DEPARTEMENT BEDRIJFSECONOMIE

VALUING THE TIMING FLEXIBILITY IN REAL OPTIONS

by

Danny CASSIMON
Jürgen VANDENBROUCKE

WORKING PAPER
96-231
July 1996

Any comments and/or suggestions can be sent to
fte.cassimon.d@alpha.ufsia.ac.be (tel. ++32/3/220.41.37) and
dbe.vandenbroucke.j@alpha.ufsia.ac.be (tel. ++32/3/220.40.44)

D/1996/2263/5

CONTENTS

INTRODUCTION

In recent years, the economic and financial theory of investment has offered us a more dynamic framework to evaluate investment projects. A new evaluation method, referred to as "real option analysis", improves classical methods in two perspectives. First, it takes into account the qualitative characteristics of the economic environment and second real option analysis values the strategic properties of the investment project itself.

Given the market conditions and the strategic characteristics of the project, an investment opportunity can be considered as an option: the right but not the obligation to buy some asset at some future time. In this paper we will focus on a real option to defer where the option value of the investment, also called the value of waiting, becomes worthless once the decision to invest is made. It will be shown that, in short, the present value of the investment need not only cover the initial expenditure but the opportunity cost as well. Subsequently, we show how this opportunity cost can be calculated, presenting different valuation models.

The structure of this paper is as follows. In the first section we give a short overview of the standard criterion of investment evaluation. We especially point out the growing relevance of uncertainty and irreversibility. The second section introduces the term 'real options' and their relation with financial option theory. The third section of this paper deals with the valuation of real options. We focus on the difference between two valuation formulas often used in the literature and then compare these option valuations with the results obtained using a quadratic approximation method. The relevance of the valuation differences with respect to the investment decision is discussed in section four. It is also shown that the choice of the appropriate valuation technique is dependent on the time to delay. A summarizing conclusion ends the paper.

1. INVESTMENT EVALUATION

Since firms have scarce resources, only the most profitable investment opportunities will be exercised. To be able to solve this optimization problem, investment theory developed several criteria to discriminate between 'good' and 'bad' projects.

One of the most frequently used evaluation criteria is known as the net present value rule. The net present value of an investment in financial or real assets is defined as the difference between the present value of all future incremental cash flows and the initial investment expenditure. In short:

$$NPV = \sum_{t=0}^{N} \frac{P_t}{(1+R)^t} - I$$

with

N = life time of the financial asset or the investment project
R = expected return on a financial asset with identical risk[1]
I = initial investment expenditure
P_t= expected incremental cash flow in period t

The net present value of financial assets should always be zero. If the net present value were positive, the financial asset would yield a higher return than its risk-equivalent. Arbitrage operations (greater demand) will increase the price of the financial asset until it reaches the expected value of its payoffs [2].

The future cash flows of an investment project should also be discounted at the expected rate of return on a financial asset with equal risk properties [3]. The calculated

---

[1] The Capital Asset Pricing Model states that the expected return on a financial asset equals the riskless rate of return augmented with a risk premium. This risk premium remunerates the investor for the non-diversable risk (market risk) inherent in the financial asset and depends on the sensitivity of the return on the asset to market movements.

[2] This reasoning is based on the assumption of efficient capital markets (financial instruments which are substitutes should have the same price) and risk-averse investors (investors prefer a higher return given a certain risk and a lower risk given a certain return).

[3] In practice, R is more often calculated as the Weighted Average Cost of Capital:

$$R = r_E \frac{E}{W} + (1-t) r_D \frac{D}{W}$$

where E = market value of the firm's equity, D = the market value of debt, W = D+E = total value of the firm, r_E = the cost of equity, r_D = the firm's current borrowing rate and t = the marginal corporate income tax rate. This measure of d is only correct to evaluate projects that will be financed according to the current composition of the firm's liabilities. That is, projects that will not change the business risk of the firm (see Brealey and Myers [1991] pp. 465-470 for more details).

net present value of an investment indicates the immediate monetary profit of the project after repayment of both the interest costs and the invested capital. According to this definition, investment is only justified if the net present value exceeds zero. The definition also reflects the economic relevance of the net present value rule. A firm facing positive net present value investment opportunities can exploit certain (temporal) economic and strategic advantages resulting in economic rents (excess returns) and is able to beat the capital market.

However, since discounting methods such as the net present value rule were first developed for the valuation of bonds and stocks, some crucial aspects of investment decision-making are neglected. Specifically, these methods ignore the value of management [Brealey and Myers, 1991, p.513]. Investors in capital markets are necessarily passive since they cannot influence interest rates or the dividend policy of the firm they invested in. Managers facing investment opportunities however, hold the option or the right to invest. These 'real options' are especially valuable when there is uncertainty[4] about the future. The net present value rule fails to quantify these additional benefits.

Furthermore, the net present value rule generally assumes that future cash flows of an investment are known with certainty. Financial managers then base their decision to invest on a simple calculation: discount future net operational cash flows at the appropriate discount rate and substract the initial investment expenditure. But how should a firm, facing uncertainty over future market conditions, decide whether to invest ? Much of these dynamics of the competitive economic environment can be incorporated in the investment evaluation by the use of option techniques.

The net present value rule is also based on the implicit assumption that either the investment is reversible or, if the investment is irreversible, it is a now or never proposition. Investment is often irreversible: once installed, capital has little or no value unless used in production [Bertola and Caballero, 1994, p.223]. Investments in research and development, marketing or advertising are other examples of firm-specific expenditures. However, most of these investment decisions need not to be taken at a predetermined moment. Managers examine both the position of the firm and the market conditions to motivate certain expenditures, especially when these expenses are lagerly irreversible and are to be justified on a qualitative basis. The possibility to delay a decision makes the dynamics of investment very sensitive to expectations about the future [Bernanke, 1983, p.86]. The investment problem involves comparing the value of the project today with the value of the project at all possible times in the future [McDonald and Siegel, 1986, p.707]. Since every project competes with itself

[4] All parameters determining the investment decision can be assumed to be uncertain in the future. Ross [1995] for example studies optionality of investment opportunities under a fluctuating interest rate. We will focus on uncertainty about the present value in general.

in succesive periods, managers have to choose between a range of mutually exclusive alternatives. The strategic value of this flexibility is ignored when using the net present value rule.

2. LINK BETWEEN REAL AND FINANCIAL OPTIONS

Investment decisions can be seen as exercising an option contract. An option is defined as the right, not the obligation, to buy (call option) or sell (put option) an asset at an agreed price during a specific period (American option) or at a predetermined future point in time (European option).

First we give an overview of some possible types of real options. Consecutively, we review the analogy between real and financial options.

2.1. Possible real options

We can distinguish between five broad types of real options (Kemna ,1989, pp.358-360).

