DEPARTEMENT BEDRIJFSECONOMIE

THE TIMING OF REPLACEMENT INVESTMENTS UNDER TECHNOLOGICAL PROGRESS AND THE IMPLICATIONS OF A FINITE PLANNING HORIZON

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WORKING PAPER
98-264
May 1998

D/1998/2263/4

The timing of replacement investments under technological progress and the implications of a finite planning horizon

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April 27, 1998

Abstract

This paper deals with the optimal timing of replacement investments under technological progress. The economic life of equipment is expressed as a function of the ratio of capital and operating costs, which allows measuring the impact of the intensity equipment is used with. A new concept of finite planning horizon is introduced within the infinite time-horizon. A numerical simulation demonstrates how these findings can be of importance for the determination of economic life and the existence of second-hand markets.

1 Introduction

Although replacement investments are at least as important as capacity investments, the focus in economic research is often drawn to capacity investments. Depreciation and replacement of existing equipment are often oversimplified in investment models (See e.g. Nickell [9] for an early contribution). Replacement investments became a research item in economic literature since the first half of this century. Pioneering research in this field has been done Preinreich [10], inspired by the work of Taylor [12] and Hotelling [4]. Although in these early studies, some very fundamental conclusions were found, it still took some time to let these ideas trickle down to practical applications in management situations. It were mainly Terborgh [13], Smith [11] and Alchian [1] who transformed the theory from a purely academic to a more practical level. Modern replacement theory is based on the concept of discounted cash-flows. The problem can be solved both in a profit-maximizing or a cost-minimizing framework. Whereas the analysis of capacity investments is often focussed on the optimal capacity, replacement analysis directs attention to the timing of the investment. For some recent interesting contributions in replacement analysis, we can refer to Meyer [8], Howe & McCabe [5] and Mauer & Ott [7].

In this paper, some new concepts and approaches to the theory of replacement investments will be presented. First a basic replacement model will be described, reviewing standard terminology. The model will be formally presented in a way that allows easy generalization to more complex situations. The model was originally designed to decide when to replace existing equipment (called the defender) by other equipment (the challenger) identical except for its age. However it can also be used to decide if existing equipment should be replaced by non-identical challengers.

The basic model will then be generalized to find the optimal timing of replacements in the case where challengers are continuously improving due to embodied technological progress. It will be demonstrated that when challenging equipment is continuously growing more efficient, the basic model of replacement timing will seriously bias the timing of replacements. This bias is caused by neglecting the value of potential growth in technological progress, forgone at the time of replacement. It could indeed be argued that installing a replacement fixes the level of technological progress temporarily for the duration of the economic life of the equipment. It isn't until the next replacement that new technology will be adopted. The model presented here will be constructed in a way that allows it to be used for a very general range of investments problems.

After discussing the impact of technological progress, we will introduce the concept of a planning horizon. For replacement problems, it is common practice to consider an infinite time-horizon, i.e. it is assumed that the present equipment will be replaced by an infinite chain of successive challengers. When confronted with rapid technological progress, it may not be sensible to assume that the present rate of technological progress will last over the entire time-horizon. Therefore it is suggested that only a finite number of challengers could be subject to technological progress. The generalized model is by its recursive structure ideally suited to solve such problems. Although the influence of technological progress is not new in the literature on replacement investments (See e.g. the work of Massé [6, p. 66 - 68], Grinyer [3, p.211] and Howe & McCabe [5, p.304]), the specification used here and the use of a finite planning horizon will allow to derive some innovating conclusions on the effect of technological progress on economic life of equipment.

In the remainder of this paper the following notation will be used. Time will be referred to by t, the time of the jth replacement by T_j. The economic life of the jth machine in a chain of replacements is ΔT_j. Evidently $T_1 = \Delta T_1$, $T_2 = T_1 + \Delta T_2$, ... When only one replacement is considered, the time of replacement and the economic life will both be referred to by T. For the immediate cashflows we will always use small letters (e.g. $k(t)$ refers to a cost occurring at time t). The present value of all costs of one machine will be expressed by a capital (e.g. $K = \int_0^T k(t)\, e^{-it} dt$) and a tilde will be used to refer to the present value of the costs of a chain of successive machines (e.g. $\widetilde{K} = \sum_j K_j e^{-iT_{j-1}}$). Where necessary, reference will be made to the defending and challenging equipment with subscripts d and c respectively.

