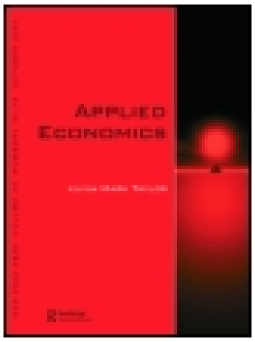

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The relationship between risk-neutral and actual default probabilities: the credit risk premium

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ABSTRACT

The study investigates empirically the relationship between the risk-neutral measure $\mathbb{Q}$ and the real-world measure $\mathbb{P}$. We study the ratio between the risk-neutral and actual default intensities, which we call the coverage ratio or the relative credit risk premium. Actual default intensities are derived from rating agencies annual transition matrices, while risk-neutral default intensities are bootstrapped from CDS quotes of European corporates. We quantify the average risk premium and its changes over time. Compared to related literature, special attention is given to the effects of the recent financial and European sovereign crises. We find that average credit risk premia rose substantially and that post-crisis levels are still higher than those observed before the financial crisis. This observation is especially true for high-quality debt and if it persists, it will have an impact on corporates funding costs. The quantification and revision of risk premia contributes to the discussion of the credit spread puzzle and could give extra insights in valuation models that start from real-world estimates. Our work is furthermore important in the context of state aid assessment. The real economic value (REV) methodology, applied by the European Commission to evaluate impaired portfolios, is based on a long-term average risk premium.

KEYWORDS

Risk-neutral; probability of default (PD); credit risk premium; real economic value (REV); coverage ratio

JEL CLASSIFICATION

G12; G13; G28

I. Introduction

In the real world given a certain time t, for every corporate there exists a probability of default (PD), which is called the actual PD. It is the probability that the company will go into default in reality between now and time t. Sometimes this PD is also called real-world PD, PD under the $\mathbb{P}$-measure ($PD^{\mathbb{P}}$) or physical PD. On the other hand, there is a risk-neutral PD, or PD under the $\mathbb{Q}$-measure ($PD^{\mathbb{Q}}$), and this PD is used to price financial instruments under the no-arbitrage condition.

The goal of this study is to examine empirically the relationship between these two kinds of PDs, which is also known in the literature as the credit spread puzzle. To this end, we quantify the credit risk premium of European corporates of various rating classes over different time periods, including the financial and sovereign crises. We find that post-crisis risk premium levels are still higher than those observed before the financial crisis, and this is particularly true for high-quality debt. If this effect persists, it will probably have a large impact on corporates' funding costs. The literature investigating risk-neutral versus historical PDs is limited, and to our knowledge we are the first developing an empirical exercise focusing explicitly on Europe and covering the full financial and sovereign crises. Our work is also relevant in the context of pricing models of default-prone financial assets. The (average) credit risk premium is in fact applied in the valuation methodology to assess state aid given to European financial institutions.

To recall and define the *credit risk premium* and the so-called *coverage ratio*, we start with a simple example of pricing a zero coupon bond (ZCB). Assume a ZCB with an uncertain payoff at maturity (T): if the bond defaults, the payoff is zero, if not the bond pays one at maturity. Let:

$$\begin{aligned} PD^{\mathbb{Q}} &= 1 - \exp\left(-\lambda^{\mathbb{Q}} T\right) = 1 - P^{\mathbb{Q}}(Surv) \\ &\approx \lambda^{\mathbb{Q}} T \end{aligned} \tag{1}$$

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$$PD^{\mathbb{P}} = 1 - \exp\left(-\lambda^{\mathbb{P}} T\right) = 1 - P^{\mathbb{P}}(Surv) \approx \lambda^{\mathbb{P}} T, \tag{2}$$

where λ is the default intensity under the $\mathbb{Q}$- or $\mathbb{P}$-measure and P($Surv$) is the survival probability under the corresponding measure. In finance, the price of a financial instrument is derived by taking the expected value of its uncertain payoffs and discount it with the appropriate factor. The price of the ZCB is calculated by taking the expectation of the payoff under the $\mathbb{Q}$-measure and discount by the risk-free rate (r). Since a recovery rate of zero is assumed, the expectation under the $\mathbb{Q}$-measure equals the survival probability under the $\mathbb{Q}$-measure:

$$\begin{aligned}\text{Price ZCB} &= \exp(-rT)E^{\mathbb{Q}}(\text{payoff}) \\ &= \exp(-rT)P^{\mathbb{Q}}(Surv) \\ &= \exp\left(-\left(r + \lambda^{\mathbb{Q}}\right)T\right).\end{aligned} \tag{3}$$

We define the ratio between the default intensities under the $\mathbb{Q}$- and $\mathbb{P}$-measures as the *relative credit risk premium* or the *coverage ratio*[1] (μ):

$$\mu = \frac{\lambda^{\mathbb{Q}}}{\lambda^{\mathbb{P}}} \approx \frac{PD^{\mathbb{Q}}}{PD^{\mathbb{P}}}. \tag{4}$$

By using the coverage ratio we can rewrite Equation (3) in terms of actual default intensities $\left(\lambda^{\mathbb{P}}\right)$:

$$\begin{aligned}\text{Price ZCB} &= \exp\left\{-\left(r + \lambda^{\mathbb{P}}\mu\right)T\right\} \\ &= \exp\left\{-\left[r + \lambda^{\mathbb{P}} + \lambda^{\mathbb{P}}(\mu - 1)\right]T\right\}.\end{aligned} \tag{5}$$

The last term of Equation (5) $\left(\lambda^{\mathbb{P}}(\mu - 1) = \lambda^{\mathbb{Q}} - \lambda^{\mathbb{P}}\right)$ is what is generally called the *(absolute) risk premium (r_p)* and hence we have:

$$\begin{aligned}\text{Price ZCB} &= \exp\left\{-\left(r + \lambda^{\mathbb{P}} + r_p\right)T\right\} \\ &= \exp\{-(r + r_p)T\}E^{\mathbb{P}}(\text{payoff}).\end{aligned} \tag{6}$$

Hence, one can price the ZCB by taking the expectation under the risk-neutral measure $\mathbb{Q}$ and discount by the risk-free interest rate or, alternatively, by taking the expectation under the real-world measure $\mathbb{P}$ and discount by the risk-free rate plus a risk premium. The relationship between the risk-neutral measure $\mathbb{Q}$ and the actual measure $\mathbb{P}$ is thus captured by the risk premium.

There are theoretical approaches determining this risk premium from two equivalent measures (Girsanov 1960), but these are usually based on assumptions such as market completeness, which in general are not satisfied in reality. In incomplete markets the martingale measure $\mathbb{Q}$ is not uniquely defined and the chosen measure $\mathbb{Q}$ depends on a certain criteria, for example, one can choose the minimal entropy martingale measure (see, e.g. Dhaene et al. (2015)). In this article, we depart from theoretical approaches and we propose to develop an empirical analysis to describe the credit risk premium, focusing on the coverage ratio μ. The literature on empirical studies is mostly conducted with US data and does not cover the recent crisis. The oldest studies are based on bond data, while most recent ones also use CDSs.

Fons (1987) was pioneer in the empirical investigation of the link between risk-neutral and actual default probabilities. He derived risk-neutral PDs from a US index of yields to maturity for noninvestment grade bonds and compared them with historical default rates. Fons concluded that holders of low-rated bonds are rewarded for bearing default risk and hence a positive risk premium was observed.

A similar result was found by Driessen (2005), who quantified the risk premium in a reduced form setting by decomposing US corporate bond returns into six components: a component describing market-wide changes, a firm-specific component, the dependency of credit spreads on risk-free interest rates, the risk premium on default jumps, the liquidity effect and the tax effect[2]. Excluding the liquidity and tax effects, Driessen estimates the value of μ between 1 and 3.