1. *Option-to-defer*: if the firm has some flexiblity to delay the investment decision, management has to determine the optimal timing of the investment. Should we invest now or wait until more information is availabe so that the investment decision can be made under less uncertainty ? The investment project itself can be seen as a call option on a dividend-paying stock. While keeping the option alive, the firm foregoes every cash flow. If the option is exercised at the cost I, the firm is entitled to all future cash flows with present value V.

Apart from the net present value of the project, the decision to invest should also incorporate the possiblity of more interesting market conditions in the future. In this way, the option value of the project can be seen as the value of waiting. If the option is exercised, its value drops to zero since the firm no longer holds the opportunity to invest. The option value (F) can be seen as the opportunity cost associated with the decision to invest now. This means that immediate investment is only justified when the NPV exceeds the option value, or $V\text{-}I>F$.

2. *Growth-option*: if a certain project is necessary to make future investment possible, this current project embodies some option characteristics that must be incorporated in its evaluation. The value of the (call) option on future investment opportunities should be added to the net present value of the initial project. This could lead to a different

investment decision than the one based solely on the net present value rule. Many expenditures associated with research programs should be viewed in this way, since these programs create the option on possible production and sales in the future.

A special case of this growth option are investment projects that consist of different sequential stages where the completion of a previous stage is a necessary, but not sufficient, condition to start the next stage. This investment problem can be value as a *compound option* or the option on an option, since in fact each stage, except the last, represents a growth option.

3. *Scale-option*: it could be possible that the investing firm has the option to complete the project at two different scales. Apart from a minimum-scale, the firm can anticipate a growing future demand by investing on a larger scale, e.g. higher production capacity, with additional costs. If future demand does rise, the firm can take advantage of this market condition more rapidly and faces lower adjustment costs than its competitives. This possibility must be incorporated as a call option in the present evaluation of the project. As indicated, the value of this call option is largely dependent on the expected changes of demand.

4. *Switch-option*: if a project creates the flexibility to switch between inputs or outputs due to a change in relative prices or market demand, than this possibility should be valued as a call option with the additional cost as the exercise price. A good example in this context is an electricity plant where it is technically possible to swich the energy source from coal to gas at low adjustment costs. Again, the value of the option is determined by the expectations about future market conditions.

5. *Stop-option*: in industries with a fast moving technology, it could be extremely valuable to be able to sell or get rid of a project which has become worthless in terms of net present value. The decision whether or not to leave the project should be based on the value of the put option with the resale value as the exercise price[5].

It should be clear that there can be several kinds of real options within a given investment opportunity. Consider for example a situation where management has to decide whether to expand a given project, sell it or delay the decision and keep production at the current level till the time the decision is made. It is most important to discover the real options and relate them to the investment decision appropriately.

---

[5] Here we assume that the value at which the (uncompleted) project can be sold stays the same over time.

2.2. The analogy between real and financial options

The link between real and financial options can best be explained using an option-to-defer. In this situation, the investing firm has the option to time the start of the whole project. While waiting, the firm is not entitled to any payoffs but can gather more information about the project itself or the market conditions in order to reduce the uncertainty under which the decision must be taken. If the project is started, the option is killed since the firm has given up the possibility to wait. This situation looks very much like a call option on a dividend-paying stock. The six variables determining the value of both the real and financial option are discussed below (see also Table 1).

Table 1. Link between real and financial options.

financial option	**real option**
stock price	present value of all future incremental cash flows
exercise price	present value of the investment expenditure(s)
volatility of the stock price	volatility of the present value of all future incremental cash flows
time to maturity	time to delay the decision
risk-free interest rate (return on a bond with the same maturity)	risk-free interest rate (return on a bond with the same maturity)
dividend	net operational cash flow per period till maturity of the real option

Source: Kemna [1989]

1. *Present value of all future incremental cash flows*: the net benefit from exercising the option is given by the difference between the present value of all future incremental cash flows and the present value of the investment expenditure(s). The value of the option increases as the stock price or the present value of all future incremental cash flows increases.

2. *Present value of the investment expenditure(s)*: because of the same reason mentioned above, the value of the option increases as the present value of the investment expenditure(s) decreases.

3. *Volatility of the present value of all future incremental cash flows*: an increase in the volatility raises the probability of high stock prices and leads to a higher option price. The probability of lower stock prices is of less importance because the value of the option can never be negative[6]. This is one of the appealing properties of options: the potential profit is unrestricted but the possible loss is limited to the initial price of the option.

4. *Time to maturity - time to delay*: to determine the influence of the duration of the option on its value, we have to distinguish between European and American options.

In the case of American options, the option is more valuable as the time till expiration increases since this raises the probability of high stock prices. However, this relation could be disturbed if high dividends are paid. Dividend payments decrease the value of the stock price, and thus the option, because the holder of the option is not entitled to them. With European options, it is even impossible to predict the influence of the time till expiration on the option value. Since European options can only be exercised at the end of their maturity, stock price movement before this point in time are irrelevant to determine the benefits that will be realised at the end of the contract.

5. *Risk-free interest rate*: the influence of the interest rate on the value of the option can best be understood by considering the option as a credit allowance. The holder of the option may already have decided to buy the stock at the maturity of the option, but he chooses to postpone the actual transaction. If he has to borrow to be able to buy the stock it is obvious that the postponement, and thus the option, is extremely valuable when current interest rates are high.

6. *Dividend - net operational cash flow per period until expiration*: dividend payments result in a lower stock price and decrease the option value. High or regular dividend payments lead to a low option price and indicate that it is more interesting to hold the stock than the option. In terms of real options, the value of waiting no longer exceeds the net present value of the investment.

---

[6] Should the value of the option be negative, one could 'buy' the option and hold it until maturiy. This would lead to riskless profits which will be eliminated by arbitrage transactions.

3. VALUING REAL OPTIONS

In this paper we will focus on three valuation formulas. First we introduce the theoretical framework developed by Merton (1973) and Black and Scholes (1973) to value European call options on a dividend-paying asset. The method used by, amongst others, Dixit and Pindyck (1994) is based on the same theoretical underpinnings, but values the option to defer as a perpetual American call. We confront these two valuation methods with the quadratic approximation of Barone-Adesi and Whaley (1987) to value American-type options.

We will use an investment project with present value V and an initial expenditure I to develop the formulas with the NPV = V-I. The investment decision can be delayed for a period (T-t) where t is the moment of evaluation and T the expiration date of the real option. We note the risk-free interest rate as i.

As is usually the case in contingent claims analysis[7], the price of the underlying asset follows a geometric Brownian motion. Since the underlying asset of our real option to defer is the present value of the project, we assume that

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where α is the expected capital gain and σ the volatility of the present value. The Wiener proces dz can also be written as $dz = \eta\sqrt{dt}$ with η a random drawing from a standardized normal distribution. The instantaneous expected drift and variance rate of this geometric Brownian motion are αV and $\sigma^2 V^2$ respectively.

