] = k z_1^F + \gamma \frac{dq}{dp_1^F} - \theta \left[ \frac{dE}{dp_1^F} + \frac{dE}{dq} \frac{dq}{dp_1^F} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_2^F} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_2^F} \right] = k z_2^F + \gamma \frac{dq}{dp_2^F} - \theta \left[ \frac{dE}{dp_2^F} + \frac{dE}{dq} \frac{dq}{dp_2^F} \right]$$

where  $\alpha_i = \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \frac{z_i^h}{z_i^H}$

$$k = \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \delta^h$$

$$\gamma = \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} (x^h - \delta^h y)$$

and, finally,

$$\theta = \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] + \lambda \frac{\partial \pi^*}{\partial E} \right\}$$

Both  $\alpha_i$  and  $k$  reflect the impact of distributional considerations on optimal prices. As is well

known,  $\alpha_i$  represents what Feldstein (1972) called the distributional characteristic of good  $i$ . In a similar way,  $k$  was interpreted by Yang (1993) as the distributional characteristic of profit. The parameter  $\gamma$  summarizes how profit shares deviate from shares in the consumption of the private good. Finally,  $\theta$  captures the full welfare impact of the externality. It consists of two parts, viz. the welfare effect the externality generates via consumers' utilities, and the welfare impact of public price changes for public sector profits. This welfare effect is measured by multiplying the changes in public profits by the shadow price of the budgetary constraint.

Finally, note that in the above formulation,  $\frac{dq}{dp_i^j}$  measures the impact of a change in  $p_i^j (j=H,F)$  on equilibrium prices for the private good, taking account of the effect of price changes on  $E$ . The appropriate expression can easily be derived from the market equilibrium restriction using the implicit function theorem. It depends on the price sensitivities of both demand and supply, and on the externality effects of prices.

The optimal pricing expressions consist of four clearly identifiable components. The first part, viz. the term on the left-hand-side, captures two channels through which public pricing affects public sector profits. First, holding private sector prices and externality levels constant, a price increase affects demands, and therefore both revenues and costs. Second, transport prices have general equilibrium effects on private sector prices, thereby affecting public sector profits. The welfare effect of the price-induced change in public sector profits is obtained by multiplying profit changes by the shadow price of the public sector budget restriction.

The second and third component reflect distributional considerations. The first term on the right-hand-side (RHS) indicates, consistent with Yang (1993), that the optimal pricing rule for the final goods is governed by their respective distributional characteristics, whereas the rules for the intermediate goods are governed by the distributional characteristic of profit. The second term on the RHS represents the indirect redistribution effect associated with the general equilibrium effects of a price change. If the distribution of consumption of the private

good deviates from the distribution of profits then  $\gamma$  differs from zero, and the general equilibrium induced private sector price changes have further redistributive effects. If consumers' profit shares equal their shares in the consumption of the private good, or if no general equilibrium effects occur, then this third effect is zero.

Finally, the last term on the RHS measures the impact of a price change on welfare through its effects (both direct and indirect via general equilibrium price changes) on the externality. An increase in  $E$  reduces welfare in three different ways: first, at constant prices the externality affects private sector profits and therefore, through the distribution of profits, consumer incomes. Second, holding prices and income constant, the externality yields negative marginal utility. Third, the externality affects public sector profits, the welfare cost of which is evaluated by the shadow price of the budget constraint. Note that, consistent with Sandmo (1975), the distributional implications of the externality do matter in the determination of optimal prices. They are captured in  $\theta$ .

### 3. Relation with previous models

In this section we briefly investigate to what extent the optimal pricing rules generate the results of previous models as special cases. First, it is clear that, if there are no external effects so that the last term on the RHS becomes irrelevant (i.e., zero), our results reduce to those reported by Yang (1993)<sup>10</sup>. Second, if no external effects exist, no distributional concerns prevail and private sector prices are exogenous (i.e., there are no general equilibrium effects of public sector prices) then it is easily shown that the model implies Ramsey prices. Third, and more importantly, the question arises as to how our results relate to those of previous pricing and taxation models with externalities. However, note that all of these models have concentrated on externalities in consumption (Sandmo (1975), Bovenberg and van der Ploeg (1994)), assumed a partial equilibrium framework (Oum and Thretheway (1988)), or assumed constant returns to scale in private sector production, implying zero profits (Sandmo (1975), Bovenberg and van der Ploeg (1994), Mayeres and Proost (1994)). Interestingly, however, imposing the assumption of constant returns to scale in our model does not substantially alter the qualitative results<sup>11</sup>. In particular, all general