Hull, Predescu, and White (2005) and Hull (2012) also calculated coverage ratios from US bond spreads and they concluded that the coverage ratio decreases when credit quality decreases. The values of the coverage ratios are around 17 for the highest rating classes and slightly above 1 for the lowest. The existence of the risk premium is according to Hull, Predescu, and White (2005) due to liquidity risk, trader's expectations ($\approx$ risk aversion) and diversifiable and nondiversifiable risk.

[1]The name*'coverage ratio'* is taken from O'Kane (2008) and originally defined as the ratio between the credit spread and the actuarial spread.
[2]The tax effect is caused by the different tax treatment of government bonds (risk-free rate) and corporates bonds in the US

Huang and Huang (2002, 2012) fit structural models with different specifications to US historical default rates (1973–1998) per rating class and compare them with the observed bond spreads. For bonds with a maturity of 10 years, they quantify the coverage ratio between six and one, while for a shorter maturity (4 years) the values range from around 31 to 1. Consistent with other papers (Elton et al. 2001; Amato and Remolona 2003; Hull, Predescu, and White 2005; Driessen 2005; Hull 2012), they found the highest coverage ratios for the AAA or AA rating classes.

Amato and Remolona (2003) proposed a different approach to assess the relationship between $\mathbb{P}$ and $\mathbb{Q}$ which focuses on expected losses *(EL)*. Expected losses for corporate bondholders are estimated as: $EL^{\mathbb{P}} = PD^{\mathbb{P}}\ (1 - R^{\mathbb{P}})$, where the recovery rate ($R^{\mathbb{P}}$) is rating dependent and the $PD^{\mathbb{P}}$ changes per maturity and rating class. The authors focus on the ratio between $EL^{\mathbb{Q}}$ and $EL^{\mathbb{P}}$ and found that it is about 350 for AAA-rated debt with a maturity between 3 and 5 years. This value reduces to *ca* 120 for AAA debt with a maturity between 7 and 10 years. For lower credit quality debt (below A-rated) their findings are in line with other studies, with a ratio between 15 and 1[3]. Amato and Remolona (2003) attribute the large premium to tax and liquidity effects, risk aversion and the difficulty to diversify bond and/or loan portfolios. In general, it is assumed that diversification can eliminate the unexpected losses in a bond portfolio. The skewness observed in bond portfolios makes it harder to diversify and many different obligor names are required to reduce unexpected losses to a minimum. Amato and Remolona (2003) argue that in practice large portfolios are not attainable and thus investors must be rewarded for this undiversifiable risk.

In a more recent study (Giesecke et al. 2011), US actual default rates and US bond yields based on a large data set covering 150 years (1866–2008) with the goal to assess how the business cycle and other financial variables relate to the behaviour of observed default rates. In this article, the authors also compare actual credit losses with market credit spreads and conclude that credit spreads are twice as large as the actual credit losses. This article also finds that credit spreads do not reflect actual default probabilities and macroeconomic conditions. The authors conclude that credit spreads are mostly determined by the risk premium and market factors such as liquidity.

Additional relevant literature on bonds' credit spreads, not directly addressing the quantification of the credit risk premium, include Sarig and Warga (1989), Kim, Ramaswamy, and Sundaresan (1993), Pedrosa and Roll (1998), Duffee (1999), Collin-Dufresne, Goldstein, and Martin (2001), Eom, Helwege, and Huang (2004), Jarrow, Lando, and Yu (2005), Davydenko and Strebulaev (2007), Bharath and Shumway (2008) and Schaefer and Strebulaev (2008).

The studies above focus on bond spreads to extract the risk-neutral PDs or hazard rates. However, by using CDS spreads the tax effect is eliminated and the liquidity effect is reduced. Therefore, coverage ratios that are derived from CDS spreads are expected to be lower than the ones derived from bonds (Longstaff, Mithal, and Neis 2005). O'Kane and Schloegl (2002) compute the coverage ratios from US CDS quotes and their results indicate that coverage ratios fluctuate between three and five. O'Kane identified three components in risk premia: liquidity, default and volatility risk. Investors ask the default risk premium to compensate the uncertainty in the predictions of the actual default and recovery rate, while the volatility risk premium compensates investors for the risk of changing risk characteristics (e.g. downgrades).

Berg (2009, 2010) published two papers assessing the link between risk-neutral and actual PDs. In his first paper, he starts with a theoretical approach based on the Merton framework (Merton 1974), where the link between the risk-neutral and actual PD is captured by the Sharpe ratio. Berg does not calibrate the Sharpe ratio on real observed spreads, and derives theoretically risk premia for given values of the Sharpe ratio. He concludes that the coverage ratio is a decreasing and convex function of the actual PD. In Berg's second paper (2010), he assumes that the firm-specific Sharpe ratio $\left(\theta_t^{\nu}\right)$ is correlated with a market Sharpe ratio $\left(\theta_t\right)$, which follows a mean-reverting CIR process (Cox, Ingersoll,

[3]If one assumes that the risk-neutral recovery rate incorporates a risk premium, that is, $R^{\mathbb{Q}} < R^{\mathbb{P}}$, the ratio estimated by Amato and Remolona (2003) represents the upper bound to the coverage ratio.

and Ross 1985). CDS spreads and Moody's EDFs, which are based on the KMV model (1993) for the period 2004–2009 were used to calibrate the average Sharpe Ratio. Berg concludes that the risk premium is time varying and that the long-term mean of the Sharpe ratio is around 40% for both Europe and the United States. Translating Berg's PDs into hazard rates and computing coverage ratios, results in the same range of values as Hull, Predescu, and White (2005).

Another study that uses Moody's EDF and US CDS spreads is the one of Berndt et al. (2008). Their data confirm that risk premia vary over time and they quantify the coverage ratios to be between 1.5 and 3. The authors also find evidence for differences across sectors.

The present article is an empirical study investigating the relationship between $\mathbb{P}$ and $\mathbb{Q}$ based on a sample of around 550 European private sector companies, including banks, other financial and nonfinancial corporates. The relationship between $\lambda^{\mathbb{P}}$ and $\lambda^{\mathbb{Q}}$ for various rating classes is examined. As we will detail in the next sections, risk-neutral default intensities are derived from CDS spread data, while actual default intensities are estimated from transition matrices computed by rating agencies. The data-set spans the period 2004–2014 and thus it is long enough to draw conclusions on the relationship between such measures.

In contrast to the previous literature, our study specifically focuses on the EU and includes the financial and sovereign crises. The only study conducted with European data was Berg (2010) and he did not cover the sovereign crisis. The quantification and revision of risk premia contributes to the discussion of the credit spread puzzle (Amato and Remolona 2003) and could give extra insights in valuation models that start from real-world estimates.

The work is furthermore relevant in the context of state aid assessment. During the financial crisis, troubled assets were transferred to government-sponsored entities (e.g. bad banks) or state guarantees were applied to toxic assets. Since state aid could distort competition and to keep a level playing field among financial institutions, the European Commission made an attempt to limit these distortions while still allowing member states to remedy serious disturbances in their economy (e.g. by rescuing a systemically relevant institution). To this end, the European Commission introduced the principle of *real economic value (REV)* in the Impaired Asset Communication (O.J.C 2009). The REV of a portfolio of impaired assets is the value which takes into account a long-term average risk premium and is the maximum transfer price or guarantee for a portfolio of impaired assets, according to EU regulations. More specifically, the REV should incorporate expected losses while ignoring potential market failures related to lack of liquidity, risk aversion or loss of confidence due to temporary distressed market conditions. In the simple case of a fixed rate instrument with cash flows CF_t, we have:

$$REV_t = \sum_{i=1}^{N} \frac{E_t^{\mathbb{P}}[CF_i]}{\left(1+r+r_p\right)^i} = \sum_{i=1}^{N} \frac{E_t^{\mathbb{Q}}[CF_i]}{\left(1+r\right)^i}, \qquad (7)$$

where N is the number of time steps to maturity, r is the risk-free rate and r_p is the long-term average risk premium paid under normal market conditions (Boudghene and Maes 2012). Impaired assets can hence be priced using the risk premium (r_p) and the real-world expected losses, assessed by banks for regulatory purposes. The risk premium, that is, relationship between $\mathbb{P}$ and $\mathbb{Q}$ is thus a crucial ingredient of the REV methodology.