3.1. The Black and Scholes formulation

To derive the differential equation which gives rise to the Black and Scholes option formula of F(V,t), we first introduce what is called Itô's lemma.

Itô's lemma is easiest to understand as a Taylor series expansion with the assumption that terms of the third or a higher degree vanish in the limit [Hull, 1993, p.243]. The

---

[7] In the case of financial options the underlying value is a tradeable asset, which is usually not true for real options. This undermines the basic idea of option analysis. For option theory to apply, the assumption is made that there exist tradeable assets which can be used to duplicate the present value of the investment opportunity. This is analogous to the hypothesis of the net present value rule that the expected cash flows can be discounted at a rate which equals the return on a financial asset with identical risk [Kemna, 1989, p.357].

lemma states that the differential of a function F of V (which follows a geometric Brownian motion) and t is:

$$dF(V,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial V}dV + \frac{1}{2}\frac{\partial^2 F}{\partial V^2}(dV)^2 \tag{2}$$

when substituting (1) in (2) we get

$$dF(V,t) = \left[\frac{\partial F}{\partial t} + \alpha V\frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V^2\frac{\partial^2 F}{\partial V^2}\right]dt + \sigma V\frac{\partial F}{\partial V}dz \tag{3}$$

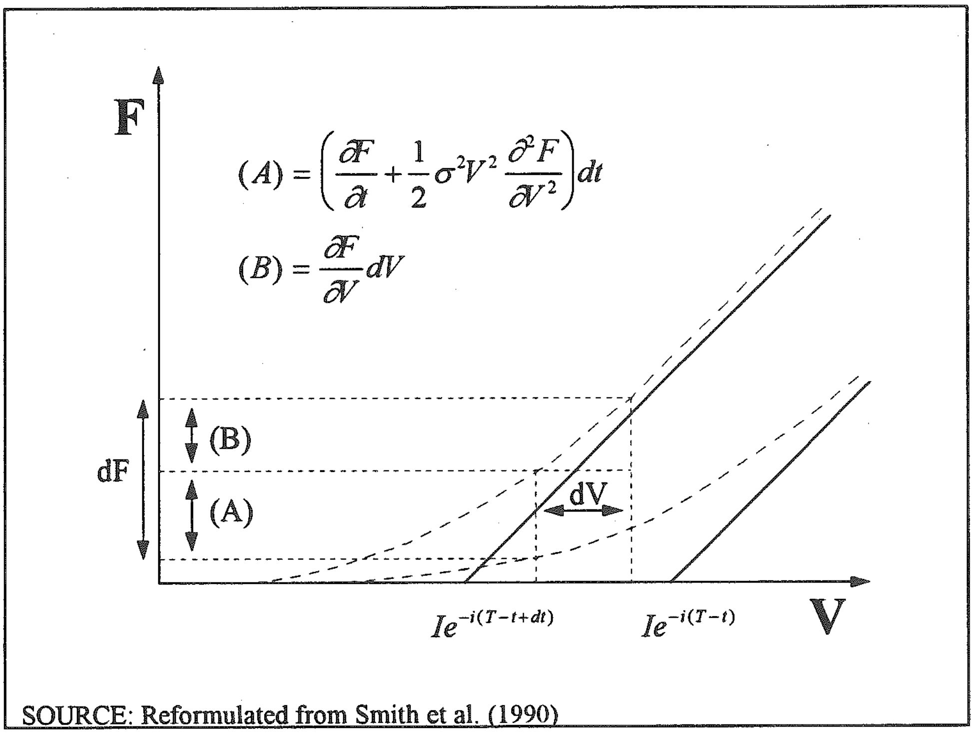

Figure 1. The change in price of a call on a non-dividend-paying stock due to an increase in present value and time to maturity[8].

In figure 1, we find a graphical illustration of equation (2) with $(dV)^2 = \sigma^2 V^2 dt$. Note that the change of the option value (A) due to the increase in time to maturity is deterministic since this change is independent of the stochastic process dV. The stochastic part of dF is represented by (B).

[8] Using a simple arbitrage model it can be shown that max [0 , $V\text{-}Ie^{-i(T-t)}$] is the lower boundary for the value of a call F with exercise price I and time to maturity (T-t) on a non-dividend paying asset V [Hull, 1993, p.156].

Now consider the portfolio consisting of one option and selling short ${\partial F}/{\partial V}$ projects[9]. The change in the value of this portfolio (dΦ) over a small interval of time equals:

$$d\Phi = dF - \frac{\partial F}{\partial V} dV$$

Using Itô's lemma, we can rewrite this expression as:

$$d\Phi = \frac{\partial F}{\partial t} dt + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} dt \tag{4}$$

By the specific construction of the portfolio, dΦ is independent of the stochastic term mentioned in figure 1. The portfolio is now deterministic or 'riskless'. Hedging is only possible because both the option value and the value of the underlying asset are influenced by the same source of uncertainty (dz) [Eales, 1995, p.148].

For holding this portfolio, we must remunerate the holder of the output at a rate of δ per time period, the portion of the return on the project not reflected in the change of the present value [Ingersoll, 1982, p.540]. Otherwise no rational investor would be willing to enter into the long side of the transaction. This δ is generally called the convenience yield. If δ is not sufficiently positive, an American call option would never be exercised before the expiry date. If δ approaches infinity, holding the present value of the investment is more interesting than holding the opportunity to invest. The option value then tends to zero and the net present value rule applies. Boundaries for the convenience yield are $0 < \delta < \infty$.

We can write the total welfare change of the portfolio holder as the change in the portfolio given by (4) minus the payment for the short position in the underlying assets:

$$\frac{\partial F}{\partial t} dt + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} dt - \delta V \frac{\partial F}{\partial V} dt$$

Since the chosen amount of V made the portfolio riskless, this welfare change must equal the riskless return $i\Phi dt$. Rearranging terms, we get the differential equation which F(V,t) must satisfy.

$$\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + (i - \delta) V \frac{\partial F}{\partial V} = iF \tag{5}$$

[9] It can be shown theoretically that this amount of underlying assets makes the portfolio riskless. If V changes, the gain or loss in the option value is totally offset by the loss or gain caused by the short position in the project.

The solution to this differential equation yields the function of a derivative security on a dividend-paying asset if the appropriate boundary conditions are imposed.

Consider an European call on a dividend-paying stock. From finance theory we know that $F(V,t) = \max[0, e^{-\delta(T-t)} V - I]$; (T-t) is the time to maturity. Solving the differential equation for this boundary condition, we find the Black and Scholes formula shown below. N() indicates the cumulative normal distribution function [Hull, 1993, p.224].

$$F(V,t) = Ve^{-\delta(T-t)}N(d_1) - Ie^{-i(T-t)}N(d_2)$$

with

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(i - \delta + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

3.2. The Dixit and Pindyck formulation[10]

Dixit and Pindyck [1994] consider the option on an investment project as an American call on a dividend-paying stock. The Black and Scholes formula derived in section 3.1. values an European call on a dividend-paying asset. From financial theory, we know that the European call will never be worth more than its American counterpart. This means that the value of waiting will be lower when the decision to invest can only be made at a certain time T.

