The main purpose of this paper is to compare the decomposition properties of rank-dependent and level-dependent indicators of income-related inequality of health. We do so by focusing on the decomposition by population groups. We show that level-dependent indices have more desirable subgroup decomposability properties than rank-dependent indices. This may prove to be an important argument in favour of the use of level-dependent indices. The difference between the subgroup decomposition results of rank-dependent and level-dependent indices is illustrated by means of an empirical study using Australian health and income data. We consider subgroups based on sex, age and employment status.

1 Introduction

In the literature on the measurement of income-related inequality of health, decomposition techniques have gained currency as methods to better understand, and if possible to explain, the observed levels of inequality at a given moment of time, as well as the changes in these levels over time. Some of these techniques have been borrowed or adapted from well-known results in the fields of income inequality measurement (e.g. Rao, 1969; Bourguignon, 1979; Shorrocks, 1980) and labour economics (e.g. Blinder, 1973; Oaxaca, 1973). A good survey of this literature has been written by Fortin et al. (2011). Other techniques have been developed specifically for the analysis of bivariate inequality (e.g. Clarke et al., 2003; Wagstaff et al., 2003; Wagstaff, 2005b; Abul Naga & Geoffard, 2006; Van Oorti

*This paper uses unit record data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey. The HILDA Project was initiated and is funded by the Australian Government Department of Families, Housing, Community Services and Indigenous Affairs (FaHCSIA) and is managed by the Melbourne Institute of Applied Economic and Social Research (Melbourne Institute). The findings and views reported in this paper, however, are those of the authors and should not be attributed to either FaHCSIA or the Melbourne Institute. We thank seminar audiences at the University of Melbourne, Monash University, the University of Canterbury and the University of Queensland, participants of the EcoMod 2016 Conference in Lisbon, and a reviewer of this journal for useful remarks and observations. A special word of thanks goes to John P. Haisken-DeNew, who prompted us to think about the decomposability of the level-dependent index. We acknowledge the Flemish Research Foundation (FWO) for Roselinde Kessels’s postdoctoral fellowship. Philip Clarke is supported by NHMRC Senior Research Fellowship (GNT1079621). All errors are our responsibility.

JEL classifications: D63, I00

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et al., 2009; Allanson & Petrie, 2013a; Erreygers & Kessels, 2013; Makdissi et al., 2013; Heckley et al., 2016; Kessels & Erreygers, 2016). Instructive surveys of the state of the art can be found in Van Doorslaer and Van Oorti (2011), O’Donnell et al. (2012), and Van Oorti et al. (2014).

Almost all of this work is centred around rank-dependent indicators of inequality such as the concentration index, which is the dominant measure of income-related inequality of health. Recently, however, a case has been made for a shift to level-dependent indicators (Erreygers & Kessels, 2017). One of the considerations which seems particularly relevant to the choice between different indicators is how amenable they are when it comes to decomposability.

In the case of measurement of income inequality, decomposability has been extensively explored over the last three decades (for a summary, see Foster & Sen, 1997, section A.5.1). What has been termed ‘subgroup’ or ‘additive’ decomposability is often seen as a highly desirable criterion for choosing univariate inequality measures, particularly by policymakers. Cowell (2011, section 3.4), for instance, has argued that decomposability means that there should be a coherent relationship between inequality in the whole of society and inequality in its constituent parts. It has been well established that the Gini coefficient is not generally subgroup decomposable, and this has been a major motivation behind the development of the class of generalised entropy measures, which do have this property. Given its similarity to the Gini coefficient, the concentration index is not subgroup decomposable, and little progress has been made to develop alternative bivariate measures which have this property.

The main purpose of this paper is to compare the subgroup decomposability properties of rank-dependent and level-dependent indicators of income-related inequality of health. We show that in contrast to rank-dependent indicators, level-dependent indicators do have the property of subgroup decomposability, which may prove to be an important argument in favour of the use of the latter.

We start with a brief presentation of the subgroup decomposability property. We then define and compare the two types of indicators of income-related inequality of health we are considering here, and analyse how they handle subgroup decomposability. We illustrate our results with an empirical study of income-related health inequality in Australia. We explore to what extent this inequality is due to differences among and between males and females, age cohorts, and groups defined by their employment status.

II Context

(i) Subgroup Decomposability

Subgroup decomposition involves three types of inequality: inequality in the population as a whole, inequality within subgroups of the population, and inequality between subgroups. One can think of subgroups based on geographical areas, age, ethnicity, etc. The basic idea of subgroup decomposability is to express the measured degree of inequality $I$ in the population as a whole as the sum of two parts: the ‘within-group’ inequality $I_W$ and the ‘between-group’ inequality $I_B$ (see Bourguignon, 1979). This exact decomposability property is expressed by the additive formula

$$I = I_W + I_B. \hspace{1cm} (1)$$

Moreover, the ‘within-group’ inequality consists of a weighted sum of the inequalities within the subgroups, while the ‘between-group’ inequality is calculated assuming that all individuals of a given subgroup are alike.

Subgroup decompositions may be used to detect whether inequality is mainly due to differences across specific groups (men and women, say) or to differences within the groups themselves (among men and among women, say). Since income-related inequality can be both positive and negative, subgroup decompositions also reveal whether the within and between components are of the same type as the overall inequality, that is, whether $I_W$ and/or $I_B$ are/is of the same sign as $I$. For instance, if $I_B/I$ is negative, the income-related inequality between the subgroups contributes ‘negatively’ to the observed level of income-related inequality in the population as a whole. In that case, the between and within components act in different directions. This type of information is potentially relevant for policymakers.

