T_c for heavy fermion superconductors linked with other physical properties at zero and applied pressure

2005 Supercond. Sci. Technol. 18 557

(http://iopscience.iop.org/0953-2048/18/4/028)

INSTITUTE OF PHYSICS PUBLISHING SUPERCONDUCTOR SCIENCE AND TECHNOLOGY
Supercond. Sci. Technol. **18** (2005) 557–560 doi:10.1088/0953-2048/18/4/028

T_c for heavy fermion superconductors linked with other physical properties at zero and applied pressure

G G N Angilella[1], **N H March**[2,3] **and R Pucci**[1]

[1] Dipartimento di Fisica e Astronomia, Università di Catania, and Istituto Nazionale per la Fisica della Materia, UdR Catania, Via S. Sofia, 64, I-95123 Catania, Italy
[2] Department of Physics, University of Antwerp, Groenenborgerlaan 171, B-2020 Antwerp, Belgium
[3] Oxford University, Oxford, UK

E-mail: Giuseppe.Angilella@ct.infn.it

Received 25 October 2004
Published 1 March 2005
Online at stacks.iop.org/SUST/18/557

Abstract
The superconducting transition temperature T_c has earlier been correlated with coherence length and effective mass for a series of heavy fermion (HF) materials at atmospheric pressure. Here, a further physical property, the dc electrical conductivity $\sigma(T_c)$, is one focal point, another being the pressure dependence of both T_c and $\sigma(T_c)$ for several HF materials. The relaxation time $\tau(T_c)$ is also studied in relation to an Uncertainty Principle limit, involving only the thermal energy $k_B T_c$ and Planck's constant.

1. Introduction

In earlier work [1, 2], the superconducting transition temperature T_c of several heavy fermion (HF) materials has been correlated with the effective mass m^* (usually $\sim 100\, m_e$, with m_e the electron mass) and the coherence length ξ by

$$k_B T_c = f(\epsilon_c), \qquad (1)$$

where ϵ_c is a characteristic energy defined as [1]

$$\epsilon_c = \frac{\hbar^2}{m^* \xi^2}. \qquad (2)$$

An approximate form of the relation between $k_B T_c$ and ϵ_c has been derived with the Bethe–Goldstone equation as the starting point [2]. One finds

$$\frac{\epsilon_F}{\epsilon_c} = \frac{4}{3} x^2 + \frac{\ell(\ell+1)}{1+x} \left(1 - \frac{x \ln x}{1+x} \right), \qquad (3)$$

where $2\epsilon_F x = |\epsilon| \simeq k_B T_c$ is the binding energy of a Cooper pair, and ϵ_F is the Fermi energy [2]. Equation (3) manifestly depends on the quantum number ℓ of the pair angular momentum, which is usually employed to parameterize the anisotropic character of the superconducting order parameter, $\ell = 0, 1, 2$ corresponding to s-, p-, and d-wave symmetry, respectively. While this expression correctly reduces to the standard one for isotropic s-wave superconductors, in the case $\ell > 0$ it agrees qualitatively with the phenomenological dependence of $k_B T_c$ on the characteristic energy ϵ_c, proposed in [1] for HF compounds as well as for high-T_c cuprates. It should be pointed out, however, that equation (3) is qualitatively insensitive to different values of $\ell > 0$, the effect of a nonzero ℓ being mainly that of having $k_B T_c$ saturating to a finite value as $\epsilon_c \to \infty$, instead of diverging, as is the case with $\ell = 0$ [2].

Since this early work, we have uncovered in the literature further relevant data, for example on $CeCoIn_5$, $CeIrIn_5$, and $CeRh_2Si_2$ (see table 1 and references therein). Figure 1 then shows an updated correlation plot of $k_B T_c$ versus ϵ_c, including the latter three new entries appearing at the two ends of the series of data, and with equation (3) used as the fitting function.

