

Subgroup Decomposability of Income-Related Inequality of Health, with an Application to Australia*

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The main purpose of this paper is to compare the decomposition properties of rank-dependent and level-dependent indicators of income-related inequality of health. We do so by focusing on the decomposition by population groups. We show that level-dependent indices have more desirable subgroup decomposability properties than rank-dependent indices. This may prove to be an important argument in favour of the use of level-dependent indices. The difference between the subgroup decomposition results of rank-dependent and level-dependent indices is illustrated by means of an empirical study using Australian health and income data. We consider subgroups based on sex, age and employment status.

1 Introduction

In the literature on the measurement of income-related inequality of health, decomposition techniques have gained currency as methods to better understand, and if possible to explain, the observed levels of inequality at a given moment of time, as well as the changes in these levels over time. Some of these techniques have been borrowed or adapted from well-known results in the

fields of income inequality measurement (e.g. Rao, 1969; Bourguignon, 1979; Shorrocks, 1980) and labour economics (e.g. Blinder, 1973; Oaxaca, 1973). A good survey of this literature has been written by Fortin *et al.* (2011). Other techniques have been developed specifically for the analysis of bivariate inequality (e.g. Clarke *et al.*, 2003; Wagstaff *et al.*, 2003; Wagstaff, 2005b; Abul Naga & Geoffard, 2006; Van Ourti

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et al., 2009; Allanson & Petrie, 2013a; Erreygers & Kessels, 2013; Makdissi *et al.*, 2013; Heckley *et al.*, 2016; Kessels & Erreygers, 2016). Instructive surveys of the state of the art can be found in Van Doorslaer and Van Ourti (2011), O'Donnell *et al.* (2012), and Van Ourti *et al.* (2014).

Almost all of this work is centred around rank-dependent indicators of inequality such as the concentration index, which is the dominant measure of income-related inequality of health. Recently, however, a case has been made for a shift to level-dependent indicators (Erreygers & Kessels, 2017). One of the considerations which seems particularly relevant to the choice between different indicators is how amenable they are when it comes to decomposability.

In the case of measurement of income inequality, decomposability has been extensively explored over the last three decades (for a summary, see Foster & Sen, 1997, section A.5.1). What has been termed 'subgroup' or 'additive' decomposability is often seen as a highly desirable criterion for choosing univariate inequality measures, particularly by policy-makers. Cowell (2011, section 3.4), for instance, has argued that decomposability means that there should be a coherent relationship between inequality in the whole of society and inequality in its constituent parts. It has been well established that the Gini coefficient is not generally subgroup decomposable, and this has been a major motivation behind the development of the class of generalised entropy measures, which do have this property. Given its similarity to the Gini coefficient, the concentration index is not subgroup decomposable, and little progress has been made to develop alternative bivariate measures which have this property.

The main purpose of this paper is to compare the subgroup decomposability properties of rank-dependent and level-dependent indicators of income-related inequality of health. We show that in contrast to rank-dependent indicators, level-dependent indicators do have the property of subgroup decomposability, which may prove to be an important argument in favour of the use of the latter.

We start with a brief presentation of the subgroup decomposability property. We then define and compare the two types of indicators of income-related inequality of health we are considering here, and analyse how they handle subgroup decomposability. We illustrate our results with an empirical study of income-related

health inequality in Australia. We explore to what extent this inequality is due to differences among and between males and females, age cohorts, and groups defined by their employment status.

II Context

(i) Subgroup Decomposability

Subgroup decomposition involves three types of inequality: inequality in the population as a whole, inequality within subgroups of the population, and inequality between subgroups. One can think of subgroups based on geographical areas, age, ethnicity, etc. The basic idea of subgroup decomposability is to express the measured degree of inequality I in the population as a whole as the sum of two parts: the 'within-group' inequality I_W and the 'between-group' inequality I_B (see Bourguignon, 1979). This exact decomposability property is expressed by the additive formula

$$I = I_W + I_B. \quad (1)$$

Moreover, the 'within-group' inequality consists of a weighted sum of the inequalities within the subgroups, while the 'between-group' inequality is calculated assuming that all individuals of a given subgroup are alike.

Subgroup decompositions may be used to detect whether inequality is mainly due to differences across specific groups (men and women, say) or to differences within the groups themselves (among men and among women, say). Since income-related inequality can be both positive and negative, subgroup decompositions also reveal whether the within and between components are of the same type as the overall inequality, that is, whether I_W and/or I_B are/is of the same sign as I . For instance, if I_B/I is negative, the income-related inequality between the subgroups contributes 'negatively' to the observed level of income-related inequality in the population as a whole. In that case, the between and within components act in different directions. This type of information is potentially relevant for policy-makers.

(ii) The Concentration Index

From the literature on income inequality we know that univariate rank-dependent indices, such as the Gini coefficient, cannot be decomposed exactly into the sum of a within-group and a between-group contribution, except in exceptional cases (Bhattacharya & Mahalanobis, 1967).

The Gini subgroup decomposition therefore involves a third or ‘residual’ component, required to balance the equation. While the origin of the residual component is clear (it is due to subgroups having overlapping income ranks), no unanimity exists on the interpretation of the term; see Mookherjee and Shorrocks (1982) and Lambert and Aronson (1993) for opposing views on the matter. The necessity of introducing a residual component carries over to rank-dependent bivariate indices such as the concentration index (Clarke *et al.*, 2003; Wagstaff, 2005b). Unless there is no overlapping whatsoever in income ranks between the subgroups, there is always a residual term.

