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2D Angle of Arrival Estimations and Bandwidth Recognition for Broadband Signals

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Abstract—In many angle of arrival (AoA) estimation algorithms for broadband signals a-prior knowledge about the impinging signals' bandwidth is required for the algorithms to function. In this paper, we present a new technique for estimating the AoA and the bandwidth of the received broadband signals without requiring any knowledge of the bandwidth of the received signals. The proposed technique consists of a uniform circular array (UCA) followed by a transversal filter. It employs variable bandwidth spatial vectors along with the signal to thermal noise ratio (STNR) estimator to estimate the AoA and the bandwidth of the received signals simultaneously. The simulation results illustrate the capabilities of the proposed technique in estimating not only the elevation and the azimuth of the impinging signals with different bandwidths, but also the bandwidths of these received signals.

Index Terms—broadband AoA estimation, Adaptive array Antenna, Transversal Filter, MVDR algorithm, UCA.

I. INTRODUCTION

Estimating the Angle of Arrival (AoA) of the received signals drew enormous attention during the last decades, both for tracking and positioning systems or for improving the performance of the communication systems. The fundamental principles of many methods that have been used in the AoA estimation of the received signals are presented in [1],[2].

Lately, due to the massive demand for high data rate transmission, broadband RF-modems took on an essential role in the modern life. Because of this, the demand for AoA estimation of the received broadband signals increased [3]-[8]. The major difference between treating the broadband signals compared to the narrowband signals is that the broadband signals require arrays that have frequency-dependent transfer functions [9]. Compton [10] tackled this issue for canceling a broadband interference signal by introducing an adaptive array antenna system followed by a tapped delay line (transversal filter).

Based on this solution presented by Compton, the bandwidth of the broadband array processor should be tuned to the bandwidth of the received signals in order to estimate the AoA of broadband signals. This approach only works when the estimator has information about the received signal's bandwidth.

In this paper, we tackled this issue in AoA estimation of broadband signals by presenting a broadband array with variable bandwidth. The proposed technique is capable

of estimating the Angle of Arrival (AoA) of the received broadband signals with different bandwidths in 2D (elevation and azimuth), and at the same time can estimate the exact bandwidths of the received signals. The proposed technique consists of a uniform circular array (UCA) followed by a transversal filter in conjunction with the minimum variance distortionless response (MVDR) beamforming algorithm [11].

II. UNIFORM CIRCULAR ARRAY UCA CONSTRUCTION

Fig.(1) shows M antenna elements that are distributed uniformly over the circumference of a circle with a radius r_c in the x-y plane. These antenna elements form a UCA. A spherical coordinate system is used to represent the AoA of the incoming plane wave, the origin of the coordinate system is located in the center of the UCA, while the angles $\theta \in [0, \frac{\pi}{2}]$ and $\phi \in [0, 2\pi]$ represent the elevation and the azimuth angles respectively. The element (m) is placed by the angle ψ_m in the UCA

$$\psi_m = 2\pi \frac{m}{M} \quad (1)$$

where $m = 1, 2, \dots, M$ elements.

Fig. 1: Uniformly Circular Array

If the received signal is impinging from an elevation θ_s and azimuth ϕ_s angles, then received signal vector is

$$\mathbf{X}(\omega, \theta_s, \phi_s) = [G_e(\omega, \theta_s, \phi_s) \exp(j\varphi_e)]^T \quad (2)$$

Where $G_e(\omega, \theta_s, \phi_s)$ is the element field pattern, T is the transpose notation, and φ_e is the phase difference vector of $(M \times 1)$ dimension

$$\varphi_e = \frac{2\pi r_c}{\lambda} \sin(\theta_s) \cos(\phi_s - \psi_m) \quad (3)$$

λ is the wavelength on the received signal. Based on the presence of λ in denominator, φ_e is a frequency dependent. The radius r_c can be expressed as

$$r_c = \frac{Md}{2\pi} \quad (4)$$

in which d is the inter element spacing.

III. THE BROADBAND ADAPTIVE PROCESSOR

Fig.(2) represents a UCA broadband beamformer with M omnidirectional antenna elements (i.e. $G_e(\omega, \theta, \phi) = 1$). There are $(J - 1)$ taps behind each element with constant delay Δ . The analog to digital converters convert $x(t)$ from time index to the $x(k)$ with discrete k index.