equilibrium effects of public sector prices remain nonzero. This is implicitly due to the fact that the externality affects both consumers and private production. An increase in public sector prices affect externality levels, and these in turn imply changes in marginal production costs and thus equilibrium prices on the private market. Note that, if the externality were a pure consumer externality and private production was characterized by constant returns to scale, an increase in the  $p_i^H$  would not affect marginal production costs, and therefore, it would have no impact on equilibrium prices.

To relate the pricing rules to more familiar expressions available in the literature, suppose that private sector prices are parametrically given. For example, assume that the model refers to a small open economy and that the private sector faces a perfectly elastic world demand for its output. It is clear that under these circumstances the general equilibrium effects of public sector pricing policies are zero. The optimal pricing rules then reduce to

$$\lambda \frac{\partial \pi^*}{\partial p_i^H} = \alpha_i z_i^H - \theta \frac{dE}{dp_i^H} \quad i=1,2$$

$$\lambda \frac{\partial \pi^*}{\partial p_i^F} = k z_i^F - \theta \frac{dE}{dp_i^F} \quad i=1,2$$

For purposes of interpretation, if we add the assumption of zero cross-price elasticities of demand, we can easily show that the rules for final goods prices are given by

$$\frac{p_1^H - MPC_1^H + \frac{\rho\theta}{\lambda(1-\eta)}}{p_1^H} = \frac{\alpha_1 - \lambda}{\lambda} \frac{1}{\epsilon_{11}^H}$$

$$\frac{p_2^H - MPC_2^H}{p_2^H} = \frac{\alpha_2 - \lambda}{\lambda} \frac{1}{\epsilon_{22}^H}$$

where  $\epsilon_{ii}^H$  is the own price elasticity of the demand for  $z_i^H$ , and  $MPC_i^H$  is the marginal private production cost associated with  $z_i^H$ . Not surprisingly, since only the first good contributes to the externality, the optimal rule for  $p_2^H$  is simply the Ramsey rule, corrected for the distributional characteristic of the good. With respect to the first good, the

term  $\frac{\rho\theta}{1-\eta}$  can be interpreted as the marginal social damage associated with an increase in the externality-generating good  $z_1^H$ . Indeed, an increase in  $z_1^H$  induces a rise in the externality level  $E$  by a factor  $\frac{\rho}{1-\eta}$ , which takes account of the direct contribution  $\rho$  and the feedback effect  $\eta$ . Therefore, the numerator of the left-hand-side of the above expression for  $p_1^H$  can be interpreted as the deviation of the price from the marginal production cost plus the marginal social damage, weighted by the inverse of the shadow price of the budget constraint. Expressed as a fraction of the price, this deviation could be inversely proportional to the own price elasticity of demand, taking account of the distributional characteristic of the first good. This illustrates that distributional issues play a double role. The distributional characteristic of the good is reflected in  $\alpha_i$ , whereas the distribution of the external effect over consumers is captured via  $\theta^{12}$ .

With respect to the pricing rules for the intermediate goods strong assumptions have to be made to derive similar simple expressions. The reason is that intermediate good prices affect not only the demand for intermediate goods, but also, via their impact on private sector profits and thus consumer incomes, the demands for all final goods. However, assuming zero cross-elasticities between private and intermediate goods we obtain<sup>13</sup>

$$\frac{p_1^F - MPC_1^F + \frac{\beta\theta}{\lambda(1-\eta)}}{p_1^F} = \frac{k-\lambda}{\lambda} \frac{1}{\varepsilon_{11}^F}$$

$$\frac{p_2^F - MPC_2^F}{p_2^F} = \frac{k-\lambda}{\lambda} \frac{1}{\varepsilon_{22}^F}$$

These expressions are formally identical to those for final goods. The interpretation is also similar, since the term  $\frac{\beta\theta}{1-\eta}$  reflects the marginal social damage due to an increase in the externality-generating good  $z_1^F$ .