2 Basic terminology

In most cases, the operating costs to keep equipment running (i.e. maintenance and repair, energy, manpower, ...) gradually increase with the age of the equipment. At a certain point in time, these costs will be considered too high and the old equipment (called the defender) will be replaced by new equipment (the challenger). The objective will be to minimize the following recursive expression:

$$\widetilde{K}_d = K_d(T) + \widetilde{K}_c e^{-iT} \tag{1}$$

in which $K_d(T)$ is the present value of all costs of the defending equipment over its economic life T, $\widetilde{K}_d$ is the cost of the defender and the cost $\widetilde{K}_c$ of the infinite chain of challengers after T. Assuming that equipment is always replaced with identical equipment (like-for-like replacement, $\widetilde{K}_d = \widetilde{K}_c$), the objective transforms into:

$$\widetilde{K}_d(T) = \frac{K_d(T)}{1 - e^{-iT}} \tag{2}$$

Necessary conditions for optimal timing of replacements are:

$$K_d'(T)\, e^{iT} = i\widetilde{K}_d(T) \qquad K_d'(T) > 0 \tag{3}$$

The first condition states that in the optimum, the marginal cost of one extra time-period of operation equals the equivalent cost of the entire replacement chain, in which an equivalent cost is defined as a perpetual cost with present value equal to the cost of the entire chain. The second condition states that the cost of a marginal period of operation must be rising.

The basic model can easily be generalized in a situation in which the challenging equipment is not identical to the defender, but more efficient (i.e. its minimal equivalent cost is lower). For this, consider a challenging equipment for which the minimum of $\widetilde{K_c}$ is lower then the minimum of $\widetilde{K_d}$. If such challenging equipment became available, the defending equipment should be replaced if $K_d'(T)\,e^{iT} = i\widetilde{K_c}$, this is the moment at which the marginal cost of the defender intersects with the minimum equivalent cost of the challenger. Nonetheless the traditional model presented here may be misguiding in some cases. The implicit assumption in the previous analysis is that the new challenger is suddenly available and does not change over time. This assumption may not be very realistic in some cases. Often, the challenging equipment itself changes continuously over time, because of technological progress. It will be demonstrated that in this case, neglecting the continuous progressive nature of technological progress as in the previous basic model can cause severe bias in the timing of optimal replacement.

3 Technological progress

Consider a form of continuous technological progress, embodied in the challenging equipment, that transforms the objective-function into:

$$\widetilde{K_d} = K_d(T) + \widetilde{K_c}e^{-(i+g)T} \tag{4}$$

in which g is the rate of technological progress. In this specification, technology reduces the costs of the challenging chain of replacements at a constant rate g. It is important to notice that all costs of the chain of replacements are affected in the same way, although the cost-structure of the challenging equipment may or may not be different from the defending equipment.

A necessary condition for the optimal timing T of the replacement for this case is:

$$K_d'(T)\,e^{iT} = (i+g)\,\widetilde{K_c}e^{-gT} \tag{5}$$

Despite apparent similarities, this condition is fundamentally different from condition (3) of the like-for-like replacement model. It states that replacement is due when the marginal cost of an extra time-period of operation of the defending equipment surpasses some adjusted equivalent cost of the challenging equipment. In the case at hand, the equivalent cost of the challenging chain of replacements $i\widetilde{K_c}e^{-gT}$ is to be adjusted by a term $g\widetilde{K_c}e^{-gT}$. Or to express it differently, the marginal cost has to be a factor $\xi = \frac{i+g}{i}$ larger then the equivalent cost of the challenger before replacement is due. It is obvious that the magnitude of ξ can be considerable, causing an important delay in the optimal timing of the replacement.

Since the first term of eq.(5) represents the normal (unadjusted) equivalent cost, the second term can be interpreted as the cost of forgone technological progress. It could indeed be argued that postponing replacement allows the operator of the equipment to increase future gains from technological progress. At the time of replacement, the technology of the challenger is fixed and the operator loses the option to choose a technology which is marginally superior. The value of this option is represented by $g\widetilde{K}_c e^{-gT}$.