Authors such as McDonald and Siegel [1985], Kemna [1987,1993] or Lint [1992a, 1992b] use the Black and Scholes formula to value real options. Majd and Pindyck [1987, 1989], Pindyck [1991], Dixit [1992], Dixit and Pindyck [1994], Teisberg [1993, 1994] and Mauer and Triantis [1994] allow the investment decision to take place at any moment within the period (T-t).

The differential equation they use to value this American call is also based on dynamic portfolio analysis (see Dixit and Pindyck [1994] p.151 and p.183). To be able to solve the equation analytically, they assume that the value of waiting is independent of the

[10] We take Dixit and Pindyck as the seminal reference because their text book "Investment under Uncertainty"[1994] gives an overview of recent advances in real option analysis.

period (T-t) or $\partial F/\partial t = 0$. Using this assumption, the original differential equation becomes[11]:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + (i-\delta)V\frac{\partial F}{\partial V} - iV = 0$$

However, this simplification can be questioned from different perspectives. In the traditional framework, a positive net present value can be obtained by exploiting market imperfections and competitive advantages. In real option analysis, the time to maturity should also be determined as the time it takes for competitors to enter the market. As such, waiting infinitely is irrealistic. From finance theory, we know that the value of an American call increases when the time to maturity increases. In the Dixit-Pindyck formulation however, the time derivative implicitely equals zero which means that the value of waiting to invest is independent of the competitive advantage the firm possesses.

Moreover, in the specific context of real options on investments, there is an even more fundamental reason why the time derivative should be included. Apart from the uncertainty about the future cash flows, which can be eliminated using dynamic portfolio analysis, the firm is confronted with another source of uncertainty aswell. Because the time to maturity of the option depends on the flexibility and inventivity of competitors, the duration of the competitive advantage is largely determined by the market structure [Smit and Ankum, 1993]. In fact, the option has an uncertain time to maturity [12]. The possible period of delay is inversely related to the strength of the competition the firm faces. In a strongly competitive industry, the firm holding the option to invest will not be left with much time to think unless it is possible to keep the information private[13]. The estimation of (T-t) will be small and since the option is American this leads to a lower valuation. The lower the option value, the more the investment decision based on real option analysis will coincide with the one based on the net present value rule. According to the net present value rule, firms invest if V-I>0 instead of V-I>F[14], inducing higher investment spending. The findings of Smit and Ankum [1993] support these arguements, since "...in comparison to perfect competition, there is a stronger tendency to postpone projects under monopoly..."

[11] Note that this differential equation is identical to the Black-Scholes differential equation (5), except for the time derivative.

[12] Referring to figure 1, this means that the position of the straight line that determines the asymptotic behaviour of the curve can not be specified.

[13] One could also argue that strong competition is an indication of the difficulty to keep strategic information private.

[14] An in-the-money option is one that would lead to a positive cash flow to the holder if it would be exercised immediately (Hull, 1993, p.140). According to this definition, a project with a positive net present value is the same as an in-the-money Amercian call option. since its intrisic value is positive (V>I). However, because of the time value of the option, that is F-max[V-I,0], it is often optimal not to exercise an in-the-money American call option immediately.

(p.246) and "...postponement under perfect competition implies loss in the expected value of the project due to anticipated competitive entry..." (p.249).

For now, let us consider the Dixit-Pindyck differential equation from the previous page. The general solution can be written as (see Chiang [1984]):

$$F(V,t) = A_1V^{\beta_1} + A_2V^{\beta_2}$$

where β_1 and β_2 are respectively the positive and negative root of the quadratic equation given below.

$$\frac{1}{2}\sigma^2\beta(\beta-1)+(i-\delta)\beta-i=0$$

To find the specific solution, financial theory imposes three boundary conditions. First, the option is worthless if the present value equals zero: F(0)=0. To assure that this condition is satisfied, we must set the coefficient of the negative power of V equal to zero, so $A_2 = 0$. With this restriction we are already able to write the specific solution as $F(V) = A_1V^{\beta_1}$. To determine the critical present value, we formulate two other boundary conditions: the 'value matching' and 'smooth pasting' condition. These state that $F(V^*) = V^* - I$ and $F'(V^*) = 1$ respectively. At the critical level V^*, the value of waiting equals the net present value of the project and the probability that the option will be exercised is then one[15].

These boundary conditions produce a simultanuous system of two equations and two unknowns: V^* and A_1. Solving the simultanuous system yields:

$$V^* = \frac{\beta_1}{\beta_1 - 1}I > I$$ [16]

with

$$\beta_1 = \frac{1}{2} - \frac{i-\delta}{\sigma^2} + \sqrt{\left[\frac{i-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2i}{\sigma^2}}$$

and

$$A_1 = \left(\frac{\beta_1 - 1}{I}\right)^{\beta_1 - 1}\left(\frac{1}{\beta_1}\right)^{\beta_1}$$

[15] The derivative F'(V) indicates the probability that the option will be exercised at a present value level V (Haugen, 1986, blz. 363).

[16] It can be shown that $\beta_1 > 1$ as long as $\delta > 0$.

3.3. A quadratic approximation

In this point we briefly discuss a quadratic approximation method for American option values used by Barone-Adesi and Whaley (1987). Their approach is also based on the Black-Scholes differential equation and their approximation results in a solution which is similar to the one we obtained by the Dixit-Pindyck formulation. The only drawback of the method for a real option-application is that the present value V^* at which immediate investment is justified can only be obtained by iteration. However, their approximation explicitely includes the expiration date of the option, which we argued to be of major importance.