(ii) The Concentration Index

From the literature on income inequality we know that univariate rank-dependent indices, such as the Gini coefficient, cannot be decomposed exactly into the sum of a within-group and a between-group contribution, except in exceptional cases (Bhattacharya & Mahalanobis, 1967).
The Gini subgroup decomposition therefore involves a third or ‘residual’ component, required to balance the equation. While the origin of the residual component is clear (it is due to subgroups having overlapping income ranks), no unanimity exists on the interpretation of the term; see Mookherjee and Shorrocks (1982) and Lambert and Aronson (1993) for opposing views on the matter. The necessity of introducing a residual component carries over to rank-dependent bivariate indices such as the concentration index (Clarke et al., 2003; Wagstaff, 2005b). Unless there is no overlapping whatsoever in income ranks between the subgroups, there is always a residual term.

In spite of this, the concentration index and its variants continue to be the standard measures of socioeconomic inequality of health, and subgroup decompositions are used in the literature, albeit sparingly. Direct applications are rather limited; a recent example concerns the exposure to second-hand tobacco smoke among children, where the decomposition is applied to the groups of urban and rural children (Hajizadeh & Nandi, 2016). Indirectly, the distinction of within and between components plays a role in the Oaxaca-type decompositions which try to analyse the changes in inequality over time (e.g. Allanson & Petrie, 2013b). Some studies of income-related inequality of health estimate inequality within subgroups, but refrain from measuring inequality between subgroups. For example, Chotikapanich et al. (2003) estimate the extent of inequality among Australian men and women, but remain silent on the inequality between the sexes.¹

It would be convenient if we had a socioeconomic inequality measure with the subgroup decomposability property. Even if we limit ourselves to the class of ‘linear’ measures of inequality, which we will define in the next section, we show that such a measure exists.

### III Methods

(i) Measures of Inequality

#### Linear Measures of Inequality

Let us consider a population consisting of $n$ individuals, where each individual $i$ ($i = 1, 2, \ldots, n$) is characterised by their levels of socioeconomic achievement ($y_i$) and of health attainment ($h_i$). For convenience, we designate socioeconomic achievement as ‘income’, but it goes without saying that socioeconomic status can be measured in a variety of ways (by consumption levels, by years of education, etc.). We assume that both $y_i$ and $h_i$ have well-defined lower bounds greater than or equal to 0. More specifically, each of these variables is either a ratio-scale variable with no upper bound, or a cardinal or ratio-scale variable with a well-defined upper bound. The two variables are not necessarily of the same nature. Income, for instance, is an unbounded ratio-scale variable. The health variable, by contrast, is often a bounded variable, and need not be of the ratio-scale type. The means of the variables are respectively

$$
\mu_y = \frac{1}{n} \sum_{i=1}^{n} y_i, \quad \mu_h = \frac{1}{n} \sum_{i=1}^{n} h_i.
$$

Inequality in the population as a whole depends on the levels of income and health of all individuals in society, that is, $I = f(y, h)$, where $y = (y_1, y_2, \ldots, y_n)$ and $h = (h_1, h_2, \ldots, h_n)$. The class of measures of inequality we consider in this paper is characterised by a function $f(y, h)$ of the additive type, and is in fact fairly broad.

Linear measures of bivariate inequality are closely related to linear measures of univariate inequality, which have been defined by Mehran (1976). We focus on bivariate indices where income serves as the weighting variable and health as the weighted variable. In formal terms, the absolute version of these indices takes the form²

$$
I = I(y, h) = \frac{1}{n} \sum_{i=1}^{n} w_i(y) h_i.
$$

What distinguishes bivariate from univariate indices is that the weights are determined by a different distribution from the one which is weighted. The concentration index, for example, defines the weights in terms of the ranks of individuals in the income distribution, and applies these weights to the distribution of health.

We also consider variants of index $I$ based on the formula $I^* = C^* I$, where $C^*$ is determined by

¹ Their results are not immediately comparable to the ones we report in Section IV, because they use a different health variable and adjust it for need and age.

² If there is no confusion possible, we will drop the arguments and simply write $I$ instead of $I(y, h)$, etc.
the characteristics of distribution \( h \) (see Heckley et al., 2016, for a similar procedure). For instance, the relative version is obtained by dividing the index by the mean of the weighted variable, that is, by taking \( C^{rel} = 1 / \mu_h \):

\[
f^{rel} = \frac{1}{\mu_h} \cdot I.
\]

(4)

Other versions of the indices have been proposed in the literature, for example in order to deal with binary variables (Wagstaff, 2005a). For bounded variables, whether they are binary or of another type, we suggest the use of the index proposed by Erreygers (2009):

\[
f^{bou} = \frac{A}{h_{max} - h_{min}} \cdot I.
\]

(5)

where \( h_{min} \) and \( h_{max} \) stand for the lower and upper bounds of the health variable, and \( A \) is a constant. Erreygers and Van Oorti (2011) have argued that the choice for a specific version of a bivariate index should be made in accordance with the nature of the health variable under consideration.