Fig. 2: Broadband Adaptive Circular Array Processor

The received signal at element m and tap j has the form

$$x_{mj}(k) = s_{mj}(k) + n_{mj}(k) \quad (5)$$

where $s_{mj}(k)$ and $n_{mj}(k)$ are the signal and the thermal noise at the m^{th} element and the j^{th} tap, respectively. The received signal vector at the m^{th} element will be

$$\mathbf{X}_m = \mathbf{S}_m + \mathbf{n}_m \quad (6)$$

where

$$\mathbf{S}_m = [s_{m1}(k) \ s_{m2}(k) \ \dots \ s_{mJ}(k)]^T \quad (7)$$

$$\mathbf{n}_m = [n_{m1}(k) \ n_{m2}(k) \ \dots \ n_{mJ}(k)]^T \quad (8)$$

The total received signal vector of $(MJ \times 1)$ dimension is

$$\mathbf{X} = [\mathbf{X}_1 \ \mathbf{X}_2 \ \dots \ \mathbf{X}_M]^T \quad (9)$$

The corresponding weight vector at the m^{th} element due to the presence of the J taps will be

$$\mathbf{W}_m = [w_{m1}(k) \ w_{m2}(k) \ \dots \ w_{mJ}(k)]^T \quad (10)$$

The total weight vector of $(MJ \times 1)$ dimension is

$$\mathbf{W} = [\mathbf{W}_1 \ \mathbf{W}_2 \ \dots \ \mathbf{W}_M]^T \quad (11)$$

The adaptive array output can be expressed as

$$y = \mathbf{W}^T \mathbf{X} \quad (12)$$

while the output power is

$$|y|^2 = \mathbf{W}^\dagger \Phi_{xx} \mathbf{W} \quad (13)$$

Where † is the transpose conjugate notation and Φ_{xx} is the system covariance matrix of $(MJ \times MJ)$ dimensions.

A. The System Covariance Matrix Φ_{xx}

Based on the assumption that the received signals and the thermal noise components are mutually independent with zero mean, the system covariance matrix Φ_{xx} decomposes into a signal covariance matrix Φ_s and noise component covariance matrix Φ_n , as follows [10]

$$\Phi_{xx} = \Phi_s + \Phi_n \quad (14)$$

where

$$\Phi_s = \begin{bmatrix} \Phi_{s(1,1)} & \Phi_{s(1,2)} & \dots & \Phi_{s(1,M)} \\ \Phi_{s(2,1)} & \Phi_{s(2,2)} & \dots & \Phi_{s(2,M)} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{s(M,1)} & \Phi_{s(M,2)} & \dots & \Phi_{s(M,M)} \end{bmatrix} \quad (15)$$

and

$$\Phi_n = \begin{bmatrix} \Phi_{n(1,1)} & 0 & \dots & 0 \\ 0 & \Phi_{n(2,2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_{n(M,M)} \end{bmatrix} \quad (16)$$

The submatrix $\Phi_{s(m,n)}$ is the signal covariance matrix associated with a pair of element signal vectors $\mathbf{S}_m$ and $\mathbf{S}_n$

$$\Phi_{s(m,n)} = E[\mathbf{S}_m \mathbf{S}_n^\dagger] \quad (17)$$

where $E[\cdot]$ is the expected value,

If the signal $s(t)$ has a flat power spectral density with an amplitude of $\left(\frac{2\pi p_s}{\Delta\omega_s}\right)$ over a limited bandwidth $\Delta\omega_s$ centered at frequency ω_o , the correlation function $R_s(\tau)$ of the received signal can be given by [10]

$$R_s(\tau) = P_s \text{sinc}\left(\frac{\Delta\omega_s \tau}{2}\right) \exp(j\omega_o \tau) \quad (18)$$

where p_s is the received signal power and τ is the time delay. Then the i^{th} row and z^{th} column of $\Phi_{s(m,n)}$ can be found by the following equation

$$[\Phi_{s(m,n)}]_{(i,z)} = R_s(T_{e(m,n)} + T_{o(i,z)}) \quad (19)$$

where $T_{e(m,n)}$ is the time difference between the two elements (m, n) and it is equal to