Finally, let us assume that the policy-maker ignores distributional issues, i.e., let the marginal social utility of income be constant across consumers. In that case  $\theta$  is independent of the distribution of  $E$ , and  $\alpha_1 = \alpha_2 = k$ . Using these results yields the optimal pricing rules, again focusing on zero cross-elasticities<sup>14</sup>

$$\frac{p_1^H - (MPC_1^H + \frac{m}{\lambda} MEC_1^H)}{p_1^H} = \frac{m - \lambda}{\lambda} \frac{1}{\epsilon_{11}^H}$$

$$\frac{p_2^H - MPC_2^H}{p_2^H} = \frac{m - \lambda}{\lambda} \frac{1}{\epsilon_{22}^H}$$

$$\frac{p_1^F - (MPC_1^F + \frac{m}{\lambda} MEC_1^F)}{p_1^F} = \frac{m - \lambda}{\lambda} \frac{1}{\epsilon_{11}^F}$$

$$\frac{p_2^F - MPC_2^F}{p_2^F} = \frac{m - \lambda}{\lambda} \frac{1}{\epsilon_{22}^F}$$

where

$$MEC_1^H = \frac{\rho\theta}{m(1-\eta)}$$

$$MEC_1^F = \frac{\beta\theta}{m(1-\eta)}$$

are the marginal external costs of an increase in  $z_1^H$  and  $z_1^F$ , respectively. They are defined as the marginal social damage normalized by the marginal utility of income.

To interpret these results note that in the empirically most relevant case  $m < \lambda$ . In other words, the welfare cost of the budget constraint exceeds the marginal utility of income, reflecting the distortionary effects of public sector revenue generation. The optimal pricing rules then resemble Ramsey pricing, in which price is a markup over marginal private cost plus a fraction of marginal external cost. This finding is analogous to those of Sandmo (1975), Oum and Thretheway (1988), and Bovenberg and van der Ploeg (1994). The finding

that the markup should capture just a fraction of the marginal external cost (and not the full marginal external cost) can be explained by the fact that the budget restriction is specified in terms of private, not social, costs, whereas of course all externality costs are captured in the objective function. A markup of price over marginal private cost, necessary to satisfy the budget constraint, itself reduces the externality level. It implies in the framework of this simplified model that it suffices to introduce a markup which only captures part of the marginal external cost, as the budget restriction itself contributes to externality reduction.



#### **4. Conclusion**

In this paper we analysed the optimal pricing rules to be followed by a welfare-maximising budget-constrained public transport sector which offers both final and intermediate goods, where one of the goods generates a general external effect on both the private production sector and on consumers. Within the framework of Yang's (1993) general equilibrium setup it was shown that the resulting rules take account of four clearly identifiable issues: the impact of price changes on public sector budgets, the distributional characteristics of both goods and profits, the general equilibrium price effects of public prices, and the marginal external welfare costs of price-induced externality changes. The rules derived in this paper yield some results of Yang (1991, 1993), Sandmo (1975), Oum and Thretheway (1988), and Bovenberg and van der Ploeg (1994) as special cases.

## References

- Baumol, W. and D. Bradford (1970), 'Optimal departures from marginal cost prices', **American Economic Review** **60**, 265-283.
- Baumol, W. and D. Bradford (1972), 'Detrimental externalities and non-convexities of the production set', **Economica** **39**, 160-176.
- Boiteux, M. (1956), 'Sur la gestion des monopoles publics restreints à l'équilibre budgétaire', **Econometrica** **24**, 22-40.
- Bös, D. (1986), **Public sector economics**, North Holland, New York. Bovenberg, L. and F. van der Ploeg (1994), 'Environmental policy, public finance and the labour market in a second-best world', **Journal of Public Economics** **55**, 349-390.
- Breutigam, R. (1979), 'Optimal pricing with intermodal competition', **American Economic Review** **69**, 38-49.
- Feldstein, M. (1972 a), 'Distributional equity and the optimal structure of public prices', **American Economic Review** **62**, 32-36.
- Feldstein, M. (1972 b), 'The pricing of public intermediate goods', **Journal of Public Economics** **1**, 45-72.
- Iwamoto, Y. and H. Konishi (1991), 'Distributional considerations of producers' profit in a commodity tax design problem', **Economics Letters** **35**, 423-428.
- Mayeres, I. and S. Proost (1994), 'Optimal tax rules with congestion type of externalities', CES-working paper, Leuven.
- Oum, T.H. and M.W. Thretheway (1988), 'Ramsey pricing in the presence of externality costs', **Journal of Transport Economics and Policy** **22**, 307-317.
- Ramsey, F. (1927), 'A contribution to the theory of taxation', **Economic Journal** **37**, 47-61.
- Sandmo, A. (1975), 'Optimal taxation in the presence of externalities', **Swedish Journal of Economics**, 86-98.
- Spencer, B.J., and J.A. Brander (1983), 'Second-best pricing of publicly produced inputs', **Journal of Public Economics** **20**, 113-119.
- Yang, C.C. (1991), 'The pricing of public intermediate goods revisited', **Journal of Public Economics** **45**, 135-141.
- Yang, C.C. (1993), 'Distributional equity and the pricing of public final and intermediate goods', **Economics Letters** **41**, 429-434.