As we stated before, the Black-Scholes differential equation must hold for any derivative on a dividend-paying asset. Depending upon the boundary conditions that are imposed, one can derive the valuation formula for an European call option (Black-Scholes) or an American call option (no analytical solution). The key insight into the quadratic approximation approach is that the differential equation also applies to the early exercise premium of the American call option. If we note this early exercise premium as ε (V,t) and write the value of the American and European call option as F(V,t) and f(V,t) respectively, this means that

$$\varepsilon(V,t) = F(V,t) - f(V,t)$$

and

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 \varepsilon}{\partial V^2} + (i-\delta)V\frac{\partial \varepsilon}{\partial V} + \frac{\partial \varepsilon}{\partial t} = i\varepsilon \qquad (6)$$

The early exercise premium is then defined as

$$\varepsilon(V,t) = (1 - e^{-i(T-t)})h(V,t) \qquad (7)$$

With no possibility to delay the investment decision, the time premium equals zero: ε(V,T)=0. For now, let $g = 1 - e^{-i(T-t)}$. As a function of (T-t) g is strictly concave as long as i>0. Substituting (7) in the original differential equation (6) and rearranging terms, we get

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 h}{\partial V^2} + (i-\delta)V\frac{\partial h}{\partial V} - \frac{i}{g}h - i[1-g]\frac{\partial h}{\partial g} = 0 \qquad (8)$$

Up to this point, the analysis has been exact. Barone-Adesi and Whaley (1987) now make the approximating assumption that the last term equals zero. For options with very short times to expiration $\partial h / \partial g$ approaches 0. With a long period of possible delay g = 1 and the last term in equation (8) disappears [Barone-Adesi and Whaley, 1987, p. 306]. As we will see, these assumptions are only correct as long as δ > 0, which was also the condition for the American option to be exercised before expiration. Taking the approximation into account, the differential equation simplifies into

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 h}{\partial V^2} + (i-\delta)V\frac{\partial h}{\partial V} - \frac{i}{g}h = 0$$

This differential equation is quite similar to the one we encountered in the Dixit-Pindyck method. Building upon the arguments we used in point 3.2., the solution can be written as

$$h(V,t) = A_1' V^{\beta_1'}$$

with

$$\beta_1' = \frac{1}{2} - \frac{i-\delta}{\sigma^2} + \sqrt{\left[\frac{i-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2i}{\sigma^2\left[1-e^{-i(T-t)}\right]}}$$

and

$$A_1' = \frac{V^*}{\beta_1'}\left\{ 1 - e^{-\delta(T-t)} N\left[d_1(V^*)\right] \right\}$$

The value of the American call option can now be calculated as follows

$$F(V,t) = f(V,t) + \frac{A_1' V^{\beta_1'}}{1-e^{-i(T-t)}}$$

where f(V,t) is determined by the Black-Scholes formula. Using the boundary conditions imposed by option theory that the option value should equal the net present value and the slope of the option should equal one (probability of exercise) at the critical price level V^*, Barone-Adesi and Whaley (1987) show that

$$V^* - I = f(V^*,t) + \frac{V^*}{\beta_1'}\left\{ 1 - e^{-\delta(T-t)} N\left[d\ (V^*)\right] \right\} \qquad (9)$$

The last term in the right-hand side of equation (9) is strictly positive as long as $\delta > 0$. If not, the American option will never be exercised early, and the investment decision has to be made using the Black-Scholes formulation for an European call.

Using this approximation method, aswell as the valuation formulas we discussed earlier, the following figures plot the values of the option as a function of V [17]. Figures 2a and 2b only differ with respect to the assumed period of possible delay: T-t = 1 and T-t = 3 respectively.

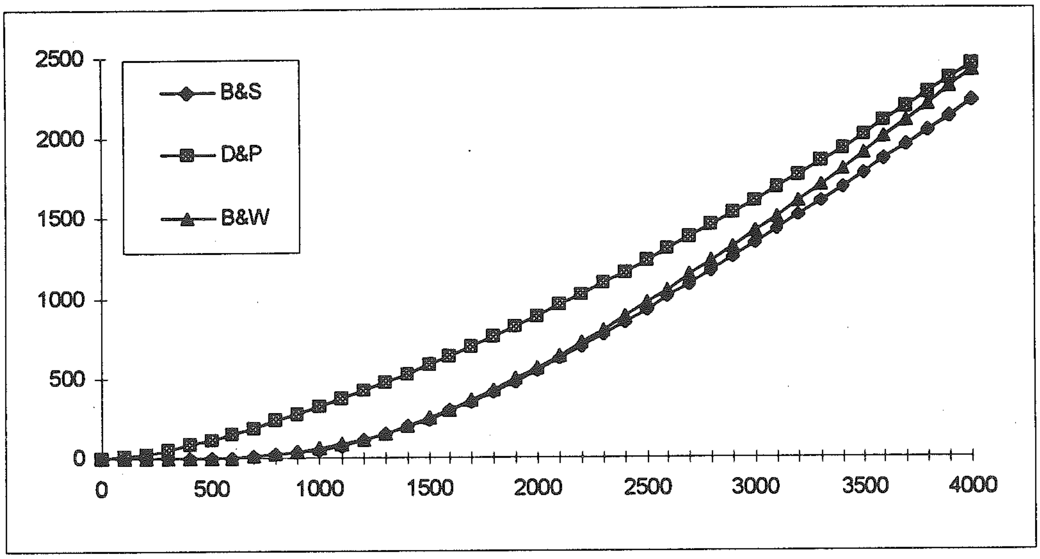

Figure 2a. Option values as a function of V.
Parameters: I = 1600, T-t = 1, $\delta = 0.09$ and $\sigma = 0.5$

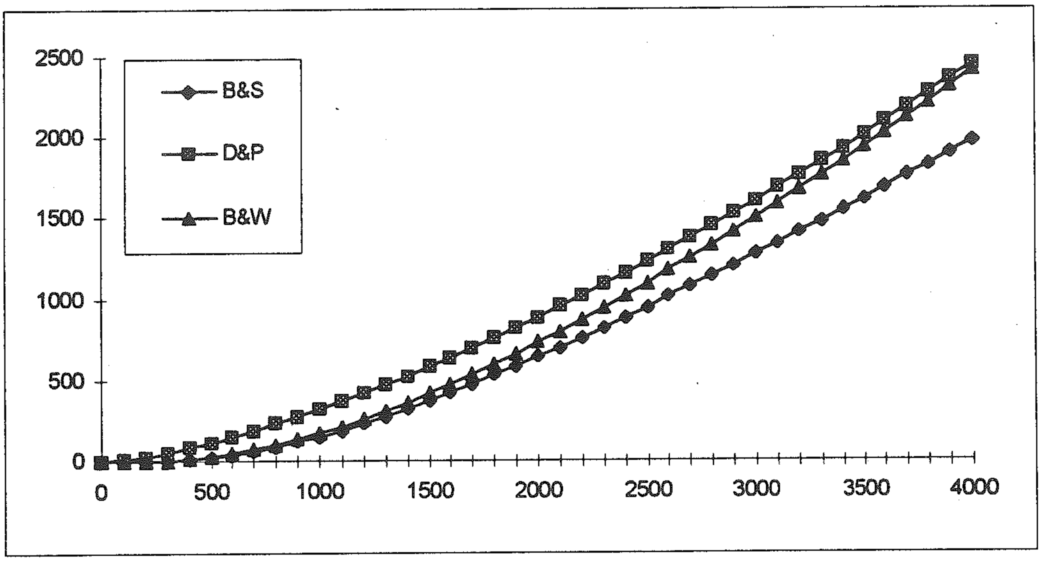

Figure 2b. Option values as a function of V.
Parameters: I = 1600, T-t = 3, $\delta = 0.09$ and $\sigma = 0.5$

[17] B&S= option valuation according to Black and Scholes ; D&P = option valuation according to Dixit and Pindyck ; B&W= approximation according to Barone-Adesi and Whaley.