The article is structured as it follows. Section II describes the applied methodology and discusses the data. Results are given in Section III and complemented with a discussion. Finally, we conclude in Section IV.

II. Methodology and data

Actual default intensities: transition matrices

This section describes how yearly transition matrices published by rating agencies are used in our context to estimate actual PDs and the corresponding intensities $\lambda^{\mathbb{P}}$.

Various approaches are used to estimate actual PDs and they all consist of at least two steps. In a first phase, corporates are grouped into bins with similar risk characteristics. Typically corporates' default risk profile is expressed in terms of rank order measures such as ratings, distance-to-defaults or Altman Z-scores (Altman 1968). In a second phase, the number of defaults per bin are counted and the default frequencies are computed. In a last and optional step, the default frequencies are smoothed (e.g. in Bluhm, Overbeck, and Wagner (2002)).

Rating agencies publish default frequencies per rating class for different maturities and different periods (yearly, 5 years, …). To examine the changes in risk premia over time, we choose the smallest time period (yearly) available in these default studies. Choosing this time frequency imposes one challenge that needs to be solved: default frequencies of high credit quality corporates (A-rated or above) are, even for longer maturities, mostly reported as zero. Zero default rates are not realistic since default rates, risk-neutral and actual, should have nonzero positive value below one ($PD \in [0, 1]$). To overcome this obstacle we apply a Markov chain with yearly changing transition matrices. This methodology results in positive default frequencies after a certain number of time steps.

The raw rating agencies' transition matrices must, however, be adjusted to apply them in our Markov chain methodology. In particular, we need to tackle the issue of not having a square matrix due to the presence of the classes *Without Rating* (*WR*) and *Default* (*D*).

Consider a 1-year transition matrix, as in Equation 8,

$$\tilde{\mathbf{TM}}_{t,1} = \begin{pmatrix} tr_{Aaa,Aaa|t,1} & tr_{Aaa,Aa|t,1} & \cdots & tr_{Aaa,Ca-C|t,1} & tr_{Aaa,WR|t,1} & tr_{Aaa,D|t,1} \\ tr_{AaAaa|t,1} & tr_{Aa,Aa|t,1} & \cdots & tr_{Aa,Ca-C|t,1} & tr_{Aa,WR|t,1} & tr_{Aa,D|t,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ tr_{Ca-C,Aaa|t,1} & tr_{Ca-C,Aa|t,1} & \cdots & tr_{Ca-C,Ca-C|t,1} & tr_{Ca-C,WR|t,1} & tr_{Ca-C,D|t,1} \end{pmatrix}, \quad (8)$$

where $tr_{rc1,rc2|t,1}$ is the observed transition rate of corporates with a rc1-rating at time t to another rating class ($rc2$) after 1 year ($t + 1$). $tr_{rc1,D|t,1}$ is the 1-year default rate of a corporate with a $rc1$-rating at time t, while $tr_{rc1,WR|t,1}$ gives the observed frequency of withdrawing the rating of $rc1$-rated corporate after 1 year. To obtain a square matrix with rows summing up to 1, we first remove the *WR*-class and rescale the remaining transition rates. Second, we add the default state as an absorbing state.

In particular, we redistribute the *WR* rating class proportionally over the other rating classes (as in Elton et al. (2001); Bangia et al. (2002); Feng, Gourieroux, and Jasiak (2008)) according to:

$$\tilde{tr}_{rc1,rc2|t,1} = \frac{tr_{rc1,rc2|t,1}}{\sum_{rc2 \in J} tr_{rc1,rc2|t,1}} \quad (9)$$

the denominator of Equation (9) is the sum of the transition rates over all rating classes, including the default state D but excluding the WR rating class ($WR \notin J$)[4]. This results in an adjusted transition matrix as in Equation 10.

$$\tilde{\mathbf{TM}}_{t,1} = \begin{pmatrix} \tilde{tr}_{Aaa,Aaa|t,1} & \tilde{tr}_{Aaa,Aa|t,1} & \cdots & \tilde{tr}_{Aaa,Ca-C|t,1} & \tilde{tr}_{Aaa,D|t,1} \\ \tilde{tr}_{Aa,Aaa|t,1} & \tilde{tr}_{Aa,Aa|t,1} & \cdots & \tilde{tr}_{Aa,Ca-C|t,1} & \tilde{tr}_{Aa,D|t,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{tr}_{Ca-C,Aaa|t,1} & \tilde{tr}_{Ca-C,Aa|t,1} & \cdots & \tilde{tr}_{Ca-C,Ca-C|t,1} & \tilde{tr}_{Ca-C,D|t,1} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \quad (10)$$

The last row of $\tilde{\mathbf{TM}}_{t,1}$ is the absorbing default state, where we assume that once a corporate is defaulted, it will not re-enter into a rating class.

We would like to stress that in a Markov chain the transition to another state only depends on the current state (rating). This implies that by applying a Markov chain the transition probability is not affected by previous rating information, only the current rating matters. It is thus irrelevant whether the current rating was preceded by a downgrade or an upgrade. In reality this assumption is violated (see Altman and Kao (1992)) and downgrades are more likely to be followed by downgrades. Transition matrices also depend on the business cycle (Nickell, Perraudin, and Varotto 2000) and hence, downgrades are more likely than upgrades in an economic downturn. Using historical

[4]For the transition matrices of the years 2004 and 2005 extra assumptions had to be made, since Moody's reports only transition rates for the entire Caa-C rating class. These assumptions are described in Appendix A

transition matrices captures this skewed behaviour better than methods using average transition matrices and adjusting them to the observed transition rates, as in Jarrow, Lando, and Turnbull (1997).

We extract yearly transition matrices from Moody's annual default studies. Since the CDS data set spans the period 2004–2014, with maturities up to 10 years (see Section Risk-neutral Default Intensities: Credit Default Swaps), the yearly transition matrices of the period 2004–2014 are adjusted according to the methodology just described. For years after 2014 we assume that the average 1-year transition matrix of the latest observation period is valid, that is, 1983–2014 for the alphanumeric (Aaa, Aa1, Aa2, …) and 1970–2014 for the letter rating classes (Aaa, Aa, A, …).

To estimate the m-years default rate from year y to year $y + m$, we compute a m-steps transition matrix $(\tilde{\mathbf{TM}}_{y,m})$ by multiplying m transition matrices starting with the one of year y. For example, we compute the default rate from 2005 to 2010 by multiplying $\tilde{\mathbf{TM}}_{2005,1} \times \tilde{\mathbf{TM}}_{2006,1} \times \cdots \times \tilde{\mathbf{TM}}_{2009,1}$. In equation:

$$\overline{\tilde{\mathbf{TM}}_{y,m}} = \prod_{t=y}^{y=m-1} \overline{\tilde{\mathbf{TM}}_{t,1}}. \tag{11}$$

The last column of $\tilde{\mathbf{TM}}_{y,m}$ are the m-year defaults rates for a given rating class ($rc1$) and year

$$\tilde{tr}_{rc1,D|y,m} = PD^{\mathbb{P}}_{rc1|y,m}. \tag{12}$$

Conversion to default intensities, also called hazard rates, is computed according Equation (2) and gives:

$$\lambda_{rc1|y,m} = -\frac{1}{m}\log\left(1 - \tilde{tr}_{rc1,D|y,m}\right). \tag{13}$$

This $\lambda_{rc1|y,m}$ represents the *actual* default intensity and to stress this, the notation $\lambda^{\mathbb{P}}_{rc1|y,m}$ will be used in the remainder of this article.