In spite of this, the concentration index and its variants continue to be the standard measures of socioeconomic inequality of health, and subgroup decompositions are used in the literature, albeit sparingly. Direct applications are rather limited; a recent example concerns the exposure to second-hand tobacco smoke among children, where the decomposition is applied to the groups of urban and rural children (Hajizadeh & Nandi, 2016). Indirectly, the distinction of within and between components plays a role in the Oaxaca-type decompositions which try to analyse the changes in inequality over time (e.g. Allanson & Petrie, 2013b). Some studies of income-related inequality of health estimate inequality within subgroups, but refrain from measuring inequality between subgroups. For example, Chotikapanich *et al.* (2003) estimate the extent of inequality among Australian men and women, but remain silent on the inequality between the sexes.¹

It would be convenient if we had a socioeconomic inequality measure with the subgroup decomposability property. Even if we limit ourselves to the class of ‘linear’ measures of inequality, which we will define in the next section, we show that such a measure exists.

III Methods

(i) Measures of Inequality

Linear Measures of Inequality

Let us consider a population consisting of n individuals, where each individual i ($i = 1, 2, \dots, n$)

¹ Their results are not immediately comparable to the ones we report in Section IV, because they use a different health variable and adjust it for need and age.

is characterised by their levels of socioeconomic achievement (y_i) and of health attainment (h_i). For convenience, we designate socioeconomic achievement as ‘income’, but it goes without saying that socioeconomic status can be measured in a variety of ways (by consumption levels, by years of education, etc.). We assume that both y_i and h_i have well-defined lower bounds greater than or equal to 0. More specifically, each of these variables is either a ratio-scale variable with no upper bound, or a cardinal or ratio-scale variable with a well-defined upper bound. The two variables are not necessarily of the same nature. Income, for instance, is an unbounded ratio-scale variable. The health variable, by contrast, is often a bounded variable, and need not be of the ratio-scale type. The means of the variables are respectively

$$\mu_y = \frac{1}{n} \sum_{i=1}^n y_i, \quad \mu_h = \frac{1}{n} \sum_{i=1}^n h_i. \quad (2)$$

Inequality in the population as a whole depends on the levels of income and health of all individuals in society, that is, $I = f(\mathbf{y}, \mathbf{h})$, where $\mathbf{y} = (y_1, y_2, \dots, y_n)$ and $\mathbf{h} = (h_1, h_2, \dots, h_n)$. The class of measures of inequality we consider in this paper is characterised by a function $f(\mathbf{y}, \mathbf{h})$ of the additive type, and is in fact fairly broad.

Linear measures of bivariate inequality are closely related to linear measures of univariate inequality, which have been defined by Mehran (1976). We focus on bivariate indices where income serves as the weighting variable and health as the weighted variable. In formal terms, the absolute version of these indices takes the form²

$$I = I(\mathbf{y}, \mathbf{h}) = \frac{1}{n} \sum_{i=1}^n w_i(\mathbf{y}) h_i. \quad (3)$$

What distinguishes bivariate from univariate indices is that the weights are determined by a different distribution from the one which is weighted. The concentration index, for example, defines the weights in terms of the ranks of individuals in the income distribution, and applies these weights to the distribution of health.

We also consider variants of index I based on the formula $I^* = C^* I$, where C^* is determined by

² If there is no confusion possible, we will drop the arguments and simply write I instead of $I(\mathbf{y}, \mathbf{h})$, etc.

the characteristics of distribution h (see Heckley *et al.*, 2016, for a similar procedure). For instance, the relative version is obtained by dividing the index by the mean of the weighted variable, that is, by taking $C^{\text{rel}} = 1/\mu_h$:

$$I^{\text{rel}} = \frac{1}{\mu_h} I. \quad (4)$$

Other versions of the indices have been proposed in the literature, for example in order to deal with binary variables (Wagstaff, 2005a). For bounded variables, whether they are binary or of another type, we suggest the use of the index proposed by Erreygers (2009):

$$I^{\text{bou}} = \frac{A}{h_{\max} - h_{\min}} I, \quad (5)$$

where h_{\min} and h_{\max} stand for the lower and upper bounds of the health variable, and A is a constant. Erreygers and Van Ourti (2011) have argued that the choice for a specific version of a bivariate index should be made in accordance with the nature of the health variable under consideration.

Rank-Dependent Indices

Rank-dependent indices rely exclusively on income ranks to define the weights $w_i(\mathbf{y})$; no other information on the income distribution enters into the calculation of the weights. Let the rank of individual i in the income distribution be $r_i(\mathbf{y})$. If there are no ties in the income distribution, the rank of the poorest person in society is equal to 1, the rank of the second poorest person equal to 2, etc., and the rank of the richest person equal to n . If a group of $m + 1$ individuals are tied in position g , the rank of each of these individuals is equal to $g + (m/2)$. For simplicity, however, we assume there are no ties in the income distribution, and that individuals are ranked according to their incomes ($y_1 < y_2 < \dots < y_n$).³

The standard concentration index, for instance, is characterised by a weighting function which is linear in the income ranks $r_i(\mathbf{y})$. This weighting function may therefore be called the ‘rank’ function and the associated bivariate index the R index:

³ The Appendix *Sample Weights and Ties* specifies how the formulas must be adapted when there are ties, and when not all individuals have the same population weight.

$$w_i^R(\mathbf{y}) = \frac{2r_i(\mathbf{y}) - n - 1}{n}, \quad (6)$$

$$R = \frac{1}{n} \sum_{i=1}^n w_i^R(\mathbf{y}) h_i. \quad (7)$$

The weights $w_i^R(\mathbf{y})$ steadily increase as $r_i(\mathbf{y})$ goes from 1 to n . If the group of the ‘poor’ is defined as those who have negative weights and the group of the ‘rich’ as those who have positive weights, the weighting function (6) puts the boundary between the two groups exactly in the middle of the population. Those with ranks smaller than or equal to $n/2$ (if n is even) or smaller than or equal to $(n - 1)/2$ (if n is odd) have negative weights, and those with ranks larger than or equal to $n/2 + 1$ (if n is even) or larger than or equal to $(n + 3)/2$ (if n is odd) have positive weights. Put differently, individuals with an income below the median income have negative weights, and individuals with an income above the median positive weights. The negative weights sum to $-n/4$ and the positive weights to $n/4$.⁴ This motivates the choice of $A = 4$ for the bounded version of the rank-dependent index, as suggested by Erreygers (2009).