$$T_{e(m,n)} = \frac{r_c}{c} \sin(\theta_s) (\cos(\phi_s - \psi_m) - \cos(\phi_s - \psi_n)) \quad (20)$$

while $T_{o(i,z)}$ is the time difference between the two taps (i, z) and it is equal to

$$T_{o(i,z)} = (i - z)T_o \quad (21)$$

T_o is the time required by the desired signal to cross the distance Δ (if we choose $\Delta = \frac{\lambda_o}{4}$) then T_o will be

$$T_o = \frac{\Delta}{c} = \frac{\pi}{2\omega_o} \quad (22)$$

where c is the speed of light, and λ_o is the wave length of the received signal, respectively. Substituting Eq.(18) in Eq.(19) leads to

$$[\Phi_{s(m,n)}]_{(i,z)} = P_s \text{sinc} \left(\frac{\Delta\omega_s (T_{e(m,n)} + T_{o(i,z)})}{2} \right) \exp(j\omega_o (T_{e(m,n)} + T_{o(i,z)})) \quad (23)$$

The product of $\Delta\omega_s T_{e(m,n)}$ and $\Delta\omega_s T_{o(i,z)}$ can be normalized over a center frequency ω_o to have a relative bandwidth rather than absolute

$$\Delta\omega_s T_{e(m,n)} = \left(\frac{\Delta\omega_s}{\omega_o} \right) \omega_o T_{e(m,n)} = \beta_s \varphi_{e(m,n)} \quad (24)$$

$$\Delta\omega_s T_{o(i,z)} = \left(\frac{\Delta\omega_s}{\omega_o} \right) \omega_o T_{o(i,z)} = \beta_s \varphi_{o(i,z)} \quad (25)$$

where β_s is the relative bandwidth of the received signals, while $\varphi_{e(m,n)}$ and $\varphi_{o(i,z)}$ are the phase difference between the two elements (m, n) and the two taps (i, z) respectively, and they are equal to

$$\varphi_{e(m,n)} = \frac{2\pi r_c}{\lambda_o} \sin(\theta_s) (\cos(\phi_s - \psi_m) - \cos(\phi_s - \psi_n)) \quad (26)$$

$$\varphi_{o(i,z)} = (i - z) \frac{\pi}{2} \quad (27)$$

Substituting Eqs.(24)-Eq.(27) in Eq.(23) gives

$$[\Phi_{s(m,n)}]_{(i,z)} = P_s \text{sinc} \left(\frac{\beta_s}{2} \tau_s \right) \exp(j\tau_s) \quad (28)$$

where τ_s is the received signal phase delay and it is equal to

$$\tau_s = \varphi_{e(m,n)} + \varphi_{o(i,z)} \quad (29)$$

where $m, n = 1, 2, \dots, M$ sensors and $i, z = 1, 2, \dots, J$ taps. If the noise component $n(t)$ has a flat spectrum density with amplitude of $\left(\frac{2\pi\sigma_n^2}{\Delta\omega_n} \right)$ over a limited bandwidth $\Delta\omega_n$, then the noise correlation matrix can be derived using the same steps as for the received signals; taking into consideration that, the noise is uncorrelated in the channels and correlated in the taps,

$$[\Phi_{n(m,n)}]_{(i,z)} = \begin{cases} R_n(T_{o(i,z)}) & \text{for } m = n \\ 0 & \text{for } m \neq n \end{cases} \quad (30)$$

leads to

$$[\Phi_{n(m,n)}]_{(i,z)} = \sigma_n^2 \text{sinc} \left(\frac{\beta_n}{2} \tau_n \right) \exp(j\tau_n) \quad (31)$$

where σ_n^2 is the thermal noise variance, β_n is the noise relative

bandwidth and τ_n is the noise phase delay and it is equal to

$$\tau_n = \varphi_{o(i,z)} \quad (32)$$

B. The Adaptive Weight Vector

The adapted weights are selected by minimizing the mean output power of the system, while maintaining unity response in the look direction. In other words, when the array spatial vector parameters $(\beta_c, \theta_c, \phi_c)$ coincide with received signals parameters $(\beta_s, \theta_s, \phi_s)$, the received signal will pass undisturbed as follows [11]

$$\min_w \mathbf{W}^\dagger \Phi_{xx}^{-1} \mathbf{W} \quad (33)$$

Subjected to

$$\mathbf{C}^\dagger \mathbf{W} = 1 \quad (34)$$

$\mathbf{C}$ is the total spatial vector. The optimum weight vector that satisfies Eq.(33) and (34) is equal to [2],[11]

$$\mathbf{W} = \frac{\Phi_{xx}^{-1} \mathbf{C}}{\mathbf{C}^\dagger \Phi_{xx}^{-1} \mathbf{C}} \quad (35)$$

