Preface

# Numerical Analysis in the 20th Century
# Vol. I: Approximation Theory

The field of numerical analysis has witnessed many significant developments in the 20th century and will continue to enjoy major new advances in the years ahead. Therefore, it seems appropriate to compile a "state-of-the-art" volume devoted to numerical analysis in the 20th century. This volume on "Approximation Theory" is the first of seven volumes that will be published in this Journal. It brings together the papers dealing with historical developments, survey papers and papers on recent trends in selected areas.

In his paper, G.A. Watson gives an *historical survey* of methods for solving approximation problems in normed linear spaces. He considers approximation in $L_p$ and Chebyshev norms of real functions and data. Y. Nievergelt describes the history of least-squares approximation. His paper surveys the development and applications of ordinary, constrained, weighted and total least-squares approximation. D. Leviatan discusses the degree of approximation of a function in the uniform or $L_p$-norm.

The development of numerical algorithms is strongly related to the type of approximating functions that are used, e.g. orthogonal polynomials, splines and wavelets, and several authors describe these different approaches.

E. Godoy, A. Ronveaux, A. Zarzo, and I. Area treat the topic of classical *orthogonal polynomials.* R. Piessens, in his paper, illustrates the use of Chebyshev polynomials in computing integral transforms and for solving integral equations.

Some developments in the use of *splines* are described by G. Nürnberger, F. Zeilfelder (for the bivariate case), and by R.-H. Wang in the multivariate case. For the numerical treatment of functions of several variables, radial basis functions are useful tools. R. Schaback treats this topic in his paper. Certain aspects of the computation of Daubechie *wavelets* are explained and illustrated in the paper by C. Taswell. P. Guillaume and A. Huard explore the case of multivariate Padé approximation.

*Special functions* have played a crucial role in approximating the solutions of certain scientific problems. N. Temme illustrates the usefulness of parabolic cylinder functions and J.M. Borwein, D.M. Bradley, R.E. Crandall provide a compendium of evaluation methods for the Riemann zeta function. S. Lewanowicz develops recursion formulae for basic hypergeometric functions. Aspects of the spectral theory for the classical Hermite differential equation appear in the paper by W.N. Everitt, L.L. Littlejohn and R. Wellman.

Many *applications* of approximation theory are to be found in linear system theory and model reduction. The paper of B. De Schutter gives an overview of minimal state space realization in

linear system theory and the paper by A. Bultheel and B. De Moor describes the use of rational approximation in linear systems and control.

For problems whose solutions may have singularities or infinite domains, *sinc approximation methods* are of value. F. Stenger summarizes the results in this field in his contribution.

G. Alefeld, and G. Mayer, provide a survey of the historical development of *interval analysis*, including several applications of interval mathematics to numerical computing.

These papers illustrate the profound impact that ideas of approximation theory have had in the creation of numerical algorithms for solving real-world scientific problems. Furthermore, approximation- theoretical concepts have proved to be basic tools in the analysis of the applicability of these algorithms.

We thank the authors of the above papers for their willingness to contribute to this volume. Also, we very much appreciate the referees for their role in making this volume a valuable source of information for the next millennium.

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