Level-Dependent Indices

In contrast to rank-dependent indices, the weights of level-dependent indices are based upon income levels rather than income ranks. The basic version of the level-dependent index L proposed by Erreygers and Kessels (2017) has a weighting function which is a simple linear function of income:⁵

$$w_i^L(\mathbf{y}) = \frac{y_i - \mu_y}{\mu_y}, \quad (8)$$

$$L = \frac{1}{n} \sum_{i=1}^n w_i^L(\mathbf{y}) h_i. \quad (9)$$

⁴ Strictly speaking, this result holds only when n is even. When n is odd, the sum of the negative weights is $-[(n - 1)(n + 1)]/(4n)$, which for large n is approximately equal to $-n/4$. And similarly for the positive weights.

⁵ The L index is related to the κ index proposed by Abul Naga and Geoffard (2006, 365). For the specific choice of their parameters $\alpha = \beta = 1$, it turns out that we have $L = (\kappa - 1)\mu_h$. It needs to be pointed out that in their social welfare framework, inspired by Tsui (1999), the parameters are constrained by the condition $\alpha + \beta \leq 1$.

The weights $w_i^L(y)$ are proportional to the deviations of the incomes from the mean. If there is a high degree of income inequality, typically there will be a lot of individuals with income levels below the mean, who will therefore have negative, but in absolute terms rather small weights. On the other side of the spectrum, those who have very high incomes will have positive, and in absolute terms quite large weights. Hence, a small change in the health level of a relatively well-off individual will probably have a more pronounced influence on the index than a comparable change in the health level of an individual who is less well-off.

All the negative weights sum to $-(n/2)D$, and all the positive weights to $(n/2)D$, where D is the relative mean deviation:

$$D = \frac{\frac{1}{n} \sum_{i=1}^n |y_i - \mu_y|}{\mu_y}. \tag{10}$$

Since D is at most equal to $2(n - 1)/n$, it follows that these two amounts are bounded by $1 - n$ and $n - 1$. For the bounded version of the level-dependent index, the natural choice for the value of A is therefore $A = 1$.

A Brief Comparison

At this stage it seems useful to highlight some of the differences between the indices R and L . We focus here on the bounds of the indices, and when these are reached.

The relative version of R varies between $-(n - 1)/n$ and $(n - 1)/n$, with the minimum value attained when the poorest person in society has all the health, and the maximum when the richest person in society has all the health. It does not matter exactly how poor or how rich the poorest and richest persons are. The relative version of L , by contrast, varies between -1 and $n - 1$. The minimum is obtained when the poorest person in society has a zero income and all the health⁶; the maximum when the richest person has all the income and all the health.

If the health variable is unbounded, the minima and maxima of the absolute versions of the indices are equal to those of the relative versions multiplied by the average of the health variable. If the health variable is bounded, the appropriate

⁶ More generally, if ties are allowed, the minimum is obtained when all the persons with positive health have zero incomes.

version of the indices is the bounded one. The minimum and maximum of the bounded version of R , taking $A = 4$, are equal to -1 and 1 , with the minimum attained when the poorest half of the population has maximum health and the richest half minimum health, and the maximum in the opposite case. The minimum and maximum of the bounded version of L , with $A = 1$, are equal to $-(1 - n)/n$ and $(n - 1)/n$. These can be reached only if one person has all the available income; the minimum is attained when that extremely rich person has minimum health, while all the others have maximum health, and the maximum when that person has maximum health, while all the others have minimum health.

(ii) *Subgroup Decomposability*

Subgroups

Suppose that we partition the population into k subsets, G_1, G_2, \dots, G_k , such that every individual i belongs to exactly one subset G_j . The number of individuals in subgroup G_j is denoted by n_j . The mean income and health attainments of subgroup G_j are equal to

$$\mu_{y_j} = \frac{1}{n_j} \sum_{i \in G_j} y_i, \quad \mu_{h_j} = \frac{1}{n_j} \sum_{i \in G_j} h_i, \tag{11}$$

and obviously we have

$$\mu_y = \sum_{j=1}^k \frac{n_j}{n} \mu_{y_j}, \quad \mu_h = \sum_{j=1}^k \frac{n_j}{n} \mu_{h_j}. \tag{12}$$

The levels of inequality in the k subgroups are denoted by I_1, I_2, \dots, I_k . Inequality in subgroup G_j depends on the levels of income and health of the individuals of that group only, that is, $I_j = f(y_j, h_j)$, where $y_j = (y_i | i \in G_j)$ and $h_j = (h_i | i \in G_j)$. The ‘within-group’ inequality consists of a weighted sum of the inequalities in the k subgroups. When the weights are equal to s_1, s_2, \dots, s_k , we have

$$I_W = \sum_{j=1}^k s_j I_j. \tag{13}$$

The ‘between-group’ inequality depends on the means of income and health in the subgroups and on the sizes of the subgroups, that is, $I_B = f(\mu_y, \mu_h)$, where $\mu_y = (\mu_{y_1}, \mu_{y_2}, \dots, \mu_{y_k})$ and $\mu_h = (\mu_{h_1}, \mu_{h_2}, \dots, \mu_{h_k})$. It is important to note that in this

case we have to use a population-weighted version of the inequality index.