Figures 2a and 2b bring forward some obvious results. First, because of the early exercise premium, the value of the European call lies below the value of both the American calls for each present value level. Second, the option value according to Dixit and Pindyck is always the highest and remains unchanged in both figures since the partial derivative of the option value with respect to time is assumed to be zero.

However, what is more important for our discussion, is the difference between the approximated option value and the value according to Dixit and Pindyck. This difference becomes smaller if we assume a longer period of possible delay (T-t). Which is what we expected since the formula of Dixit and Pindyck actually values a perpetual call option. The fact that the difference between the perpetual and the approximated option value is especially large when (T-t) is small supports our argument that the investment rule based on D&P postpones investment too long in a competitive market.

4. THE DECISION TO INVEST

The decision to invest is based on the comparison between the net present value and the option value of the investment project. In order to invest, the net present value should not exceed zero, but should exceed the opportunity cost of killing the option instead.

In the subsections that follow we compare the investment decisions based on the different valuation methods we discussed. We especially concentrate on the relevance of the time to delay.

4.1. Comparing the results of different valuation techniques

When evaluating investments, including optionality is unavoidable but the way real option analysis should be integrated in the decision to invest depends on the specific properties of the project.

Management has to determine if the real option inherent in the investment project is American rather than European. If the option to invest in a certain project depends on an exploration phase, e.g. drills in order to exploit an oil field [Kemna, 1993, p.260], than the option can only be exercised when this phase is completed. As such, the investment opportunity should be viewed as an European option. On the other hand, it is also possible that the firm facing the option does not have to make such preparations. In this case the time until maturity of the option is used to gather information, which will enable the firm to make the investment decision under less uncertainty. Since under this scenario the project can be started at any moment, the valuation formula for an American call should be used.

In figures 3a and 3b the critical level V^* , i.e. the present value for which the net present value of the project equals the option value, is shown as a function of the level of uncertainty.

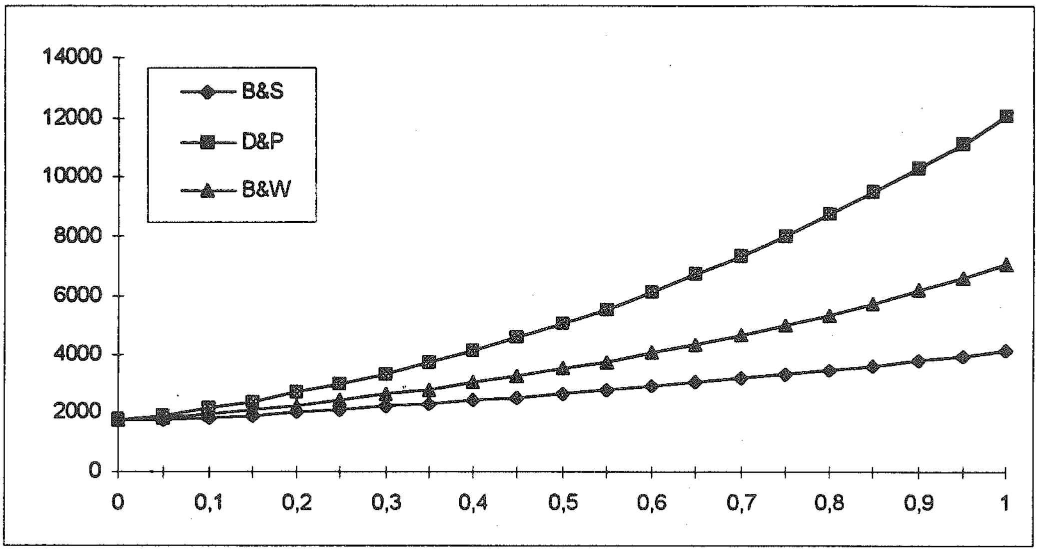

Figure 3a. The critical V^* as a function of the level of uncertainty.
Parameters: I = 1600, T-t = 1 and δ = 0,09

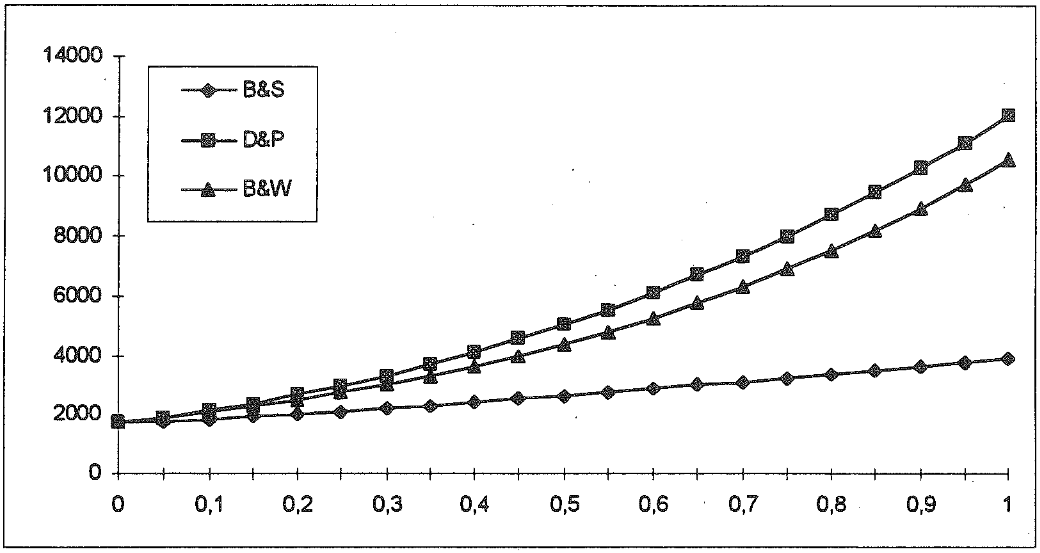

Figure 3b. The critical V^* as a function of the level of uncertainty.
Parameters: I = 1600, T-t = 3 and δ = 0,09

We first notice that the Dixit and Pindyck method always results in the highest V^*, leading to the longest postponement of the decision to invest. This is explained by the fact that the formula of Dixit and Pindyck fails to capture the price effect of short periods of possible delay. Second, since a higher volatility leads to a higher option value, an increase in uncertainty reduces the incentive to invest. The critical price according to Dixit and Pindyck however is more sensitive to changes in the level of uncertainty, especially when (T-t) is small. Again, because the time variable is omitted in the model of Dixit and Pindyck uncertainty is accumulated over a much longer period.

Since the difference between the valuation methods is largely due to this time to delay effect, we study the impact of the period of possible delay more closely in the next section.