It is important to notice that eq.(5) expresses a necessary condition for a very general class of problems. There are indeed no assumptions on the nature of the replacement chain, except that its cost is continuously decreasing. Whatever, the nature of the cost of the chain of challengers, it has to outperform the defender by a factor ξ before replacement is due. In what follows, some special cases will be analyzed.

4 Time horizon and planning horizon

A typical problem with replacement under technological progress is often that there is little or no information on the technological progress in the more distant future. The only information available is often the rate of ongoing technological progress, i.e. the rate at which the immediate challenger of existing equipment grows more efficient. However it is often practically impossible to predict the impact of technological progress on evolution of costs in the long run. The decision-maker is thus confronted with a dilemma. Although he may want to consider all costs over an infinite period of time (the time-horizon), he lacks the correct information about the more distant future.

Therefore he will be forced to make some assumptions, attempting to approximate the cost of future challengers. Such assumption could be that chain of challengers consists of an infinite chain of identical machines (See Grinyer [3] for a formal model using this assumption). In that case, it is likely that the assumed cost of the challengers overestimates the true cost, since technological progress after the first replacement is neglected. A different assumption could be that technology continues to progress at the same rate after the first replacement, in which case each successive machine in the replacement chain is different from its predecessor only by a factor e^{-gT} and $\widetilde{K}_d = \widetilde{K}_c$. The objective then is to minimize $\widetilde{K}_d = \frac{K_d(T)}{1-e^{-(i+g)T}}$. Such an assumption would be valid if it is fair to postulate a constant rate of technological progress over an infinite horizon. However, if the present rate of technological progress is considerable, it may be reasonable to expect that such high rate of progress will not continue perpetually. If progress slows down in a more distant future, costs of the challengers will be underestimated using the latter assumption. Therefore it may be interesting to consider also some mixed cases.

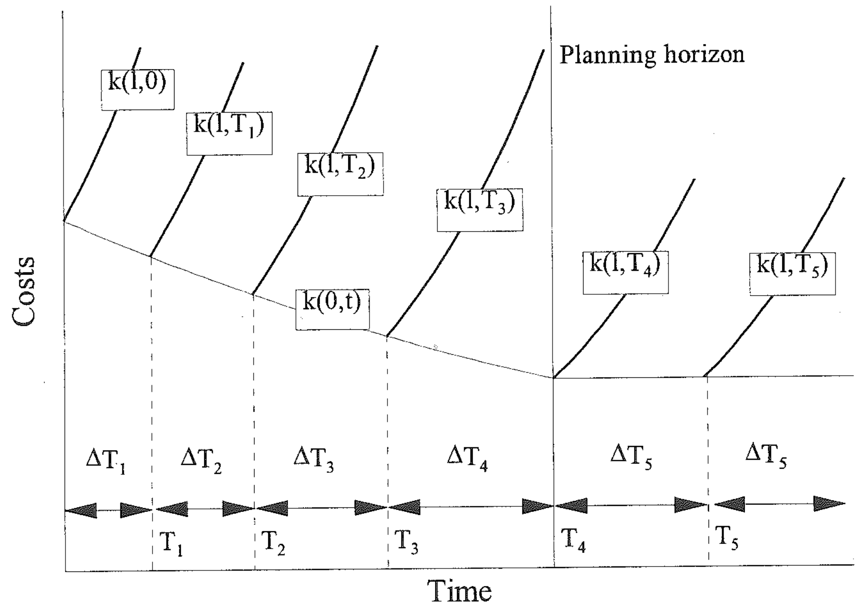

Figure 1: Time-horizon and planning horizon

To explain these cases, we refer to figure 1. It represents the evolution of the immediate cost of equipment $k(l,t)$ in function of its age l and the time (read: the level of technological progress) it was installed t. Starting from the left, the upward sloping line represents the cost of the first defender $k(l,0)$ (installed at $t=0$), which rises as the equipment ages. Meanwhile, technology advances and challengers become more efficient, represented by the downward sloping line $k(0,t)$. At the time of replacement T_1, the challenger will be a factor $e^{-g\Delta T_1}$ more efficient. Notice that in figure 1 there is no technological progress after the fourth replacement. This does not necessarily refer to the fact that technology stops evolving after T_4, but simply that the operator does not take into account further progress after this point. Costs after the fourth replacement are simply approximated by the cost of an infinite like-for-like replacement chain. We will refer to this as the planning horizon of the operator (indicated by the vertical line). In this case the planning horizon covers 4 replacements, although the time horizon is infinite. We could refer to the behavior of an operator using a finite planning horizon as partially myopic. The work of Grinyer cited earlier uses an extremely short planning horizon of only one replacement. Evidently, as indicated before, time- and planning-horizon can also coincide.