Linear Measures

For the calculation of the within component I_W , we need to know the levels of inequality in all subgroups (i.e. I_1, I_2, \dots, I_k) and the weights of all subgroups (i.e. s_1, s_2, \dots, s_k). By analogy with (3), inequality in subgroup G_j can be defined as

$$I_j = \frac{1}{n_j} \sum_{i \in G_j} w_i(\mathbf{y}_j) h_i. \quad (14)$$

The notation $w_i(\mathbf{y}_j)$ indicates that the weights of individuals depend on their position within the group to which they belong. As far as the subgroup weights are concerned, no general formula exists: these are indicator-specific.

The ‘between-group’ component I_B is calculated assuming that every individual of group G_j has income level μ_{y_j} and health level μ_{h_j} . It can therefore be defined as

$$I_B = \frac{1}{n} \sum_{j=1}^k n_j w_j(\boldsymbol{\mu}_y) \mu_{h_j}. \quad (15)$$

The weights $w_j(\boldsymbol{\mu}_y)$ are applied to every individual of the group G_j and reflect the average situation of an individual of that group.

As far as the other versions of the index are concerned, their decompositions can be related easily to those of the absolute version. Let us consider index $I^* = C^* I$. The ‘between-group’ component I_B^* is equal to $C^* I_B$. The ‘within-group’ component, on the other hand, is equal to $I_W^* = \sum_{j=1}^k s_j^* I_j^*$, where $I_j^* = C_j^* I_j$. If we define the subgroup weights as $s_j^* = (C^*/C_j^*) s_j$, we obtain $I_W^* = C^* I_W$. This means that whatever version of the index we use (absolute, relative, bounded, ...), the ratio of the ‘within-group’ to the ‘between-group’ component is always the same.

Rank-Dependent Measures

Let us begin by defining the weights $w_i(\mathbf{y}_j)$ and $w_j(\boldsymbol{\mu}_y)$ of expressions (14) and (15) for the rank-dependent index R . To simplify the expressions, we assume that the groups are labelled in accordance with average incomes, that is, $\mu_{y_1} < \mu_{y_2} < \dots < \mu_{y_k}$. For the calculation of the within-group inequalities R_j , the relevant ranks are those within each group. If we designate individual i 's rank within group G_j , for any $i \in G_j$, by $r_i(\mathbf{y}_j)$, the weights $w_i^R(\mathbf{y}_j)$ are equal to

$$w_i^R(\mathbf{y}_j) = \frac{2r_i(\mathbf{y}_j) - n_j - 1}{n_j}. \quad (16)$$

Next, we need to define the subgroup weights s_j in order to calculate the weighted sum $R_W = \sum_{j=1}^k s_j R_j$. In accordance with the literature on the decomposition of the Gini coefficient, we assume the weights s_j are equal to the squares of the population shares, $s_j = (n_j/n)^2$ (Lambert & Aronson, 1993, p. 1221).

For the calculation of the between-group inequality R_B , the rank assigned to every individual of a given group must coincide with the average rank of the individuals of that group in the whole population. The average rank $r_j(\boldsymbol{\mu}_y)$ of the individuals of group G_j can be calculated by means of this recursive formula:

$$r_j(\boldsymbol{\mu}_y) = \frac{n_j + 1}{2} + \sum_{l=0}^{j-1} n_l, \quad (17)$$

with $n_0 = 0$ and $j = 1, \dots, k$. The weights $w_j^R(\boldsymbol{\mu}_y)$ are then equal to

$$w_j^R(\boldsymbol{\mu}_y) = \frac{2r_j(\boldsymbol{\mu}_y) - n - 1}{n}. \quad (18)$$

In general, the sum of the within and between components differs from the overall index R . Hence, we are confronted with a residual term $R_X = R - R_W - R_B$, which is almost always different from zero and can be both positive and negative. The residual term is zero ‘‘if the subgroup income ranges do not overlap’’ (Lambert & Aronson, 1993, p. 1221). In most cases, however, we end up with a non-zero residual term equal to

$$R_X = \frac{1}{n} \sum_{j=1}^k \sum_{i \in G_j} \left[w_i^R(\mathbf{y}_j) h_i - \frac{n_j}{n} w_i^R(\mathbf{y}_j) h_i - w_j^R(\boldsymbol{\mu}_y) \mu_{h_j} \right]. \quad (19)$$

Not surprisingly, then, the rank-dependent index R does not have the property of subgroup decomposability.

It is worth noting that the subgroup weights for the calculation of the within component R_W do not sum to 1. Similar results hold for the relative and bounded versions of the index. While the subgroup weights for the bounded version are the

same as those for the absolute version, the subgroup weights for the relative version are equal to $s_j = \left(\frac{n_j}{n}\right)^2 \frac{\mu_{y_j}}{\mu_y}$.

