

DEPARTMENT OF ECONOMICS

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commuting and optimal congestion taxes**

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The tax treatment of company cars, commuting and optimal congestion taxes

Bruno De Borger and Bart Wuyts (*)

Abstract

In Europe, the preferential tax treatment of company cars implies that many employees receive a company car as part of their compensation package. In this paper, we consider a model in which wages and the decision whether or not to provide a company car are the result of direct negotiation between employer and employee. Using this framework, we theoretically and numerically study first- and second-best optimal tax policies on labour and transport markets, focusing on the role of the tax treatment of company cars. We show that higher labour taxes and a more favourable tax treatment of company cars raise the fraction employees that receives a company car; congestion and congestion tolls reduce it. More importantly, we find that earlier models that ignored the preferential tax treatment of company cars may have substantially underestimated optimal congestion tolls in Europe. The numerical illustration, calibrated using Belgian data, suggests that about one third of the optimal congestion toll is due to the current tax treatment of company cars. We further find that eliminating the preferential tax treatment of company cars is an imperfect -- but easy to implement -- substitute for currently unavailable congestion tolls: it yields about half the welfare gain attainable through optimal congestion taxes. Finally, the favourable tax treatment of company cars justifies large public transport subsidies; the numerical results are consistent with zero public transport fares.

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1. Introduction

In developed market economies, it is not uncommon for private firms to provide a number of fringe benefits to their employees as part of an overall compensation package. Frequently observed fringe benefits include, among others, health care benefits, pension plans, subsidized meals, and transport-related benefits. Not surprisingly, an extensive literature deals with the effects of providing such benefits. The early literature focused on the role of the tax system in the provision of non-wage compensation (e.g., Long and Scott (1982), Clotfelter (1983), Zax (1988)), and on the related question of whether and how fringe benefits should be taxed (see Katz and Mankiw (1985), Adamache and Sloan (1985)). More recently, a number of studies have analyzed the implications of fringe benefits for the outcomes on the labour market, such as the effects on wages, labour turnover and unemployment (see, e.g., Hashimoto and Zhao (2000) and Dale-Olsen (2004)). Moreover, a series of papers have concentrated on specific fringe benefits, including health insurance (Olson (2002), Royalty (2000), Gruber (2001)), pension advantages (Bernstein (2002)), and transport-related benefits such as parking and company cars (Shoup (2005), van Ommeren, van der Vlist and Nijkamp (2006), Gutiérrez-i-Puigarnau and van Ommeren (2010)).

Despite the recent interest in transport-related fringe benefits it is fair to say that, with the exception of Gutiérrez-i-Puigarnau and van Ommeren (2010), the literature has not paid attention to the implications of the widespread provision of company cars in Europe¹. They are most common in the UK, but they are also a popular fringe benefit in most European countries, including Belgium, the Netherlands, Germany, Italy, and Greece. For example, in 1995, it was estimated that 50% of new cars in the UK were company cars (Economist Intelligence Unit (1996)). For the Netherlands, recent estimates suggest that some 15% of all employees have a company car (Gutiérrez-i-Puigarnau and van Ommeren (2010)). Moreover, a large survey among 62,500 Belgian workers found that 21% had received a company car as part of their compensation package (Vacature (2006)); the share of company cars in newly registered cars was even much higher, over 40% in 2007 (Wuyts, (2009)).

Although it could be argued that for some jobs company cars may enhance productivity, it is now common wisdom that the large number of company cars in Europe is a direct consequence of their favourable fiscal treatment on both the demand and supply sides

¹ Although they are also observed in other regions of the world (e.g., Israel, New Zealand), company cars are largely a European phenomenon. Note that studies focusing on optimal tax policies for the US (such as Parry and Bento (2001)) have good reasons not to pay specific attention to company cars, since in the US they are not taxed preferentially.

of the labour market². The tax code in many countries makes it cheaper for employers to provide a car than to grant an otherwise equivalent wage increase (Wuyts (2009)). Moreover, for employees the tax imputed value of the car that is used for income tax purposes is typically much less than its market value. The consequence is that company cars are provided to workers at an effective price that is much lower than when bought privately on the market. For example, Gutiérrez-i-Puigarnau and van Ommeren (2010) estimate that in the Netherlands this effective price is 20-60% below market prices. The annual welfare cost of this preferential tax treatment of company cars on car ownership alone is estimated at some 600-800 euro per car. For Europe as a whole, the estimated cost amounts to 12 billion per year. Note that progressive income taxes further imply that, at the individual level, the tax benefit rises with income.

In this paper we focus on a different implication of the preferential tax treatment of company cars. We introduce the provision of company cars by firms into a model of optimal taxation of the markets for labour and commuting transport, and we analyze the interaction between the tax treatment of company cars, taxes on wages, congestion taxes and public transport fares. Accounting for the phenomenon of company cars in optimal tax models of transport and labour markets (see, among others, Parry and Bento (2001), Van Dender (2003), De Borger (2009)) may be highly relevant for several reasons. First, the decision whether or not to give employees a company car strongly depends on the government's tax policies with respect to wages, commuting transport, and company cars. Second, the tax treatment of company cars will have potentially strong implications for optimal congestion taxes and public transport fares, because providing company cars affects employees' modal choice for the journey-to-work. For example, in Belgium public transport has a modal share in the morning commute of approximately 33% (Statbel (2000)); however, employees that receive a company car almost automatically use it for commuting purposes (Vacature (2006), Wuyts (2009)). As a consequence, even granted that people with a company car may have weaker than average preferences for public transport use, the provision of company cars will raise overall peak-

² The job-specific productivity effect of providing a company car may be substantial for at least some groups of employees (e.g., sales representatives), but the widespread provision of company cars cannot be explained by such productivity effects alone. For example, in the Netherlands it turns out that 78% of workers that received a company car had not used it at all for business purposes over the last three months before they were interviewed (Gutiérrez-i-Puigarnau and van Ommeren (2010)). The Belgian data show that more than 13% of lower level administrative staff – for whom productivity effects can hardly be invoked -- received a company car (Vacature (2006), Wuyts (2009)). Note that -- apart from favourable tax treatment -- there are other potential explanations for the use of transport-related fringe benefits by firms. For example, search models of imperfectly competitive labour markets can explain compensation for longer commutes (e.g., Burdett and Mortensen (1998), Zax (1991), Manning (2003)). Again, however, it is unlikely that search arguments alone justify the very high numbers of company cars observed in some countries.

period car use and reduce public transport ridership. Third, the fiscal treatment of company cars offers a potentially useful policy instrument to control labour and transport markets, especially in a second-best world where optimal congestion taxes may be unavailable.

We consider a model in which wages and company cars, following Katz and Mankiw (1985), are the result of direct negotiation between employer and employee. All employees commute to work. Those that do not receive a company car commute by car or public transport, employees with a company car are assumed to always use it for commuting purposes. Road transport by car contributes to congestion; public transport is assumed to be congestion-free. The government has four instruments to affect the working of the labour and transport markets: a tax on wages, the tax imputed value of company cars, a tax on commuting car transport, and the public transport fare. Within this setting, we first analyze how the fraction of employees that receives a company car depends on the various taxes considered. We then study the government's optimal tax problem under various restrictions on the tax instruments, focusing explicitly on the role of the fiscal treatment of company cars. A numerical version of the theoretical model, calibrated to Belgian data, illustrates the theoretical results.

Our findings include the following. First, not surprisingly, both a higher tax on wages and a more favourable tax treatment of company cars raises the fraction of the labour force that gets a company car as part of the compensation package. Higher congestion and higher congestion tolls have a negative effect, as they make public transport more attractive and reduce the relative value of having a company car. Second, if congestion tolls can be set optimally, the optimal tax treatment of company cars is to set the imputed annual value for income tax purposes equal to the net annual cost to the firm of providing the car to the employee. Third, the currently observed preferential tax treatment of company cars raises optimal congestion taxes; this suggests that previous models have underestimated optimal congestion tolls in Europe. The numerical illustration implies that about 3.5 euro of the optimal congestion toll of 11 euro (for a 20 kilometre roundtrip) is due to the current preferential tax treatment of company cars. Implementation of optimal labour and congestion taxes is estimated to reduce the number of company cars by more than 50 percent. Fourth, suppose that for technical or political reasons optimal congestion tolls are unavailable. We find that, at current congestion taxes (which do not fully cover the marginal external congestion cost), the tax imputed value of company cars should be set higher than the net cost of the car to the employer. Numerical results suggest that the second-best optimal tax structure would make company cars disappear from firms' compensation packages, and

congestion would decline substantially. We find that optimal taxation of company cars is clearly an (imperfect) substitute for unavailable congestion tolls; it would generate more than half the welfare improvement that could be realized by optimal congestion tolls. Finally, the current preferential tax treatment implies a convincing argument for public transport subsidies. In fact, the numerical results are consistent with zero public transport fares.

This paper has several obvious limitations. First, it focuses on company cars, the labour market and peak-period congestion, but it does not explicitly deal with the car ownership decision. In this sense, it complements the recent analysis of the welfare implications of the tax treatment of company cars through changes in household car ownership by Gutiérrez-i-Puigarnau and van Ommeren (2010). They did not consider the relation with the labour market, nor did they study the relation between the fiscal treatment of company cars and optimal congestion taxes. Second, given the emphasis of the paper on commuting transport and congestion, we ignore non-commuting transport as well as other externalities (e.g., pollution)³. Third, we follow earlier work (e.g., Parry and Bento (2001)) and assume a proportional tax on labour. Although explicitly capturing the progressivity of income taxes is essential if one is interested in explaining the relation between company car provision and wages, our aim here is to study the aggregate trade-offs between taxes on labour, company cars and commuting transport⁴. Finally, in line with earlier work, the model does not have an explicit spatial dimension but uses a one-link transport network between residence and workplace.

The rest of the paper is organized as follows. In Section 2, we present the structure of the analytical model and study how the government's policy instruments affect wages and the share of workers obtaining a company car as part of their remuneration package. In Section 3 we then analyze various second best optimal tax policies, focusing on the role of preferential company car taxation for labour and congestion policies. A numerical version of the analytical model is described and implemented in Section 4, using Belgian data. Section 5 concludes.

³ It is well known that company cars are also used for non-commuting purposes. However, the focus is on the relation between company cars and peak-period congestion which is largely due to commuting trips; to simplify the analysis, we therefore ignore non-commuting transport. As for pollution, it has been argued that the provision of company cars speeds up the process of making the car fleet more environmentally-friendly, because company cars are typically replaced at a faster rate than regular private cars. In a dynamic setting, a lower tax on company cars could then be justified even if it generates more congestion. This potential positive welfare effect is not captured by our static model.

⁴ A previous version of the paper considered a model with progressive income taxes distinguishing between average and marginal tax rates. The qualitative implications for increases in marginal tax rates obtained were similar to the results reported in the current paper, but the technical analysis became highly complex without offering extra insight on the issue at hand.

2. Structure of the model

In this section, we present the structure of the model. To keep the optimal tax problem studied in the next sections tractable, we make several simplifying assumptions. First, as in most European countries decisions on company cars as part of labour compensation are not included in collective bargaining agreements between firms and unions, we assume that such decisions are the result of individual negotiation between worker and employer in the sense of, e.g., Katz and Mankiw (1985) and Hashimoto and Zhao (2000). Competitive firms maximize profits subject to offering the worker at least a reservation utility that induces her to accept the position. Except for their intrinsic valuation of the company car, worker preferences are identical. Moreover, the negotiation process concerns a particular, homogeneous type of job so that, in the absence of company car provision, all workers in this job would earn the same wage. Workers that attach great utility to the possession of a company car will accept a lower wage if the firm offers to provide a company car. Second, we assume that a worker who receives a company car as part of his compensation package will automatically use this car as the only transport mode for commuting purposes; this assumption is largely consistent with empirical observations (for example, it holds for more than 95% of the cases in the Belgian survey, see Vacature (2006)). For employees who do not receive a company car, the model allows a choice between car and public transport. Finally, car traffic is assumed to generate congestion, but public transport neither suffers from, nor contributes to, congestion (it is, say, rail or metro).

