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Reference:

Jia Meisheng, Chen Guohua, Reniers Genserik.- An innovative framework for determining the damage probability of equipment exposed to fire
Fire safety journal - ISSN 0379-7112 - 92(2017), p. 177-187
Full text (Publisher's DOI): <https://doi.org/10.1016/J.FIRESAF.2017.05.015>
To cite this reference: <https://hdl.handle.net/10067/1440560151162165141>

An innovative framework for determining the damage probability of equipment exposed to fire

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Abstract

When considering a quantitative risk assessment of domino effects in chemical process facilities, the Damage Probability of equipment exposed to Fire (DPF) is a key element. In this paper, an innovative framework is proposed to determine the DPF, and the approach is demonstrated using a vertical plate. Being different from the current Probit model, structural reliability methods are applied to a pre-established lumped temperature model to obtain the DPF. Moreover, the static and dynamic DPF are distinguished, where the static one is obtained through the first-order reliability method and response surface method with an innovation in the pre-analysis of stable state to derive the limit state equation, while the dynamic one is obtained solving the Kolmogorov backward equation using the finite difference method based on stochastic diffusion process and first passage failure theories. The vertical plate demonstration shows the feasibility and availability of the proposed framework. A more practical case study with a horizontal LPG tank is also discussed to validate the suggested approach.

Keywords

Domino effect; Damage probability of equipment; Heat radiation; Structural reliability method; Kolmogorov backward equation

Acronyms	
DPF	damage probability of equipment exposed to fire
FDM	finite difference method
FORM	first-order reliability method
FPF	first passage failure
KBE	Kolmogorov backward equation
LSE	limit state equation
LTM	lumped temperature model
MFPT	mean first passage time
PASS	pre-analysis of stable state
RSM	response surface method
RTI	rising time interval
SDE	stochastic differential equation
SDP	stochastic diffusion process
<i>tte</i>	estimated time for an effective mitigation
<i>tff</i>	time to failure of equipment

1 Introduction

Domino accidents in chemical and process industries have been drawing attention for several decades, and many catastrophes strengthened their importance [1]. A lot of research and papers have been made in the field. It can be witnessed by the recent encompassing volume on this topic by Reniers and Cozzani [2]. Therein, quantitative risk assessment is a widely used technique whose key components include the damage probabilities of equipment exposed to heat radiation, blast wave overpressure and/or blast fragments [3-5]. In this paper, the damage probability of equipment exposed to heat radiation from fire (DPF) is focused upon.

Industrial fire scenarios may cause a severe heat load on various equipment and result in a bucking, cracking or rupture of containment. The mechanism might be a decreased

tensile strength of the material of the shell wall, an increased internal pressure, a generated high local thermal stress, or the melting of the nonmetallic parts. These issues are usually and specifically settled via experiments [6,7] or cumbersome finite element modeling [8,9]. On the positive side, both the experiment and finite element modeling are reliable and accurate. However, they may be too heavy to integrate into a technical framework for quantitative risk assessment of domino effect.

To determine the DPF, a square of the ratio of two distances was used earlier [10]: one is the maximum distance at which the source fire could damage the target unit based on a heat radiation threshold, and the other is the spatial distance between the source and target unit. Reviewing this ratio method, the interpretation of the derived DPF is ambiguous, and the uncertainty source is unclear. In Khan and Abbasi's work [3], though the combination of build-up high pressure and material failure of pressure vessels was considered specifically, the determination of DPF still used the ratio method. Currently, the Probit model, linking the DPF with time to failure of equipment (t_{tf}), is widely used [4,5]. The Probit parameters are obtained by applying a lognormal probability density function to the estimated time for an effective mitigation (t_{te}). The t_{tf} is calculated using a fitted relation to heat radiation intensity and equipment volume [11]. Essentially, the Probit model is a binary-class regression that the coefficients are just determined by t_{te} and absolutely irrelevant to t_{tf} .