Motivated by the study of Homes *et al* [3] on high-T_c materials (plus elemental metals Nb and Pb with relatively high T_c values for such superconductors) and of Zaanen [4], it is useful in connection with table 1 to define a relaxation time $\tau(T_c)$ through [5]:

$$\sigma(T_c) = \frac{n_n e^2 \tau(T_c)}{m^*}, \qquad (4)$$

where $\sigma(T_c) = \rho^{-1}(T_c)$ is the dc electrical conductivity at the transition temperature T_c, and n_n is the carrier density

0953-2048/05/040557+04$30.00 © 2005 IOP Publishing Ltd Printed in the UK

Table 1. Selected physical properties for uranium and cerium based HF materials. Where available, multiple entries separated by slashes refer to properties along different crystallographic directions. T_N is the magnetic ordering (Néel) temperature, γ denotes the Sommerfeld specific-heat coefficient, λ_0 the superconducting penetration depth extrapolated at $T = 0$, and ω_{pn} is the plasma frequency in the normal state. The last two columns are the 'Uncertainty Principle' relaxation time τ_{UP}, equation (5), and the relaxation time $\tau(T_c)$ at T_c, equation (4) [4].

Compound	T_c (K)	T_N (K)	ξ (Å)	m^*/m_e	γ (J mol^{-1} K^{-2})
UPt$_3$	0.52, 0.48 [13]	5.0 [14, 13]	100/120 [14]	180 [14]	0.450 [14]
UBe$_{13}$	0.87 [13]	—	100 [14]	260 [14]	1.100 [14]
UNi$_2$Al$_3$	1.0 [14, 13]	4.3–4.6 [14, 13]	240 [14]	48 [14]	0.120 [14]
UPd$_2$Al$_3$	2.0 [14, 13]	14.5 [13]	85 [14]	66 [14]	0.145 [14]
URu$_2$Si$_2$	1–1.5 [13, 15]	17–17.5 [14, 15]	100/150 [14]	140 [14]	0.065–0.18 [14, 15]
CeCu$_2$Si$_2$	0.65 [13]	1.3 [14]	90 [14]	380 [14]	0.73–1.1 [14]
CeRh$_2$Si$_2$	0.35[a] [16]	35–36 [16, 17]	370 [16]	220 [16]	0.08 [16]
CePd$_2$Si$_2$	0.4[b] [18]	10.2[c] [18]	150 [18]		0.13 [19]
CeCu$_2$Ge$_2$	0.64[d] [20]	4.1 [20]			
CeNi$_2$Ge$_2$	0.22[e] [19]				0.4 [18]
CeRu$_2$Ge$_2$	7.40 [21]	8.55 [21]			
CePt$_3$Si	0.75 [22]	2.2 [22]	81–97 [22]		0.39 [22]
CeNiGe$_2$	—	3, 4 [23, 17]	—		0.22 [23]
CeNiGe$_3$	0.48[f] [24]	5.5[g] [24]	130[h] [24]		0.034 [24]
CeCoIn$_5$	2.3 [25]		58 [26], 35/82 [27]	83 [26], 5/49/87 [27]	0.29 [25, 26]
CeRhIn$_5$	2.1[i] [28]	3.8 [28]	57[j] [7]		0.40 [28]
CeIrIn$_5$	0.40 [29]	0 [30]	241 [26]	140 [26], 20/30 [31]	0.72–0.75 [29, 25]
CeIn$_3$	0.25[k] [28]	10[l] [28]			$\leqslant$0.13[m] [32, 6]
CePd$_3$	—		—	36 [15]	0.037 [15]