The remainder of this section has the following structure. We first describe how employees' preferences were modelled and derive the effects of the government's tax policy instruments (labour tax, congestion toll, imputed tax value of the company car, public transport fare) on wages (section 2.1) and on the fraction of employees that receive a company car (section 2.2). We then briefly discuss the characteristics of the transport market (section 2.3). The firm's employment decisions are considered in section 2.4. Finally, we consider the objective function and the budget constraint of the government (section 2.5).

2.1. Preferences and the effect of tax instruments on wages

The model assumes that all workers will in equilibrium attain the same reservation utility level, but they will receive different wages depending on whether or not they receive a company car and, in the latter case, on their intrinsic preference for a company car. To model preferences, we specify very simple utility functions that capture the main ingredients for the

analysis that follows (preference for company cars, modal choice for people not having a company car) in the simplest possible way. Although this restricts the generality of the model, it does not affect any of the qualitative conclusions to be derived.

First, consider the problem faced by a worker that does not receive a company car. He cares about a general consumption good (X), transport trips by car (T) and public transport (P), and leisure (l). Let utility be given by the following quasi-linear utility specification:

$$U = X + u(T, P) + v(l) \quad (1)$$

where $u(\cdot)$ and $v(\cdot)$ are quasi-concave and continuous. The employee faces a budget and a time restriction. Let us denote his wage for the period considered as w_{nc} (the subscript “nc” refers to “no company car”). This wage is taxed at a flat rate t . The cost per commuting trip by car consists of a monetary cost (fuel, maintenance, etc.) c_T , plus a congestion tax τ_T imposed by the government. The worker pays a fare p_f for each trip he makes by public transport. We further denote by a and ϕ the time needed for a commuting trip by car and by public transport, respectively. These times are exogenous from the viewpoint of the worker. Finally, to save on notation we normalize the fixed number of days the individual has to work for the contract period considered at one. This implies that T and P sum to one. With these assumptions, we can express the employee’s budget and time restrictions as follows:

$$\begin{aligned} X + (\tau_T + c_T)T + p_f P &= w_{nc}(1-t) \\ l + (1+a)T + (1+\phi)P &= D \\ T + P &= 1 \end{aligned} \quad (2)$$

Here D is exogenous time available.

Maximizing (1) subject to (2) yields the indirect utility function

$$w_{nc}(1-t) + V^{nc}[\tau_T + c_T, p_f, a, \phi]$$

It has the following characteristics:

$$\frac{\partial V^{nc}}{\partial \tau_T} = -T; \quad \frac{\partial V^{nc}}{\partial p_f} = -P; \quad \frac{\partial V^{nc}}{\partial a} = -v' T \quad (3)$$

where $v' = \frac{dv(l)}{dl}$; given the quasi-linear specification (1), it is the worker’s value of time.

Since bargaining will ultimately result in the fixed reservation utility level, tax and congestion effects on the wage w_{nc} can easily be derived by the implicit function theorem.

Using (3), it follows that:

$$\frac{\partial w_{nc}}{\partial t} = \frac{w_{nc}}{1-t}; \frac{\partial w_{nc}}{\partial \tau_T} = \frac{T}{1-t}; \frac{\partial w_{nc}}{\partial p_f} = \frac{P}{1-t}; \frac{\partial w_{nc}}{\partial a} = \frac{T}{1-t} v' \quad (4)$$

These results are plausible. Higher labour taxes, higher congestion taxes, higher public transport prices and more congestion all lead to a higher wage.

Similarly, consider a worker who does receive a company car. The worker is by assumption always commuting by car. Preferences are specified in the simplest possible way as:

$$U = X + v(l) + (z + \varepsilon) \quad (5)$$

The first two terms on the right hand side are the same as before. The terms $z + \varepsilon$ represent the intrinsic monetary value of the utility the employee attaches to the possession (status effects, the comfort not to be worried about maintenance and reparations of the car, etc.) and use (for commuting) of a company car. We assume the parameter z is a common component equal for all workers, but ε is individual-specific.

Consider a particular employee receiving a company car. Denote the person' wage over the period considered by w_c . Consistent with current practice in most countries (e.g. Austria, Belgium, Denmark, Germany, the Netherlands, Spain, UK, USA, etc.), the use of a company car is regarded as a benefit in-kind, so that an imputed value is added to taxable income. We denote this imputed value by ρ . This parameter is an important tax instrument for the government to manipulate company car attractiveness, and it will play a crucial role in the optimal tax analyses below. Under our assumptions, the budget and time constraint are then simply given by:

$$\begin{aligned} X &= w_c - t(w_c + \rho) - \tau_T \\ l + (1+a) &= D \end{aligned} \quad (6)$$

Note that use has been made of the fact that people receiving a company car automatically use it for commuting; moreover, we used the normalization of working time at one.

Combining (5) and (6) and noting that the worker will attain his reservation utility level (denoted \bar{U}) in equilibrium, we easily obtain the wage as :

$$w_c = w_c^* - \frac{\varepsilon}{1-t} \quad (7)$$

Here

$$w_c^* = \left[\bar{U} + t\rho + \tau_T - z - v(D-1-a) \right] / (1-t) \quad (8)$$

Note from (7) that workers with a high preference for a company car, and that actually get one, will negotiate a lower wage, *ceteris paribus*.

As before, tax and congestion effects on wage w_c are easily derived. We have, using (7) and (8), the following results:

$$\frac{\partial w_c}{\partial t} = \frac{w_c + \rho}{1-t}; \quad \frac{\partial w_c}{\partial \tau_T} = \frac{1}{1-t}; \quad \frac{\partial w_c}{\partial \rho} = \frac{t}{1-t}; \quad \frac{\partial w_c}{\partial a} = \frac{v'}{1-t} \quad (9)$$

The effect of a labour tax increase rises in the imputed tax value of the company car; this is because the tax is paid on income that includes the imputed value of the car. Moreover, a less favourable tax treatment of the company car at the level of the worker (higher ρ) implies a higher equilibrium wage. Also observe that the impact of a congestion toll is higher for people receiving a company car than for those that don't (compare (4) and (9), and note that $T < I$). Those having a company car are necessarily affected, whereas those that don't to some extent commute by public transport. These substitution possibilities imply less responsiveness of wages to congestion tolls.

2.2. Taxes and company cars

Providing a worker with a company car has costs as well as potential benefits to the firm. We denote by m the extra productivity induced by a company car; we assume the employer knows this extra productivity for the job under consideration and assume it is equal for all potential employees⁵. If a company car is provided, the firm faces an extra cost, denoted β . Largely in line with current practice in a many countries, we assume that β is all-inclusive: it captures the leasing price of the car, the cost of fuel consumption, and possible taxes on company cars at the firm level. However, congestion tolls are assumed to be fully at the expense of the employee⁶. Moreover, for simplicity the cost per car is assumed to be independent of the number of workers that receive one⁷.

Within this setting, the firm will provide a company car if it is profitable to do so. Consider a firm interested in hiring an extra worker. In the absence of company car provision,

⁵ This is a strong assumption, but it is rather innocuous for the results of the analysis. In an alternative version of the model, we assumed workers all had the same preference for a company car and let the productivity of providing a company car follow a given distribution. This yielded identical results for all practical purposes. However, the current setup facilitates the derivations of the results.

⁶ These assumptions are in fact quite realistic. Operational leasing, where maintenance and insurance are borne by the leasing company in exchange for a fixed lease payment by the employer, is the most popular leasing formula in several European countries, including Belgium. Moreover, the survey among Belgian employees (Vacature (2006)) shows that 93 percent of employees receiving a company car are also provided with a fuel card by their employer. Finally, given the assumption of exogenous reservation utility, whether the employer or the employee (partially) pays for the congestion tax does not matter in this model; it produces identical number of company cars and does not affect the optimal tax results.

⁷ This may not be the case due to economies of scale and contract negotiations between the firm and leasing companies. We make this assumption for simplicity; see Hashimoto and Zhang (2000) for a model where the unit cost of the fringe benefit depends on the number of employees that receive them.

all workers are equally productive; the relation between output and employment n is given by $f(n)$. The marginal product $f'(n)$ is strictly positive and decreasing in n . Employment n will be determined endogenously, as explained later. Normalizing output price at one, the marginal profit generated by hiring an extra worker and not providing a company car is then:

$$MP_{nc} = f'(n) - w_{nc}$$

Similarly, in case the extra worker is given a company car, marginal profit is:

$$MP_c = f'(n) + m - w_c - \beta$$

Therefore, it is profitable to provide the extra worker with a company car if:

$$w_{nc} - w_c - \beta + m > 0 \quad (10)$$

Using (7) and (10) then implies that a company car will be provided to the worker under consideration if:

$$\varepsilon > \mu \quad (11)$$

where

$$\mu = (w_c^* - w_{nc} + \beta - m)(1 - t) \quad (12)$$

We assume ε to be distributed uniformly over the $[-b, +b]$ interval⁸. The fraction F of employees that are provided with a company car can then be written as:

$$F = \frac{1}{2} \left(1 - \frac{\mu}{b} \right) \quad (13)$$

Of course, (13) implies that the fraction of employees receiving a company car depends on all tax instruments, on congestion, and on both the cost of providing a company car and the extra productivity of doing so. Differentiating (13) and using (12), (4), (7), (8) and (9), we find after straightforward manipulations:

$$\begin{aligned} \frac{\partial F}{\partial t} &= -\frac{\rho - \beta + m}{2b}; & \frac{\partial F}{\partial \tau_T} &= -\frac{P}{2b}; & \frac{\partial F}{\partial p_f} &= \frac{P}{2b}; & \frac{\partial F}{\partial \rho} &= -\frac{t}{2b} \\ \frac{\partial F}{\partial a} &= -v' \frac{P}{2b}; & \frac{\partial F}{\partial \beta} &= -\frac{1-t}{2b}; & \frac{\partial F}{\partial m} &= \frac{1-t}{2b} \end{aligned} \quad (14)$$

The impact of a higher labour tax on the fraction of employees obtaining a company car is ambiguous; it depends on the tax imputed value of the car. It is negative if the value imputed for tax purposes is higher than the net-of-productivity cost to the firm of providing the car to the employee (i.e., if $\rho > \beta - m$); it is positive if the opposite holds. The reason for

⁸ The uniform distribution simplifies the derivations; however, other distributions give qualitatively the same results.

this finding follows from two observations. First, a higher labour tax will raise the wage of company car owners more than that of other workers if $w_c + \rho > w_{nc}$ (see (4) and (9)). Second, the firm trades off these relative increases in wages against the extra net cost of providing the car, $\beta - m$.

Higher congestion taxes or increased congestion unambiguously lead to a reduced share of employees with a company car. This negative impact is larger when there is more public transport demand. The intuition for this finding is easy. We observed above that an increase in the congestion toll raises wages more for people having a company car, because of the extra substitution possibilities offered by public transport for employees not having a company car. Higher congestion tolls make it therefore relatively more expensive to provide a company car⁹. A similar argument holds for increases in congestion.

In line with intuition, the effect of the tax imputed value ρ on company car provision is negative. Higher net costs of providing a company car to the firm (higher β) similarly reduce the fraction of cars provided. Finally, if company cars are more productive (higher m), this obviously raises F .