The optimum adaptive output power due to impinging signals $s(t)$ and the thermal noise $n(t)$ can be expressed as:

$$P_{opt}(\beta, \theta, \phi) = \mathbf{W}^\dagger \Phi_s \mathbf{W} \quad (36)$$

$$P_n = \mathbf{W}^\dagger \Phi_n \mathbf{W} \quad (37)$$

then the output Signal to Thermal Noise Ratio (STNR) is

$$\text{STNR}(\beta, \theta, \phi) = \frac{P_{opt}(\beta, \theta, \phi)}{P_n} \quad (38)$$

IV. THE PROPOSED TECHNIQUE

The spatial vector for the m^{th} element due to the presence of the J taps can be expressed as

$$\mathbf{C}_m = [c_{(m,1)} \ c_{(m,2)} \ \dots \ c_{(m,J)}]^T \quad (39)$$

The total spatial vector of $(MJ \times 1)$ dimension is

$$\mathbf{C} = [\mathbf{C}_1 \ \mathbf{C}_2 \ \dots \ \mathbf{C}_M]^T \quad (40)$$

If the spatial vector tuned to have a flat power spectral density with an amplitude of $\left(\frac{2\pi}{\Delta\omega_c} \right)$ over a limited bandwidth $\Delta\omega_c$ centered at frequency ω_o , the correlation function $R_c(\tau)$ of the spatial vector can be given by

$$R_c(\tau) = \text{sinc} \left(\frac{\Delta\omega_c \tau}{2} \right) \exp(j\omega_o \tau) \quad (41)$$

Then the i^{th} element of $\mathbf{C}_m$ can be found by the following equation

$$[\mathbf{C}_{(m)}]_{(i)} = R_c(T_{e(m,1)} + T_{o(i,1)}) \quad (42)$$

Substituting Eq.(41) in Eq.(42) leads to

$$[\mathbf{C}_{(m)}]_{(i)} = \text{sinc} \left(\frac{\Delta\omega_c}{2} (T_{e(m,1)} + T_{o(i,1)}) \right) \exp(j\omega_o (T_{e(m,1)} + T_{o(i,1)})) \quad (43)$$

TABLE I: impinging signals information

Signal	θ_s	ϕ_s	SNR
s1	20°	300°	5dB
s2	60°	100°	5dB
s3	30°	50°	5dB

after applying the same procedure in section III; the spatial vector will be equal to

$$[\mathbf{C}_{(m)}]_{(i)} = \text{sinc}\left(\frac{\beta_c}{2}\tau_c\right) \exp(j\tau_c) \quad (44)$$

where β_c is the spatial vector relative bandwidth, while τ_c is a spatial vector phase delay and it is equal to

$$\tau_c = \varphi_{e(m,1)} + \varphi_{o(i,1)} \quad (45)$$

The STNR estimation Eq.(38) gives a maximum indication towards the AoA of the received signals when the spatial vector Eq.(44) bandwidth tuned to the bandwidth of the received signals ($\beta_c = \beta_s$), while the STNR estimation will suffer a distortion when ($\beta_s \neq \beta_c$).

If we apply Q spatial vectors with Q different bandwidths to the processor, as follows

$$\mathbf{C} = \begin{bmatrix} \text{sinc}\left(\frac{\beta_c(1)}{2}\tau_c\right) \exp(j\tau_c) \\ \text{sinc}\left(\frac{\beta_c(2)}{2}\tau_c\right) \exp(j\tau_c) \\ \vdots \\ \text{sinc}\left(\frac{\beta_c(Q)}{2}\tau_c\right) \exp(j\tau_c) \end{bmatrix} \quad (46)$$

then we will get Q AoA estimations. If the q^{th} estimate gives the maximum STNR response, that means $\beta_c(q) = \beta_s$.