In this paper, structural reliability methods are introduced [12]. The static DPF, which is independent of time, is firstly researched using the first-order reliability method (FORM) and response surface method (RSM) [13] coupling with an innovation in the

pre-analysis of stable state (PASS) of the lumped temperature model (LTM) to derive the limit state equation (LSE). The dynamic DPF that depends on time is obtained by solving the Kolmogorov backward equation (KBE) using the finite difference method (FDM) based on stochastic diffusion process (SDP) and first passage failure (FPF) theories [14,15]. The static and dynamic DPF offer complementary perspectives to state the same problem. To display the landscape of the proposed framework and approach clearly, a vertical plate is used to make a demonstration. A more practical case study with a horizontal LPG tank is also discussed. Though the research is carried out within a quantitative risk assessment of domino effect in mind, inherently safer design of target equipment and layout optimization of plants or clusters could also be elaborated.

In the following, Section 2 briefly introduces some preliminary theories and methods. The framework of the approach and its demonstration using a vertical plate to guide the application and interpretation are presented in Section 3. The practical case study of a horizontal LPG tank is discussed in Section 4. Finally, some conclusions are made in Section 5.

2 Preliminary theories and methods

2.1 The Probit method and model

According to Cozzani et al.'s theory and method [4,11], the Probit model is essentially an empirical binary-class regression which is the following:

$$\Phi^{-1}(DPF) = k_1 + k_2 \ln(ttf) \quad (1)$$

where $\Phi^{-1}(\cdot)$ is the inverse of standard normal cumulative distribution function, and

k_1 and k_2 are regression coefficients.

The basic assumption is that the target equipment exposed to fire is absolutely safe as long as t_{tf} is greater than t_{te} . Thus two special values of t_{te} are taken instead of t_{tf} to calculate k_1 and k_2 . Statistics show that an effective mitigation could start in less than 5 min only in 10% of cases, and less than 20 min in 90% cases [11]. In Eq. 1, 5 min corresponds to the inverse value of 3.71, while 20 min corresponds to the inverse value of 6.27. Then k_1 and k_2 are calculated as 9.25 and -1.85, respectively.

As for the t_{tf} , it is determined by fitting two sets of numerically simulated data, and the fitting relation is as follows [4]

$$\begin{cases} \ln(t_{tf}) = -1.13\ln I - 2.67 \times 10^{-5}V + 9.9, & \text{Atmospheric} \\ \ln(t_{tf}) = -0.95\ln I + 8.845V^{0.032}, & \text{Pressurized} \end{cases} \quad (2)$$

where I is the heat radiation intensity of fire and V is the equipment volume.

2.2 Structural reliability and first-order reliability method (FORM)

By definition, the performance of components or systems can be described using a performance function $g(\mathbf{X})$, where $\mathbf{X} = [X_1, \dots, X_i, \dots, X_n]^T$ is a vector of basic random variables. $g(\mathbf{X})$ is usually explicitly or implicitly derived from LSE. Failure corresponds to $g(\mathbf{X})$ taking non-positive values. Thus the failure probability P_f can be calculated integrating the joint probability density function of $\mathbf{X}$, $f_{\mathbf{X}}(\mathbf{x})$, over the failure domain

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (3)$$

In general cases, there is no analytical solution to Eq. 3, and the integral is potentially high-dimensional. For this reason, various structural reliability methods are developed including FORM. The key idea of FORM is to approximate $g(\mathbf{X})$ using a first-order

Taylor expansion and the expansion point, which is also called design point, is chosen on the failure or limit state surface. A detailed FORM procedure is the following [13]:

<p>(i) Assign the initial design point $\mathbf{x}^*$ with the mean $\boldsymbol{\mu}_X$;</p> <p>(ii) Calculate the sensitivity coefficient $\cos\theta_{X_i}$</p> $\cos\theta_{X_i} = - \frac{\frac{\partial g(\mathbf{x}^*)}{\partial X_i} \sigma_{X_i}}{\sqrt{\sum_{i=1}^n \left[\frac{\partial g(\mathbf{x}^*)}{\partial X_i} \right]^2 \sigma_{X_i}^2}} \quad (4)$ <p>where σ_{X_i} is the standard deviation of X_i.</p> <p>(iii) Calculate the reliability index β</p> $\beta = \frac{g(\mathbf{x}^*) + \sum_{i=1}^n \frac{\partial g(\mathbf{x}^*)}{\partial X_i} (\mu_{X_i} - x_i^*)}{\sqrt{\sum_{i=1}^n \left[\frac{\partial g(\mathbf{x}^*)}{\partial X_i} \right]^2 \sigma_{X_i}^2}} \quad (5)$ <p>(iv) Calculate the new design point $\mathbf{x}^*$</p> $x_i^* = \mu_{X_i} + \beta \sigma_{X_i} \cos\theta_{X_i} \quad (6)$ <p>(v) Repeat steps from (ii) to (iv) until the difference of $\ \mathbf{x}\$ is less than 10^{-6};</p> <p>(vi) Calculate P_f</p> $P_f = 1 - \Phi(\beta) \quad (7)$ <p>where $\Phi(\cdot)$ is the standard normal cumulative distribution function.</p>
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In this paper, if $\mathbf{X}$ is characterized with a non-normal distribution and is not mutually independent, the JC method and orthogonal transformation method are suggested [13].