2.3. Road transport and the marginal external cost of congestion

It is instructive at this point to briefly describe the road transport market and the marginal external cost of congestion in this model. The time it takes to make a commuting trip by car a depends on total commuting demand by car, which we denote T_{tot} . Since we normalized the number of days worked at one and given our assumption that people with a company car always commute by car, it is defined as:

$$T_{tot} = n[F + (1 - F)T] \quad (15)$$

The marginal external congestion cost of an increase in car transport is given by:

$$MEC = (v')(T_{tot})(a') \quad (16)$$

⁹ Van Ommeren et al. (2006) empirically estimate a positive relation between workers' commuting time and the probability of receiving a company car, using UK data. This may seem to contradict our finding that fewer company cars are provided when congestion tolls or congestion rises. This is not the case, however. In Van Ommeren et al. (2006) commuting distances differ substantially across individuals. In the current paper commuting distance is the same for all individuals, but commuting time and costs depend on congestion and congestion tolls, respectively. Extending our model to include distance we show elsewhere (see Wuyts (2009)) that the impact of distance (holding tolls and congestion constant) on the fraction of employees receiving a company car is indeed plausibly positive. Conditional on distance, however, more congestion reduces F also in this extended setting.

where $a' = \frac{\partial a(T_{tot})}{\partial T_{tot}}$. To understand the definition (16), note that an increase in total car transport demand raises the time it takes to complete a commuting trip (a'). This extra time loss is experienced by all road users (T_{tot}), whose value of time is v' .

Note that the marginal congestion cost also equals the monetary value of the utility losses a marginal increase in traffic imposes on all other road users; moreover, given the quasi-linear utility functions used and the assumption of exogenous reservation utility, *MEC* can be expressed in terms of the wage increases firms will have to pay their employees in order for the latter to accept to work for the firm. To see this, consider an individual without company car. The welfare loss he suffers from a congestion increase can be expressed as, using (3):

$$-\frac{\partial V^{nc}}{\partial a} a' = v' T a'$$

But this can be rewritten, using (4), as:

$$-\frac{\partial V^{nc}}{\partial a} a' = (1-t) \frac{\partial w_{nc}}{\partial a} a'$$

A similar analysis holds for people that do receive a company car. Taking into account the total number of people with and without company cars (nF and $n(1-F)$, respectively), noting the definition of total transport demand given in (15), and using the wage derivatives with respect to congestion a (see (4) and (9)), it then immediately follows that the marginal external cost of congestion can be written in three equivalent ways:

$$MEC = (v')(T_{tot})(a') = n \left[F \frac{\partial V^c}{\partial a} a' + (1-F) \frac{\partial V^{nc}}{\partial a} a' \right] = n(1-t) a' \left[F \frac{\partial w_c}{\partial a} + (1-F) \frac{\partial w_{nc}}{\partial a} \right] \quad (17)$$

2.4. Taxes, company cars and employment

To determine the effects of the various tax parameters on employment, assume the firm sells on a competitive market at a price equal to one; it determines optimal employment so as to maximize profit. Profit is given by

$$f(n) + m(Fn) - n \left[F(\bar{W}_c) + (1-F)w_{nc} \right] - \beta(Fn) \quad (18)$$

In this expression \bar{W}_c is the average wage paid to a worker with a company car. Given the uniform distribution of the company car preference parameter ε , it equals:

$$\bar{W}_c = w_c^* - \frac{1}{1-t} E(\varepsilon | \varepsilon > \mu) = w_c^* - \frac{(\mu + b)}{2(1-t)} \quad (19)$$

The first two terms of (18) capture revenues, including the extra revenue due to the productivity effect of providing company cars to a fraction F of the labour force. The firm's labour costs consist of wages of employees with and without company cars, plus the cost of providing the cars.

Profit can be more compactly rewritten as:

$$f(n) - n\hat{w}$$

where \hat{w} is the average 'net' cost of labour per employee:

$$\hat{w} = F(\bar{W}_c) + (1-F)w_{nc} + F(\beta - m) \quad (20)$$

To see the interpretation of \hat{w} , note that employees not receiving a company car are paid w_{nc} , and there is a fraction $(1-F)$ of such workers. The 'net' cost to the firm of employees that do get a car consists of the cost of providing the car (β) corrected for the induced productivity effect (m), plus their wage. It is easily shown, as one would expect, that tax increases unambiguously lead to a higher net labour cost per worker. More precisely, in Appendix 1 we derive the following results:

$$\begin{aligned} \frac{\partial \hat{w}}{\partial t} &= \frac{\hat{w} + F(\rho - \beta + m)}{1-t} > 0; & \frac{\partial \hat{w}}{\partial \tau_T} &= \frac{T_{tot}}{n(1-t)} > 0; & \frac{\partial \hat{w}}{\partial p_f} &= \frac{P(1-F)}{(1-t)} > 0; \\ \frac{\partial \hat{w}}{\partial a} &= \frac{v'(T_{tot})}{n(1-t)} > 0; & \frac{\partial \hat{w}}{\partial \rho} &= \frac{Ft}{1-t} > 0 \end{aligned} \quad (21)$$

Finally, using these results, the employment implications of tax parameters are now easily determined. The first-order condition for maximum profit is just:

$$f'(n) - \hat{w} = 0$$

The solution of this expression gives the labour demand function $n(\hat{w})$, expressing employment as a function of the average net wage. Consequently, the effects of taxes and congestion on employment can be written as:

$$\frac{\partial n}{\partial t} = \frac{\partial n}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial t}; \quad \frac{\partial n}{\partial \tau_T} = \frac{\partial n}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial \tau_T}; \quad \frac{\partial n}{\partial p_f} = \frac{\partial n}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial p_f}; \quad \frac{\partial n}{\partial a} = \frac{\partial n}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial a}; \quad \frac{\partial n}{\partial \rho} = \frac{\partial n}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial \rho} \quad (22)$$

Assuming a downward sloping labour demand function, it follows from (21) and (22) that higher labour taxes, congestion taxes, more congestion, a higher taxable basis of company cars and higher public transport fares for commuting all unambiguously lead to less employment.

2.5. The government

The government maximizes social welfare subject to a budgetary constraint. Social welfare takes account of both workers' utilities (which in equilibrium are, however, equal to the fixed reservation utilities) and the firm's profits. The government has four tax instruments available: taxes on labour and on car transport, the tax imputed value of company cars, and public transport fares.

Denote the exogenously determined amount of revenue to be generated as R . The budget constraint can then be written as:

$$t \left\{ n \left[F \left(\bar{W}_c \right) + (1-F)w_{nc} + \rho F \right] \right\} + \tau_T (T_{tot}) + \tau_p (P_{tot}) = R \quad (23)$$

The labor tax applies to wages of both employees with and without company car, and to the imputed value of company cars for the fraction of the labor force that receives such a car. The tax on road use applies to total road transport, defined by (15). As for public transport, we include any fixed costs of providing public transport into R , and define the 'tax' on public transport as the difference between the fare and the marginal production cost:

$$\tau_p = p_f - c_f$$

The tax applies to all public transport trips $P_{tot} = n(1-F)P$.

Using the definition of the average net labour cost given in (20), the budget constraint (23) can be compactly rewritten as:

$$t \left\{ n \left[\hat{w} + (\rho - \beta + m)F \right] \right\} + \tau_T (T_{tot}) + \tau_p (P_{tot}) = R \quad (24)$$

3. Optimal tax policies

In this section, we turn to the optimal tax problem faced by a benevolent government; it optimizes social welfare subject to a financing constraint. The government has in principle four policy instruments (t , τ_T , τ_p , and ρ). However, given the structure of the model, the perfect complementarity between commuting and employment implies that not all instruments can be independently optimally selected¹⁰.

In subsection 3.1 we consider the optimal tax structure when the government can optimally choose the labour tax, the congestion toll and the tax imputed value of company cars; following Parry and Bento (2001), we assume the public transport fare is given. This

¹⁰ This is a well known phenomenon in the optimal congestion tax literature (see, e.g., Van Dender (2003)). Parry and Bento (2001) implicitly resolve it by assuming free public transport.

case will allow us to illustrate the first best tax structure in a straightforward way. In subsection 3.2 we then consider several second-best tax structures, each of which yields, although for very different reasons, useful results. First, we derive optimal congestion and labour taxes, conditional on a given tax treatment of company cars, and for given public transport fares. This is a useful exercise because it allows us to directly relate our results to the earlier literature that studies congestion and labour taxes but ignores the tax preferential tax treatment of company cars (e.g., Parry and Bento (2001), Van Dender (2003), De Borger (2009)). Moreover, it provides information on how optimal congestion tolls have to be adjusted in countries where company cars are taxed preferentially. Second, we analyze optimal taxation of labour compensation (taxes on wage and imputed value of the company car for tax purposes) if transport taxes, for political or technical reasons, cannot be optimally varied in function of external cost considerations. This provides information on the potential role of the fiscal treatment of company cars as a substitute for unavailable congestion tolls in coping with un-priced congestion. Third, we study optimal congestion taxes and public transport fares, conditional on given labour market policies (labour and company car tax treatment). This may be quite relevant in practice, because in many countries labour and transport policies are governed by different parts of the government administration.

3.1. Illustrating the first-best tax structure

As a benchmark case, in this subsection we treat the public transport fare as exogenously given but allow all remaining taxes to be optimally selected. The optimal tax problem is to, using (24):

$$\begin{aligned} & \underset{t, \tau_T, \rho}{\text{Max}} \quad f(n) - n\hat{w} \\ & \text{s.t.} \quad t \left\{ n[\hat{w} + (\rho - \beta + m)] \right\} + \tau_T(T_{tot}) + \tau_p(P_{tot}) = R \end{aligned}$$

Associating a multiplier γ with the budget restriction, the first-order conditions are derived and further elaborated in Appendix 2. It is shown that the optimal tax expressions are given by:

$$\begin{aligned} t &= -\frac{\tau_p}{\hat{w}} + \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} \\ \tau_T &= MEC + \tau_p \\ \rho &= \beta - m \end{aligned} \tag{25}$$

Interpretation of this benchmark case is straightforward. First, suppose the (exogenously given) public transport fare equals the resource cost of public transport

provision ($\tau_p = 0$), and assume the government budget constraint does not bind, so that the shadow cost of public funds γ equals 1. The optimal tax structure then boils down to $t = 0$, $\tau = MEC$, $\rho = \beta - m$. This is the first-best tax structure: the implied availability of a lump-sum tax instrument makes labour taxes unnecessary, and congestion is taxed at marginal external cost. The imputed tax value of company cars equals the net cost to the firm of providing the car. Second, continue to assume $\tau_p = 0$ but suppose the budget constraint does bind ($\gamma > 1$); we then face second-best conditions. The labour tax now has a revenue-raising function and is positive; it inversely depends on the labour demand elasticity. Third, both the congestion toll and the labour tax correct for potential suboptimal pricing of public transport. If public transport is subsidized this reduces the congestion toll below MEC , and it raises the labour tax to generate the extra revenues to finance the subsidies. Interestingly, note that the optimal tax imputed value of company cars is not affected by the unavailability of a lump-sum tax; we still have $\rho = \beta - m$. This is consistent with the argument of Katz and Mankiw (1985), viz., that the tax system should not distort the firm's decision with respect to the composition of labour compensation between wages and fringe benefits. In our framework, variations in the labour tax have no impact on the probability of receiving a company car only when $\rho = \beta - m$, see (14).

3.2. The fiscal treatment of company cars and optimal transport taxes

In this subsection, we focus on the implications of the widely observed preferential tax treatment of company cars for the design of optimal labour and transport tax policies. As the derivations are lengthy and uninteresting in their own right, we focus on the results and defer calculations to technical appendices.

3.2.1 The fiscal treatment of company cars: implications for labour and congestion taxes

What is the relation between optimal taxes on labour and road transport and the preferential tax treatment of company cars? What is the 'bias' in optimal congestion tolls that have been derived in models ignoring the preferential tax treatment of company cars? To answer these questions, we derive optimal labour and congestion taxes, taking the fiscal treatment of company cars and public transport fares as exogenously given.

Consider a benevolent government that maximizes social welfare using the labour tax t and the congestion tax τ_T as instruments, assuming that the imputed tax values of company

cars and public transport prices are treated as given. In line with European tax codes, we assume preferential tax treatment of company cars. Under these conditions, we find (see Appendix 3) that the optimal second-best tax structure is described by the following expressions:

$$\tau_T = MEC + \tau_p + tn\delta[Z_\tau]$$

$$t = \left[-\frac{\tau_p}{\hat{w}} + \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} \right] \left[\frac{1}{\frac{\hat{w} + \delta F}{\hat{w}} - \delta[Z_t] \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}}} \right] \quad (26)$$

In these expressions, the terms Z_τ and Z_t depend on various elasticities but, importantly, preferential tax treatment of company cars implies they are both unambiguously negative (see Appendix 3). Moreover, we defined

$$\delta = \rho - \beta + m \quad (27)$$

If the tax imputed value of the company car (ρ) equals the net cost to the firm of providing the car ($\beta - m$) then $\delta = 0$. Preferential tax treatment of company cars implies $\delta < 0$.