Level-Dependent Indices

The situation is different for the level-dependent index. Let us begin by looking at the calculation of the within component. For the inequality in group G_j , the weight of any individual $i \in G_j$ is defined in terms of the deviation of this individual's income from the mean income in group G_j . More formally, the weights $w_i^L(\mathbf{y}_j)$ are equal to

$$w_i^L(\mathbf{y}_j) = \frac{y_i - \mu_{y_j}}{\mu_{y_j}}. \quad (20)$$

The subgroup weights for the calculation of the within component $L_W = \sum_{j=1}^k s_j L_j$ are assumed to be equal to the income shares of the subgroups, $s_j = (n_j \mu_{y_j}) / (n \mu_y)$.

With regard to the calculation of the between component L_B , each individual of a given group receives a weight proportional to the deviation of the mean income of that group from the mean income of the whole population. This means that the weights $w_j^L(\boldsymbol{\mu}_y)$ are equal to

$$w_j^L(\boldsymbol{\mu}_y) = \frac{\mu_{y_j} - \mu_y}{\mu_y}. \quad (21)$$

It is straightforward to show that the sum of the within and between components is always equal to L . Observe that we have

$$L = \frac{1}{n \mu_y} \sum_{i=1}^n y_i h_i - \mu_h, \quad (22)$$

and likewise

$$L_j = \frac{1}{n_j \mu_{y_j}} \sum_{i \in G_j} y_i h_i - \mu_{h_j}. \quad (23)$$

It follows that

$$s_j L_j = \frac{1}{n \mu_y} \sum_{i \in G_j} y_i h_i - s_j \mu_{h_j}, \quad (24)$$

and therefore that the within component is equal to

$$L_W = \frac{1}{n \mu_y} \sum_{i=1}^n y_i h_i - \sum_{j=1}^k s_j \mu_{h_j}. \quad (25)$$

The between component, on the other hand, can be expressed as

$$L_B = \sum_{j=1}^k s_j \mu_{h_j} - \mu_h. \quad (26)$$

Hence we obtain that

$$L_W + L_B = \frac{1}{n \mu_y} \sum_{i=1}^n y_i h_i - \mu_h = L. \quad (27)$$

The conclusion is that the level-dependent index L has the property of subgroup decomposability.

Since the subgroup weights used for the calculation of the within component L_W are equal to the shares of the groups in total income, these weights add up to 1, which may be perceived as an attractive property. For the bounded version of the index, the subgroup weights are the same as those for the absolute version. For the relative version, the subgroup weights are

$$s_j = \frac{n_j \mu_{y_j} \mu_{h_j}}{n \mu_y \mu_h}. \quad (28)$$

IV Data and Results

(i) Description of the Data

Our data come from the Household, Income and Labour Dynamics in Australia (HILDA) Survey, wave 13.⁷ We consider only individuals aged 15 or higher. As our income variable we take equivalised income, calculated using the modified OECD equivalence scale (Australian Bureau of Statistics, 2013, Appendix 3). The equivalised income of a person is equal to the disposable income of this person's household divided by the household's equivalence factor. This factor is obtained by giving the first adult of the household a score of 1, every other adult a score of 0.5, every child a score of 0.3, and then adding up all the scores. It should be noted that even though children below the age of 15 are not part of our sample, they are taken into account for the

⁷ More information on the HILDA database can be found at <http://www.melbourneinstitute.com/hilda>.

TABLE 1
Frequency Statistics of the Sample

Variable	Share (%)
Sex	
Female	53.17
Male	46.83
Age group	
Age 15–24	17.43
Age 25–34	16.97
Age 35–44	16.00
Age 45–54	17.66
Age 55–64	14.64
Age 65–74	10.65
Age 75+	6.63
Employment status	
Employed	63.49
Unemployed	4.04
Not in labour force	32.47

calculation of the equivalised incomes. As our health variable we take the SF-6D health score. Since this is a bounded variable, we use the bounded version of the inequality indices, R^{bou} and L^{bou} . Given that the bounds of the variable are $h_{\min} = 0$ and $h_{\max} = 1$, it is easy to check that $R^{\text{bou}} = 4R$ and $L^{\text{bou}} = L$.

Our sample includes 14,729 individuals with valid, non-missing values for all variables under study.⁸ Since this is a fairly large and, to the best of our knowledge, representative sample, we decided not to apply sample weights in our calculations.⁹ The composition of our sample is described in Table 1. We consider subgroup decompositions based on sex, age and employment status.

(ii) *Decomposition by Sex*

For the first subgroup decomposition we partitioned the population into two groups, females and males. As shown in Table 2, there is some variation among these groups, with respect to both health and income. Women tend to have slightly worse health and lower income than men. The rank-dependent index measures a slightly higher extent of income-related inequality among women than among men, while according to the level-dependent index there is just as much income-related inequality among men as there is among women.

The results of the subgroup decomposition according to sex can be found in Table 3. Let us begin by looking at the results for the level-dependent index. The observed inequality comes overwhelmingly (98.3%) from heterogeneity within the groups; the

TABLE 2
Summary Statistics for the Sex Decomposition

Group	Health		Income (\$)		Indices	
	Mean	SD	Mean	SD	R^{bou}	L^{bou}
Female	0.7508	0.1244	50,014	34,725	0.0661	0.0141
Male	0.7718	0.1216	52,389	35,102	0.0629	0.0141
All	0.7606	0.1236	51,126	34,921	0.0657	0.0144

TABLE 3
Decomposition According to Sex

	R^{bou}		L^{bou}	
	Values	%	Values	%
Within	0.0325	49.47	0.0141	98.30
Between	0.0210	31.92	0.0002	1.70
Residual	0.0122	18.61	–	–
Total	0.0657	100.00	0.0144	100.00

between-component accounts for a meager part (1.7%) of the total. The results for the rank-dependent index also point in the direction of the predominance of the within-group component. Nevertheless, the estimated share of the within-component is much

⁸ This means in particular that individuals with a negative income are excluded.