## Appendix

### Derivation of optimal pricing rules

In this appendix we provide more details on the derivation of the optimal pricing rules presented and interpreted in the main body of the paper. Consider the planner's problem

$$\begin{aligned} \text{Max}_{q, p_1^H, p_2^H, p_1^F, p_2^F} \quad & W \left\{ v^1(q, p_1^H, p_2^H, \delta^H \pi, E), \dots, v^H(q, p_1^H, p_2^H, \delta^H \pi, E) \right\} \\ \text{s.t.} \quad & \pi^*(q, p_1^H, p_2^H, p_1^F, p_2^F, E) = 0 \quad (\lambda) \\ & \sum_h x^h(q, p_1^H, p_2^H, \delta^H \pi, E) = y(q, p_1^F, p_2^F, E) \quad (\mu) \end{aligned}$$

where  $\pi(q, p_1^F, p_2^F, E)$  is the profit function.

First consider the first-order condition w.r. to  $p_1^H$ . This can be written as, denoting the consumer's nonlabor income as  $\Gamma^h = \delta^h \pi$

$$\begin{aligned} \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial p_1^H} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^H} + \frac{\partial v^h}{\partial E} \frac{dE}{dp_1^H} \right] + \lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^H} \right] \\ + \mu \left\{ \frac{\partial y}{\partial E} \frac{dE}{dp_1^H} - \sum_h \left[ \frac{\partial x^h}{\partial p_1^H} + \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial x^h}{\partial E} \right) \frac{dE}{dp_1^H} \right] \right\} = 0 \end{aligned}$$

But Roy's identity implies

$$\sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial p_1^H} = - \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} z_1^h = - \sum_h \left[ \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \frac{z_1^h}{z_1^H} \right] z_1^H = - z_1^H \alpha_1$$

$$\text{where } \alpha_1 = \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \frac{z_1^h}{z_1^H}$$

is the well-known 'distributional characteristic' of good 1. The first-order condition can therefore be rewritten as

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^H} \right] = \alpha_1 z_1^H + \mu \left\{ \sum_h \frac{\partial x^h}{\partial p_1^H} - \left[ \frac{\partial y}{\partial E} - \sum_h \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial x^h}{\partial E} \right) \right] \frac{dE}{dp_1^H} \right\} - \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial E} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} \right] \frac{dE}{dp_1^H} \quad (1)$$

Next consider the first-order condition w.r. to  $q$ . This yields

$$\begin{aligned} & \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial q} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dq} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial q} + \frac{\partial v^h}{\partial E} \frac{dE}{dq} \right] \\ & + \lambda \left[ \frac{\partial \pi^*}{\partial q} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dq} \right] + \mu \left\{ \frac{\partial y}{\partial q} + \frac{\partial y}{\partial E} \frac{dE}{dq} - \sum_h \left[ \frac{\partial x^h}{\partial q} + \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial q} \right) + \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial x^h}{\partial E} \right) \frac{dE}{dq} \right] \right\} \\ & = 0 \end{aligned}$$

But Roy's identity implies

$$\frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial q} = - \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} x^h$$

And Hotelling's lemma yields

$$\frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial q} = \frac{\partial v^h}{\partial I^h} \delta^h y, \text{ since } \delta^h \pi = I^h$$

Using these results implies

$$\begin{aligned}
\sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial q} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial q} \right] &= - \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} x^h + \frac{\partial v^h}{\partial I^h} \delta^h y \right] \\
&= - \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} (x^h - \delta^h y) \\
&= - \gamma
\end{aligned}$$

where  $\sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h}$  is marginal social utility of income of person h, and  $\gamma$  reflects the distribution of profit shares.