The tax rules are complex, and numerical analysis will provide more detailed intuition in the next section. A few implications do stand out from the theoretical results in (26), however. For example, if no preferential tax treatment for company cars exists ($\delta = 0$), it is easily verified that we again find the results of the previous section 3.1. To see this, note that the first term between square brackets on the right hand side of the labour tax expression is the optimal first best labour tax rule (25); the final term between square brackets equals one if $\delta = 0$.

Consistent with the situation in many European countries, let us assume that company cars are taxed preferentially relative to wages ($\delta < 0$). Then system (26) implies, provided the optimal labour tax is positive, that this unambiguously raises the optimal congestion tax. The reason for the higher congestion tax is that preferential taxes on company cars yield an inefficiently large number of employees receiving a company car, implying too much traffic and inefficiently low public transport use. The higher tax on car use reduces the fraction of people receiving one, see (14). Moreover, it stimulates demand for public transport and discourages car commuting by people not obtaining a company car.

We further note that the impact of the preferential taxation of company cars on optimal labour taxes is ambiguous in general; expression (26) suggests that it depends in a complex way on the sensitivity of labour demand and on the fraction of employees with a company car¹¹. Subsidizing company cars distorts markets and has revenue effects. If the number of company cars F is not too large, revenue losses of the preferential treatment of company cars are limited, and (26) implies a lower optimal labour tax. This is quite intuitive: as labour compensation is in the form of wages and company cars, the preferential tax treatment on company cars implies that employees without a company car are in comparison taxed inefficiently high; this is ‘corrected’ by reducing the labour tax. However, if many company cars are provided and they are treated favourably by the tax system, then the tax revenue losses are substantial and have to be compensated by higher labour taxes.

The main general implication of this exercise is that studies that have numerically modelled optimal congestion tolls in countries where company cars are treated very favourably (the UK, Netherlands, Belgium, France, etc.) have underestimated optimal congestion tolls by ignoring the existence of company cars. Numerical evaluation in the next section will provide information on how large this ‘bias’ is.

3.2.2 Is optimal taxation of company cars a substitute for unavailable congestion tolls?

Although economists have advocated the use of congestion pricing for many years, actual implementation is still quite limited¹². In this subsection, we study optimal taxes on the labour market (labour tax and tax treatment of company cars), assuming that transport taxes (congestion taxes and public transport fares) for some reason cannot be set optimally. We are specifically interested in (i) the impact of sub-optimally low congestion taxes on the optimal tax treatment of company cars, and (ii) the extent to which optimal taxation of company cars may serve as a substitute for an unavailable congestion tax.

The derivation of the optimal tax rules is reported in Appendix 4. Given the nonlinearity of the government’s budget constraint in t and ρ (see (23)) no closed-form expressions are available. However, it is straightforward to characterize the effect of suboptimal taxes on the transport market for labour taxes and the tax imputed value of company cars. It is shown that the tax parameters satisfy the following expressions:

¹¹ Note that Z_t itself depends on elasticities and the fraction of employees with a company car, see Appendix 3.

¹² Exceptions are Singapore, London and Stockholm. Congestion taxes have been seriously considered but finally rejected in at least some other places (e.g., New York). In some European countries (e.g., the Netherlands), congestion pricing has been on the political agenda for a long time, but implementation has been postponed several times.

$$\begin{aligned}
t &= \left[-\frac{\tau_p}{\hat{w}} + \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} \right] - [\tau_T - MEC - \tau_p] \left[\frac{T}{\hat{w} \left(1 - n(1-F) \frac{\partial T}{\partial a} \right)} \right] \\
t\delta &= -[\tau_T - MEC - \tau_p] \left[\frac{P}{1 - n(1-F) \frac{\partial T}{\partial a}} \right]
\end{aligned} \tag{28}$$

To interpret these rules, separately consider the role of suboptimal pricing of congestion and suboptimal public transport fares. First, let car transport be priced below *MEC*. Since congestion reduces demand for car transport by people not having a company car (hence $\frac{\partial T}{\partial a} < 0$), it follows from (28) that this necessarily raises the labour tax. More importantly, it implies a tax imputed value of company cars above the net cost to the firm: $\delta > 0 \rightarrow \rho > \beta - m$. This effect will be larger the more public transport demand there is. Taxing company cars above the tax neutral level makes providing such fringe benefits highly unattractive, as only people with extremely strong preference for a company car will be interested in having one. Fewer employees with a company car then implies that there is less commuting by car, so that congestion goes down. In this sense, high tax imputed values for company cars serve as an imperfect substitute for unavailable optimal congestion taxes.

Second, suppose public transport is subsidized. It follows from (28) that this has ambiguous effects on the labour tax: it raises the first but reduces the second term on the right hand side of the expression for the optimal labour tax. However, it necessarily reduces the tax imputed value of company cars; this is a standard second-best finding.

3.2.3 Company cars and optimal commuting taxes

Finally, suppose different government agencies are responsible for pricing transport services and for deciding on labour market policies. We therefore briefly consider optimal transport policies (congestion taxes and public transport fees) for given labour market policies (labour taxes and treatment of company cars). We find the following rules (see Appendix 5):

$$(\tau_T - MEC) = \left(\frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\tilde{w}}{n}} - t \right) \tilde{w} + t\delta \left[\frac{P^2(1-F)}{2b(1-F) \frac{\partial T}{\partial \tau_T} - P^2} \right] \quad (29)$$

$$\tau_p = \left(\frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\tilde{w}}{n}} - t \right) \tilde{w} + t\delta \left[\frac{(F + (1-F)T)P}{P^2 - 2b(1-F) \frac{\partial T}{\partial \tau_T}} \right]$$

where $\tilde{w} = \hat{w} + \delta F$ is the labour tax base per worker.

Interpretation of (29) is easy. First, assume there is no preferential tax treatment of company cars ($\delta = 0$). The first term on the right-hand-side of both expressions then just says that, depending on the value of the exogenously fixed labour tax, both the congestion tax and the public transport fare may have to contribute to generating revenues. If the labour tax is set relatively high, then the congestion toll can be set below *MEC* and public transport can be subsidized. However, if the labour tax does not generate sufficient revenues to cover the government budget restriction, then both modes have a revenue-raising role. Note that they then raise equal revenues per commuting trip. Second, if company cars are taxed preferentially ($\delta < 0$), this unambiguously reduces the public transport fare and raises the optimal congestion tax. Indeed, the terms between square brackets on the right-hand-side of the optimal congestion toll and public transport fare are necessarily negative and positive, respectively. These effects are larger the less elastic is car transport demand. Note the clear redistribution from owners to non-owners of a company car through the optimal commuter taxes: the system increases road taxes and recycles additional revenues to public transport users.

4. Numerical illustration

The numerical model optimally determines taxes on transport and labour markets under various different scenarios on the available policy instruments. We first (subsection 4.1) discuss the structure and the calibration of the numerical model and provide some information of the Belgian data we used for the application. Unless otherwise noted, data were provided by Statbel, a division of the Federal Government Service FOD-Economics. Next we report on the numerical results obtained for various second-best optimal tax exercises (subsection 4.2).

4.1 Structure, calibration and data

The numerical model very closely follows the structure of the theoretical model presented earlier in the paper. Some minor changes are introduced to facilitate calibration and solution of the model.

Consider preferences. Utility of employees without and with a company car was specified as suggested in (1) and (5), respectively. However, to facilitate the numerical optimisation and the interpretation of the results, we assumed constant time values. A flexible functional form was used for the transport sub-utility function $u(T, P)$ in (1).

To determine parameters of the utility functions and to collect the necessary information to detail the different budgetary constraints we combined information from various different sources. Using data from the Belgian Mobility Survey (Pollet (2000)), we assumed an average commuting trip of 20 kilometre; moreover, as suggested by these data, in the reference situation two thirds of commuting trips were car trips, one third of all commuting was by public transport.

The cost structure of car transport was taken from recent key data from the Belgian Automobile Association (VAB). They report the average user cost, including taxes -- but not capturing currently nonexistent tolls -- to be 0.363 euro/km. The public transport fare was set at 2.07 euro per round-trip, which is consistent with an average fare of 5.6 eurocents per kilometre observed for Belgian public railroads¹³. In other words, in our illustration we -- realistically -- assume that public transportation is heavily subsidized: the Belgian data show that 43.4% of the costs of a passenger rail trip is covered by the government¹⁴. In the numerical example, the round-trip cost thus equals 3.66 euro of which 1.59 euro is subsidized by the government.

The data on car commuting transport flows are a mixture of urban and non-urban (i.e., mainly highways) road traffic. In Belgium, approximately 75.1% of all vehicle kilometres were driven on non-urban roads in 2000, and 24.9% on urban roads¹⁵. To capture congestion, separate aggregate speed-flow relations were determined for urban and non-urban conditions. Analysing a wide range of specifications, Kirwan et al. (2001) suggest that a specific exponential type of aggregate congestion function is the most plausible. We used updated information from Mayeres et al. (1996) for Brussels to calibrate the aggregate congestion

¹³ The source is the national railroad company NMBS, Statistical Abstract, 2005.

¹⁴ NMBS Statistical Abstract 2005.

¹⁵ Total vehicle kilometres driven in Belgium in 2000 amounted to 18.03 billion on urban roads and 54.47 billion on non-urban roads.

function for urban conditions. It gives the time (in minutes) needed to drive 1 km as a function of the total traffic flow. The congestion function obtained was:

$$60/s = 1.199376 + 0.000624 \times \exp(14.0203 \times V)$$

where s is the speed (km/hr) and V is the total traffic flow. In a similar way, the non-urban (or highway) congestion function was estimated from data presented in De Borger and Proost (2001). The congestion function is:

$$60/s = 0.499146 + 0.000853 \times \exp(15.5925 \times V)$$

Both calibrated congestion functions are shown in Figure 1.

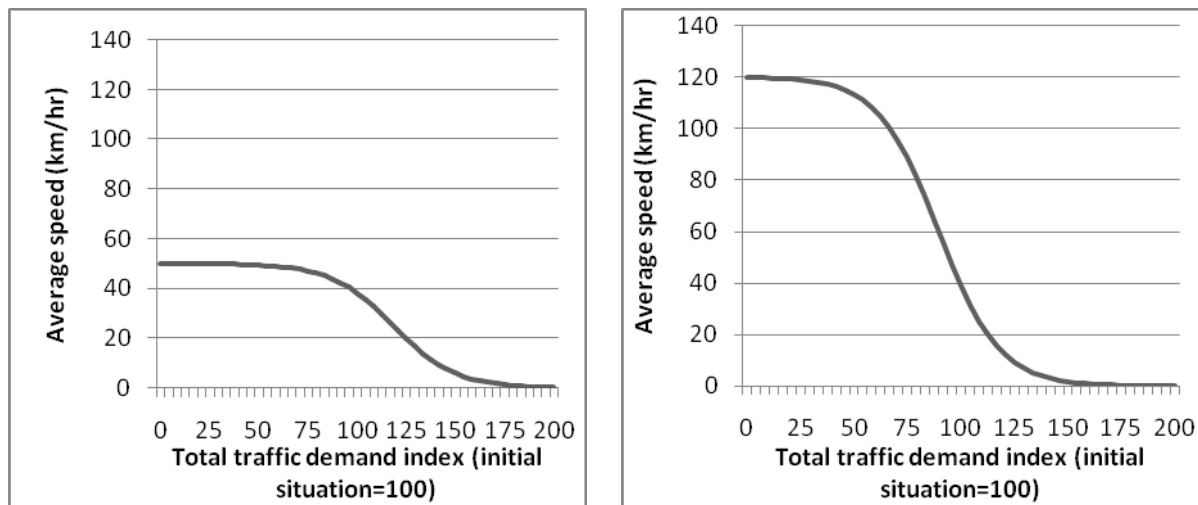


Figure 1: The aggregate congestion function: urban roads (left), non-urban roads (right).