⁹ The sample-weighted results are very close to the results presented below. They can be obtained upon request from the authors.

TABLE 4
Summary Statistics for the Age Decomposition

Group	Health		Income (\$)		Indices	
	Mean	SD	Mean	SD	R^{bou}	L^{bou}
Age 15–24	0.7837	0.1179	45,376	26,065	0.0334	0.0080
Age 25–34	0.7841	0.1132	55,126	29,572	0.0490	0.0106
Age 35–44	0.7741	0.1204	54,101	31,592	0.0555	0.0114
Age 45–54	0.7586	0.1218	56,699	33,753	0.0654	0.0130
Age 55–64	0.7438	0.1266	55,639	40,009	0.0818	0.0192
Age 65–74	0.7333	0.1270	45,305	46,554	0.0581	0.0167
Age 75+	0.6937	0.1249	33,381	35,607	0.0330	0.0110
All	0.7606	0.1236	51,126	34,921	0.0657	0.0144

TABLE 5
Decomposition According to Age

	R^{bou}		L^{bou}	
	Values	%	Values	%
Within	0.0084	12.85	0.0128	88.86
Between	0.0132	20.17	0.0016	11.14
Residual	0.0440	66.98	–	–
Total	0.0657	100.00	0.0144	100.00

smaller (nearly 50% of the total) and that of the between-component much larger (almost 32%). These results must be treated with caution, however, since the residual component amounts to about 19%.

(iii) Decomposition by Age

For our second subgroup decomposition we divided the population into seven age groups, with the youngest group consisting of those aged 15–24, and the oldest group consisting of those aged 75 or older. As can be seen from Table 4, older people tend to have worse health and lower income than younger people. According to both indices, the level of socioeconomic inequality is the highest in the group aged 55–64.

The decomposition results are reported in Table 5. According to the level-dependent index the within-component is again by far the largest. In comparison to the results for the decomposition by sex, the level-dependent index now indicates a substantially larger between component (11.14% instead of 1.70%). The results for the rank-dependent index are completely different. This index suggests that the share of the

between component exceeds that of the within component. The estimated share of the within component is surprisingly small, not even 13% of the total. Amazingly, the residual component amounts to two-thirds of the total.¹⁰

(iv) Decomposition by Employment Status

For the third subgroup decomposition we partitioned the population into three groups according to their employment status. Table 6 indicates that there are large differences in health and income between the employed, the unemployed and those who are not in the labour force, and also that the level of inequality is the highest among the last of these.

Table 7 gives the decomposition results for both indices. The level-dependent index attributes 44.11% of the observed inequality to the between-group component, which is much larger than what we obtained for the two previous decompositions. The rank-dependent index, by contrast, suggests that the between component is much larger than the within component, and moreover that it is almost equal in size to the overall inequality. Once again, given the substantial residual term (–18.45% of the observed inequality), this result must be treated with caution.

¹⁰ We also calculated the within and between components for a combined sex and age subgroup classification, based on 14 sex–age groups. The level-dependent index estimates that the between component amounts to 12.48%, which is close to the sum of 1.70% and 11.14%; the rank-dependent index, by contrast, estimates that the between component is equal to 37.12%, which is nowhere near the sum of 31.92% and 20.17%.

TABLE 6
Summary Statistics for the Employment Decomposition

Group	Health		Income (\$)		Indices	
	Mean	SD	Mean	SD	R^{bou}	L^{bou}
Employed	0.7857	0.1088	58,553	34,503	0.0213	0.0051
Unemployed	0.7406	0.1313	37,811	23,340	0.0353	0.0096
Not in labour force	0.7140	0.1352	38,263	32,638	0.0645	0.0165
All	0.7606	0.1236	51,126	34,921	0.0657	0.0144

TABLE 7
Decomposition According to Employment

	R^{bou}		L^{bou}	
	Values	%	Values	%
Within	0.0154	23.49	0.0080	55.89
Between	0.0624	94.96	0.0063	44.11
Residual	-0.0121	-18.45	-	-
Total	0.0657	100.00	0.0144	100.00

V Discussion

Several interesting conclusions can be drawn from the three subgroup decompositions we have just presented. These relate to both methodological and empirical aspects.

First, there is a striking divergence between the decomposition results of the rank-dependent and level-dependent indicators. In comparison to the level-dependent indicator, the rank-dependent indicator has much lower estimates for the within component (the absolute differences amount respectively to -48.33%, -76.01% and -32.40%) and much higher estimates for the between component (respectively 30.22%, 9.03% and 50.85%). These differences are substantial. The main cause is not that rank-dependent and level-dependent indicators measure inequality differently (in fact, they give broadly similar results),¹¹ but that they handle the within-between decomposition in a completely different way.

Second, the presence of a substantial residual term complicates the interpretation of the results for the rank-dependent index. The magnitude of the residual term is in all three cases considerable

¹¹ The correlation coefficient between the values of the two indicators for the population as a whole and for the 12 subgroups we considered above is 0.91.

(respectively 18.61%, 66.98% and -18.45%), and moreover it can be both positive and negative. All of this means that a lot of caution must be exercised when one tries to estimate the within and between components of income-related inequality by means of a rank-dependent index.