5 Costs and intensity of utilization

So far it was always avoided making any assumption on the exact specification of the underlying cost functions. However, without losing generality, costs can be split in an operating cost that occurs over the entire economic life and capital costs that occur only at the time of replacement. The total present cost of the defending equipment can then be expressed as:

$$K_d(T) = \int_0^T m(t)\, e^{-it} dt + V(0) - V(T)\, e^{-(i+g)T} \tag{6}$$

The first term is the present value of operating cost m over the economic life, the second term is the capital cost of new equipment $V(0)$ and the last term is the present value of the salvage value $(V(T)\, e^{-gT})$, regained when the equipment is replaced. Notice that we assume that there is an additional loss in salvage value due to technological progress in the challenging equipment. Using this specification, the cost of a marginal increase of the economic life is:

$$K_d'(T) = \left[m(T) - v(T)\, e^{-gT} + (i+g)\, V(T)\, e^{-gT}\right] e^{-iT} \tag{7}$$

in which $v(t) = \frac{dV}{dt}$ is the rate of depreciation at t. The marginal cost consists of the cost of operation in the final time-period, the additional depreciation and the opportunity cost of the capital invested. From eq.(5), it follows that the optimal time of replacement can be found as the solution of:

$$m(T) - v(T)\, e^{-gT} + (i+g)\, V(T)\, e^{-gT} = (i+g)\, \widetilde{K}_c e^{-gT} \tag{8}$$

In order not to overload the complexity of the notation and to concentrate to the problem at hand, we will assume for the rest of the exposition that the equipment loses all its value above scrap-value immediately once it has been installed. Also for notational convenience we will assume that V_0 represents the net-capital cost, i.e. the cost of new equipment less the scrap-value of the old equipment.

Now assume that operating costs can be expressed as the product of the cost of new equipment and a factor expressing the influence of age: $m(t) = m_0 u(t)$. A similar assumption can be made regarding the evolution of the salvage value. Then the previous cost function can be expressed as:

$$K_d(t, m_{0d}, V_{0d}) = m_{0d} K_d(t, \Omega_d) \quad (9)$$

$$\widetilde{K}_c(m_{0c}, V_{0c}) = m_{0c} \widetilde{K}_c(\Omega_c) \quad (10)$$

in which $\Omega = \frac{V_0}{m_0}$. Notice that in general, V_0 and m_0 need not to be equal for the defending and challenging equipment. In what follows however, it will be assumed that the costs defender and challengers are identical, except for the effect of technological progress. Eq.(8) then transforms into:

$$u(T) = (i + g)\widetilde{K}_c(\Omega) e^{-gT} \quad (11)$$

Since V_0 and m_0 appear only as ratio $\Omega = \frac{V_0}{m_0}$ in eq.(11), the economic life will be a homogenous function of V_0 and m_0 of degree zero.

Assume further that the equipment can be used at different intensities, as most equipment can; machines can vary their speed, trucks and train can haul light or heavy cargo, equipment can be used for variable fractions of the day, ... Let φ be the intensity of utilization, expressed as a fraction of the maximum capacity of the equipment ($0 \leq \varphi \leq 1$). The value of new equipment is unaffected by the intensity it will be used with, and so will the salvage value since we assumed instant depreciation. Now, assume that the effect of the intensity of utilization and age on operating costs can be separated in the following way: $m(\varphi, t) = w(\varphi)m(t) = \Psi m(t)$ (let $m(t)$ be the operating cost when the equipment is used at full capacity). The function $w(\varphi) = \Psi$ can be normalized without loss of generality so that $w(1) = 1$. It is most likely that w is monotonically increasing. In that case the ratio of the initial capital cost and operating cost depends on the intensity of utilization as in $\Omega(\varphi) = \Omega\Psi^{-1}$ and economic life will be determined solely by $\Omega\Psi^{-1}$.