Risk-neutral default intensities: credit default swaps

The risk-neutral default intensities are derived from single-name CDS spreads by adopting a simplified version of the standard CDS pricing model.[5] In this model a piecewise constant credit curve is assumed. Mathematically this gives:

$$P^{\mathbb{Q}}_t(Surv) = \begin{cases} e^{-\lambda^{\mathbb{Q}}_1 t} & \textit{for} \quad 0 \leq t < t_1 \\ P^{\mathbb{Q}}_{t_1}(Surv)e^{-\lambda^{\mathbb{Q}}_{i+1}(t-t_1)} & \textit{for} \quad t_i \leq t < t_{i+1} \\ P^{\mathbb{Q}}_{t_n}(Surv)e^{-\lambda^{\mathbb{Q}}_n(t-t_n)} & \textit{for} \quad t > t_n, \end{cases}, \tag{14}$$

where $P^{\mathbb{Q}}_t(Surv)$ are the survival probabilities under the risk-neutral measure and $\lambda^{\mathbb{Q}}_i$ is the default intensity or hazard rate in between two jumps (t_i and t_{i+1}) in the credit curve. Graphically, this is shown in Figure 1. We assume a recovery rate *(R)* of 40% for bootstrapping the CDS spreads.

The single-name CDS spreads are provided by CMA Datavision, part of the S&P Capital IQ, for the period April 2004–2014. The CDS spreads are daily Bid and Ask quotes for a total of 582 corporates headquartered in Europe and European sovereigns. Both short-term (3, 6, 9 months) and long-term (1–10 years) CDS contracts are covered in the database, which results into *ca* 12,000,000 daily CDS quotes in total. CMA Datavision distinguishes within this data set 11 different industries and also makes a distinction between CDS written on senior or subordinated debt. Banks are not reported as a separate industry and, therefore, the CMA data set was complemented with the information in

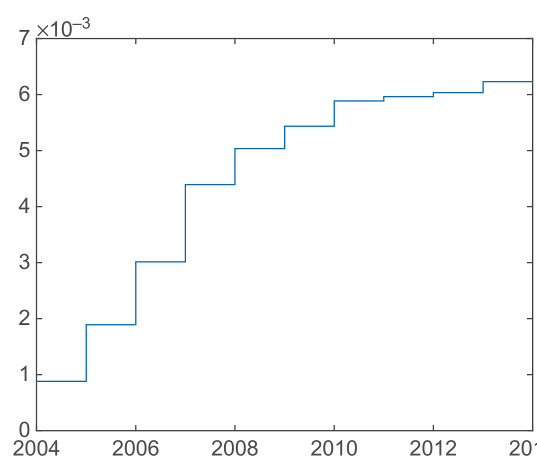


Figure 1. Credit curve derived from Median CDS spreads in 2004 with rating A1.

[5]See www.cdsmodel.com

Bankscope. Hereby, the category 'Financials' was split into 'Banks' and 'Other Financials'[6].

In Appendix B, multiple pie charts (Figure B1) describe the main characteristics of the CDS data. CDS are more often written on senior than subordinated debt. The current analysis focuses on CDS written on senior debt, which represents 83.4% of our database. Moreover, the industry 'Sovereigns/States/Agencies' is excluded.

To link CDS data with transition matrices, we need the rating information of the corporates. Corporates' ratings were gathered from Moody's and for corporates without a Moody's rating, CMA Datavision provided the S&P rating. The evolution over time of the relative importance of the rating classes is shown in Figure 2.

Descriptive statistics of the CDS spreads' mid quotes and bid-ask spreads over the entire sample are given in the first part of Table 1. The median mid CDS price is almost 90 bps, while the average mid prices equals 180 bps. The distribution of the mid prices is thus highly skewed. The bid-ask spreads behave very similar as the mid prices. The median bid-ask spread is equal to 9 bps, while the mean is double as large.

The other parts of Table 1 give more descriptive statistics of the CDS mid prices and their bid-ask spreads by year, tenor and rating. From this table, it is clear that in the pre-crisis period lower CDS and bid-ask spreads were observed than during the financial and sovereign crisis. In the period 2013–2014, the market eased which resulted in lower bid-ask and CDS spreads. Another observation is that the CDS mid prices clearly follow the expected increasing trend over the different rating classes. Only the median CDS mid price of the Ca-C rating class does not fit this trend. Since there are only two corporates with a Ca-C rating class, this raises doubt about the representativeness of this rating class. Hence, we exclude the Ca-C rating class from the analysis. Examining the bid-ask spreads for different tenors, one can see that there are no large differences between different tenors. Bid-ask spreads of CDS with shorter tenors have a slightly higher average, but the median is almost the same. The 5-years CDS has the lowest mean and median bid-ask spread, but they are very comparable with the bid-ask spreads of the longer maturities (>5 years). Looking at the bid-ask spreads per rating class, the liquidity decreases going from investment grade to noninvestment grade (rating below Ba). Especially, for the CDS of Caa-rated corporates the bid-ask spreads are rather high and thus results for this rating class should be carefully read.

Since rating agency data allow for estimating actual hazard rates per rating class, year and maturity $\left(\lambda^{\mathbb{P}}_{rc|y,m}\right)$, we need risk-neutral default intensities by rating class, year and maturity. To this end, the yearly median CDS mid spread per rating class and maturity $\left(CDS_{rc,y,m}\right)$ are calculated. These median mid prices are supplied to the bootstrapping algorithm, which

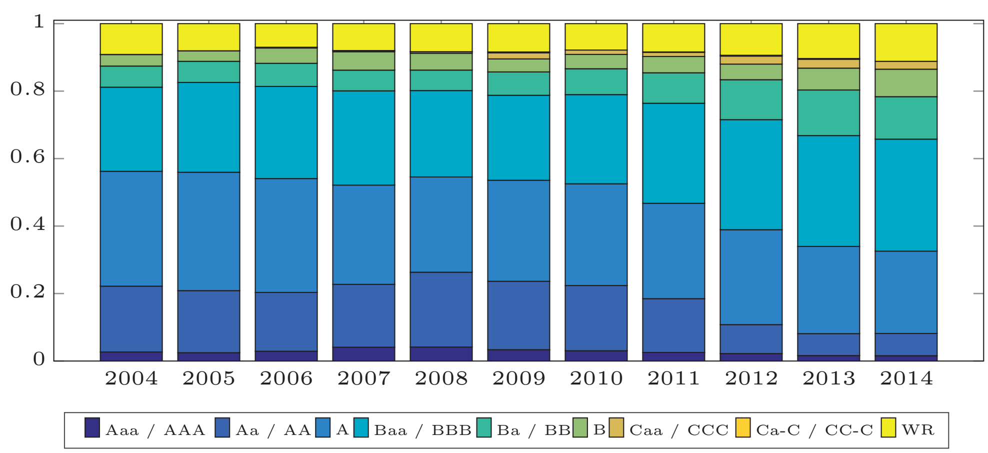


Figure 2. Evolution over time of ratings. Source: CMA Datavision and Moody's.

[6]We used the 'specialization' field in Bankscope to determine whether the financial is a bank or not. commercial banks, savings bank, bank holding companies, (Specialized) governmental credit institution, real estate and mortgage bank, cooperative banks and investment banks are categorized as banks.

Table 1. CDS spreads and bid-ask spreads statistics.