2.3 Response surface method (RSM)

FORM is feasible if $g(\mathbf{X})$ is explicit. On the contrary, if $g(\mathbf{X})$ is implicit, a combination of RSM and FORM is available. The key idea of RSM is to approximate $g(\mathbf{X})$ using a response surface function, where a second-degree polynomial is always widely used [13]

$$\hat{g}(\mathbf{X}) = a + \sum_{i=1}^n b_i X_i + \sum_{i=1}^n c_i X_i^2 \quad (8)$$

where $\hat{g}(\mathbf{X})$ is the response surface function and a , b_i and c_i are undetermined coefficients.

A central composite design can be used to estimate $\hat{g}(\mathbf{X})$. The sample points, except

the central one, can be determined as follows

$$x_i = \mu_{X_i} \pm 2\sigma_{X_i} \quad (9)$$

A detailed RSM procedure is the following [13]:

- (i) Assign the initial iterative point $\mathbf{x}$ with the mean $\boldsymbol{\mu}_X$;
- (ii) Calculate the estimated value $\hat{g}_i$ at every sample point using $g(\mathbf{X})$, and then derive a , b_i and c_i by solving the linear equations

$$a + \sum_{i=1}^n b_i x_i + \sum_{i=1}^n c_i x_i^2 = \hat{g}_i \quad (10)$$

- (iii) Execute FORM using $\hat{g}(\mathbf{X})$ to obtain the new design point $\mathbf{x}^*$;
- (iv) Calculate the new iterative point $\mathbf{x}$ with the linear interpolation

$$\mathbf{x} = \boldsymbol{\mu}_X + \frac{g(\boldsymbol{\mu}_X)}{g(\boldsymbol{\mu}_X) - g(\mathbf{x}^*)} (\mathbf{x}^* - \boldsymbol{\mu}_X) \quad (11)$$

- (v) Repeat steps from (ii) to (iv) until the difference of $\|\mathbf{x}\|$ is less than 10^{-6} .

2.4 Stochastic diffusion process (SDP)

Generally, FORM and RSM are feasible for static or stable systems whose responses are time-independent. For dynamic systems, they could only be applied to some specific moments at which always the extreme responses are captured. In mathematics, SDP and FPF can be used to obtain a time-dependent structural reliability. SDP is a continuous-time Markov process with almost surely continuous sample paths. It is also a solution to a stochastic differential equation (SDE). In the following, the modeling of SDP is presented [15].

Firstly, a discrete dynamic system can be described as

$$\dot{X}_i = g_i(\mathbf{X}, t) + f_{ir}(\mathbf{X}, t) \xi_r(t), \quad \begin{matrix} i = 1, 2, \dots, n \\ r = 1, 2, \dots, s \end{matrix} \quad (12)$$

where $X_i(t)$ is the i^{th} component of the n -dimensional stochastic response process vector $\mathbf{X}(t)$; $\dot{\cdot}$ is the time derivative operation; $\xi_r(t)$ is the r^{th} component of the s -dimensional stochastic excitation process vector $\boldsymbol{\xi}(t)$; $g_i(\cdot)$ and $f_{ir}(\cdot)$ are deterministic functions.