**Rank-Dependent Indices**

Rank-dependent indices rely exclusively on income ranks to define the weights \( w_i(y) \); no other information on the income distribution enters into the calculation of the weights. Let the rank of individual \( i \) in the income distribution be \( r_i(y) \). If there are no ties in the income distribution, the rank of the poorest person in society is equal to 1, the rank of the second poorest person equal to 2, etc., and the rank of the richest person equal to \( n \). If a group of \( m + 1 \) individuals are tied in position \( g \), the rank of each of these individuals is equal to \( g + (m/2) \). For simplicity, however, we assume there are no ties in the income distribution, and that individuals are ranked according to their incomes \( y_1 < y_2 < \ldots < y_n \).

The standard concentration index, for instance, is characterised by a weighting function which is linear in the income ranks \( r_i(y) \). This weighting function may therefore be called the ‘rank’ function and the associated bivariate index the \( R \) index:

\[
w_i^R(y) = \frac{2r_i(y) - n - 1}{n},
\]

(6)

\[
R = \frac{1}{n} \sum_{i=1}^{n} w_i^R(y)h_i.
\]

(7)

The weights \( w_i^R(y) \) steadily increase as \( r_i(y) \) goes from 1 to \( n \). If the group of the ‘poor’ is defined as those who have negative weights and the group of the ‘rich’ as those who have positive weights, the weighting function (6) puts the boundary between the two groups exactly in the middle of the population. Those with ranks smaller than or equal to \( n/2 \) (if \( n \) is even) or smaller than or equal to \( (n - 1)/2 \) (if \( n \) is odd) have negative weights, and those with ranks larger than or equal to \( n/2 + 1 \) (if \( n \) is even) or larger than or equal to \( (n + 3)/2 \) (if \( n \) is odd) have positive weights. Put differently, individuals with an income below the median income have negative weights, and individuals with an income above the median positive weights. The negative weights sum to \(-n/4\) and the positive weights to \(n/4\). This motivates the choice of \( A = 4 \) for the bounded version of the rank-dependent index, as suggested by Erreygers (2009).

**Level-Dependent Indices**

In contrast to rank-dependent indices, the weights of level-dependent indices are based upon income levels rather than income ranks. The basic version of the level-dependent index \( L \) proposed by Erreygers and Kessels (2017) has a weighting function which is a simple linear function of income:

\[
w_i^L(y) = \frac{y_i - \mu_y}{\mu_y},
\]

(8)

\[
L = \frac{1}{n} \sum_{i=1}^{n} w_i^L(y)h_i.
\]

(9)

4 Strictly speaking, this result holds only when \( n \) is even. When \( n \) is odd, the sum of the negative weights is \(-[(n - 1)(n + 1)]/4n\), which for large \( n \) is approximately equal to \(-n/4\). And similarly for the positive weights.

5 The \( L \) index is related to the \( k \) index proposed by Abul Naga and Geoffard (2006, 365). For the specific choice of their parameters \( x = \beta = 1 \), it turns out that we have \( L = (k - 1)\mu_h \). It needs to be pointed out that in their social welfare framework, inspired by Tsui (1999), the parameters are constrained by the condition \( x + \beta \leq 1 \).

---

3 The Appendix *Sample Weights and Ties* specifies how the formulas must be adapted when there are ties, and when not all individuals have the same population weight.
The weights \( w^L_i(y) \) are proportional to the deviations of the incomes from the mean. If there is a high degree of income inequality, typically there will be a lot of individuals with income levels below the mean, who will therefore have negative, but in absolute terms rather small weights. On the other side of the spectrum, those who have very high incomes will have positive, and in absolute terms quite large weights. Hence, a small change in the health level of a relatively well-off individual will probably have a more pronounced influence on the index than a comparable change in the health level of an individual who is less well-off.

All the negative weights sum to \(-\frac{1}{2}D\), and all the positive weights to \((n/2)D\), where \( D \) is the relative mean deviation:

\[
D = \frac{\frac{1}{n} \sum_{i=1}^{n} |y_i - \mu_y|}{\mu_y}.
\]

Since \( D \) is at most equal to \( 2(n - 1)/n \), it follows that these two amounts are bounded by \( 1 - n \) and \( n - 1 \). For the bounded version of the level-dependent index, the natural choice for the value of \( A \) is therefore \( A = 1 \).

**A Brief Comparison**

At this stage it seems useful to highlight some of the differences between the indices \( R \) and \( L \). We focus here on the bounds of the indices, and when these are reached.

The relative version of \( R \) varies between \(-\frac{1}{n} \sum_{i=1}^{n} y_i - \frac{1}{n} \sum_{i=1}^{n} \mu_y \), with the minimum value attained when the poorest person in society has all the health, and the maximum when the richest person in society has all the health. It does not matter exactly how poor or how rich the poorest and richest persons are. The relative version of \( L \), by contrast, varies between \(-1 \) and \( n - 1 \). The minimum is obtained when the poorest person in society has a zero income and all the health; the maximum when the richest person has maximum health, while all the others have minimum health, and the maximum when that person has maximum health, while all the others have minimum health.