Compound	$\rho(T_c)$ ($\mu\Omega$ cm)	λ_0 (Å)	ω_{pn} (eV)	n_n (10^{22} cm^{-3})	τ_{UP} (ps)	$\tau(T_c)$ (ps)
UPt$_3$	0.3–3 [14]	>15 000 [13]	2.6 [15]	1.8 [33]	13.9	11.8–118
UBe$_{13}$	18 [14]	11 000 [13]			8.8	
UNi$_2$Al$_3$	7 [14]	3300 [13]	2.9 [34]	0.61 [34]	7.6	4.0
UPd$_2$Al$_3$	4 [14]	4000 [13]	5.5 [35]	1.9 [36]	3.8	3.1
URu$_2$Si$_2$	12–70 [14]	10 000 [13]			5.1–7.6	
CeCu$_2$Si$_2$	2–65 [14]	5000 [13]		11.4 [16]	11.8	0.2–6.0
CeRh$_2$Si$_2$	2 [16]			19.7 [16]	21.8	2.0
CePd$_2$Si$_2$	1.4 [18]				19.1	
CeCu$_2$Ge$_2$	$\approx$6 [20]				11.9	
CeNi$_2$Ge$_2$	$\approx$3 [18]				34.7	
CeRu$_2$Ge$_2$					1.0	
CePt$_3$Si	6.5 [22]				10.2	
CeNiGe$_2$	—	—			—	—
CeNiGe$_3$					15.9	
CeCoIn$_5$	7.21 [6]	2810 [37]		1.15 [26]	3.3	3.55
CeRhIn$_5$	5–7.5 [28, 7]				3.6	
CeIrIn$_5$	$\leqslant$1 [29, 26]			2.67 [26]	19.1	18.6
CeIn$_3$					30.6	
CePd$_3$	—	—	2.3 [15]		—	—

[a] At 0.9 GPa.

[b] At 2.71 GPa.

[c] At 0 GPa. T_N vanishes at $p_c = 2.86$ GPa [18].

[d] At 10.1 GPa.

[e] At 1.5 GPa.

[f] At 4–10 GPa.

[g] At 0 GPa. T_N vanishes at $p_c = 5.5$ GPa [24].

[h] At 6.5 GPa.

[i] At 1.7 GPa. T_c reaches 2.2 K at 2.5 GPa [7].

[j] At 2.5 GPa.

[k] At 2.5 GPa.

[l] At 0 GPa.

[m] At 0 GPa.

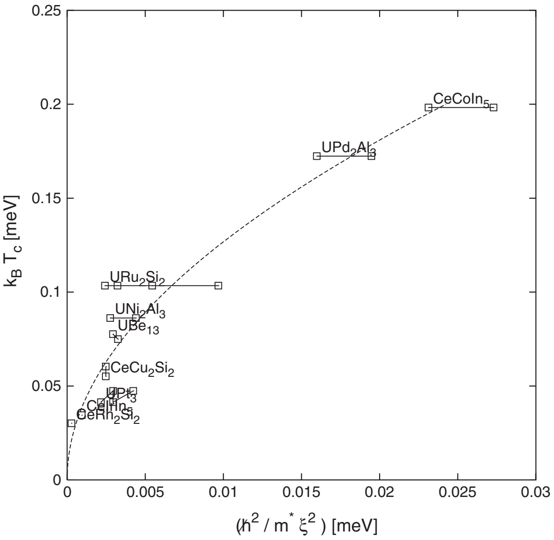

Figure 1. Measured superconducting transition temperatures T_c for a variety of HF materials plotted against the characteristic energy ϵ_c defined in equation (2). Data for T_c, m^*, ξ have been taken from table 1. The dashed curve is a phenomenological fit based on equation (3) [2].

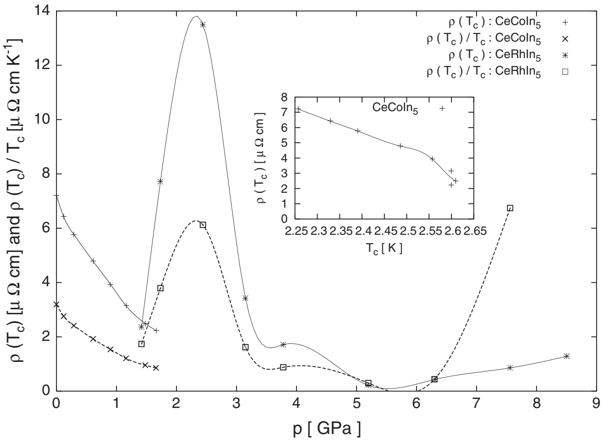

Figure 2. Resistivity $\rho(T_c)$ at T_c (solid curves) and ratio $\rho(T_c)/T_c$ (dashed curves) of CeCoIn$_5$ [6] and CeRhIn$_5$ [7]. Inset: plots $\rho(T_c)$ versus T_c for CeCoIn$_5$ [6]. The curves are guides to the eye.