Therefore, the first-order condition for q can be written as follows :

$$\begin{aligned}
\mu \left\{ \frac{\partial y}{\partial q} - \sum_h \frac{\partial x^h}{\partial q} - \sum_h \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial q} + \left[ \frac{\partial y}{\partial E} - \sum_h \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial x^h}{\partial E} \right) \right] \frac{dE}{dq} \right\} \\
= \gamma - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \frac{dE}{dq} - \lambda \left[ \frac{\partial \pi^*}{\partial q} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dq} \right]
\end{aligned} \tag{2}$$

Using (2), the condition for  $p_1^H$ , equation (1), can be simplified. To see this, first note that the market equilibrium restrictions

$$\sum_h x^h(q, p_1^H, p_2^H, \delta^h \pi, E) = y(q, p_1^F, p_2^F, E)$$

and the implicit function theorem imply

$$-\frac{dq}{dp_1^H} = \frac{\sum_h \left[ \frac{\partial x^h}{\partial p_1^H} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^H} + \frac{\partial x^h}{\partial E} \frac{dE}{dp_1^H} \right] - \frac{\partial y}{\partial E} \frac{dE}{dp_1^H}}{\sum_h \left[ \frac{\partial x^h}{\partial q} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial q} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dq} + \frac{\partial x^h}{\partial E} \frac{dE}{dq} \right] - \frac{\partial y}{\partial q} - \frac{\partial y}{\partial E} \frac{dE}{dq}}$$

This expression gives the effect of  $p_1^H$  on the equilibrium prices q, taking into account the impact of  $p_1^H$  and q on the externality E.

Using the definition of  $-\frac{dq}{dp_1^H}$  given above, (2) can be rewritten as

$$\mu \left\{ \sum_h \frac{\partial x^h}{\partial p_1^H} - \left[ \frac{\partial y}{\partial E} - \sum_h \left( \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial x^h}{\partial E} \right) \right] \frac{dE}{dp_1^H} \right\} =$$

$$\gamma \frac{dq}{dp_1^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \frac{dq}{dp_1^H} \frac{dE}{dq} - \lambda \left[ \frac{\partial \pi^*}{\partial q} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dq} \right] \frac{dq}{dp_1^H}$$

Finally, take equation (1) and substitute this result to get

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^H} \right] = \alpha_1 z_1^H + \gamma \frac{dq}{dp_1^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \frac{dq}{dp_1^H} \frac{dE}{dq}$$

$$- \lambda \left[ \frac{\partial \pi^*}{\partial q} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dq} \right] \frac{dq}{dp_1^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial E} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} \right] \right\} \frac{dE}{dp_1^H}$$

This ultimately yields

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^H} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_1^H} + \frac{dE}{dq} \frac{dq}{dp_1^H} \right] \right] =$$

$$\alpha_1 z_1^H + \gamma \frac{dq}{dp_1^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_1^H} + \frac{dE}{dq} \frac{dq}{dp_1^H} \right]$$

A completely analogous derivation yields the rule for the price of the second transport mode.

With respect to the prices of the intermediate goods, consider the first-order condition for  $p_1^F$

$$\sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial p_1^F} + \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^F} + \frac{\partial v^h}{\partial E} \frac{dE}{dp_1^F} \right]$$

$$+\lambda \left\{ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^F} \right\} + \mu \left\{ \frac{\partial y}{\partial p_1^F} + \frac{\partial y}{\partial E} \frac{dE}{dp_1^F} - \sum_h \left[ \frac{\partial x^h}{\partial I^h} \left( \frac{\partial I^h}{\partial p_1^F} + \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^F} \right) + \frac{\partial x^h}{\partial E} \right] * \frac{dE}{dp_1^F} \right\} = 0$$