Third, there is an obvious reason why the rank-dependent index systematically underestimates the within component of inequality: the sum of the subgroup weights it uses for the calculation of the within component is always smaller than 1.¹² This means that even if the amount of inequality is the same in every subgroup (which is virtually the case in the decomposition according to sex), the value of the within component is smaller than this level of inequality. The underestimation of the within component is exacerbated when the number of subgroups increases, as in the case of the decomposition according to age. The issue does not arise when we use the level-dependent index.¹³

In our view, these arguments reduce the confidence policy-makers should place in estimates of the within and between components of income-related inequality of health generated by rank-dependent indices such as the concentration index. If perfect subgroup decomposability is regarded as a desirable property of inequality measures (a position we defend), it seems more prudent to rely on the estimates generated by level-dependent indices. This is not a purely academic matter: the choice of indicator may have policy implications. If policies were based on the results of the level-dependent indicator, the focus would be on

¹² For a discussion of a similar issue in the literature on income inequality, see Chakravarty (2001). He distinguished subgroup decomposability and population share weighted decomposability.

¹³ See also Erreygers *et al.* (2017), who applied the within-between decomposition to the global socioeconomic inequality of life expectancy.

inequalities within the subgroups in each of the three cases we examined. Policies based on the results of the rank-dependent indicator would probably be different, however. Especially when it comes to employment status, it is to be expected that the emphasis would be put on inequality between the groups of the employed, the unemployed and those not in the labour force, rather than on inequality within each of these groups.

VI Conclusion

In this paper, we explored the subgroup decomposability properties of both rank-dependent and level-dependent indices of socioeconomic inequality of health. Whereas the level-dependent index can be decomposed perfectly into a within and between component, the decomposition of the rank-dependent index involves a residual component, which is zero in exceptional cases only. By means of three empirical examples, we showed that the two indices may generate widely different estimates of the within and between components. We have good reasons to believe that the subgroup decomposition results of the level-dependent index are more reliable than those of the rank-dependent index. We are convinced that this decomposability property constitutes a strong argument in favour of using level-dependent indices alongside, and maybe even instead of, the still dominant rank-dependent indices such as the concentration index. Given the wide range of topics to which this index has been applied (e.g. to check whether out-of-pocket payments for health care are progressive or regressive), it is important to be aware of its limitations and to explore whether level-dependent indices might perform better.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix A: Robustness checks for Subgroup decomposability of income-related inequality of health, with an application to Australia

Appendix B: Results adjusted for sample weights for Subgroup decomposability of income-related inequality of health, with an application to Australia

REFERENCES

- Abul Naga, R.H. and Geoffard, P.-Y. (2006), 'Decomposition of Bivariate Inequality Indices by Attributes', *Economics Letters*, **90**, 362–7.
- Allanson, P. and Petrie, D. (2013a), 'Longitudinal Methods to Investigate the Role of Health Determinants in the Dynamics of Income-Related Health Inequality', *Journal of Health Economics*, **32**, 922–37.
- Allanson, P. and Petrie, D. (2013b), 'Accounting for Population Change in the Longitudinal Analysis of Income-Related Health Inequalities', in Rosa Dias, P. and O'Donnell, O. (eds), *Health and Inequality*. Emerald, Bingley; 193–225.
- Australian Bureau of Statistics (2013), *Household Income and Income Distribution, Australia, 2011–12*, catalogue no. 6523.0. ABS, Canberra.
- Bhattacharya, N. and Mahalanobis, B. (1967), 'Regional Disparities in Household Consumption in India', *Journal of the American Statistical Association*, **62**, 143–61.
- Blinder, A. (1973), 'Wage Discrimination: Reduced Form and Structural Estimates', *Journal of Human Resources*, **8**, 945–57.
- Bourguignon, F. (1979), 'Decomposable Income Inequality Measures', *Econometrica*, **47**, 901–20.
- Chakravarty, S.R. (2001), 'The Variance as a Subgroup Decomposable Measure of Inequality', *Social Indicators Research*, **53**, 79–95.
- Chotikapanich, D., Creedy, J. and Hopkins, S. (2003), 'Income and Health Concentration in Australia', *Economic Record*, **79**, 297–305.
- Clarke, P., Gerdtham, U.-G. and Connelly, L.B. (2003), 'A Note on the Decomposition of the Health Concentration Index', *Health Economics*, **12**, 511–16.
- Cowell, F. (2011), *Measuring Inequality*, 3rd edn. Oxford University Press, Oxford.
- Erreygers, G. (2009), 'Correcting the Concentration Index', *Journal of Health Economics*, **28**, 504–15.
- Erreygers, G. and Kessels, R. (2013), 'Regression-Based Decompositions of Rank-Dependent Indicators of Socioeconomic Inequality of Health', in Rosa Dias, P. and O'Donnell, O. (eds), *Health and Inequality*. Emerald, Bingley; 227–59.
- Erreygers, G. and Kessels, R. (2017), 'Socioeconomic Status and Health: A New Approach to the Measurement of Socioeconomic Inequality of Health', *International Journal of Environmental Research and Public Health*, **14**, 673.
- Erreygers, G. and Van Ourti, T. (2011), 'Measuring Socioeconomic Inequality in Health, Health Care, and Health Financing by means of Rank-Dependent Indices: A Recipe for Good Practice', *Journal of Health Economics*, **30**, 685–94.
- Erreygers, G., Clarke, P. and Zheng, Q. (2017), 'On the Measurement of Socioeconomic Inequality of Health between Countries', *Journal of Economic Inequality*, **15**, 175–93.
- Fortin, N., Lemieux, T. and Firpo, S. (2011), 'Decomposition Methods in Economics', in Card, D. and Ashenfelter, O. (eds), *Handbook of Labor Economics*, 4th edn. Elsevier, North-Holland; 1–104.
- Foster, J.E. and Sen, A. (1997), 'On Economic Inequality after a Quarter Century', in Sen, A. *On Economic Inequality*, enlarged edn. Clarendon, Oxford; 107–219.