(ii) **Subgroup Decomposability**

**Subgroups**

Suppose that we partition the population into \( k \) subsets, \( G_1, G_2, \ldots, G_k \), such that every individual \( i \) belongs to exactly one subset \( G_j \). The number of individuals in subgroup \( G_j \) is denoted by \( n_j \). The mean income and health attainments of subgroup \( G_j \) are equal to

\[
\mu_{y_j} = \frac{1}{n_j} \sum_{i \in G_j} y_i, \quad \mu_{h_j} = \frac{1}{n_j} \sum_{i \in G_j} h_i,
\]

and obviously we have

\[
\mu_y = \sum_{j=1}^{k} \frac{n_j}{n} \mu_{y_j}, \quad \mu_h = \sum_{j=1}^{k} \frac{n_j}{n} \mu_{h_j}.
\]

The levels of inequality in the \( k \) subgroups are denoted by \( I_1, I_2, \ldots, I_k \). Inequality in subgroup \( G_j \) depends on the levels of income and health of the individuals of that group only, that is, \( I_j = f(y_j, h_j) \), where \( y_j = (y_{ij} | i \in G_j) \) and \( h_j = (h_{ij} | i \in G_j) \). The ‘within-group’ inequality consists of a weighted sum of the inequalities in the \( k \) subgroups. When the weights are equal to \( s_1, s_2, \ldots, s_k \), we have

\[
I_W = \sum_{j=1}^{k} s_j I_j.
\]

The ‘between-group’ inequality depends on the means of income and health in the subgroups and on the sizes of the subgroups, that is, \( I_B = f(\mu_y, \mu_h) \), where \( \mu_y = (\mu_{y_1}, \mu_{y_2}, \ldots, \mu_{y_k}) \) and \( \mu_h = (\mu_{h_1}, \mu_{h_2}, \ldots, \mu_{h_k}) \). It is important to note that in this

\[6\]

More generally, if ties are allowed, the minimum is obtained when all the persons with positive health have zero incomes.
case we have to use a population-weighted version of the inequality index.

**Linear Measures**

For the calculation of the within component $I_W$, we need to know the levels of inequality in all subgroups (i.e., $I_1, I_2, \ldots, I_k$) and the weights of all subgroups (i.e., $s_1, s_2, \ldots, s_k$). By analogy with (3), inequality in subgroup $G_j$ can be defined as

$$I_j = \frac{1}{n_j} \sum_{i \in G_j} w_j(y_i) h_i. \tag{14}$$

The notation $w_j(y_i)$ indicates that the weights of individuals depend on their position within the group to which they belong. As far as the subgroup weights are concerned, no general formula exists: these are indicator-specific.

The ‘between-group’ component $I_B$ is calculated assuming that every individual of group $G_j$ has income level $\mu_{y_j}$ and health level $\mu_{h_j}$. It can therefore be defined as

$$I_B = \frac{1}{n} \sum_{j=1}^{k} n_j w_j(\mu_j) h_j. \tag{15}$$

The weights $w_j(\mu_j)$ are applied to every individual of the group $G_j$ and reflect the average situation of an individual of that group.

As far as the other versions of the index are concerned, their decompositions can be related easily to those of the absolute version. Let us consider index $I^* = C'I$. The ‘between-group’ component $I_B$ is equal to $C'I_B$. The ‘within-group’ component, on the other hand, is equal to $I_W = \sum_{j=1}^{k} s_j I'_j$, where $I'_j = C'I_j$. If we define the subgroup weights as $s_j = (C_j'/C_j)^2 s_j$, we obtain $I_W = C'I_W$. This means that whatever version of the index we use (absolute, relative, bounded, . . .), the ratio of the ‘within-group’ to the ‘between-group’ component is always the same.

**Rank-Dependent Measures**

Let us begin by defining the weights $w_j(y_i)$ and $w_j(\mu_i)$ of expressions (14) and (15) for the rank-dependent index $R$. To simplify the expressions, we assume that the groups are labelled in accordance with average incomes, that is, $\mu_{y_1} < \mu_{y_2} < \ldots < \mu_{y_n}$. For the calculation of the within-group inequalities $R_j$, the relevant ranks are those within each group. If we designate individual $i$’s rank within group $G_j$, for any $i \in G_j$, by $r_i(y_i)$, the weights $w^R_j(y_j)$ are equal to

$$w^R_j(y_j) = \frac{2r_i(y_i) - n_j - 1}{n_j}. \tag{16}$$

Next, we need to define the subgroup weights $s_j$ in order to calculate the weighted sum $R_W = \sum_{j=1}^{k} s_j R_j$. In accordance with the literature on the decomposition of the Gini coefficient, we assume the weights $s_j$ are equal to the squares of the population shares, $s_j = (n_j/n)^2$ (Lambert & Aronson, 1993, p. 1221).

For the calculation of the between-group inequality $R_B$, the rank assigned to every individual of a given group must coincide with the average rank of the individuals of that group in the whole population. The average rank $r_j(\mu_j)$ of the individuals of group $G_j$ can be calculated by means of this recursive formula:

$$r_j(\mu_j) = \frac{n_j + 1}{2} + \sum_{l=0}^{j-1} n_l, \tag{17}$$

with $n_0 = 0$ and $j = 1, \ldots, k$. The weights $w^R_j(\mu_j)$ are then equal to

$$w^R_j(\mu_j) = \frac{2r_j(\mu_j) - n - 1}{n}. \tag{18}$$

In general, the sum of the within and between components differs from the overall index $R$. Hence, we are confronted with a residual term $R_X = R - R_W - R_B$, which is almost always different from zero and can be both positive and negative. The residual term is zero “if the subgroup income ranges do not overlap” (Lambert & Aronson, 1993, p. 1221). In most cases, however, we end up with a non-zero residual term equal to

$$R_X = \frac{1}{n} \sum_{j=1}^{k} \sum_{i \in G_j} \left[ w^R_j(y_i) h_i - \frac{n_j}{n} w^R_j(y_j) h_i - w^R_j(\mu_j) h_i \right]. \tag{19}$$

Not surprisingly, then, the rank-dependent index $R$ does not have the property of subgroup decomposability.