The utility function parameters for people not having a company car were calibrated by combining the observed quantity and price information with assumptions on various elasticities. An uncompensated demand elasticity with respect to the generalized transport cost of -0.33 was used. This is within the range of estimates reported in the recent literature (see Gunn and de Jongh (2001)). The uncompensated net wage elasticity of labour supply was set to 0.2, in line with estimates used by Parry and Bento (2001). The value of time was set at 7.5 euro per hour. It was carefully checked whether the calibrated parameter values satisfied concavity conditions within a wide and reasonable range of values for transport and leisure demands.

To calibrate utility function parameters and the appropriate budget constraint for employees that did get a company car, we combined data from the Belgian survey (Vacature

(2006)) with a number of, hopefully plausible, assumptions¹⁶. The survey has 62,284 respondents, of which 20.9 % had a company car. The average monthly leasing price of a company car in the sample was 539 euro per month¹⁷. Moreover, in our empirical application we assume for simplicity that all employees with a company car are provided with a fuel card. This assumption is, in fact, not far from the truth. The dataset shows that 93 percent of employees that receive a company car as part of the compensation package actually do receive a fuel card. The cost of a fuel card to the firm depends on the total number of kilometres the employee uses the car. Since no data on non-commuting is available we assume for simplicity that the non-commuting share is one third of the kilometres for commuting purposes. In total, the average company car is then assumed to be used for 1170 km/month (i.e. 40 commuting kilometres/day * 22 working days * 1.33). At a fuel cost of 1.5 euro/liter and an average fuel consumption of 6 liter per 100 km, fuel expenses add up to 105 euro/month.

In line with the Belgian tax code (see Wuyts (2009)) and within the range of values reported by Gutiérrez-i-Puigarnau and van Ommeren (2009) we set ρ , the imputed value for tax purposes, equal to 60 percent of the total annual value of the company car (i.e. leasing cost plus fuel card)¹⁸. The net annual cost of the company car to the firm (i.e. $\beta - m$) is set at 90% of its annual leasing cost; this implies we assume a very low extra productivity effect induced by giving employees a car.

Recall that the value of z indicates to which extent the average employee values ownership of a company car (additional to the cost savings attributable to free commuting). In the numerical exercise, we assume that z equals 10 percent of the total value of the company car. The parameter b of the uniform distribution was calibrated based on (14), using the estimated fraction of people with a company car. The calibrated value for b implied that people with the strongest preferences for a company car value the possession and use of a company car (apart from the pure monetary benefit) at almost 20 euro per day more than the average of the population.

¹⁶ Accurate estimates for the parameter values that relate to company car taxation (i.e., ρ , β and z) in a numerical exercise like the one of this paper are not obvious to determine. In practice, the tax treatment of company cars obviously depends on various characteristics not explicitly included in our model (the type of company car, daily commuting distances, the importance of non-commuting transport, etc.). Indeed, the model assumes one type of company car and a given commuting distance of 20 kilometres one way.

¹⁷ The data set contains no information on the value or the leasing price of the company car provided to the respondent. However, based on the company car type given by the respondent, we added the corresponding leasing price to the data (source: <http://directlease.be>).

¹⁸ Simulations of the Belgian tax code suggest that 60% is a reasonable approximation for the median tax advantage.

Finally, to facilitate calibration of the production function, a flexible form was used¹⁹. The gross wage was set at 16.25 euro per hour, with a flat labour tax rate of 40 percent (as in Van Dender (2003)), so that the net wage in the reference situation is 9.75 euro per hour. To calibrate the parameters we used information provided in Konings and Roodhooft (1997). They produced extensive empirical evidence on the demand for labour in Belgian enterprises, reporting a labour cost elasticity of -0.60. This information was combined with reference wages and employment levels (according to Statbel, the Belgian population was slightly over 10.25 million, of which approximately 35% was employed) and with the observed labour share in the Belgian economy to calibrate the production function parameters. Again it was carefully checked that the resulting expression satisfied all requirements imposed by microeconomic theory.

4.2. Numerical results

In this subsection, we report numerical results on three second-best optimal tax exercises. We consecutively report on the bias in optimal congestion tolls due to ignoring the tax treatment of company cars, the potential role of taxing company cars as a substitute for unavailable congestion tolls, and the implications for public transport fares.

4.2.1. The tax treatment of company cars and congestion taxes

We used the numerical model to determine congestion and labour taxes, conditional on the observed preferential tax treatment of company cars. To avoid interdependency between tax instruments, we also assume exogenously given public transport fares.

The information in the second column of Table 1 gives the tax structure and some other relevant pieces of information for the calibrated reference situation. Note that, for expositional purposes, we included existing indirect taxes into the user cost figures for car use, so that the congestion charge in the reference is zero. The tax imputed value of a company car is 0.6, so there is substantial preferential treatment.

Results of the optimal tax exercise are also provided in Table 1. Not surprisingly, since congestion is under-priced in the reference, the congestion toll drastically rises. Interestingly, however, despite public transport subsidies (which imply lower congestion tolls, see (26) in the theoretical section), it actually exceeds marginal external cost. This can be explained by

¹⁹ Specifically, we used $f(n) = \varphi_0 + \varphi_1 \log(n) + \varphi_2 (\log(n))^2$

the preferential treatment of company cars, which raises congestion taxes (again, see (26)). The importance of this ‘company car effect’ can readily be derived from Table 1. Indeed, at the optimum the congestion toll amounts to 11.01 euro, and marginal external cost equals 9.04 euro. Expression (26) given above shows that public transport subsidies alone would reduce the toll below external cost by the size of the subsidy (equal to 1.59 euro) to 7.45 euro per trip. The tax treatment of company cars raises the optimal congestion tax by 3.56 euro per trip to 11.01 euro. These figures are a first indication that ignoring existing company car tax treatment may give rise to a strongly underestimated optimal congestion tax.

The labour tax at the optimum is below its reference value. Part of the reason is simple that current reference labour taxes in Belgium are high. Note that the optimal labour tax follows from three separate effects (see (26)): its revenue-raising role, public transport subsidies (their financing requires raising the labour tax), and the treatment of company cars. Under-taxed company cars raise congestion toll revenues and put downward pressure on optimal labour taxes.

The implications of the optimal tax structure are clear. First, despite favourable tax treatment of company cars, the drastic increase in congestion tolls implies that much fewer employees actually get one. The reason is that higher congestion tolls raise negotiated wages (see (4) and (9)), and reduce the willingness of employers to provide company cars (see (14)). The fraction employees with a company car falls from 21 percent to less than 10 percent. Second, the demand for public transport by employees without company car increases from one third to 41.8% of commuters. Fewer company cars and more public transport obviously further contribute to less congested roads: average speed increases by almost 23 km per hour. Third, higher congestion tolls and lower labour taxes jointly slightly raise employment. This is consistent with what Parry and Bento (2001) suggested, viz. that internalizing marginal external costs of road use and recycling additional revenues to lower labour taxes may in fact have positive employment effects. Finally, note that moving from the reference situation to the optimal tax structure yields a welfare gain of 2% .

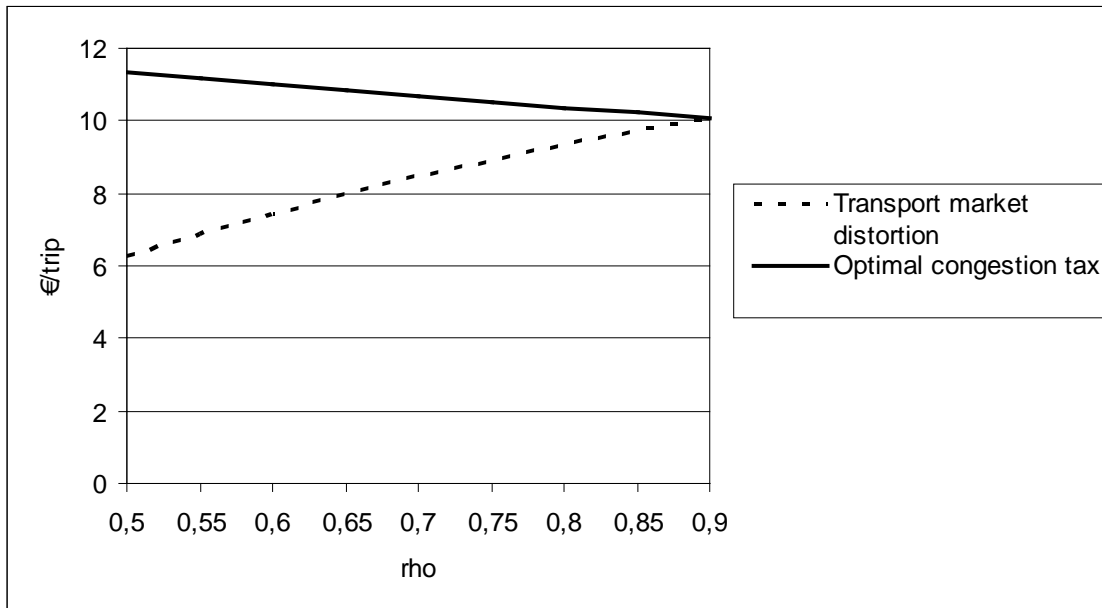
The results are obviously sensitive to the assumed imputed tax value for company cars. Figure 2 gives the optimal congestion tax for various different values of ρ . Higher values of ρ imply less favourable tax treatment of company cars so that we observe correspondingly lower congestion taxes. Moreover, on Figure 2 we have also tried to clarify the decomposition of the optimal congestion tax in its different components. The optimal congestion tax corrects for the distortions on the transport market due to un-priced congestion

and public transport subsidies (i.e. $\tau_p + MEC$), and it adjusts for the preferential treatment of company cars. For a tax imputed value of the company car equal to its tax neutral level (i.e. $\rho = 0.9$, and as a consequence $\delta = 0$) the congestion tax only corrects for the transport market distortion. For values of ρ below the tax neutral level, the optimal congestion tax will also correct for the labour market distortion induced by the tax preferential treatment of company cars (i.e. the vertical difference between the optimal congestion tax and the transport market distortion in Figure 3).

Table 1. Second best optimal labour and congestion taxes: numerical results

	Reference equilibrium	Optimal labour and congestion taxes
Welfare index	100	102.05
Congestion tax (euro/trip)	0.00	11.01
Labour tax (%)	40.00	34.67
Employment (%)	36.00	36.33
Fraction of employees with company car	20.90	9.05
Modal choice of employees without company car:		
% car	66.67	58.2
% public transport	33.33	41.8
Marginal external cost (euro/trip)	30.68	9.04
Average speed (km/hr)	38.80	61.73
Distribution of tax revenues (Total revenues=100)	Labour tax=97.92 Congestion tax=0.00 Company car tax=2.91 Public transport tax=-0.83	Labour tax=86.45 Congestion tax=13.65 Company car tax=1.10 Public transport tax=-1.21
Public transportation fare is 2.07 euro per round trip (resource cost is 3.66 and subsidy is 1.59). The tax imputed value of company cars, ρ , is equal to 0.6		

Figure 2: Sensitivity of optimal congestion taxes to the tax treatment of company cars



4.3. The tax on company cars as a substitute for unavailable congestion tolls

Suppose that for whatever reason congestion tolls cannot be implemented, then to what extent can a tax reform of the labour market (wages and company cars) be a -- potentially politically more acceptable -- substitute for unavailable optimal tolls?

In Table 2, we report the results of determining optimal values of t and ρ , conditional on exogenous congestion tolls and public transport fares. Given that congestion is under-priced in the reference situation, the tax imputed value of company cars is almost twice as high as its initial level; it is well above the tax neutral level (equal to 0.9). The consequence is that company cars disappear entirely from employees' remuneration package²⁰. The optimal labour tax is not strongly affected.