in the normal state. For several materials, experimental data collected in table 1 for all the physical quantities appearing in equation (4) exist with the exception of the relaxation time $\tau(T_c)$. Table 1 therefore records the value of $\tau(T_c)$ extracted from equation (4) using experimental values for $\rho(T_c)$, n_n and m^*. For comparison, we have also recorded the 'Uncertainty Principle' (UP) estimate τ_{UP} given by Zaanen [4], following the study of Homes *et al* [3]:

$$\tau_{UP} = \frac{\hbar}{k_B T_c}. \tag{5}$$

In most cases, the values of $\tau(T_c)$ entered in table 1 are of the same order of magnitude of τ_{UP} given by equation (5), but no simple correlation exists between $1/\tau(T_c)$ and T_c in the HF materials.

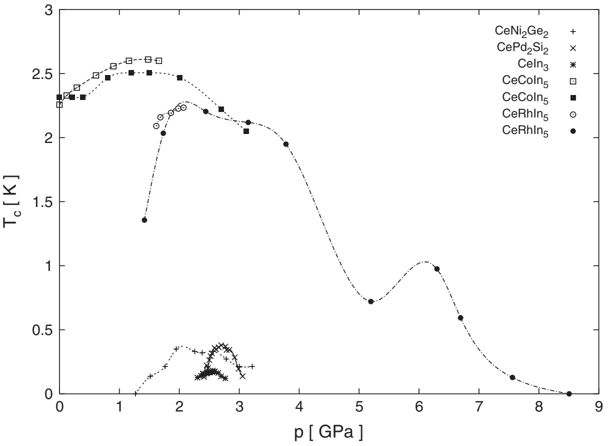

Figure 3. Experimental superconducting transition temperatures T_c for CeNi$_2$Ge$_2$ (+ [38]), CePd$_2$Si$_2$ (× [18]), CeIn$_3$ (∗ [18]), CeCoIn$_5$ (□ [6], ■ [8]), CeRhIn$_5$ (○ [28], ● [7]), plotted as a function of pressure p. The curves are guides to the eye.

2. Pressure dependence of T_c and $\rho(T_c)$

Of the HF materials referred to above, we next note that CeCoIn$_5$, which is a superconductor at atmospheric pressure ($p = 0$), has been studied over a pressure range out to about 3 GPa [6]. Figure 2 plots the variation of the normal state resistivity $\rho(T_c)$, together with the ratio $\rho(T_c)/T_c$, as a function of pressure. In contrast to the almost monotonic decrease of both these quantities with increasing pressure, we have also plotted available experimental data for CeRhIn$_5$ [7], which however starts superconducting at 1.7 GPa. The structure of both $\rho(T_c)$ and $\rho(T_c)/T_c$ as a function of pressure is marked, and they correlate. The inset shows $\rho(T_c)$ versus T_c constructed from the set of data for CeCoIn$_5$ [6]. Over a range of T_c from 2.2 to 2.5 K, the behaviour is rather linear, followed by a more sudden decrease of resistivity with increasing T_c.

3. Summary and future directions

Our findings to this point may be summarized as follows. The interlink between T_c and the characteristic energy ϵ_c in figure 1 appears relatively robust, as evidenced by the addition of quite recent data. The classification of HF materials is clearly quite different from the high-T_c regularity plus Nb and Pb discussed by Homes *et al* [3] and also by Zaanen [4]. However, the Uncertainty Principle relaxation time τ_{UP} is found to be of the same order of magnitude as that extracted from measured dc conductivity data plus effective masses at atmospheric pressure, namely $\tau(T_c)$. However, no inverse correlation between $\tau(T_c)$ and T_c is found for an admittedly limited number of HF materials. In the same context, we have used the superconducting penetration depth λ_0 in table 1 to estimate the superfluid density ρ_s as λ_0^{-2}. Then, following Homes *et al* [3], if we construct $\rho_s/T_c\sigma(T_c)$, then for all but one of the HF materials for which data are recorded in table 1, this ratio is at least a factor of seven greater than for high-T_c materials, and with a huge scatter, confirming the above conclusion that HF materials are in a quite different category from high-T_c materials plus the elemental BCS superconductors Nb and Pb.