It is worth noting that the subgroup weights for the calculation of the within component $R_W$ do not sum to 1. Similar results hold for the relative and bounded versions of the index. While the subgroup weights for the bounded version are the
same as those for the absolute version, the subgroup weights for the relative version are equal to
\[ s_j = \left( \frac{n_j}{n} \right)^2 \frac{\mu_{y_j}}{\mu_y}. \]

**Level-Dependent Indices**

The situation is different for the level-dependent index. Let us begin by looking at the calculation of the within component. For the inequality in group \( G_j \), the weight of any individual \( i \in G_j \) is defined in terms of the deviation of this individual’s income from the mean income in group \( G_j \). More formally, the weights \( w_{L_i}(y_j) \) are equal to
\[ w_{L_i}(y_j) = \frac{y_i - \mu_{y_j}}{\mu_{y_j}}. \]

The subgroup weights for the calculation of the within component \( L_W = \sum_{j=1}^{k} s_j L_j \) are assumed to be equal to the income shares of the subgroups, \( s_j = (n_j \mu_{y_j}) / (n \mu_y) \).

With regard to the calculation of the between component \( L_B \), each individual of a given group receives a weight proportional to the deviation of the mean income of that group from the mean income of the whole population. This means that the weights \( w_{L_j}(\mu_y) \) are equal to
\[ w_{L_j}(\mu_y) = \frac{\mu_{y_j} - \mu_y}{\mu_y}. \]

It is straightforward to show that the sum of the within and between components is always equal to \( L \). Observe that we have
\[ L = \frac{1}{n \mu_y} \sum_{i=1}^{n} y_i h_i - \mu_h, \]
and likewise
\[ L_j = \frac{1}{n_j \mu_{y_j}} \sum_{i \in G_j} y_i h_i - \mu_{h_j}. \]

It follows that
\[ s_j L_j = \frac{1}{n \mu_y} \sum_{i \in G_j} y_i h_i - s_j \mu_{h_j}, \]
and therefore that the within component is equal to
\[ L_W = \frac{1}{n \mu_y} \sum_{i=1}^{n} y_i h_i - \sum_{j=1}^{k} s_j \mu_{h_j}. \] (25)

The between component, on the other hand, can be expressed as
\[ L_B = \sum_{j=1}^{k} s_j \mu_{h_j} - \mu_h. \] (26)

Hence we obtain that
\[ L_W + L_B = \frac{1}{n \mu_y} \sum_{i=1}^{n} y_i h_i - \mu_h = L. \] (27)

The conclusion is that the level-dependent index \( L \) has the property of subgroup decomposability.

Since the subgroup weights used for the calculation of the within component \( L_W \) are equal to the shares of the groups in total income, these weights add up to 1, which may be perceived as an attractive property. For the bounded version of the index, the subgroup weights are the same as those for the absolute version. For the relative version, the subgroup weights are
\[ s_j = \frac{n_j \mu_{y_j}}{n \mu_y} \frac{\mu_{h_j}}{\mu_h}. \] (28)

**IV Data and Results**

(i) Description of the Data

Our data come from the Household, Income and Labour Dynamics in Australia (HILDA) Survey, wave 13.\(^7\) We consider only individuals aged 15 or higher. As our income variable we take equivalised income, calculated using the modified OECD equivalence scale (Australian Bureau of Statistics, 2013, Appendix 3). The equivalised income of a person is equal to the disposable income of this person’s household divided by the household’s equivalence factor. This factor is obtained by giving the first adult of the household a score of 1, every other adult a score of 0.5, every child a score of 0.3, and then adding up all the scores. It should be noted that even though children below the age of 15 are not part of our sample, they are taken into account for the

---

\(^7\) More information on the HILDA database can be found at http://www.melbourneinstitute.com/hilda.
calculation of the equivalised incomes. As our health variable we take the SF-6D health score. Since this is a bounded variable, we use the bounded version of the inequality indices, $R_{bou}$ and $L_{bou}$. Given that the bounds of the variable are $h_{min} = 0$ and $h_{max} = 1$, it is easy to check that $R_{bou} = 4R$ and $L_{bou} = L$.

Our sample includes 14,729 individuals with valid, non-missing values for all variables under study. Since this is a fairly large and, to the best of our knowledge, representative sample, we decided not to apply sample weights in our calculations. The composition of our sample is described in Table 1. We consider subgroup decompositions based on sex, age and employment status.

(ii) Decomposition by Sex

For the first subgroup decomposition we partitioned the population into two groups, females and males. As shown in Table 2, there is some variation among these groups, with respect to both health and income. Women tend to have slightly worse health and lower income than men. The rank-dependent index measures a slightly higher extent of income-related inequality among women than among men, while according to the level-dependent index there is just as much income-related inequality among men as there is among women.