²⁰ Given the low productivity effect we assumed for company cars, the fraction of employees with a company car was actually marginally negative; we imposed it to be non-negative in order to produce the results of Table 4.

Table 2. Second best optimal labour and company car taxes: numerical results

	Reference equilibrium	Optimal labour and company car taxes
Welfare index	100	101.33
Tax imputed value of company car (i.e. ρ)	0.6	1.18
Labour tax (%)	40.00	39.6
Employment (%)	36.00	36.21
Fraction of employees with company car	20.90	0.00
Employees without company car:		
% of car commuting	66.67	69.05
% of public transport commuting	33.33	31.95
Marginal external cost (euro/trip)	30.68	19.43
Average speed (km/hr)	38.80	47.62
Distribution of tax revenues (Total revenues=100)	Labour tax=97.92 Congestion tax=0.00 Company car tax=2.91 Public transport tax=-0.83	Labour tax=100.98 Congestion tax=0.00 Company car tax=0.00 Public transport tax=-0.98
Public transportation fare is 2.07 euro per round trip (resource cost is 3.66 and subsidy is 1.59). no congestion taxes are charged.		

Interestingly, a substantial reduction in congestion can be obtained by adapting the tax treatment of company cars. It follows from Table 2 that elimination of the current preferential tax treatment of company cars leads to a reduction of marginal external costs by more than one third. This finding is to some extent due to the nonlinearity of the congestion function, to some extent it follows from the assumption that at least some employees that no longer have a company car will commute by public transport.

Finally, not surprisingly, the increase in welfare is considerably smaller than in Table 1. Implementation of optimal congestion and labour taxes resulted in a welfare gain of 2.05%; the increase attained by substituting optimal tax treatment of company cars for an unavailable congestion tax is limited to 1.33%. Given our assumptions this does suggest that more than

half the welfare gain of an optimal congestion toll can be achieved by optimally adjusting the tax base of company cars.

4.4. The tax treatment of company cars and public transport subsidies

In a final illustration, we assume that policymakers on the labour market operate independently of policymakers that set transport taxes; we derive optimal transport taxes (car and public transport) for given levels of labour market policies (taxes on labour tax and company cars).

The results are reported in Table 3. First, they confirm the increase in congestion tax required to correct for the under-pricing of congestion and the preferential tax treatment of company cars. However, the high labour tax in the reference situation implies that congestion tolls remain below marginal external cost. Second, the most striking finding is that optimal public transport subsidies are very large; in fact, the result is consistent with zero public transport fares²¹. One reason for this is again the high labour tax; this implies that relatively few revenues have to be generated on transport services. The other reason is the tax treatment of company cars. It induces subsidies to public transport to reduce the number of employees with a company car (see (14)).

²¹ Again, the unconstrained optimum produced fares that were marginally negative. We constrained the fare to be non-negative to find the results in Table 3.

Table 3: Second best optimal transport policies

	Reference equilibrium	Optimal congestion tax and public transport fare
Welfare index	100	100.98
Congestion tax (euro/trip)	0.00	1.04
Public transportation fare (euro/trip)	2.07	0
Employment (%)	36.00	36.16
Fraction of employees with company car	20.90	18.49
Employees without company car:		
% of car commuting	66.67	63.85
% of public transport commuting	33.33	36.15
Marginal external cost (euro/trip)	30.68	22.63
Average speed (km/hr)	38.80	44.64
Distribution of tax revenues (Total revenues=100)	Labour tax=97.92 Congestion tax=0.00 Company car tax=2.91 Public transport tax=-0.83	Labour tax=98.10 Congestion tax=1.46 Company car tax=2.59 Public transport tax=-2.15
Labour taxes are exogenously set to 40% and the tax imputed value of company cars is 0.6.		

5. Conclusion

This paper has considered the implications of the preferential tax treatment of company cars that is so widespread in a number of European countries. We considered a model of employees that commute to work by car or public transport; car transport generates congestion. Wages and whether or not a company car will be provided was modelled as the results of direct negotiation between employer and employee. Company cars may have productivity effects and employees differ in their preference for having such a car. Within this framework we first derive the effect of labour and transport market tax policies on the number of employees provided with a company car, on wages, and on employment. We then study the government's problem of designing optimal tax policies on labour compensation (labour tax and imputed tax base for company cars) and on transport flows (congestion tolls and public transport fares).

We first show that higher labour taxes and a more favourable tax treatment of company cars raises the fraction of the labour force that gets a company car as part of the compensation package. Congestion and congestion tolls have a negative effect; they make public transport more attractive and reduce the relative value of having a company car. We then show that the currently observed preferential tax treatment of company cars raises optimal congestion taxes, suggesting that previous models may have substantially underestimated optimal congestion tolls. Our numerical illustration based on Belgian data suggests that about 3.5 euro of the optimal congestion toll of 11 euro (for a 20 kilometre roundtrip) is due to the current preferential tax treatment of company cars. The optimal tax structure substantially reduces the number of company cars firm provide to workers. Furthermore, if for technical or political reasons optimal congestion tolls are unavailable we find that, at current transport taxes that do not fully cover marginal external congestion costs the optimal tax imputed value of company cars should be set higher (and not lower as is currently the case) than the net cost of the car to the employer. If productivity effects of company cars are small, this wipes out such cars from compensation packages altogether. Moreover, optimal taxation of company cars is found to be an imperfect, but quite powerful, substitute for unavailable congestion tolls. Finally, current preferential tax treatment is an argument for public transport subsidies. In fact, the numerical results are consistent with zero public transport fares.

Although results were derived in a simple setting, the main results may have quite general relevance for revising transport policies in Europe. As argued by other authors, the

welfare cost of excessive company car provision that operates through changes in car ownership is estimated to be substantial (Gutierrez-i-Puigarnau and Van Ommeren (2010)). The current paper now suggests that reducing the preferential tax treatment of company cars has also large implications for congestion and for the design of transport policies directed at reducing traffic delays.

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Appendix 1. Effects of taxes and congestion on the net average wage

In this appendix, we derive the effects of tax parameters on the net average wage \hat{w} . This was given by (see (20)):

$$\hat{w} = F(\bar{W}_c) + (1-F)w_{nc} + F(\beta - m) \quad (\text{A1.1})$$

where

$$\bar{W}_c = w_c^* - \frac{1}{1-t} E(\varepsilon | \varepsilon > \mu) = w_c^* - \frac{(\mu + b)}{2(1-t)} \quad (\text{A1.2})$$

To derive the tax effects, first note that we can rewrite (A1.2) as follows, using (12):

$$\bar{W}_c = \frac{(\mu - b)}{2(1-t)} + w_{nc} - \beta + m$$

By substituting and rearranging we then have:

$$\hat{w} = w_{nc} + F \left(\frac{(\mu - b)}{2(1-t)} \right)$$

Next, use (13) to find²²:

$$\hat{w} = w_{nc} - \left(\frac{bF^2}{1-t} \right) \quad (\text{A1.3})$$

Straightforward algebra then establishes the derivative results. For example,

$$\frac{\partial \hat{w}}{\partial t} = \frac{\partial w_{nc}}{\partial t} - \frac{2(1-t)bF \frac{\partial F}{\partial t} + bF^2}{(1-t)^2}$$

Using (4), (14) and (A1.3), we obtain after straightforward algebra:

$$\frac{\partial \hat{w}}{\partial t} = \frac{\hat{w} + F(\rho - \beta + m)}{1-t}$$

Starting from (A1.3), entirely analogous derivations yield the other results reported in the main body of the paper (see (21)):

$$\begin{aligned} \frac{\partial \hat{w}}{\partial \tau} &= \frac{T_{tot}}{n(1-t)} > 0 \quad ; \quad \frac{\partial \hat{w}}{\partial p} = \frac{P(1-F)}{(1-t)} > 0; \\ \frac{\partial \hat{w}}{\partial a} &= \frac{v'(T_{tot})}{n(1-t)} > 0; \quad \frac{\partial \hat{w}}{\partial \rho} = \frac{Ft}{1-t} > 0 \end{aligned}$$

²² This expression (A1.3) seems to suggest that an exogenous increase in the fraction of employees receiving a company car reduces the net average labor cost per employee. However, this interpretation is inappropriate. To see this, note that conditional on b , an increase in F necessarily implies a reduction in the net cost $(\beta - m)$ of providing a car, see (11)-(12). This cost reduction at the same time raises the fraction of employees with a company car and it reduces the net average labor cost.

Appendix 2. Derivation of the optimal tax structure

Noting that our normalizations imply $P_{tot} = n - T_{tot}$, we can reformulate the optimal tax problem defined in Section 3.1 as²³:

$$\begin{aligned} \underset{t, \tau_T, \rho}{Max} \quad & f(n(\hat{w})) - (n(\hat{w}))\hat{w} \\ \text{s.t.} \quad & t \left\{ n(\hat{w}) [\hat{w} + (\rho - \beta + m)F] \right\} + [\tau_T - \tau_p](T_{tot}) + \tau_p(n(\hat{w})) = R \end{aligned}$$

In this formulation, we have made explicit the dependence of employment on the average net cost of labor through the labor demand function $n(\hat{w})$. This is obtained as the solution of the firm's first-order condition for optimal employment ($f'(n) - \hat{w} = 0$).

Associating a multiplier γ with the government budget restriction, the first-order condition with respect to the labor tax t can be written (using $f' - \hat{w} = 0$) as:

$$-n \frac{d\hat{w}}{dt} + \gamma \left\{ n\tilde{w} + t \left(n + \tilde{w} \frac{\partial n}{\partial \hat{w}} \right) \frac{d\hat{w}}{dt} + t\delta n \frac{dF}{dt} + [\tau_T - \tau_p] \frac{dT_{tot}}{dt} + \tau_p \frac{\partial n}{\partial \hat{w}} \frac{d\hat{w}}{dt} \right\} = 0$$

In this expression, we have defined

$$\tilde{w} = \hat{w} + \delta F = \hat{w} + (\rho - \beta + m)F \quad (\text{A2.1})$$

and

$$\delta = \rho - \beta + m \quad (\text{A2.2})$$

Note that \tilde{w} is the labor tax base per employee for the government. It differs from the labor cost per employee to the firm to the extent that the tax imputed value of the company car ρ differs from the net cost to the firm of providing the company car ($\beta - m$). Observe that the sign of δ tells us whether there is preferential tax treatment of company cars; there is if $\delta < 0$.

Simple manipulations show that the first-order condition can be rearranged as:

$$\left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{dt} - n(1-t) \frac{d\hat{w}}{dt} + [\tau_T - \tau_p] \frac{dT_{tot}}{dt} + (t\delta) \left(n \frac{dF}{dt} \right) + n\tilde{w} = 0 \quad (\text{A2.3})$$

Now note that, in general:

$$\frac{d\hat{w}}{dt} = \frac{\partial \hat{w}}{\partial t} + \frac{\partial \hat{w}}{\partial a} a' \frac{dT_{tot}}{dt}$$

Using the partial derivatives presented in (21), we have:

$$\frac{d\hat{w}}{dt} = \left[\frac{\tilde{w}}{1-t} + \frac{MEC}{n(1-t)} \frac{dT_{tot}}{dt} \right]; \quad MEC = v' T_{tot} a'$$

²³ Indeed, we have $P_{tot} = n(1-F)P = n(1-F)(1-T) = n - n[F + (1-F)T] = n - T_{tot}$.