The proposed technique combines the variable spatial vector Eq.(46) and the STNR estimation Eq.(38) to detect the AoA and the relative bandwidth of the received broadband signals.

V. SIMULATION AND RESULTS

In the following simulation cases, we assumed the presence of 3 signals (Table I) impinging on the UCA. The UCA is constructed of 8 elements followed by 5 delay taps ($J = 6$). The radius r_c of the UCA equals to $0.6366 \lambda_o$ with inter element spacing d equals to $0.5 \lambda_o$ between each successive elements.

A. Capon Estimator

In order to address the challenges of estimating the AoA of broadband signals with different bandwidths, we deployed the Capon Estimator [12] with the system covariance matrix Eq.(14) and the fixed spatial vector Eq.(44)

$$\text{Capon}(\theta, \phi) = \frac{1}{\mathbf{C}^\dagger \Phi_{xx}^{-1} \mathbf{C}} \quad (47)$$

The bandwidth of the fixed spatial vector β_c is equal to 0.2. In Fig.(3) all the received signals have the same bandwidth as the spatial vector bandwidth, while in Fig.(4) the bandwidths of the impinging signals are different and only the signal **s3** its bandwidth is equal to the spatial vector bandwidth. It is obvious that the Capon Estimator can only estimate the AoA of the received signals when the bandwidth of the spatial vector is equal to the bandwidth of the received signals.

Fig. 3: Capon AoA estimation. The bandwidth of the impinging signals $\beta_{s1}, \beta_{s2}, \beta_{s3}$ and the spatial vector β_c are equal to 0.2. The Capon Estimator can estimate the AoA efficiently when the estimator bandwidth is tuned to the bandwidth of the received signals.

Fig. 4: Capon AoA estimation. The bandwidth of the impinging signals and the spatial vector are $\beta_{s1} = 0.05$, $\beta_{s2} = 0.1$, $\beta_{s3} = 0.2$ and $\beta_c = 0.2$, respectively. The Capon Estimator could only estimate the source **s3** (see Table I) since $\beta_{s3} = \beta_c$, while it fails in estimating the other two signals since $\beta_{s1} \neq \beta_{s2} \neq \beta_c$.

B. The Proposed Technique

The STNR estimation as expressed in Eq.(38) and the variable adaptive UCA spatial vector (Eq. 46) were deployed. The spatial vector of the proposed technique operates with four estimations (i.e. $Q = 4$), starts from $\beta_c(1) = 0$ to $\beta_c(5) = 0.2$ with 0.05 step.

Fig.(5) shows that the proposed technique could estimate the AoA of all the received broadband signals. Fig.(6) represents the estimated relative bandwidth of the received signals (β_s) along with the AoA estimation. Obviously the proposed technique can estimate the AoA and the bandwidth of the received signals efficiently.

Fig. 5: The proposed AoA estimation technique. The bandwidths of the impinging signals are $\beta_{s1} = 0.05$, $\beta_{s2} = 0.1$ and $\beta_{s3} = 0.2$, respectively. The proposed technique could estimate all the received signals (see table I) efficiently.

Fig. 6: The Bandwidth Recognition Indication. The bandwidth of the impinging signals are $\beta_{s1} = 0.05$, $\beta_{s2} = 0.1$ and $\beta_{s3} = 0.2$, respectively. The proposed technique can estimate the bandwidth of the received signals.

VI. CONCLUSION

In this paper, we present a new technique for estimating the angle of arrival (AoA) in 2D (the elevation and azimuth domains) and the bandwidth of the received broadband signals in one step. The proposed technique consists of a uniform circular array (UCA) followed by a transversal filter in conjunction with the minimum variance distortionless response (MVDR) beamforming algorithm. It deploys variable bandwidth spatial vectors along with signal to thermal noise ratio (STNR) response. The simulation results show that:

- 1) The proposed technique can estimate the AoA of multiple broadband impinging signals with different bandwidths efficiently, while the fixed bandwidth spatial vector (i.e. fixed β_c) can only estimate the AoA of the received signals when $\beta_s = \beta_c$.
- 2) The proposed technique can give an accurate estimate of the bandwidth of the received signals.

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