But Hotelling's lemma yields

$$\frac{\partial \pi}{\partial p_1^F} = -z_1^F$$

which implies, using  $I^h = \delta^h \pi$

$$\begin{aligned} \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial p_1^F} &= - \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I^h} \delta^h z_1^F \\ &= -k z_1^F \end{aligned}$$

, where  $k$  can be interpreted as the distributional characteristic of profit. Using these results the first-order condition can be written

$$\begin{aligned} \lambda \left\{ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^F} \right\} &= k z_1^F - \mu \left\{ \frac{\partial y}{\partial p_1^F} + \frac{\partial y}{\partial E} \frac{dE}{dp_1^F} - \sum_h \left[ \frac{\partial x^h}{\partial I^h} \left( \frac{\partial I^h}{\partial p_1^F} + \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^F} \right) + \frac{\partial x^h}{\partial E} \right] \frac{dE}{dp_1^F} \right\} \\ &\quad - \sum_h \frac{\partial W}{\partial v^h} \left( \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right) \frac{dE}{dp_1^F} \end{aligned}$$

To simplify, use same procedure as before. Note that

$$-\frac{dq}{dp_1^F} = \frac{\sum_h \left[ \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial p_1^F} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dp_1^F} + \frac{\partial x^h}{\partial E} \frac{dE}{dp_1^F} \right] - \frac{\partial y}{\gamma p_1^F} - \frac{\partial y}{\partial E} \frac{dE}{dp_1^F}}{\sum_h \left[ \frac{\partial x^h}{\partial q} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial q} + \frac{\partial x^h}{\partial I^h} \frac{\partial I^h}{\partial E} \frac{dE}{dq} + \frac{\partial x^h}{\partial E} \frac{dE}{dq} \right] - \frac{\partial y}{\partial q} - \frac{\partial y}{\partial E} \frac{dE}{dq}}$$

Appropriate substitution in the first-order condition for  $p_1^F$  yields

$$\lambda \left\{ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dp_1^F} \right\} = k z_1^F + \gamma \frac{dq}{dp_1^F} - \sum_h \left\{ \frac{\partial W}{\partial v^h} \left( \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right) \right\} \frac{dq}{dp_1^F} \frac{dE}{dq}$$

$$-\lambda \left[ \frac{\partial \pi^*}{\partial q} + \frac{\partial \pi^*}{\partial E} \frac{dE}{dq} \right] \frac{dq}{dp_1^F} - \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \frac{dE}{dp_1^F}$$

or, finally

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^F} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_1^F} + \frac{dE}{dq} \frac{dq}{dp_1^F} \right] \right] = k z_1^F + \gamma \frac{dq}{dp_1^F}$$

$$- \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_1^F} + \frac{dE}{dq} \frac{dq}{dp_1^F} \right]$$

Completely analogous derivations apply with respect to  $p_2^F$ .

The obtained results can be summarized as

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^H} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^H} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_1^H} + \frac{dE}{dq} \frac{dq}{dp_1^H} \right] \right] = \alpha_1 z_1^H + \gamma \frac{dq}{dp_1^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_1^H} + \frac{dE}{dq} \frac{dq}{dp_1^H} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_2^H} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_2^H} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_2^H} + \frac{dE}{dq} \frac{dq}{dp_2^H} \right] \right] = \alpha_2 z_2^H + \gamma \frac{dq}{dp_2^H} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_2^H} + \frac{dE}{dq} \frac{dq}{dp_2^H} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_1^F} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_1^F} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_1^F} + \frac{dE}{dq} \frac{dq}{dp_1^F} \right] \right] = k z_1^F + \gamma \frac{dq}{dp_1^F} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_1^F} + \frac{dE}{dq} \frac{dq}{dp_1^F} \right]$$

$$\lambda \left[ \frac{\partial \pi^*}{\partial p_2^F} + \frac{\partial \pi^*}{\partial q} \frac{dq}{dp_2^F} + \frac{\partial \pi^*}{\partial E} \left[ \frac{dE}{dp_2^F} + \frac{dE}{dq} \frac{dq}{dp_2^F} \right] \right] = k z_2^F + \gamma \frac{dq}{dp_2^F} - \left\{ \sum_h \frac{\partial W}{\partial v^h} \left[ \frac{\partial v^h}{\partial I^h} \frac{\partial I^h}{\partial E} + \frac{\partial v^h}{\partial E} \right] \right\} \left[ \frac{dE}{dp_2^F} + \frac{dE}{dq} \frac{dq}{dp_2^F} \right]$$

which yields, after minor rearrangements, the expressions reported in the main body of the paper.