Substituting this result in (A2.3), we obtain:

$$\left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{dt} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{dt} + (t\delta) \left(n \frac{dF}{dt} \right) = 0 \quad (\text{A2.4})$$

The first-order conditions with respect to the toll and the imputed tax value of company cars can be written in a similar way. To see this, note that they are given by:

$$\begin{aligned} -n \frac{d\hat{w}}{d\tau_T} + \gamma \left\{ n\tilde{w} + t \left(n + \tilde{w} \frac{\partial n}{\partial \hat{w}} \right) \frac{d\hat{w}}{d\tau_T} + t\delta n \frac{dF}{d\tau_T} + [\tau_T - \tau_p] \frac{dT_{tot}}{d\tau_T} + \tau_p \frac{\partial n}{\partial \hat{w}} \frac{d\hat{w}}{d\tau_T} \right\} &= 0 \\ -n \frac{d\hat{w}}{d\rho} + \gamma \left\{ n\tilde{w} + t \left(n + \tilde{w} \frac{\partial n}{\partial \hat{w}} \right) \frac{d\hat{w}}{d\rho} + t\delta n \frac{dF}{d\rho} + [\tau_T - \tau_p] \frac{dT_{tot}}{d\rho} + \tau_p \frac{\partial n}{\partial \hat{w}} \frac{d\hat{w}}{d\rho} \right\} &= 0 \end{aligned}$$

Observe that:

$$\frac{d\hat{w}}{d\tau_T} = \frac{\partial \hat{w}}{\partial \tau_T} + \frac{\partial \hat{w}}{\partial a} a' \frac{dT_{tot}}{d\tau_T}; \quad \frac{d\hat{w}}{d\rho} = \frac{\partial \hat{w}}{\partial \rho} + \frac{\partial \hat{w}}{\partial a} a' \frac{dT_{tot}}{d\rho}$$

and again using the derivative results of (21), we find after analogous steps:

$$\begin{aligned} \left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{d\tau_T} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{d\tau_T} + (t\delta) \left(n \frac{dF}{d\tau_T} \right) &= 0 \\ \left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{d\rho} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{d\rho} + (t\delta) \left(n \frac{dF}{d\rho} \right) &= 0 \end{aligned} \quad (\text{A2.5})$$

It then easily follows that the solution to the system consisting of (A2.4)-(A2.5) satisfies:

$$\begin{aligned} t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} &= 0 \\ \tau_T - MEC - \tau_p &= 0 \\ \delta &= 0 \end{aligned}$$

Finally, using (A2.1)-(A2.2) yields the result reported in the main body of the paper.

$$\begin{aligned} t &= -\frac{\tau_p}{\hat{w}} + \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} \\ \tau_T &= MEC + \tau_p \\ \rho &= \beta - m \end{aligned}$$

Appendix 3. Derivation of optimal labour and congestion taxes

Suppose only taxes on labour and car transport can be freely determined. The two relevant first-order conditions are the same as in Appendix 2. We write them as follows to capture the exogeneity of company car imputed tax values:

$$\begin{aligned} \left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{dt} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{dt} &= -(t\delta) \left(n \frac{dF}{dt} \right) \\ \left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{d\tau_T} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{d\tau_T} &= -(t\delta) \left(n \frac{dF}{d\tau_T} \right) \end{aligned}$$

Reformulate this system as:

$$\begin{aligned} \{Q_t\} \frac{d\hat{w}}{dt} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{dt} &= -(t\delta) \left(n \frac{dF}{dt} \right) \\ \{Q_t\} \frac{d\hat{w}}{d\tau_T} + [\tau_T - MEC - \tau_p] \frac{dT_{tot}}{d\tau_T} &= -(t\delta) \left(n \frac{dF}{d\tau_T} \right) \end{aligned} \tag{A3.1}$$

where

$$Q_t = \left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \tag{A3.2}$$

Solving for the optimal tax on labour

We first solve the system (A3.1) for Q_t and then use (A3.2) to get t . Cramer's rule yields:

$$Q_t = tn\delta \left[\frac{\frac{dF}{d\tau_T} \frac{dT_{tot}}{dt} - \frac{dF}{dt} \frac{dT_{tot}}{d\tau_T}}{\frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{dt}} \right] \tag{A3.3}$$

Consider the denominator of (A3.3). Note that

$$\frac{d\hat{w}}{dt} = \frac{\partial \hat{w}}{\partial t} + \frac{\partial \hat{w}}{\partial a} a' \frac{dT_{tot}}{dt}; \quad \frac{d\hat{w}}{d\tau_T} = \frac{\partial \hat{w}}{\partial \tau_T} + \frac{\partial \hat{w}}{\partial a} a' \frac{dT_{tot}}{d\tau_T} \tag{A3.4}$$

A change in the labour or transport tax has a direct effect on total transport demand (holding congestion constant) and an indirect effect via changes in congestion:

$$\frac{dT_{tot}}{dt} = \frac{\partial T_{tot}}{\partial t} + \frac{\partial T_{tot}}{\partial a} a' \frac{dT_{tot}}{dt}; \quad \frac{dT_{tot}}{d\tau_T} = \frac{\partial T_{tot}}{\partial \tau_T} + \frac{\partial T_{tot}}{\partial a} a' \frac{dT_{tot}}{d\tau_T}$$

Hence

$$\frac{dT_{tot}}{dt} = \chi \frac{\partial T_{tot}}{\partial t}, \quad \frac{dT_{tot}}{d\tau_T} = \chi \frac{\partial T_{tot}}{\partial \tau_T}; \quad \chi = \frac{1}{1 - \frac{\partial T_{tot}}{\partial a} a'} \quad (\text{A3.5})$$

where χ is the congestion feedback effect on transport demand (see Mayeres and Proost (1997), Sandmo (2000)). Using (A3.4)-(A3.5) immediately yields:

$$\frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{dt} = \chi \left(\frac{\partial \hat{w}}{\partial t} \frac{\partial T_{tot}}{\partial \tau_T} - \frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T_{tot}}{\partial t} \right)$$

Starting from the definition of total car transport demand $T_{tot} = n[F + (1-F)T]$ and taking account of the fact that employment depends on the net real wage (i.e., $n = n(\hat{w})$), we work out the term on the right hand side to find:

$$\frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{dt} = \chi \left[n(1-T) \left(\frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} - \frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} \right) + n(1-F) \left(\frac{\partial \hat{w}}{\partial t} \frac{\partial T}{\partial \tau_T} - \frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T}{\partial t} \right) \right]$$

Finally, assuming for simplicity that the labour tax does not affect modal choice for people not having a company car, we easily show, using (14) and (21):

$$\frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{dt} = \chi \left[n(1-T) \left(\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right) + n(1-F) \left(\frac{\tilde{w}}{1-t} \frac{\partial T}{\partial \tau_T} \right) \right] \quad (\text{A3.6})$$

Given preferential tax treatment of company cars ($\delta < 0$), this expression is necessarily negative.

Then consider the numerator of (A3.3). We again have, after simple algebra:

$$\frac{dF}{d\tau_T} \frac{dT_{tot}}{dt} - \frac{dF}{dt} \frac{dT_{tot}}{d\tau_T} = \chi \left[\frac{\partial F}{\partial \tau_T} \frac{\partial T_{tot}}{\partial t} - \frac{\partial F}{\partial t} \frac{\partial T_{tot}}{\partial \tau_T} \right]$$

Working out the term between brackets and using the definition of total transport demand, shows that:

$$\frac{dF}{d\tau_T} \frac{dT_{tot}}{dt} - \frac{dF}{dt} \frac{dT_{tot}}{d\tau_T} = \chi \left[n(1-F) \left(\frac{\partial F}{\partial \tau_T} \frac{\partial T}{\partial t} - \frac{\partial F}{\partial t} \frac{\partial T}{\partial \tau_T} \right) + \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \left(\frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} - \frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} \right) \right]$$

Finally, again assuming that the labour tax does not affect modal choice and using (14) and (21) we find:

$$\frac{dF}{d\tau_T} \frac{dT_{tot}}{dt} - \frac{dF}{dt} \frac{dT_{tot}}{d\tau_T} = \chi \left[n(1-F) \left(\frac{\delta}{2b} \frac{\partial T}{\partial \tau_T} \right) + \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \left(\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right) \right] \quad (\text{A3.7})$$

This is positive, provided preferential tax treatment of company cars.

Substituting (A3.6) and (A3.7) in (A3.3), we can write the solution for Q , as:

$$Q_t = tn\delta[Z_t] \quad (\text{A3.8})$$

where

$$Z_t = \left\{ \frac{n(1-F) \left(\frac{\delta}{2b} \frac{\partial T}{\partial \tau_T} \right) + \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \left(\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right)}{n(1-T) \left(\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right) + n(1-F) \left(\frac{\tilde{w}}{1-t} \frac{\partial T}{\partial \tau_T} \right)} \right\} < 0$$

The inequality follows from assuming preferential tax treatment ($\delta < 0$). Finally, use (A3.2) and (A3.8) to solve for the optimal labour tax. Using, $\tilde{w} = \hat{w} + \delta F$, the result can be written as:

$$t = \left[-\frac{\tau_p}{\hat{w}} + \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} \right] \left[\frac{1}{\frac{\hat{w} + \delta F}{\hat{w}} - \delta[Z_t] \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}}} \right]$$

Solving for the optimal congestion tax

Using Cramer's rule to solve (A3.1), the optimal congestion tax satisfies:

$$[\tau_T - MEC - \tau_p] = tn\delta \left[\frac{\frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt}}{\frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{dt}} \right] \quad (\text{A3.9})$$

Consider the numerator in this expression. Working out yields:

$$\begin{aligned} \frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt} &= \left(\frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} \right) \\ &+ \chi \left[\left(\frac{\partial \hat{w}}{\partial a} \frac{\partial F}{\partial t} - \frac{\partial F}{\partial a} \frac{\partial \hat{w}}{\partial t} \right) \frac{\partial T_{tot}}{\partial \tau_T} + \left(\frac{\partial F}{\partial a} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial a} \right) \frac{\partial T_{tot}}{\partial t} \right] \end{aligned} \quad (\text{A3.10})$$

The final term on the right hand side is easily shown to equal zero, using (14) and (21).

Moreover, again using (14), we have:

$$\left(\frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} \right) = \frac{1}{v'} \left(\frac{\partial \hat{w}}{\partial a} \frac{\partial F}{\partial t} - \frac{\partial F}{\partial a} \frac{\partial \hat{w}}{\partial t} \right)$$

Using these findings in (A3.10) implies:

$$\frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt} = \left(\frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} \right) \left[1 + v' \frac{\partial T_{tot}}{\partial \tau_T} \chi \right]$$

This can be rewritten, using the definition of χ :

$$\frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt} = \left(\frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} \right) \left[\frac{1 - \frac{\partial T_{tot}}{\partial a} + v' \frac{\partial T_{tot}}{\partial \tau_T}}{1 - \frac{\partial T_{tot}}{\partial a}} \right]$$

Work out the expression between square brackets, making use of the definition of total transport demand and expressions (14) and (21); this leads to:

$$\frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt} = \chi \left(\frac{\partial F}{\partial t} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial t} \right)$$

Finally, working out the right hand side, using (14) and (21), yields:

$$\frac{dF}{dt} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{dt} = \chi \left[\frac{Pb\tilde{w} - \delta T_{tot}}{2bn(1-t)} \right] \quad (\text{A3.11})$$

So we have, using (A3.6), (A3.9) and (A3.11):

$$\tau_T = MEC + \tau_p + t\delta[Z_\tau]$$

where

$$Z_\tau = \left\{ \frac{- \left[\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right]}{n(1-T) \left[\frac{\delta T_{tot} - Pb\tilde{w}}{2bn(1-t)} \right] + n(1-F) \left[\frac{\tilde{w}}{1-t} \frac{\partial T}{\partial \tau_T} \right]} \right\} < 0$$

Appendix 4. Optimal taxation of worker compensation, conditional on transport taxes

Suppose only the labour market taxes (labour and company car imputed value) can be freely determined. The first-order conditions are as in Appendix 2, but we now rearrange to capture the exogeneity of transport taxes:

$$\left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{dt} + (t\delta) \left(n \frac{dF}{dt} \right) = - \left[\tau_T - MEC - \tau_p \right] \frac{dT_{tot}}{dt}$$

$$\left\{ t\tilde{w} \frac{\partial n}{\partial \hat{w}} - n \left(\frac{1-\gamma}{\gamma} \right) + \tau_p \frac{\partial n}{\partial \hat{w}} \right\} \frac{d\hat{w}}{d\rho} + (t\delta) \left(n \frac{dF}{d\rho} \right) = - \left[\tau_T - MEC - \tau_p \right] \frac{dT_{tot}}{d\rho}$$