	CDS Spreads			Bid-ask Spread			#
	Mean	Median	σ	Mean	Median	σ	Corporates
Entire sample	178.85	88.40	497.16	17.52	9.18	36.35	546
Year							
2004	63.33	32.50	98.58	7.34	5.00	7.54	273
2005	59.52	30.97	93.58	6.96	5.00	6.83	294
2006	59.80	27.60	98.50	6.28	4.90	5.67	409
2007	71.75	34.10	113.12	6.70	5.00	5.57	421
2008	229.94	111.00	374.84	22.23	10.00	48.89	403
2009	333.81	133.96	1294.91	30.91	15.00	70.19	374
2010	177.09	111.20	187.51	15.46	9.72	19.23	377
2011	238.20	137.12	340.27	21.32	11.00	37.66	379
2012	260.71	148.41	339.94	26.06	15.09	40.99	378
2013	180.66	110.45	414.20	20.09	14.00	28.42	433
2014	134.72	93.94	150.13	17.04	11.14	19.98	452
Tenor							
1	125.17	35.61	603.22	25.79	9.54	62.81	546
2	143.59	52.30	537.62	22.06	9.94	45.90	546
3	160.64	67.60	511.56	19.27	10.00	36.74	546
4	175.62	82.84	494.70	17.00	9.75	31.57	546
5	184.56	92.24	478.75	14.76	9.00	28.23	546
6	193.40	101.83	475.19	15.04	9.00	27.87	546
7	197.61	107.00	468.36	15.32	9.00	28.29	546
8	200.42	110.80	462.90	15.60	9.11	28.45	546
9	202.66	113.60	458.63	15.58	9.16	28.19	546
10	204.55	115.98	455.24	14.92	9.00	26.97	546
Rating							
Aaa	53.23	17.40	65.67	5.94	4.06	4.89	9
Aa	86.63	61.06	108.06	9.85	6.00	16.63	102
A	101.84	61.14	145.77	11.53	6.06	18.88	235
Baa	133.23	88.87	169.16	13.98	8.64	21.90	263
Ba	325.83	244.86	323.02	30.54	20.00	44.83	131
B	553.53	408.03	738.23	40.06	24.19	61.71	92
Caa	1269.38	780.51	3312.23	98.22	51.51	148.09	26
Ca-C	8984.65	251.85	11617.14	581.07	18.22	758.41	2
WR	234.67	115.54	404.74	25.00	15.38	39.29	147

returns the risk-neutral default probability $\left(PD^{\mathbb{Q}}_{rc|y,m}\right)$. Using Equation (1), we get the risk-neutral default intensities by rating class, year and maturity $\left(\lambda^{\mathbb{Q}}_{rc|y,m}\right)$.

Coverage ratio

Given the two default intensities $\lambda^{\mathbb{P}}_{rc|y,m}$ and $\lambda^{\mathbb{Q}}_{rc|y,m}$, we calculate the relative risk premium or the coverage ratio (μ), as:

$$\mu_{rc|y,m} = \frac{\lambda^{\mathbb{Q}}_{rc|y,m}}{\lambda^{\mathbb{P}}_{rc|y,m}}. \tag{15}$$

In the remainder of this study we drop the subscripts $(rc|y, m)$ to simplify the notation.

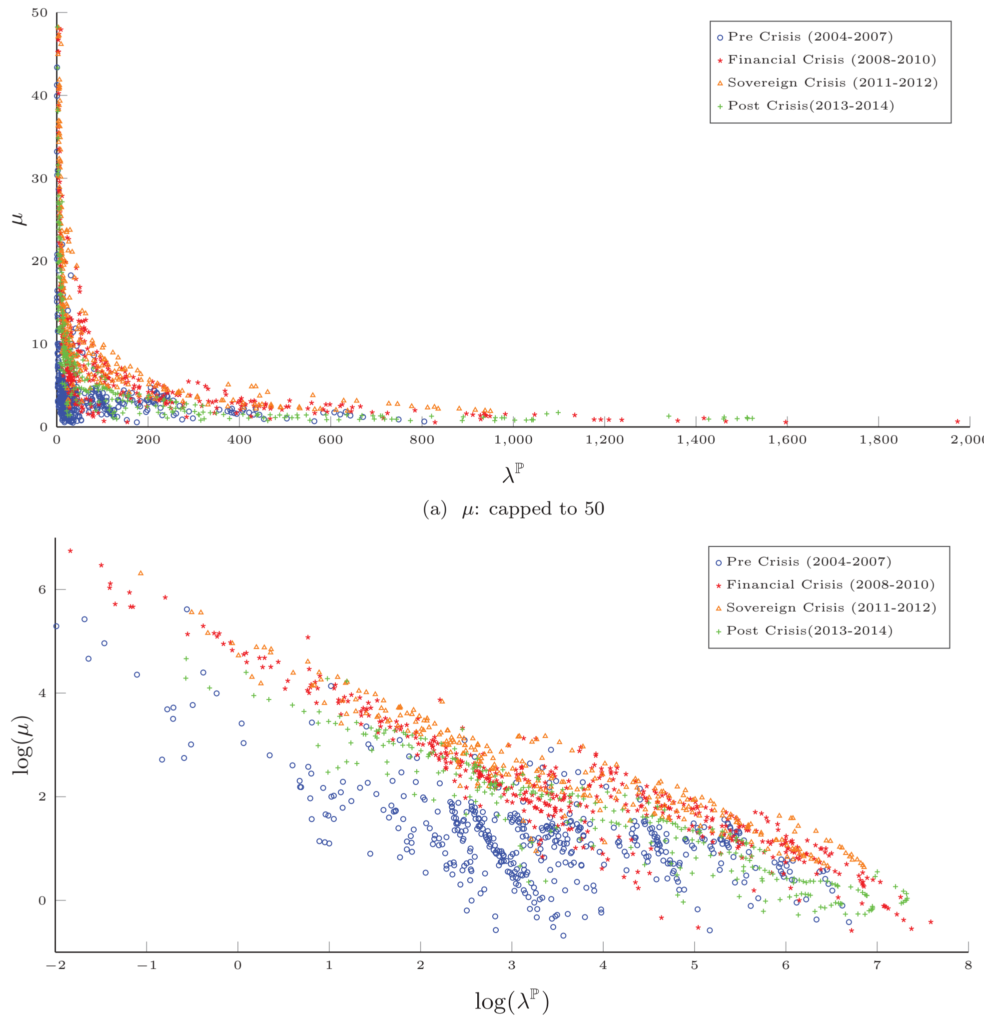
III. Results: risk premia

Coverage ratios

Applying the described methodology and employing the data set results in *ca* 1500 coverage ratios. To ensure the representativeness per rating class, we exclude the coverage ratios which are backed by less than five corporates. The computed coverage ratios (μ) are shown in Figure 3(a) as a function of $\lambda^{\mathbb{P}}$. The expected convex decreasing relationship is supported by the data as one can see in the graph. This confirms the findings of Berg (2009, 2010). The analysis of different sub-periods, represented by different colours, shows that the coverage ratios changed substantially over time.

In Figure 3(b), the data points are presented with a logarithmic scale, which clearly shows the upward and downward shifts. Risk premia were low before the financial crisis and during the financial and sovereign crises the coverage ratios shifted upwards, while returning to lower levels after the crises. The post-crisis levels are still above the initial levels (pre-crisis) and this could indicate a permanent upward shift in risk premia.

We average the coverage ratios per maturity and per rating class over different sub-periods to increase the readability of the results and to compare them



(a) μ: capped to 50

(b) μ: logarithmic scale

Figure 3. Coverage ratios.

with prior studies. In Table 2, the average 7-year coverage ratios per letter rating are shown. Over the entire period, the decreasing trend is evident and the coverage ratios are higher than the ones reported by Hull (2012) (last column of Table 2). This is especially true for coverage ratios derived from high-rated corporates (Aaa and Aa). For the other rating classes we find an increase of the coverage ratios between 30 and 80%.