6 A numerical simulation

For practical applications, a specific functional form for the cost-function has to be determined based on the data available. A suitable specification will often be an exponential function, which is both mathematically elegant to handle and easy to interpret due to constant growth rates. Therefore suppose that the operating costs grow at a constant rate: $\frac{dm/m}{dl} = \mu$. In that case $m(t) = \Psi m_0.e^{\mu t}$. At any stage (e.g. the jth machine), the objective is to minimize:

$$\widetilde{K}_j = V_0 + \frac{\Psi m_0}{i-\mu}\left[1 - e^{-(i-\mu)\Delta T_j}\right] + e^{-(i+g)\Delta T_j}\widetilde{K}_{j+1} \tag{12}$$

First-order conditions for optimal replacement are now:

$$m_0 e^{\mu \Delta T_j} = (i+g)\, e^{-g\Delta T_j}\widetilde{K}_{j+1} \tag{13}$$

which is the equivalent of eq.(8).

To demonstrate the numerical characteristics of the model, we can insert some values in this equation and analyze the corresponding replacement decision. Let $V_0 = 1000$, $m_0 = 100$, or $\Omega = 10$, which is more relevant to economic life. Suppose also that the operating cost increases at a continuous rate of 8%. Finally, let the opportunity cost of capital be 5% (continuous discounting) and assume that technology grows at a rate of $g = 3\%$ per unit of time. Let $\widetilde{K}_0$ be the cost of a like-for-like replacement cycle, the $\widetilde{K}_j$ is the cost of a cycle with a planning horizon of j replacements and an infinite time-horizon. To demonstrate the effect of a planning horizon and the principle, the numerical value of economic life and minimal costs were determined in a 10-period model ($j = 0 \rightarrow 9$). The main results are presented in figures 2 and 3. Figure 2 represents the economic life of equipment for values of Ψ ranging from 1 to .2. The horizontal lines reflect the economic life using an infinite planning horizon. The first observation is the economic life in an infinite like-for-like replacement cycle, the second is the economic life using a planning horizon of one replacement, etc. It is interesting to notice that in some cases, using a planning horizon of only one replacement can cause an increased economic life as compared to the like-for-like replacement cycle (See also Grinyer [3] and the author [2]). For instance, for the case with full intensity of utilization an economic life of 12.45 units of time was found for the like-for-like replacement cycle and 13.33 units of time for defender in a cycle with planning horizon at the first replacement. As it appears, the rise in economic life due to technological progress, disappears quickly when the planning horizon is expanded. In this example the endpoint effect of nearing planning horizon faints quickly after the fifth period. It could be concluded that if technological progress goes on indefinitely at a constant rate, truncating the planning horizon unduly after a very limited number of periods could lead to serious biases in the determination of economic life. Nonetheless, if technological progress in the more distant future is likely to drop substantially, a limited planning horizon may be the better solution.

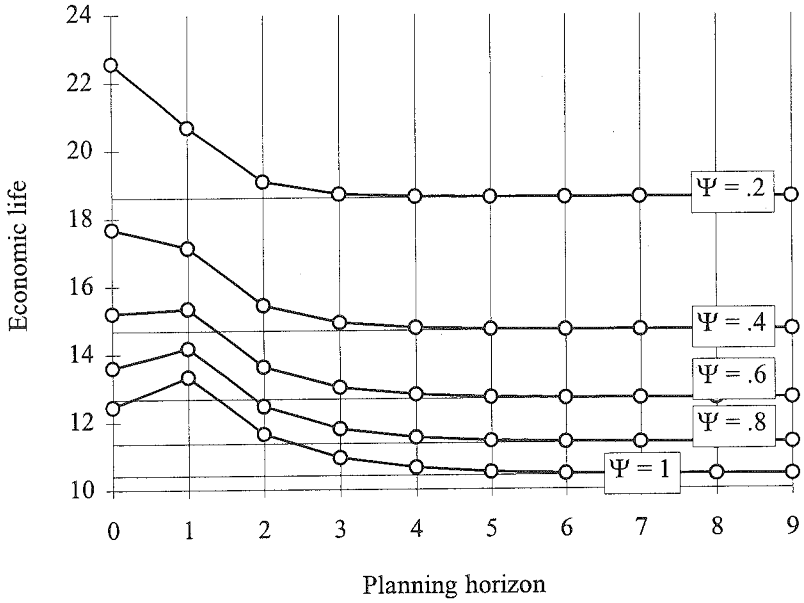

Figure 2: Economic life in function of planning horizon for different values of Ψ