Reformulate this system as follows:

$$\{Q_t\} \frac{d\hat{w}}{dt} + (t\delta) \left(n \frac{dF}{dt} \right) = - \left[\tau_T - MEC - \tau_p \right] \frac{dT_{tot}}{dt}$$

$$\{Q_t\} \frac{d\hat{w}}{d\rho} + (t\delta) \left(n \frac{dF}{d\rho} \right) = -[\tau_T - MEC - \tau_p] \frac{dT_{tot}}{d\rho}$$

Notation is as before. Solving by Cramer's rule we have:

$$Q_t = [\tau_T - MEC - \tau_p] \left[\begin{array}{c} \frac{dF}{dt} \frac{dT_{tot}}{d\rho} - \frac{dF}{d\rho} \frac{dT_{tot}}{dt} \\ \frac{d\hat{w}}{dt} \frac{dF}{d\rho} - \frac{d\hat{w}}{d\rho} \frac{dF}{dt} \end{array} \right] \quad (A4.1)$$

$$t\delta = [\tau_T - MEC - \tau_p] \left[\begin{array}{c} \frac{d\hat{w}}{d\rho} \frac{dT_{tot}}{dt} - \frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\rho} \\ \frac{d\hat{w}}{dt} \frac{dF}{d\rho} - \frac{d\hat{w}}{d\rho} \frac{dF}{dt} \end{array} \right] \quad (A4.2)$$

Substantial but straightforward algebra yields, using the same type of derivations as in Appendix 3:

$$\frac{d\hat{w}}{dt} \frac{dF}{d\rho} - \frac{d\hat{w}}{d\rho} \frac{dF}{dt} = \left[\frac{\partial \hat{w}}{\partial t} \frac{\partial F}{\partial \rho} - \frac{\partial \hat{w}}{\partial \rho} \frac{\partial F}{\partial t} \right] [\chi] \left[1 - n(1-F) \frac{\partial T}{\partial a} \right] = -[\chi] \left[1 - n(1-F) \frac{\partial T}{\partial a} \right] \frac{t\hat{w}}{2b(1-t)}$$

$$\frac{dF}{dt} \frac{dT_{tot}}{d\rho} - \frac{dF}{d\rho} \frac{dT_{tot}}{dt} = \chi \left[\frac{\partial F}{\partial t} \frac{\partial T_{tot}}{\partial \rho} - \frac{\partial F}{\partial \rho} \frac{\partial T_{tot}}{\partial t} \right] = \chi \left\{ \frac{t\hat{w}}{2b(1-t)} \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \right\}$$

$$\frac{d\hat{w}}{d\rho} \frac{dT_{tot}}{dt} - \frac{d\hat{w}}{dt} \frac{dT_{tot}}{d\rho} = \chi n(1-T) \left[\frac{\partial \hat{w}}{\partial \rho} \frac{\partial T_{tot}}{\partial t} - \frac{\partial \hat{w}}{\partial t} \frac{\partial T_{tot}}{\partial \rho} \right] = \chi n(1-T) \left[\frac{\hat{w}}{2b(1-t)} \right]$$

Here we have again assumed that the labour tax t does not affect modal choice for people not having a company car. Using these results in (A4.1) and (A4.2) gives:

$$Q_t = -[\tau_T - MEC - \tau_p] \left[\begin{array}{c} \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \\ 1 - n(1-F) \frac{\partial T}{\partial a} \end{array} \right] \quad (A4.3)$$

$$t\delta = -[\tau_T - MEC - \tau_p] \left[\begin{array}{c} (1-T) \\ 1 - n(1-F) \frac{\partial T}{\partial a} \end{array} \right] \quad (A4.4)$$

Use (A3.2) in (A4.3) and solve for the labour tax:

$$t = \frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\tilde{w}}{n}} - \frac{\tau_p}{\tilde{w}} - [\tau_T - MEC - \tau_p] \left[\frac{T_{tot}}{n\tilde{w} \left[1 - n(1-F) \frac{\partial T}{\partial a} \right]} \right] \quad (A4.5)$$

Finally, multiply both sides of (A4.5) by (\tilde{w}/\hat{w}) , note that $\tilde{w} = \hat{w} + \delta F$, use (A4.4) and the definition of total transport demand to find:

$$t = \left[\frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\hat{w}}{n}} - \frac{\tau_p}{\hat{w}} \right] - [\tau_T - MEC - \tau_p] \left[\frac{T}{\hat{w} \left(1 - n(1-F) \frac{\partial T}{\partial a} \right)} \right] \quad (\text{A4.6})$$

Appendix 5. Optimal transport pricing conditional on labour market compensation

The first-order conditions are:

$$(\tau_T - MEC) \frac{dT_{tot}}{d\tau_T} + \tau_p \left(\frac{\partial n}{\partial \hat{w}} \frac{d\hat{w}}{d\tau_T} - \frac{dT_{tot}}{d\tau_T} \right) = \{S\} \frac{d\hat{w}}{d\tau_T} - (t\delta n) \left(\frac{dF}{d\tau_T} \right) \quad (\text{A5.1})$$

$$(\tau_T - MEC) \frac{dT_{tot}}{d\tau_p} + \tau_p \left(\frac{\partial n}{\partial \hat{w}} \frac{d\hat{w}}{d\tau_p} - \frac{dT_{tot}}{d\tau_p} \right) = \{S\} \frac{d\hat{w}}{d\tau_p} - (t\delta n) \left(\frac{dF}{d\tau_p} \right)$$

where $S = n \left(\frac{1-\gamma}{\gamma} \right) - t\tilde{w} \frac{\partial n}{\partial \hat{w}}$.

Solving for the congestion tax

Writing (A5.1) in matrix notation and solving by Cramer's rule gives:

$$(\tau_T - MEC) = \frac{S}{\frac{\partial n}{\partial \hat{w}}} + tn\delta \left[\frac{\frac{\partial n}{\partial \hat{w}} \left(\frac{dF}{d\tau_p} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{d\tau_p} \right) - \left(\frac{dF}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{dT_{tot}}{d\tau_p} \right)}{\frac{\partial n}{\partial \hat{w}} \left(\frac{d\hat{w}}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{d\tau_p} \right)} \right]$$

(A5.2)

Consider first the denominator of this expression. Straightforward algebra shows:

$$\begin{aligned} \frac{d\hat{w}}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{d\tau_p} &= \chi \left[\frac{\partial \hat{w}}{\partial \tau_p} \frac{\partial T_{tot}}{\partial \tau_T} - \frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T_{tot}}{\partial \tau_p} \right] \\ &= \chi \left[n(1-T) \left(\frac{\partial \hat{w}}{\partial \tau_p} \frac{\partial F}{\partial \tau_T} - \frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial F}{\partial \tau_p} \right) + n(1-F) \left(\frac{\partial \hat{w}}{\partial \tau_p} \frac{\partial T}{\partial \tau_T} - \frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T}{\partial \tau_p} \right) \right] \end{aligned} \quad (\text{A5.3})$$

Perfect complementarity ($P+T=1$) and (14) further imply:

$$\frac{\partial T}{\partial \tau_T} = -\frac{\partial T}{\partial \tau_p} \quad \frac{\partial F}{\partial \tau_T} = -\frac{\partial F}{\partial \tau_p} \quad (\text{A5.4})$$

Moreover, it follows from (21) that:

$$\left(\frac{\partial \hat{w}}{\partial \tau_T} + \frac{\partial \hat{w}}{\partial \tau_p} \right) = \frac{1}{1-t}$$

Using this result together with (A5.4) in (A5.3) implies:

$$\frac{d\hat{w}}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{d\tau_p} = \chi \frac{1}{1-t} \left[n(1-T) \frac{\partial F}{\partial \tau_T} + n(1-F) \frac{\partial T}{\partial \tau_T} \right] \quad (\text{A5.5})$$

Next take the first term in the numerator of (A5.2). It can be written, working out the total differentials:

$$\begin{aligned} \frac{dF}{d\tau_p} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{d\tau_p} &= \frac{\partial F}{\partial \tau_p} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial \tau_p} \\ &+ \chi \frac{\partial F}{\partial a} \left(\frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T_{tot}}{\partial \tau_p} - \frac{\partial \hat{w}}{\partial \tau_p} \frac{\partial T_{tot}}{\partial \tau_T} \right) + \chi \frac{\partial \hat{w}}{\partial a} \left(\frac{\partial F}{\partial \tau_p} \frac{\partial T_{tot}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial T_{tot}}{\partial \tau_p} \right) \end{aligned} \quad (\text{A5.6})$$

Using the definition of total car transport and (21) we easily show:

$$\begin{aligned} \left(\frac{\partial \hat{w}}{\partial \tau_T} \frac{\partial T_{tot}}{\partial \tau_p} - \frac{\partial \hat{w}}{\partial \tau_p} \frac{\partial T_{tot}}{\partial \tau_T} \right) &= -\frac{n}{1-t} \left[(1-T) \frac{\partial F}{\partial \tau_T} - (1-F) \frac{\partial T}{\partial \tau_T} \right] \\ \left(\frac{\partial F}{\partial \tau_p} \frac{\partial T_{tot}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial T_{tot}}{\partial \tau_p} \right) &= -\frac{\partial F}{\partial \tau_T} \frac{T_{tot}}{n} \frac{\partial n}{\partial \hat{w}} \frac{1}{1-t} \end{aligned}$$

Substituting in (A5.6) and working out we obtain:

$$\frac{dF}{d\tau_p} \frac{d\hat{w}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{d\hat{w}}{d\tau_p} = \chi \left(\frac{\partial F}{\partial \tau_p} \frac{\partial \hat{w}}{\partial \tau_T} - \frac{\partial F}{\partial \tau_T} \frac{\partial \hat{w}}{\partial \tau_p} \right) = \chi \left[\frac{P}{2b(1-t)} \right] \quad (\text{A5.7})$$

Finally, the last term of the numerator in (A5.2) is shown to be, using similar techniques:

$$\frac{dF}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{dF}{d\tau_T} \frac{dT_{tot}}{d\tau_p} = \chi \left[\frac{\partial n}{\partial \hat{w}} \frac{T_{tot}}{n} \frac{P}{2b(1-t)} \right] < 0 \quad (\text{A5.8})$$

Substituting (A5.5), (A5.7) and (A5.8) in (A5.2) yields:

$$(\tau_T - MEC) = \left(\frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\tilde{w}}{n}} - t \right) \tilde{w} + t\delta \left[\frac{P^2(1-F)}{2b(1-F) \frac{\partial T}{\partial \tau_T} - P^2} \right]$$

Solving for the public transport fare

Finally, turn to the public transport fee. From Cramer's rule we have, see (A5.1):

$$\tau_p = \frac{S}{\frac{\partial n}{\partial \hat{w}}} + tn\delta \left[\frac{\left(\frac{dF}{d\tau_T} \frac{dT_{tot}}{d\tau_p} - \frac{dF}{d\tau_p} \frac{dT_{tot}}{d\tau_T} \right)}{\frac{\partial n}{\partial \hat{w}} \left(\frac{d\hat{w}}{d\tau_p} \frac{dT_{tot}}{d\tau_T} - \frac{d\hat{w}}{d\tau_T} \frac{dT_{tot}}{d\tau_p} \right)} \right]$$

Using (A5.5) and (A5.7) and working out, we find:

$$\tau_p = \left(\frac{1-\gamma}{\gamma} \frac{1}{\frac{\partial n}{\partial \hat{w}} \frac{\tilde{w}}{n}} - t \right) \tilde{w} + t\delta \left[\frac{(F + (1-F)T)P}{P^2 - 2b(1-F) \frac{\partial T}{\partial \tau_T}} \right]$$