The results of the subgroup decomposition according to sex can be found in Table 3. Let us begin by looking at the results for the level-dependent index. The observed inequality comes overwhelmingly (98.3%) from heterogeneity within the groups; the between-component accounts for a meager part (1.7%) of the total. The results for the rank-dependent index also point in the direction of the predominance of the within-group component. Nevertheless, the estimated share of the within-component is much

### Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>53.17</td>
</tr>
<tr>
<td>Male</td>
<td>46.83</td>
</tr>
<tr>
<td>Age group</td>
<td></td>
</tr>
<tr>
<td>Age 15–24</td>
<td>17.43</td>
</tr>
<tr>
<td>Age 25–34</td>
<td>16.97</td>
</tr>
<tr>
<td>Age 35–44</td>
<td>16.00</td>
</tr>
<tr>
<td>Age 45–54</td>
<td>17.66</td>
</tr>
<tr>
<td>Age 55–64</td>
<td>14.64</td>
</tr>
<tr>
<td>Age 65–74</td>
<td>10.65</td>
</tr>
<tr>
<td>Age 75+</td>
<td>6.63</td>
</tr>
<tr>
<td>Employment status</td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>63.49</td>
</tr>
<tr>
<td>Unemployed</td>
<td>4.04</td>
</tr>
<tr>
<td>Not in labour force</td>
<td>32.47</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Group</th>
<th>Health</th>
<th>Income ($)</th>
<th>Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>Female</td>
<td>0.7508</td>
<td>0.1244</td>
<td>50,014</td>
</tr>
<tr>
<td>Male</td>
<td>0.7718</td>
<td>0.1216</td>
<td>52,389</td>
</tr>
<tr>
<td>All</td>
<td>0.7606</td>
<td>0.1236</td>
<td>51,126</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th></th>
<th>$R_{bou}$</th>
<th>$L_{bou}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>%</td>
<td>Values</td>
</tr>
<tr>
<td>Within</td>
<td>0.0325</td>
<td>49.47</td>
</tr>
<tr>
<td>Between</td>
<td>0.0210</td>
<td>31.92</td>
</tr>
<tr>
<td>Residual</td>
<td>0.0122</td>
<td>18.61</td>
</tr>
<tr>
<td>Total</td>
<td>0.0657</td>
<td>100.00</td>
</tr>
</tbody>
</table>

8 This means in particular that individuals with a negative income are excluded.
9 The sample-weighted results are very close to the results presented below. They can be obtained upon request from the authors.
smaller (nearly 50% of the total) and that of the between-component much larger (almost 32%). These results must be treated with caution, however, since the residual component amounts to about 19%.

(iii) Decomposition by Age

For our second subgroup decomposition we divided the population into seven age groups, with the youngest group consisting of those aged 15–24, and the oldest group consisting of those aged 75 or older. As can be seen from Table 4, older people tend to have worse health and lower income than younger people. According to both indices, the level of socioeconomic inequality is the highest in the group aged 55–64.

The decomposition results are reported in Table 5. According to the level-dependent index the within-component is again by far the largest. In comparison to the results for the decomposition by sex, the level-dependent index now indicates a substantially larger between component (11.14% instead of 1.70%). The results for the rank-dependent index are completely different. This index suggests that the share of the between component exceeds that of the within component. The estimated share of the within component is surprisingly small, not even 13% of the total. Amazingly, the residual component amounts to two-thirds of the total.10

Table 4
Summary Statistics for the Age Decomposition

<table>
<thead>
<tr>
<th>Group</th>
<th>Health Mean</th>
<th>SD</th>
<th>Income ($) Mean</th>
<th>SD</th>
<th>$R_{bou}$</th>
<th>$L_{bou}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 15–24</td>
<td>0.7837</td>
<td>0.1179</td>
<td>45,376</td>
<td>26,065</td>
<td>0.0334</td>
<td>0.0080</td>
</tr>
<tr>
<td>Age 25–34</td>
<td>0.7841</td>
<td>0.1132</td>
<td>55,126</td>
<td>29,572</td>
<td>0.0490</td>
<td>0.0106</td>
</tr>
<tr>
<td>Age 35–44</td>
<td>0.7741</td>
<td>0.1204</td>
<td>54,101</td>
<td>31,592</td>
<td>0.0555</td>
<td>0.0114</td>
</tr>
<tr>
<td>Age 45–54</td>
<td>0.7586</td>
<td>0.1218</td>
<td>56,699</td>
<td>33,753</td>
<td>0.0654</td>
<td>0.0130</td>
</tr>
<tr>
<td>Age 55–64</td>
<td>0.7438</td>
<td>0.1266</td>
<td>55,639</td>
<td>40,009</td>
<td>0.0818</td>
<td>0.0192</td>
</tr>
<tr>
<td>Age 65–74</td>
<td>0.7333</td>
<td>0.1270</td>
<td>45,305</td>
<td>46,554</td>
<td>0.0581</td>
<td>0.0167</td>
</tr>
<tr>
<td>Age 75+</td>
<td>0.6937</td>
<td>0.1249</td>
<td>33,381</td>
<td>35,607</td>
<td>0.0330</td>
<td>0.0110</td>
</tr>
<tr>
<td>All</td>
<td>0.7606</td>
<td>0.1236</td>
<td>51,126</td>
<td>34,921</td>
<td>0.0657</td>
<td>0.0144</td>
</tr>
</tbody>
</table>