## ENDNOTES

1. Moreover, Brander and Spencer (1983) showed that the Ramsey rules can easily be corrected to account for possible downstream monopoly power.
2. The analysis of Bovenberg and van der Ploeg (1994) is not limited to the derivation of optimal tax rules. Their paper contributes to a better understanding of the interrelation between environmental and tax policies, government revenues, and employment. Moreover, they investigate the implications of changes in social priorities (e.g., a tendency towards 'greener' preferences) on these crucial variables.
3. This paper is conceptually related to Mayeres and Proost (1994) who also allow, although in a slightly different way, for external effects with explicit feedback effects within an optimal taxation framework. Unfortunately, the optimal taxation rules they derive are not straightforward to interpret. Moreover, the emphasis of the paper is largely on an empirical simulation exercise investigating the possibility of a double dividend of externality taxes.
4. Note that in practice the government does not itself produce all transport services, but uses a combination of public prices (e.g., for public transport services) and taxes (e.g., in the private trucking industry) to control the transport sector. We do not explicitly make that distinction but instead treat the transport sector as if all prices were directly controlled by the government.
5. Although it could be justified to induce the proper incentives, it has been argued that a formal budget restriction on the public sector may be inappropriate from a policy viewpoint. The implied welfare cost of public funds may be substantially different from the one obtained when funds have to be generated from other sources (Laffont and Tirole (1990)). However, in this paper we remain within the Ramsey tradition.
6. It is well-known that many externalities (e.g., congestion) are a highly nonlinear function of transport volumes. This would suggest to explicitly specify  $E$  as a function of the weighted use of mode 1. Although this could easily be done, from a notational viewpoint it would complicate some of the expressions to be derived below without giving any additional insight.
7. To accommodate such cases, the externality  $E$  may be assumed to enter additively in the utility function.
8. Note that, to save on notation, we assume a joint cost function for all transport modes. The introduction of separate cost functions for the different modes does not affect the interpretation of the optimal pricing results.
9. The derivations of the pricing rules are available in an unpublished appendix.
10. Due to a slightly different formulation of the pricing problem, minor notational differences do exist between our results and those reported in Yang. In particular, Yang specifies the private sector equilibrium prices as explicit reduced-form expressions which depend on public sector prices only. Our formulation makes the dependency of equilibrium prices on private sector profits explicit.
11. Imposing constant returns to scale does require a somewhat different approach, because neither the profit function nor the supply function and the unconditional factor demand functions are defined in that case. However, it is easy to show that imposing constant returns to scale and assuming an externality in consumption only, we find qualitatively similar results to those obtained by Sandmo (1975).

12. Note that this result is qualitatively similar to the finding in Sandmo (1975, p. 93). This may be surprising, since Sandmo's model was a general equilibrium setup with a constant returns to scale production technology, whereas we assumed zero general equilibrium effects and nonconstant returns to scale to derive the above result. Again, the reason is due to the fact that Sandmo considered a consumer externality only. As pointed out before, in that case and assuming constant returns, the prices  $p_i^H$  have no general equilibrium price effects.

13. A similar simplifying assumption is made in Yang (1991). Note, however, that it is necessarily somewhat artificial. As intermediate good prices affect private sector profits and consumer incomes, it is easy to show that it amounts to assuming zero income elasticities for the  $z_i^H$ . Without this assumption the results are less straightforward to interpret. They are obviously available from the author.

14. Assuming nonzero cross-elasticities of demand, the pricing rules for the final goods are described by

$$\lambda \frac{\partial \pi^*}{\partial p_i^H} = m z_i^H - \frac{\theta \rho}{1-\eta} \frac{\partial z_1^H}{\partial p_i^H}, \quad i=1, 2.$$

Note that these relations are formally similar to Bovenberg and van der Ploeg (1994, p. 359).

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