The pre-crisis coverage ratios, which overlap partially with Hull's and Driessen's sample period, are lower than those of Hull (2012) and are located within the interval of Driessen (2005). In this period, the coverage ratio of Aa-corporates (2.33) does not fit the expected decreasing trend. This coverage ratio can be explained by examining the sample more thoroughly. In the pre-crisis sample, 63% of the Aa-rated corporates are banks, which were perceived as low-risk before the crisis. The derived risk-neutral default intensity is, therefore, low. Comparing the risk-neutral default intensity with the corresponding actual value results in a low coverage ratio for this rating class. For the rating classes below Aa, banks' proportion is smaller and hence higher risk-neutral default intensities for these rating classes are found in the pre-crisis period.

For the highly stressed periods (financial and sovereign crises), we find coverage ratios that are higher than the ones reported previously. Coverage ratios are more than doubled when using Hull's ratios as a reference. In the post-crisis these ratios decreased, but for high credit quality debt these ratios are still substantially high.

Table 2. Coverage ratio: maturity 7 years.

Period	Entire period 2004–2014	Pre-crisis 2004–2007	Financial crisis 2008–2009	Sovereign crisis 2010–2012	Post-crisis 2013–2014	Hull (2012) 1996–2007
Aaa	78.18	19.00	137.36			17.2
Aa	20.67	2.33	22.70	37.64	29.88	11.5
A	9.99	3.52	11.01	16.00	12.88	6.5
Baa	6.31	2.67	7.88	9.79	6.78	4.8
Ba	3.87	2.14	5.62	5.38	3.30	2.1
B	2.32	1.83	3.10	3.05	1.40	1.3
Caa	1.55		1.34	2.01	1.06	1.4

By comparing the coverage ratios over the different sub-periods, one can see that for high credit-quality companies these ratios are volatile, while for the lower-rated corporates the coverage ratios fluctuate less over time. This observation is a signal of a systemic crisis and could result from the additional risk arising from a market-wide failure. If one interprets the post-crisis coverage ratios as a permanent shift in risk premia, this will impact corporates' funding costs especially those firms with an excellent credit quality.

Going from letter to alphanumeric rating classes and changing the maturity to 5 years, results are mixed. For these coverage ratios the decreasing trend is broadly speaking still valid, as shown in Table 3. However, there are multiple coverage ratios that do not fit this decreasing trend in all selected sub-periods. Especially in the pre-crisis period the coverage ratios fluctuate around a flat mean of *ca* 3.5. This fluctuation around a mean was also found by O'Kane and Schloegl (2002) (last column of Table 3), but their mean is somewhat higher than the one we find. For the sub-periods after 2008, multiple observations do not follow the decreasing trend. Nevertheless, the highest coverage ratios are still found for the corporate with the lowest (default) risk profiles and lower coverage ratios are found for noninvestment grade corporates.

Estimation

Next, risk premia are fitted parametrically to find a relationship between the default intensity and the coverage ratio (both logged) that can be used to derive the coverage ratio for rating classes with a few or without observations (e.g. Ca-C). We already have some insight in the kind of relationship that we are trying to fit. First, the relationship between the coverage ratios and $\lambda^{\mathbb{P}}$ should be decreasing and convex, consistent with prior literature by Berg (2009). Second, coverage ratios must be larger than one, since risk premia are in general positive $\left(\lambda^{\mathbb{Q}} \geq \lambda^{\mathbb{P}}\right)$. Finally, coverage ratios should converge towards one for high values of $\lambda^{\mathbb{P}}$, since both $\mathrm{PD}^{\mathbb{P}}$ and $\mathrm{PD}^{\mathbb{Q}}$ are bounded ($\mathrm{PD} \in [0, 1]$).

Table 3. Coverage ratio: maturity 5 years.

Period	Entire period 2004–2014	Pre-crisis 2004–2007	Financial crisis 2008–2009	Sovereign crisis 2010–2012	Post-crisis 2013–2014	O'Kane and Schloegl (2002)
Aaa	242.70	33.22	452.19			
Aa1	57.42	4.92	167.68	52.18		4.47
Aa2	34.78	2.84	58.81	57.33	40.82	3.12
Aa3	22.62	0.90	22.57	51.31	23.06	3.52
A1	9.71	1.45	13.15	19.31	8.38	4.05
A2	22.56	3.98	32.58	42.14	20.30	4.67
A3	13.82	4.64	8.62	28.18	15.86	4.77
Baal	8.18	4.07	6.04	14.53	9.03	5.33
Baa2	10.02	2.82	8.22	20.78	10.07	5.54
Baa3	11.00	4.10	12.07	22.54	6.42	5.02
Ba1	6.44	3.68	6.56	10.08	6.37	3.56
Ba2	9.12	4.31	14.79	15.14	4.06	3.19
Ba3	5.31	4.57	4.34	8.11	3.21	
B1	5.55	3.88	5.78	9.55	2.66	
B2	3.06	1.54	3.96	5.15	1.28	
B3	2.85	1.21	2.91	4.49	1.16	
Caa1	1.99		1.11	3.21	1.03	
Caa2	1.77		1.95	2.00	1.14	

Due to the nature of the construction, coverage ratios exhibit a power specification ($y = ax^b$), with b negative (see Figure 3). To transform this nonlinear relationship, we apply a log–log specification like in Berndt et al. (2008):

$$\log(\mu) = \tilde{a} + \tilde{b}\log\left(\lambda^{\mathbb{P}}\right) \tag{16}$$

After removing outliers[7] and expressing the actual default intensities in basis points, $\tilde{a}$ and $\tilde{b}$ are estimated for the different predefined sub-periods with ordinary least squares (OLS). Panel A of Table 4 summarizes the results. All coefficients are significantly different from zero and the adjusted-R^2 are relatively high. One exception is the R^2 of the pre-crisis period, which is lower than 50%. The R^2 are even above 80% for the periods of distress. Figure 4 shows the fit graphically for the entire observation period and shows that the estimated relationship is underestimating the coverage ratios for low values of actual default intensities.

Although the log–log transformation is capturing the fit rather well, extrapolating this linear relation returns coverage ratios smaller than one for default intensities that are higher than exp $\left(\frac{-\tilde{a}}{\tilde{b}}\right)$. For the entire sample period $\left(\frac{-\tilde{a}}{\tilde{b}}\right) = 7.10$, as one can see in Figure 4, and translating this into basis points leads to a value of 1215.

To deal with this issue another specification of the relationship between coverage ratios and actual default intensities is needed. Therefore, we opt for a log-transform of the dependent variable:

$$\log(\mu) = a\left(\lambda^{\mathbb{P}}\right)^{b}. \tag{17}$$

This specification has the advantage that the estimated coverage ratios will be larger than one and for high actual default intensities the coverage ratios will converge towards one. The main drawback of this specification is that the relationship is nonlinear in the coefficients and thus a different estimation procedure is required. Hence, we apply the nonlinear least-squares approach using the Levenberg–Marquardt (Levenberg 1944; Marquardt 1963) algorithm with starting values (0, 0) to obtain estimates for a and b.

Estimated coefficients and adjusted-R^2 can be found in the Panel B of Table 4. The results are similar to the ones with the log–log transform. All coefficients are significantly different from zero and

Table 4. Estimates coefficients: no industry distinction.