Table 5
Decomposition According to Age

<table>
<thead>
<tr>
<th></th>
<th>$R_{bou}$ Values</th>
<th>% Values</th>
<th>$L_{bou}$ Values</th>
<th>% Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within</td>
<td>0.0084</td>
<td>12.85</td>
<td>0.0128</td>
<td>88.86</td>
</tr>
<tr>
<td>Between</td>
<td>0.0132</td>
<td>20.17</td>
<td>0.0016</td>
<td>11.14</td>
</tr>
<tr>
<td>Residual</td>
<td>0.0440</td>
<td>66.98</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Total</td>
<td>0.0657</td>
<td>100.00</td>
<td>0.0144</td>
<td>100.00</td>
</tr>
</tbody>
</table>

10 We also calculated the within and between components for a combined sex and age subgroup classification, based on 14 sex–age groups. The level-dependent index estimates that the between component amounts to 12.48%, which is close to the sum of 1.70% and 11.14%; the rank-dependent index, by contrast, estimates that the between component is equal to 37.12%, which is nowhere near the sum of 31.92% and 20.17%.

(iv) Decomposition by Employment Status

For the third subgroup decomposition we partitioned the population into three groups according to their employment status. Table 6 indicates that there are large differences in health and income between the employed, the unemployed and those who are not in the labour force, and also that the level of inequality is the highest among the last of these.

Table 7 gives the decomposition results for both indices. The level-dependent index attributes 44.11% of the observed inequality to the between-group component, which is much larger than what we obtained for the two previous decompositions. The rank-dependent index, by contrast, suggests that the between component is much larger than the within component, and moreover that it is almost equal in size to the overall inequality. Once again, given the substantial residual term (−18.45% of the observed inequality), this result must be treated with caution.

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Several interesting conclusions can be drawn from the three subgroup decompositions we have just presented. These relate to both methodological and empirical aspects.

First, there is a striking divergence between the decomposition results of the rank-dependent and level-dependent indicators. In comparison to the level-dependent indicator, the rank-dependent indicator has much lower estimates for the within component (the absolute differences amount respectively to $\frac{1}{4}$8.33%, $\frac{1}{2}$6.01% and $\frac{1}{3}$2.40%) and much higher estimates for the between component (respectively 30.22%, 9.03% and 50.85%). These differences are substantial. The main cause is not that rank-dependent and level-dependent indicators measure inequality differently (in fact, they give broadly similar results), but that they handle the within–between decomposition in a completely different way.

Second, the presence of a substantial residual term complicates the interpretation of the results for the rank-dependent index. The magnitude of the residual term is in all three cases considerable (respectively 18.61%, 66.98% and −18.45%), and moreover it can be both positive and negative. All of this means that a lot of caution must be exercised when one tries to estimate the within and between components of income-related inequality by means of a rank-dependent index.

Third, there is an obvious reason why the rank-dependent index systematically underestimates the within component of inequality: the sum of the subgroup weights it uses for the calculation of the within component is always smaller than 1.12 This means that even if the amount of inequality is the same in every subgroup (which is virtually the case in the decomposition according to sex), the value of the within component is smaller than this level of inequality. The underestimation of the within component is exacerbated when the number of subgroups increases, as in the case of the decomposition according to age. The issue does not arise when we use the level-dependent index.13

In our view, these arguments reduce the confidence policy-makers should place in estimates of the within and between components of income-related inequality generated by rank-dependent indices such as the concentration index. If perfect subgroup decomposability is regarded as a desirable property of inequality measures (a position we defend), it seems more prudent to rely on the estimates generated by level-dependent indices. This is not a purely academic matter: the choice of indicator may have policy implications. If policies were based on the results of the level-dependent indicator, the focus would be on

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
Group & Health Mean & SD & Income ($) Mean & SD & Indices $R^{\text{bou}}$ & $L^{\text{bou}}$ \\
\hline
Employed & 0.7857 & 0.1088 & 58,553 & 34,503 & 0.0213 & 0.0051 \\
Unemployed & 0.7406 & 0.1313 & 37,811 & 23,340 & 0.0353 & 0.0096 \\
Not in labour force & 0.7140 & 0.1352 & 38,263 & 32,638 & 0.0645 & 0.0165 \\
All & 0.7606 & 0.1236 & 51,126 & 34,921 & 0.0657 & 0.0144 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
 & $R^{\text{bou}}$ & & $L^{\text{bou}}$ & \\
\hline
Values & % & & Values & % \\
Within & 0.0154 & 23.49 & 0.0080 & 55.89 \\
Between & 0.0624 & 94.96 & 0.0063 & 44.11 \\
Residual & −0.0121 & −18.45 & & \\
Total & 0.0657 & 100.00 & 0.0144 & 100.00 \\
\hline
\end{tabular}
\end{table}

\section{V Discussion}

Several interesting conclusions can be drawn from the three subgroup decompositions we have just presented. These relate to both methodological and empirical aspects.