It can clearly be observed that economic life of equipment can be prolonged when using it at a lower intensity. This effect is completely independent from the effect of use on the wear of the equipment (since we assumed constant the growth-rate of operating costs and immediate depreciation). In the model developed here, the prolonged economic life is solely the consequence from a change in the ratio of capital and operating costs. Such a potential increase in economic life can be an explanation for the existence of second-hand markets for certain equipment. Obviously, for equipment that can be used at various rates of intensity, the economic life is not uniquely determined by the characteristics of the equipment, but also by the characteristics of operation. The extent at which economic life can be prolonged by lowering the intensity of utilization could be used to explain the degree in which second-hand markets are developed. Based on the previous findings, it can be argued that second-hand markets are most likely to exist for equipment that can be used at a wide range of intensities. Also, it appears that the economic life is most sensitive to changes in the intensity of utilization when there is no technological progress or when using an infinite planning horizon. Close to the planning horizon ($j = 2$ or 3), economic life is considerably less sensitive to changes in intensity. Changing Ψ from 1 to .2 in steps of .2 reveals that the elasticity of the economic life with respect to Ψ ($\varepsilon_{\Psi}^{T} = \frac{\Delta T/T}{\Delta\Psi/\Psi}$) ranges from $-.274$ to $-.281$ for $j = 2$, and from $-.390$ to $-.353$ for an infinite planning horizon. In similar situations when it is expexred that the present technological progress will not last, second-hand markets are likely to be less developed. Regardless of the planning horizon the economic life of the equipment is always prolonged when used at lower intensity, even in this case where the depreciation process is unaffected by the intensity of utilization. It was demonstrated elsewhere [2] that prolonged economic life can be expected as a result of a reduction in the intensity of utilization for a fairly general class of cost-specifications.