As to future directions, we feel that further work, both experimental and theoretical, on applying pressure to HF materials should be illuminating. Thus, we have collected in figure 3 some available experimental data for T_c as a function of pressure. The simplest example, and the only one shown which superconducts at $p = 0$, is $CeCoIn_5$ [6, 8], which has a relatively smooth variation of T_c with pressure, exhibiting a single maximum. It is tempting for the future to study whether a link can be forged, for $p < p_{max}$, the latter corresponding to the maximum of T_c, with figure 1. However, whereas figure 1 relates T_c to a single variable, i.e. the characteristic energy ϵ_c defined in equation (2), it may be that one must add further variables to describe the pressure dependence of T_c. For example, the detail of spin fluctuation [9–12], believed at present to be at least partially responsible for Cooper pair formation in this class of materials, may need inclusion. However, of course, some account is already present through the coherence length ξ, in which the size of the Cooper pair is manifested. Of course, for the remaining materials in figure 3, the pressure dependence of T_c is more complex, including the fact that pressure is already needed to induce superconductivity.

Acknowledgments

We thank N J Curro, G Sparn, and H Wilhelm for helpful discussions and correspondence. NHM wishes to acknowledge that his contribution to this study was brought to fruition during a visit to the University of Catania in 2004. He wishes to thank the Department of Physics and Astronomy for the stimulating atmosphere and for generous hospitality.

References

[1] Angilella G G N, March N H and Pucci R 2000 *Phys. Rev.* B **62** 13919

[2] Angilella G G N, Leys F E, March N H and Pucci R 2004 *Phys. Lett.* A **322** 475

[3] Homes C C *et al* 2004 *Nature* **430** 539

[4] Zaanen J 2004 *Nature* **430** 512

[5] Ashcroft N W and Mermin N D 1976 *Solid State Physics* (Fort Worth: Saunders)

[6] Nicklas M, Borth R, Lengyel E, Pagliuso P G, Sarrao J L, Sidorov V A, Sparn G, Steglich F and Thomson J D 2001 *J. Phys.: Condens. Matter* **13** L905

[7] Muramatsu T, Tateiwa N, Kobayashi T C, Shimizu K, Amaya K, Aoki D, Shishido H, Haga Y and Ōnuki Y 2001 *J. Phys. Soc. Japan* **70** 3362

[8] Shishido H, Ueda T, Hashimoto S, Kubo T, Settai R, Harima H and Ōnuki Y 2003 *J. Phys.: Condens. Matter* **15** L499

[9] Monthoux P and Lonzarich G G 1999 *Phys. Rev.* B **59** 14598

[10] Abanov Ar, Chubukov A V and Finkel'stein A M 2001 *Europhys. Lett.* **54** 488

[11] Angilella G G N, March N H and Pucci R 2002 *Phys. Rev.* B **65** 092509

[12] Curro N J, Sarrao J L, Thompson J D, Pagliuso P G, Kos Š, Abanov A and Pines D 2003 *Phys. Rev. Lett.* **90** 227202

[13] Amato A 1997 *Rev. Mod. Phys.* **69** 1119

[14] Heffner R H and Norman M R 1996 *Comment. Condens. Matter Phys.* **17** 361 see also refs therein

[15] Degiorgi L 1999 *Rev. Mod. Phys.* **71** 687

[16] Movshovich R, Graf T, Mandrus D, Thompson J D, Smith J L and Fisk Z 1996 *Phys. Rev.* B **53** 8241