4.2. Relevance of time to delay

The Dixit-Pindyck formulation is derived under the hypothesis of perpetuality and values a perpetual call option. The firm evaluates the investment project assuming it has a monopoly on the competitive advantage it created, where in fact it has not. Real option analysis based on this investment criterion will result in a critical present value V^*, independent of the period during which future developments can be anticipated. The criterion leads to an overvaluation of the value of waiting, especially in highly competitive markets.

The valuation formula introduced by Barone-Adesi and Whaley (1987) stresses the role of the time premium of the American call option. Consequently, the time to maturity (T-t) influences the value of both the option and the critical level V^*. It is thereby straightforward to show that the present value V^* of equation (9) simplifies to the Dixit-Pindyck formulation if the period of possible delay approaches infinity.

First, we note that $\lim_{T-t\to\infty} \beta_1' = \beta_1$. If the perpetual option can only be exercised at the expiration date, the foregone earnings during this period make the investment opportunity worthless: $\lim_{T-t\to\infty} f(V^* t) = \lim_{T-t\to\infty} \max[\ 0, e^{-\delta(T-t)}V^* - I\ ] = 0$. Substituting in equation (9) yields

$$V^* - I = \lim_{T-t\to\infty}\left[f(V^*,t) + \frac{V^*}{\beta_1'}\left\{ 1 - e^{-\delta(T-t)} N\left[d_1(V^*)\right] \right\}\right]$$

$$\Leftrightarrow V^* - I = \frac{V^*}{\beta_1} \quad \text{if } \delta > 0$$

$$\Leftrightarrow \; V^{*} = \frac{\beta_1}{\beta_1 - 1} I > \mathrm{I}$$

This again emphasizes the irrealistic optimism of the Dixit-Pindyck criterion.

Another appealing property of the critical value based on equation (9) is that it equals the net present value rule when there exists no possibility to postpone the investment decision. We know that $\lim_{T-t \to 0} N[d_1(V^*)] = 1$ since $V^* \geq I$. Further more, because of the lack of time value the European option f(V,t) is only worth its intrisic value max[0, V^* - I].

$$V^* - I = \lim_{T-t \to 0} \left[f(V^*,t) + \frac{V^*}{\beta_1^{'}} \{ \; 1 - e^{-\delta(T-t)} N[d_1(V^*)] \; \} \right]$$

$$\Leftrightarrow \; V^* - I = \max[\; 0, V^* - I \;]$$

The lowest value V^* that satisfies this equation is I, the trigger value given by the net present value criterion. A graphical representation is given in figure 4.

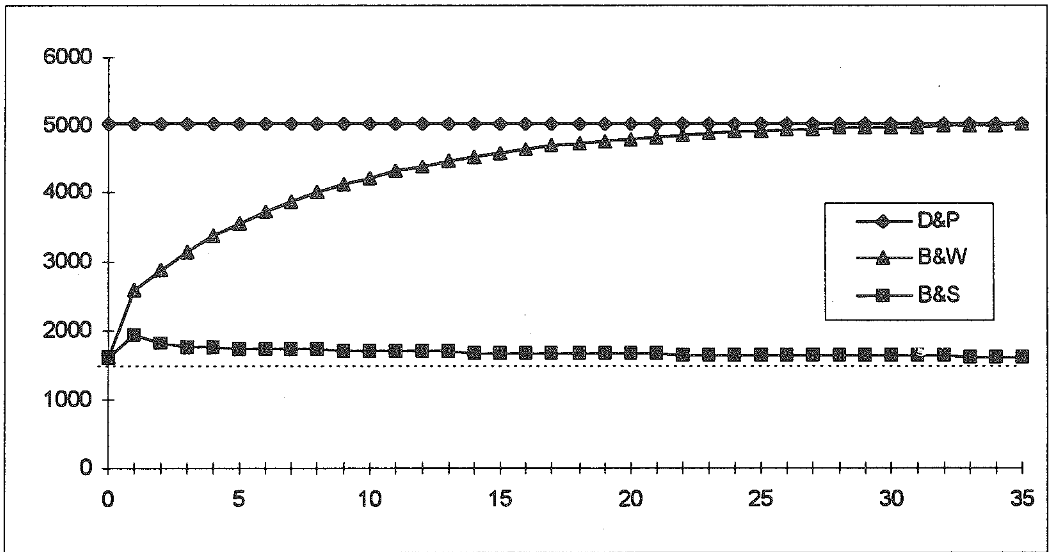

Figure 4. The critical V^* as a function of time to delay [18]

Parameters: I = 1600, δ = 0.09, σ = 0.5

[18] Whether the time to delay is expressed in terms of years, months or days has no effect on the results. Note however, that the paramaters i, δ and σ should be measured accordingly.

The uncertainty about future cash flows and the irreversibility of the investment expenditures drive a widge between the net present value rule and the investment criterion based on real option analysis. According to Dixit and Pindyck (1994) this widge becomes larger (β_1 decreases) as the convenience yield decreases or the uncertainty about future cash flows increases. More uncertainty or a lower rate of foregone earnings if the investment is delayed result in a longer postponement of the decision to invest. Both relations are known from financial option theory and are also valid for the V^* based on the quadratic approximation.

However, under the assumption of perpertuality an increase in σ or a decrease in δ is cumulated over an infinite horizon. Especially when the actual period of possible delay is assumed to be small, e.g. in a highly competitive market, it is very likely that the required V^* will not be attained. The investment decision will be postponed for too long or there may be no investment at all.

The Black-Scholes formulation for European calls is also shown in figure 4. As the net present value rule assumes that investment can only take place at time t, an investment opportunitity of the European-type can only be exercised at time T. Because of the convenience yield, the pay off of the investment will be decreased considerably. Under the hypothesis of perpetuality, the V^* of the European-type investment is equal to the net present value criterion.

4.3. Classifying valuation techniques according to option characteristics

In table 2 we present a classification of the appropriate valuation technique to be used according to two option characteristics, being the type of the option (European or American) and the time to delay (T-t).

Table 2. Classification of the valuation methods.

time to delay / option type	$T\text{-}t = 0$	$0 < T\text{-}t < \infty$	$T\text{-}t = \infty$
European	(B&S simplifies to) NPV - rule	**Black and Scholes**	(B&S simplifies to) NPV - rule
American	(B&W simplifies to) NPV - rule	**Barone-Adesi and Whaley**	(B&W simplifies to) Dixit and Pindyck

Real options of the European-type should be valued using the Black-Scholes formula for options on a dividend-paying asset. As is shown in section 4.2. and figure 4, this formula simplifies to the net present value rule in the extreme situation where there exists no time to delay or when (T-t) approaches infinity.

If a firm has the flexibility to make the investment decision at any moment during the period of possible delay, the critital present value V^* on which this decision is based should be determined according to the approximation of Barone-Adesi and Whaley. Again, the approximation simplifies to the net present value rule when there is no possibility to delay the investment. The critical value V^* used by, amongst others, Dixit and Pindyck, only applies when the time to delay approaches infinity in the formula of Barone-Adesi and Whaley.