Panel A: linear: $\log(\mu) = \tilde{a} + \tilde{b}\log\left(\lambda^{\mathbb{P}}\right)$			
Period	$\tilde{a}$	$\tilde{b}$	R^2-adj
Entire period	3.6516***	−0.5099***	0.5839
Pre-crisis	2.4241***	−0.3580***	0.3775
Financial crisis	4.3709***	−0.6075***	0.8524
Sovereign crisis	4.5015***	−0.5727***	0.8969
Post-crisis	3.8979***	−0.5728***	0.8884
Panel B: power: $(\mu) = a\left(\lambda^{\mathbb{P}}\right)^{b}$			
Period	a	b	R^2-adj
Entire period	4.1649***	−0.2588***	0.6162
Pre-crisis	2.9242***	−0.2975***	0.5036
Financial crisis	4.7708***	−0.2460***	0.8671
Sovereign crisis	5.1012***	−0.2263***	0.9126
Post-crisis	4.7200***	−0.3015***	0.8276

*** for p-values < 0.001, ** for p-values < 0.01, * for p-values < 0.05, . for p-values < 0.1

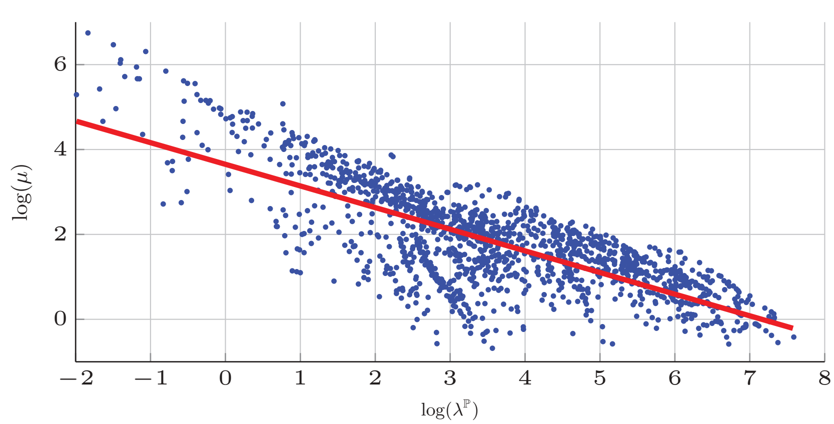
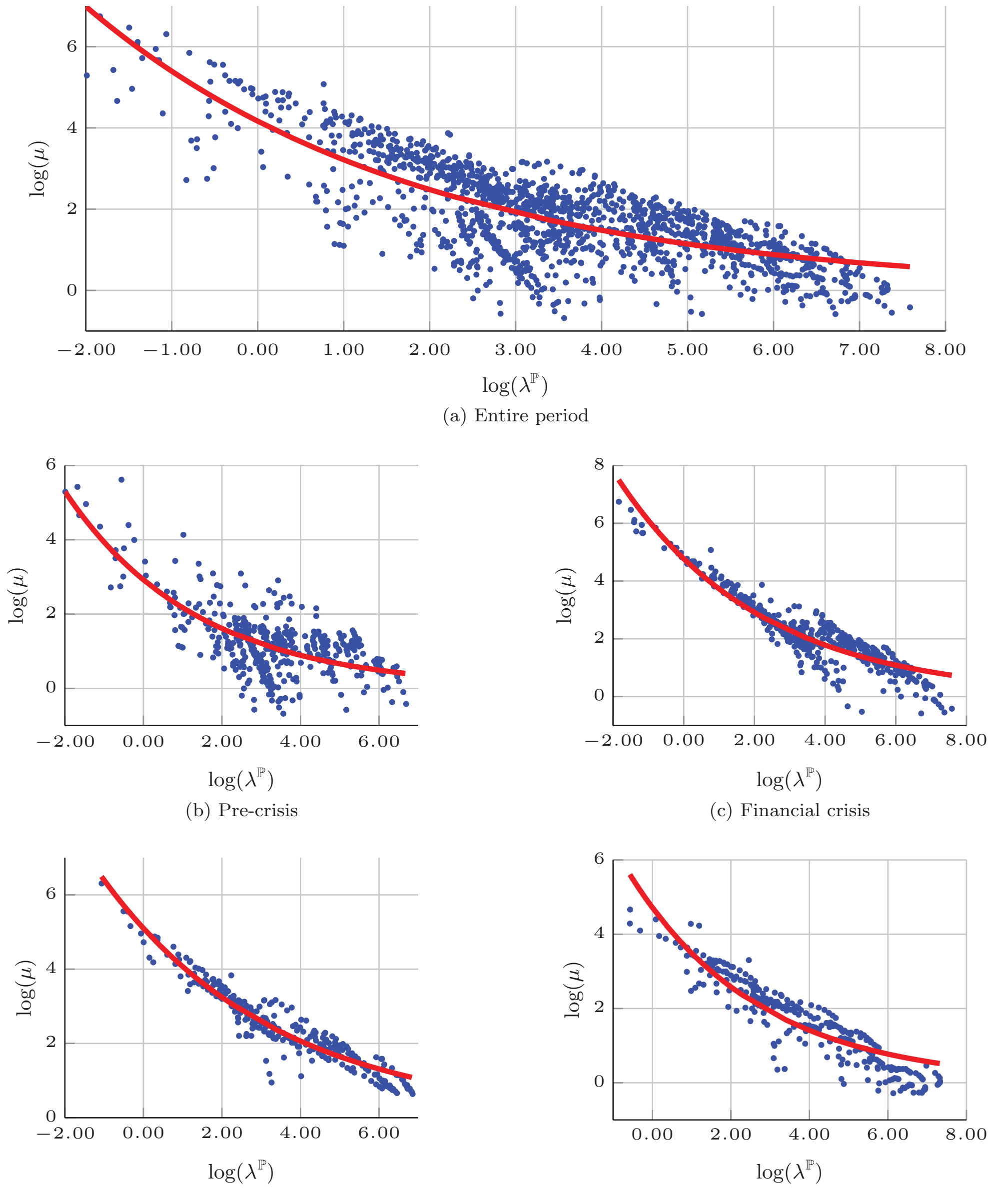


Figure 4. Entire period: log–log transform.

[7]In total, we identify 28 observations as outliers, which represent 1.86% of the sample.



(a) Entire period
(b) Pre-crisis
(c) Financial crisis
(d) Sovereign crisis
(e) Post crisis

Figure 5. $\log(\mu) = a\left(\lambda^{\mathbb{P}}\right)^{b}$.

also in terms of the adjusted-R^2 the differences between the log–log and the log-transform are not that large. For the pre-crisis period an improvement of the R^2 is found, while for the post-crisis period the R^2 declines compared to the log–log transform. The fitted relationships are also graphically shown in Figure 5. For the entire sample period, the fitted line in Figure 5(a) still underestimates the risk premia for low default intensities. Splitting the entire period into the sub-periods shows that this is mainly due to the pre-crisis period, where the relationship between coverage ratios and $\lambda^{\mathbb{P}}$ is less straightforward. For the other sub-periods, the fit performs better and thus extrapolation of these results appears to be more credible.

The results above are calculated with the alphanumeric rating classes, but when applying the same procedure for letter rating classes results are similar. In Table 5, the estimated coefficients and fitted coverage ratios per letter rating are summarized. With the actual default intensity of the Ca-C rating class, derived from the Markov chain, and the estimated coefficients, we can calculate its coverage ratio. Comparing the fitted coverage ratios of the entire sample period with the ones of Hull (2012), we can conclude that they are in the same order of

Table 5. Estimation results: letter rating.

Panel A: estimates-fit: $\log(\mu) = a\left(\lambda^{\mathbb{P}}\right)^{b}$					
Period		a	b		R^2-adj
Entire period		4.1733***	−0.2839***		0.6902
Pre-crisis		2.9179***	−0.3558***		0.5550
Financial crisis		4.6197***	−0.2559***		0.9019
Sovereign crisis		4.8554***	−0.2214***		0.9586
Post-crisis		4.4020***	−0.3013***		0.8293
Panel B: Fitted coverage ratios: maturity 7 years					
Period	Entire period	Pre-crisis	Financial crisis	Sovereign crisis	Post-crisis
Aaa	61.65	13.51	99.91	160.34	120.92
Aa	10.10	3.40	13.68	29.43	17.99
A	7.30	3.07	10.22	16.16	7.86
Baa	4.76	2.26	6.90	10.58	4.51
Ba	2.78	1.64	3.76	5.30	2.57
B	2.14	1.40	2.73	3.81	1.99
Caa	1.77	1.27	2.09	2.93	1.70
Ca-C	1.51	1.18	1.75	2.24	1.43

*** for p-values < 0.001, ** for p-values < 0.01, * for p-values < 0.05, for p-values < 0.1

magnitude. Only for the Aaa-rating class the estimated coverage ratios are three times higher than the one of Hull (2012).