[17] Alsmadi A M, Nakotte H, Sarrao J L, Hung M H and Lacerda A H 2003 *J. Appl. Phys.* **93** 8343

[18] Grosche F M, Walker I R, Julian S R, Mathur N D, Freye D M, Steiner M J and Lonzarich G G 2001 *J. Phys.: Condens. Matter* **13** 2845

[19] Lister S J S, Grosche F M, Carter F V, Haselwimmer R K W, Saxena S S, Mathur N D, Julian S R and Lonzarich G G 1997 *Z. Phys.* B **103** 263

[20] Jaccard D, Behnia K and Sierro J 1992 *Phys. Lett.* A **163** 475

[21] Wilhelm H and Jaccard D 2004 *Phys. Rev.* B **69** 214408

[22] Bauer E, Hilscher G, Michor H, Paul C, Scheidt E W, Gribanov A, Seropegin Y, Noël H, Sigrist M and Rogl P 2004 *Phys. Rev. Lett.* **92** 027003

[23] Hung M H, Lacerda A H, Pagliuso P G, Sarrao J L and Thompson J D 2002 *J. Appl. Phys.* **91** 8522

[24] Nakashima M, Tabata K, Thamizhavel A, Kobayashi T C, Hedo M, Uwatoko Y, Shimizu K, Settai R and Ōnuki Y 2004 *J. Phys.: Condens. Matter* **16** L255

[25] Petrovic C, Pagliuso P G, Hundley M F, Movshovich R, Sarrao J L, Thompson J D, Fisk Z and Monthoux P 2001 *J. Phys.: Condens. Matter* **13** L337

[26] Movshovich R, Jaime M, Thompson J D, Petrovic C, Fisk Z, Pagliuso P G and Sarrao J L 2001 *Phys. Rev. Lett.* **86** 5152

[27] Settai R, Shishido H, Ikeda S, Murakawa Y, Nakashima M, Aoki D, Haga Y, Harima H and Ōnuki Y 2001 *J. Phys.: Condens. Matter* **13** L627

[28] Hegger H, Petrovic C, Moshopoulou E G, Hundley M F, Sarrao J L, Fisk Z and Thompson J D 2000 *Phys. Rev. Lett.* **84** 4986

[29] Petrovic C, Movshovich R, Jaime M, Pagliuso P G, Hundley M F, Sarrao J L, Fisk Z and Thompson J D 2001 *Europhys. Lett.* **53** 354

[30] Zheng G, Yamaguchi N, Kan H, Kitaoka Y, Sarrao J L, Pagliuso P G, Moreno N O and Thompson J D 2004 *Phys. Rev.* B **70** 014511 (*Preprint* cond-mat/0408006)

[31] Haga Y *et al* 2001 *Phys. Rev.* B **63** R060503

[32] Nasu S, van Diepen A M, Neumann H H and Craig R S 1971 *J. Phys. Chem. Solids* **32** 2772

[33] Sulewski P E, Sievers A J, Maple M B, Torikachvili M S, Smith J L and Fisk Z 1988 *Phys. Rev.* B **38** 5338

[34] Cao N, Garrett J D, Timusk T, Liu H L and Tanner D B 1996 *Phys. Rev.* B **53** 2601

[35] Dressel M, Gorshunov B, Kasper N, Nebendahl B, Huth M and Adrian H 2000 *J. Phys.: Condens. Matter* **12** L633

[36] Degiorgi L, Dressel M, Gruner G, Wachter P, Sato N, Komatsubara T and Uemura Y 1994 *Europhys. Lett.* **25** 311

[37] Özcan S, Broun D M, Morgan B, Haselwimmer R K W, Sarrao J L, Kamal S, Bidinosti C P, Turner P J, Raudsepp M and Waldram J R 2003 *Europhys. Lett.* **62** 412

[38] Grosche F M, Agarwal P, Julian S R, Wilson N J, Haselwimmer R K W, Lister S J S, Mathur N D, Carter F V, Saxena S S and Lonzarich G G 2000 *J. Phys.: Condens. Matter* **12** L533