CONCLUSION

In this paper we first formulated the important arguments used in real option analysis to modify the traditional net present value investment rule. The role of uncertainty and irreversibility is neglected or undervalued in NPV-evaluations but can be quantified by the use of option techniques. To illustrate the relevance of option characteristics in investment opportunities, we mentioned some possible real options often encountered in practice.

We then concentrated on a real option to defer to study the valuation more closely. When the decision to invest can be postponed but has to be made at a specific moment in the future, the Black-Scholes formula for European call options on a dividend-paying stock can be used. If the investment decision can be made at any time during the period of possible delay, the option to defer need to be valued as an American call option on a dividend-paying stock for which there exists no analytical solution.

Two approximations for the American call option were discussed in this paper. The approximation of, amongst others, Dixit and Pindyck assumes that the option value is not influenced by the period of possible delay i.e. the option is perpetual. By confronting this valuation with the quadratic approximation of Barone-Adesi and Whaley we showed that the assumption of perpetuality leads to an overvaluation of the option value to wait. The decision to invest is postponed longer than necesarry or there may be no investment at all. This is especially the case when the market is competitive and the period of possible delay is small. In short, the quadratic approximation of Barone-Adesi and Whaley is always valid for American-type real options since it summarizes to the formula of Dixit and Pindyck if (T-t) approaches infinity and to the net present value rule when there is no possibility to delay.

References

ABEL, A.B., DIXIT, A.K., EBERLY, J.C. and R.S. PINDYCK (1995), "Options, the value of capital and investment", NBER Working Paper Series n°5227, 38 p.

BARONE-ADESI, G. and R. WHALEY (1987), "Efficient analytic approximation of American option values", Journal of Finance vol.42 n°2, blz. 301-320.

BERNANKE, B.S. (1983), "Irreversibility, uncertainty and cyclical investment", Quarterly Journal of Economics (February), pp.85-106.

BERTOLA, G. and R.J. CABALLERO (1994), "Irreversibility and Aggregate Investment", Review of Economic Studies vol.61, pp.223-246.

BLACK, F. and M. SCHOLES (1973), "The pricing of options and corporate liabilities", Journal of Political Economy vol.81(1) n°3, pp.637-659.

BREALEY, R.A. and S.C. MYERS (1991), "Principles of Corporate Finance - fourth edition", McGraw-Hill Inc., USA, 924 p.

BRENNAN, M.J. and E.S. SCHWARTZ (1985), "Evaluating Natural Resource Investments", Journal of Business vol.58 n°2, pp.135-157.

BUCKLEY, A. (1995), "From Domestic to International Investment with Real Operating Options", Doctoral Dissertation, Amsterdam University Press, pp.324.

CABALLERO, R.J. (1991), "On the Sign of the Investment-Uncertainty Relationship", American Economic Review (March), pp.279-288.

CHIANG, A.C. (1984), "Fundamental Methods of Mathematical Economics", McGraw-Hill Inc., Singapore, pp.788.

COX, J.C., ROSS, S.A. and M. RUBINSTEIN (1979), "Option pricing: A simplified approach", Journal of Financial Economics vol.7 n°3 (September), pp. 229-263.

DIXIT, A. (1992), "Investment and Hysteresis", Journal of Economic Perspectives vol.6 n°1 (Winter), pp.107-132.

DIXIT, A. and R. PINDYCK (1994), "Investment under Uncertainty", Princeton University Press, New Jersey, pp.468.

EALES, B.A. (1995), "Financial Risk Management", McGraw-Hill, London, pp. 278.

HAUGEN, R. (1986), "Modern investment theory", Prentice Hall, New Jersey, 541 p.

HULL, J. (1993), "Options, Futures and Other Derivative Securities", Prentice-Hall Inc., New Jersey, pp.492.

INGERSOLL, J. (1982), "The pricing of commodity-linked bonds (by E. Schwartz): Discussion", Journal of Finance vol.37 n°2, p. 540-541.

KEMNA, A. (1987), "Options in real and financial markets", Doctoral Dissertation, Erasmus University Press, Rotterdam, pp.205.

KEMNA, A. (1989), "Reële opties in investeringsprojecten", Maandblad voor Accountancy en Bedrijfseconomie (September), pp. 355-364.

KEMNA, A. (1993), "Case Studies on Real Options", Financial Management vol.22 n°3 (Autumn), pp.259-270.

LINT, L.J.O. (1992a), "De investeringsanalyse van geavanceerde flexibele produktietechnologieën", De Constructeur (December), pp.58-80.

LINT, L.J.O. (1992b), "Options in advanced energy-related production processes", International Journal of Production Economics vol.27, pp.257-264.

MAJD, S. and R. PINDYCK (1987), "Time to build, option value and investment decisions", Journal of Financial Economics vol.18, pp.7-27.

MAJD, S. and R. PINDYCK (1989), "The learning curve and optimal production under uncertainty", RAND Journal of Economics vol.20 n°3 (Autumn), pp.331-343.

MAUER, D.C. and A.J. TRIANTIS (1994), "Interactions of corporate financing and investment decisions: a dynamic framework", Journal of Finance vol.49 n°4 (September), pp.1253-1278.

McDONALD, R. and D. SIEGEL (1985), "Investment and the valuation of firms when there is an option to shut down", International Economic Review vol.26 n°2, pp.331-349.

McDONALD, R. and D. SIEGEL (1986), "The value of waiting to invest", The Quarterly Journal of Economics (November), pp. 707-728.

MERTON, R. (1973), "Theory of rational option pricing", Bell Journal of Economics and Management Science, p. 141-183.

PINDYCK, R. (1991), "Irreversibility, Uncertainty and Investment", Journal of Economic Literature vol.29 (September), pp.1110-1148.

ROSS, S.A., WESTERFIELD, R.W. and J.F. JAFFE (1990), "Corporate Finance - second edition", Richard D. Irwin Inc., USA, 833p.

ROSS, S.A. (1995), "Uses, Abuses and Alternatives to the Net Present Value Rule", Financial Management vol.24 n°3 (Autumn), pp.96-102.

SMIT, H. and L. ANKUM (1993), "A Real Options and Game-Theoretic Approach to Corporate Investment Strategy Under Competition", Financial Management (Autumn), pp.241-250.

SMITH, C.W., SMITHSON, C.W. and D.S. WILFORD (1990), "Managing Financial Risk", Ballinger Publishing Company, Cambridge, pp.416.

TEISBERG, E.O. (1993), "Capital investment strategies under uncertain regulation", RAND Journal of Economics vol.24 n°4 (Winter), pp.591-604.

TEISBERG, E.O. (1994), "An option valuation analysis of investment choices by a regulated firm", Management Science vol.40 n°4 (April), pp.535-548.