First, there is a striking divergence between the decomposition results of the rank-dependent and level-dependent indicators. In comparison to the level-dependent indicator, the rank-dependent indicator has much lower estimates for the within component (the absolute differences amount respectively to $\frac{1}{4}$8.33%, $\frac{1}{2}$6.01% and $\frac{1}{3}$2.40%) and much higher estimates for the between component (respectively 30.22%, 9.03% and 50.85%). These differences are substantial. The main cause is not that rank-dependent and level-dependent indicators measure inequality differently (in fact, they give broadly similar results), but that they handle the within–between decomposition in a completely different way.

Second, the presence of a substantial residual term complicates the interpretation of the results for the rank-dependent index. The magnitude of the residual term is in all three cases considerable (respectively 18.61%, 66.98% and −18.45%), and moreover it can be both positive and negative. All of this means that a lot of caution must be exercised when one tries to estimate the within and between components of income-related inequality by means of a rank-dependent index.

Third, there is an obvious reason why the rank-dependent index systematically underestimates the within component of inequality: the sum of the subgroup weights it uses for the calculation of the within component is always smaller than 1.12 This means that even if the amount of inequality is the same in every subgroup (which is virtually the case in the decomposition according to sex), the value of the within component is smaller than this level of inequality. The underestimation of the within component is exacerbated when the number of subgroups increases, as in the case of the decomposition according to age. The issue does not arise when we use the level-dependent index.13

In our view, these arguments reduce the confidence policy-makers should place in estimates of the within and between components of income-related inequality generated by rank-dependent indices such as the concentration index. If perfect subgroup decomposability is regarded as a desirable property of inequality measures (a position we defend), it seems more prudent to rely on the estimates generated by level-dependent indices. This is not a purely academic matter: the choice of indicator may have policy implications. If policies were based on the results of the level-dependent indicator, the focus would be on

11 The correlation coefficient between the values of the two indicators for the population as a whole and for the 12 subgroups we considered above is 0.91.

12 For a discussion of a similar issue in the literature on income inequality, see Chakravarty (2001). He distinguished subgroup decomposability and population share weighted decomposability.

13 See also Erreygers et al. (2017), who applied the within–between decomposition to the global socio-economic inequality of life expectancy.
inequalities within the subgroups in each of the three cases we examined. Policies based on the results of the rank-dependent indicator would probably be different, however. Especially when it comes to employment status, it is to be expected that the emphasis would be put on inequality between the groups of the employed, the unemployed and those not in the labour force, rather than on inequality within each of these groups.

VI Conclusion

In this paper, we explored the subgroup decomposability properties of both rank-dependent and level-dependent indices of socioeconomic inequality of health. Whereas the level-dependent index can be decomposed perfectly into a within and between component, the decomposition of the rank-dependent index involves a residual component, which is zero in exceptional cases only. By means of three empirical examples, we showed that the two indices may generate widely different estimates of the within and between components. We have good reasons to believe that the subgroup decomposition results of the level-dependent index are more reliable than those of the rank-dependent index. We are convinced that this decomposability property constitutes a strong argument in favour of using level-dependent indices alongside, and maybe even instead of, the still dominant rank-dependent indices such as the concentration index. Given the wide range of topics to which this index has been applied (e.g. to check whether out-of-pocket payments for health care are progressive or regressive), it is important to be aware of its limitations and to explore whether level-dependent indices might perform better.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix A: Robustness checks for Subgroup decomposability of income-related inequality of health, with an application to Australia

Appendix B: Results adjusted for sample weights for Subgroup decomposability of income-related inequality of health, with an application to Australia

REFERENCES


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Appendix: Sample Weights and Ties

This appendix describes how the weights \( w_i \) must be defined when working with datasets in which not all individuals have the same sample weight, and when there are ties between individuals in the income distribution.

Let us assume that the individuals of our dataset \((1, 2, \ldots, n)\) are ranked according to their individual income, \( y_1 \leq y_2 \leq \ldots \leq y_n \). We denote the sample weight of individual \( i \) by \( \sigma_i \), and we assume that \( \sum_{i=1}^{n} \sigma_i = 1 \).

If there are ties in the income distribution, we define \( k \) groups of individuals \( G_1, G_2, \ldots, G_k \) such that everyone in group \( G_j \) has income \( y_{G_j} \), and that \( y_{G_1} < y_{G_2} < \ldots < y_{G_k} \). The sample weight of group \( G_j \) is \( \sigma_{G_j} = \sum_{i \in G_j} \sigma_i \). The cumulative sample weight of group \( G_j \) is defined by the recursive formula \( \pi_{G_j} = \pi_{G_{j-1}} + \sigma_{G_j} \), where we take \( \pi_{G_0} = 0 \) and \( j = 1, \ldots, k \).

### A.1. Rank-Dependent Weights

Let individual \( i \) belong to group \( G_j \). Then the weight of this individual is equal to

\[
w_i^R = n \sigma_i (\pi_{G_{i-1}} + \pi_{G_j} - 1).
\]

(A1)

Working out the terms between brackets, we obtain

\[
w_i^R = n \sigma_i \left[ \pi_{G_j} + 2 \sum_{l=0}^{j-1} \sigma_{G_l} - 1 \right].
\]

(A2)

### A.2. Level-Dependent Weights

When working with level-dependent weights, there is no need to consider group weights. The weight of individual \( i \) is equal to

\[
w_i^L = n \sigma_i \frac{y_i - \mu_y}{\mu_y}.
\]

(A3)