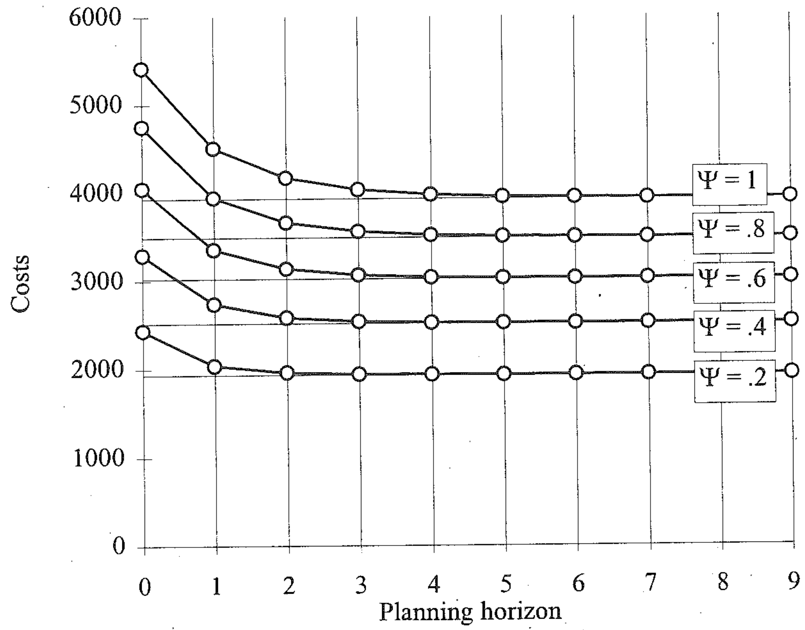

Figure 3: Minimal cost in function of planning horizon for different values of Ψ

Finally, the minimal cost related to this equipment, using an infinite time horizon and a limited planning horizon, is represented in figure 3. Notice that costs fall with longer planning horizons, which is the expected result because then more can be gained from technological progress. It is also obvious that costs should fall with the intensity of utilization (as they do), but it is interesting to see that the impact of a reduction in the intensity of utilization is stronger at high values of Ψ and near the planning horizon. When changing Ψ from 1 to .2 in steps of .2, the elasticity of the minimal cost with respect to Ψ ($\varepsilon_{\Psi}^{K} = \frac{\Delta\widetilde{K}/\widetilde{K}}{\Delta\Psi/\Psi}$) ranges from .599 to .436 for $j = 2$, and from .537 to .391 for an infinite planning horizon.

7 Conclusion

This paper dealt with deterministic single-machine replacement under technological progress and variable intensity of utilization. It was demonstrated that when technological progress is a continuous growth-process, replacement of existing equipment will be delayed even when the marginal cost of the equipment is equal to the minimum cost of the challenger. The delay is caused by the value of forgone technological progress at the time of replacement. It can indeed be argued that replacing the defender with the challenger forces the operator to stick to some type of technology for the duration of the economic life of the challenger. A formal expression for the value of this option was presented for very general conditions.

Next, the concept of a planning horizon was introduced. It was argued that although an infinite time-horizon is used, it may be necessary to consider the possibility that technological progress does not continue indefinitely at the same constant rate. Therefore costs of future challengers may be better approximated using a finite planning horizon, after which like-for-like replacement occurs. The recursive nature of the optimization procedure developed before is ideally suited to handle such problems. The model with ever lasting technological progress and the model using technological progress ending after the first replacement - both of which occur frequently in the literature - can be considered as special cases of the model developed here.

Splitting the cost of equipment into operating costs and capital costs allowed to detect the influence of a variable intensity of utilization. The intensity equipment is used with influences both the wear of the equipment, but also the ratio between capital and operating costs. By assumption, the influence of use on the salvage value was left out of the model. Such simplification is likely to have only a minor and purely numerical impact on results, since it is generally agreed in economic literature that the impact of the salvage value on the replacement decision is rather marginal. This practice simplified the analysis considerably and allowed to focus on the impact of the ratio of capital and operating cost, which is surprisingly important. In these conditions, the timing of the replacement does not depend on the absolute value of capital costs and operating costs, but solely on the ratio of both costs. In order to construct a numerical simulation, it was further assumed that operating costs grow exponentially over time. Although numerical values used in this simulation are of course partly arbitrary, it is possible for this type of problem to specify reasonable values of all parameters. Because of the well-chosen model specification, no absolute cost figures are needed to draw conclusions on the behavior of economic life. Furthermore it was found that the general conclusions of the simulation remain stable for a wide range of values of the simulation parameters.

Optimal economic life was determined for arbitrary finite and infinite planning-horizons. Near the planning horizon, a special endpoint effect was observed that consisted of a pronounced prolongation of economic life, presumably because of the increased opportunity cost of forgone technological progress. However such endpoint effect seems to be only significant very close to the planning horizon and for low values of the ratio of capital cost and operating cost. In any case, the fundamental conclusion that equipment should not be replaced the moment its costs rise above the minimum equivalent cost of the challengers when there is ongoing technological progress, but much later. The ideal planning horizon depends on the nature of the technological progress. When progress is expected to last for a very long time, it may be the simplest just to assume an infinite planning horizon, since the economic life of the first defender is nearly identical using a long planning horizon and an infinite planning horizon. If progress is not expected to last, a finite planning horizon of only 2 or 3 machines may be the better option, to take fully into account the value of future technological progress forgone by replacement. Finally, when variable intensity of utilization and technological progress are melted together in one model it becomes clear that the intensity of utilization influences the impact of technological progress on the replacement process and vice versa. It appears that a reduction of the intensity of utilization has the least impact on economic life near the planning horizon. Since an increased economic life due to lowering the intensity of utilization may give the equipment some value to other operators, the former conclusions may be of interest for the existence of second-hand markets for some types of equipment. It is argued that second-hand markets are more likely to appear for equipment that can be used at different intensities and which economic life is the most sensitive to changes in the intensity of utilization.

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