- Hajizadeh, M. and Nandi, A. (2016), 'The Socioeconomic Gradient of Secondhand Smoke Exposure in Children: Evidence from 26 Low-Income and Middle-Income Countries', *Tobacco Control*, **25**, e146–55.
- Heckley, G., Gerdtham, U.-G. and Kjellsson, G. (2016), 'A General Method for Decomposing the Causes of Socioeconomic Inequality in Health', *Journal of Health Economics*, **48**, 89–106.
- Kessels, R. and Erreygers, G. (2016), 'Structural Equation Modeling for Decomposing Rank-Dependent Indicators of Socioeconomic Inequality of Health: An Empirical Study', *Health Economics Review*, **6**, 56.
- Lambert, P.J. and Aronson, J.R. (1993), 'Inequality Decomposition Analysis and the Gini Coefficient Revisited', *Economic Journal*, **103**, 1221–7.
- Makdissi, P., Sylla, D. and Yazbeck, M. (2013), 'Decomposing Health Achievement and Socioeconomic Health Inequalities in Presence of Multiple Categorical Information', *Economic Modelling*, **35**, 964–8.
- Mehran, F. (1976), 'Linear Measures of Income Inequality', *Econometrica*, **44**, 805–9.
- Mookherjee, D. and Shorrocks, A.F. (1982), 'A Decomposition Analysis of the Trend in UK Income Inequality', *Economic Journal*, **92**, 886–902.
- O'Donnell, O., Van Doorslaer, E. and Wagstaff, A. (2012), 'Decomposition of Inequalities in Health and Health Care', in Jones, A.M. (ed.), *The Elgar Companion to Health Economics, Second Edition*. Edward Elgar, Cheltenham; 179–91.
- Oaxaca, R.L. (1973), 'Male-Female Wage Differentials in Urban Labor Markets', *International Economic Review*, **14**, 693–709.
- Rao, V.M. (1969), 'Two Decompositions of Concentration Ratio', *Journal of the Royal Statistical Society, Series A*, **132**, 418–25.
- Shorrocks, A.F. (1980), 'The Class of Additively Decomposable Inequality Measures', *Econometrica*, **48**, 613–25.
- Tsui, K. (1999), 'Multidimensional Inequality and Multidimensional Generalized Entropy Measures: An Axiomatic Derivation', *Social Choice and Welfare*, **16**, 145–57.
- Van Doorslaer, E. and Van Ourti, T. (2011), 'Measuring Inequality and Inequity in Health and Health Care', in Glied, S. and Smith, P.C. (eds), *The Oxford Handbook of Health Economics*. Oxford University Press, Oxford; 837–69.
- Van Ourti, T., Van Doorslaer, E. and Koolman, X. (2009), 'The Effect of Income Growth and Inequality on Health Inequality: Theory and Empirical Evidence from the European Panel', *Journal of Health Economics*, **28**, 525–39.
- Van Ourti T., Erreygers, G. and Clarke, P. (2014), 'Measuring Equality and Equity in Health and Health Care', in Culyer, A.J. (ed.), *Encyclopedia of Health Economics*. Elsevier, Amsterdam, Vol. **2**, 234–9.
- Wagstaff, A. (2005a), 'The Bounds of the Concentration Index When the Variable of Interest is Binary, with an Application to Immunization Inequality', *Health Economics*, **14**, 429–32.
- Wagstaff, A. (2005b), 'Inequality Decomposition and Geographic Targeting with Applications to China and Vietnam', *Health Economics*, **14**, 649–53.
- Wagstaff, A., Van Doorslaer, E. and Watanabe, N. (2003), 'On Decomposing the Causes of Health Sector Inequalities with an Application to Malnutrition Inequalities in Vietnam', *Journal of Econometrics*, **112**, 207–23.

Appendix: Sample Weights and Ties

This appendix describes how the weights w_i must be defined when working with datasets in which not all individuals have the same sample weight, and when there are ties between individuals in the income distribution.

Let us assume that the individuals of our dataset (1, 2, ..., n) are ranked according to their individual income, $y_1 \leq y_2 \leq \dots \leq y_n$. We denote the sample weight of individual i by σ_i , and we assume that $\sum_{i=1}^n \sigma_i = 1$.

If there are ties in the income distribution, we define k groups of individuals G_1, G_2, \dots, G_k such that everyone in group G_j has income y_{G_j} , and that $y_{G_1} < y_{G_2} < \dots < y_{G_k}$. The sample weight of group G_j is $\sigma_{G_j} = \sum_{i \in G_j} \sigma_i$. The cumulative sample weight of group G_j is defined by the recursive formula $\pi_{G_j} = \pi_{G_{j-1}} + \sigma_{G_j}$, where we take $\pi_{G_0} = 0$ and $j = 1, \dots, k$.

A.1. Rank-Dependent Weights

Let individual i belong to group G_j . Then the weight of this individual is equal to

$$w_i^R = n\sigma_i(\pi_{G_{j-1}} + \pi_{G_j} - 1). \quad (A1)$$

Working out the terms between brackets, we obtain

$$w_i^R = n\sigma_i \left[\sigma_{G_j} + 2 \sum_{l=0}^{j-1} \sigma_{G_l} - 1 \right]. \quad (A2)$$

A.2. Level-Dependent Weights

When working with level-dependent weights, there is no need to consider group weights. The weight of individual i is equal to

$$w_i^L = n\sigma_i \frac{y_i - \mu_y}{\mu_y}. \quad (A3)$$