IV. Concluding remarks

This research contributes to the discussion of risk premia in credit markets. We quantify empirically the relationship between default probabilities under the risk-neutral measure (Q) and the actual measure (P) for European corporates, over a sample period spanning 2004–2014. In line with existing literature (Hull 2012; Hull, Predescu, and White 2005; Driessen 2005; O'Kane and Schloegl 2002; Berndt et al. 2008; Huang and Huang 2012), we conclude that in general the ratio between PDs under $\mathbb{Q}$ and P, which we call the coverage ratio, is larger than one. For average credit quality (Aa-Baa) the coverage ratio lays, roughly speaking, between two and five before the financial and sovereign crises. For highly distress bonds (high actual default intensity or PD) it converges towards one. The credit risk premium is a decreasing function of credit quality: the higher the credit quality, the higher the coverage ratio.

Most of the empirical studies were conducted before the financial crisis with US data and none of these studies included the European sovereign crisis. To our knowledge, we are the first to assess the behaviour of credit risk premia for European corporates along the financial and sovereign crises. Our results demonstrate that the coverage ratio substantially increased since 2008 and that its post-crisis levels (after 2012) are still higher compared to pre-crisis values. We find for instance that for high-rated corporates (Aa) post-crisis levels are more than double as high. If one interprets this increase as a permanent shift in risk premia, this will impact corporates' funding costs. The increase is less pronounced for low rated corporates, whose time-varying behaviour is less extreme over the considered sub-periods.

We also fit a linear and nonlinear relationship between the actual default intensity and the coverage ratio, to extrapolate coverage ratio values for rating classes which are not very well-populated after the financial and sovereign crisis (Aaa and Ca-C). Berg (2009) derived from a Merton model (Merton 1974) that the coverage ratio as a function of actual PD ($PD^{\mathbb{P}}$) is a convex function. Keeping this observation in mind, we fit the relationship in two ways. First, we estimate a power model by applying a log–log transformation. This transformation implies coverage ratios below one for low credit quality bonds. Second, to deal with this drawback, we apply only the log-transformation of the dependent variable. Both models performed equally well in terms of fit, but the second approach has the advantage that the coverage ratios converge towards one for highly distressed bonds.

Our findings are furthermore relevant in the context of state aid assessment, which is based on the concept of REV (cf. the European Commissions Impaired Asset Communication (O.J.C 2009)). The

REV of a portfolio of impaired assets is the value which takes into account a long-term average risk premium and not the risk-premium of a distressed market at the pricing date. REV estimates in that sense expected losses under normal circumstances, ignoring potential market failures related to lack of liquidity, risk aversion or loss of confidence in distressed situations. The methodology of coverage ratios can be deployed to estimate the (transfer) price of a portfolio ignoring the market distortions in crisis situations, and thus, could serve policy makers in assessing whether state aid is compliant with the REV principle. If assets are transferred to a state-sponsored vehicle at a price which exceeds the REV, the Commission will order recovery in the form of a claw back and/or in depth restructuring measures to limit distortion of competition[8].

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No potential conflict of interest was reported by the authors.

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[8]Moreover, the 2013 Banking communication (O.J.C 2013) restricts impaired asset measures further from the perspective of shareholder burden sharing, so that it becomes unlikely that a EU-state can demonstrate that a transfer above the REV would be aid limited to the minimum necessary.

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Appendix A. 2004–2005 transtion matrices assumptions

The reported transition matrices for the years 2004 and 2005 include only the rating class Caa-C. Since the transition matrices published after 2006 have also transition rates for the rating classed Caal, Caa2, Caa3, Ca-C, it was chosen to adjust the 2004 and the 2005 transition matrices to the same dimensions (20×20). Since there are no corporates with a credit rating of Caa or lower in 2004–2005, it will have a very low impact on the obtained numbers. These assumptions concern both rows and columns.

The transition rates from a certain rating class to the rating class Caa-C must be split up into four subcategories (columns). To do this, we used the transition matrices of 2006–2013. First, we summed up all transition rates per subcategory (e.g. 25.04 in Table A1) and calculated their relative importance ($14.37/25.04 = 0.5739$). This was done for every rating category separately and per year. Afterwards, the average per rating class over the years was taken and these percentages were then applied to split up the reported transition rate (68.05) in the four subcategories ($68.06 \times 0.5739 = 39.06$). Table 6 explains the underlying idea of this procedure.

For the extra rows a more restricted assumption was made. The transition rates are assumed to be equal to the average transition rates for the period 1983–2008. Since ratings are lagged these transition rates represent the average pre-financial crisis transition rates.

Table A1. Assumptions transition matrices 2004–2005.

Rating	Sum Caa-C	Caa1	Caa2	Caa3	Ca-C
B3 2006	25.04	14.37	8.06	1.52	1.09
Relative importance (%)	100	57.39	32.19	6.07	4.35
B3 2004	68.06	39.06	21.91	4.13	2.96

Appendix B. Statistics CDS Spreads

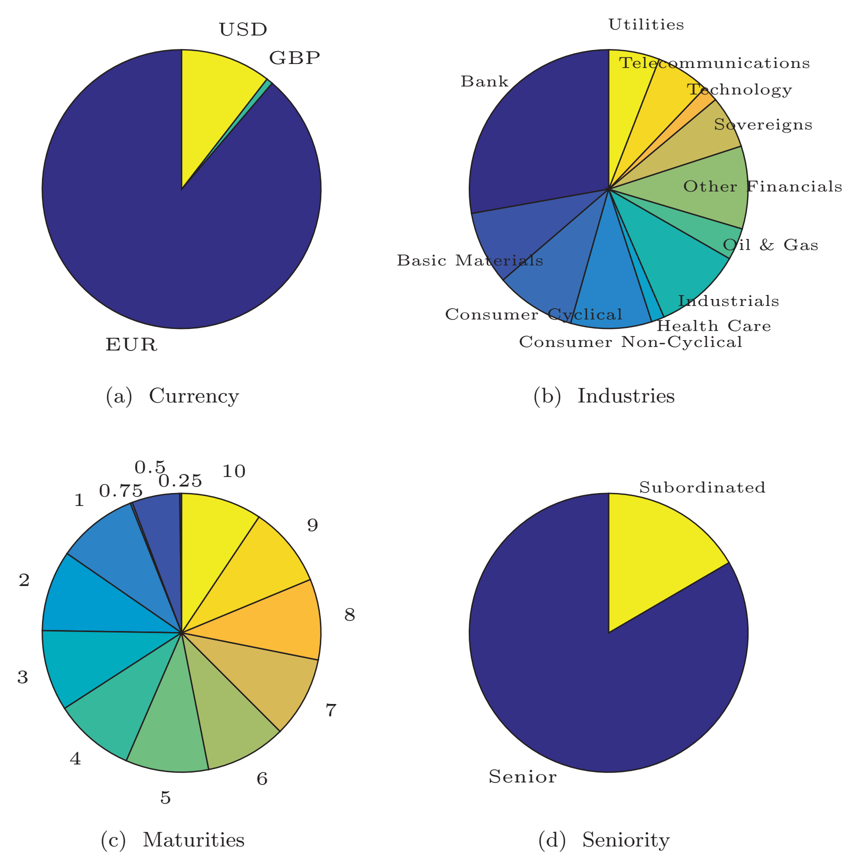

Figure B1. CDS Data. Source: CMA Datavision.