Assume $\xi(t)$ is of zero-mean Gaussian white noise:

$$\begin{aligned} E[\xi(t)] &= 0 \\ E[\xi(t)\xi^T(t+\tau)] &= 2\mathbf{D}\delta(\tau) \end{aligned} \quad (13)$$

where $E[\cdot]$ is the mean operation, τ is a time interval, $2\mathbf{D}$ is an intensity matrix of $\xi(t)$ which is symmetric but non-diagonal in general, and $\delta(\tau)$ is the Dirac function. Define $\mathbf{W}(t)$ as a mutually independent m -dimensional zero-mean Gaussian white noise vector with unit intensity. Then $\xi(t)$ can be expressed as

$$\xi(t) = \mathbf{L}\mathbf{W}(t), \quad \mathbf{L}\mathbf{L}^T = 2\mathbf{D} \quad (14)$$

where $\mathbf{L}$ is a $s \times m$ -dimensional constant matrix.

Then Eq. 12 can be converted to the following

$$\dot{X}_i = g_i(\mathbf{X}, t) + \sigma_{ik}(\mathbf{X}, t)W_k(t), \quad \begin{matrix} i = 1, 2, \dots, n \\ k = 1, 2, \dots, m \end{matrix} \quad (15)$$

where $W_k(t)$ is the k^{th} component of $\mathbf{W}(t)$ and $\sigma_{ik} = (\mathbf{f}\mathbf{L})_{ik}$.

Consider $W_k(t)$ as a generalized Gaussian process, and substitute it with the derivative of the generalized Wiener process. Then Eq. 15 can be transformed to the Stratonovich SDE as follows

$$dX_i = g_i(\mathbf{X}, t)dt + \sigma_{ik}(\mathbf{X}, t) \circ dB_k(t), \quad \begin{matrix} i = 1, 2, \dots, n \\ k = 1, 2, \dots, m \end{matrix} \quad (16)$$

where $\circ$ is the definition of Stratonovich stochastic integral and $B_k(t)$ is the standard Wiener process.

The Ito SDE can be expressed as

$$dX_i = m_i(\mathbf{X}, t)dt + \sigma_{ik}(\mathbf{X}, t)dB_k(t), \quad \begin{matrix} i = 1, 2, \dots, n \\ k = 1, 2, \dots, m \end{matrix} \quad (17)$$

where the expression of $m_i(\mathbf{X}, t)$ is

$$m_i(\mathbf{X}, t) = g_i(\mathbf{X}, t) + \sum_{j,k} \frac{1}{2} \sigma_{jk}(\mathbf{X}, t) \frac{\partial \sigma_{ik}(\mathbf{X}, t)}{\partial X_j} \quad (18)$$

Define the transition probability density function of $\mathbf{X}(t)$ as $p(\mathbf{x}, t | \mathbf{x}_0, t_0)$, which

satisfies the Kolmogorov forward and backward equation. Their drift coefficient $a_i(\mathbf{x}, t)$ and diffusion coefficient $b_{ij}(\mathbf{x}, t)$ can be expressed as

$$a_i(\mathbf{x}, t) = m_i(\mathbf{X}, t)|_{\mathbf{X}=\mathbf{x}} \quad (19)$$

$$b_{ij}(\mathbf{x}, t) = \sum_k \sigma_{ik}(\mathbf{X}, t) \sigma_{jk}(\mathbf{X}, t)|_{\mathbf{X}=\mathbf{x}} = \sum_{k,l} 2D_{kl} f_{ik}(\mathbf{X}, t) f_{jl}(\mathbf{X}, t)|_{\mathbf{X}=\mathbf{x}} \quad (20)$$

2.5 First passage failure (FPF) and dynamic reliability

The FPF occurs when the response process of the system crosses some critical barrier at least once in a given time interval. For the n -dimensional time-homogeneous diffusion process $\mathbf{X}(t)$ defined in space R^n , a_i and b_{ij} will not explicitly contain t . Define Ω as an open safety domain in R^n , and its piecewise smooth boundary is Γ . To research the probability that $\mathbf{X}(t)$ stays in Ω , i.e., the dynamic structural reliability, a conditional reliability function needs to be defined [15]:

$$R(t|\mathbf{x}_0) = P\{\mathbf{X}(s) \in \Omega, s \in (0, t] | \mathbf{X}(0) = \mathbf{x}_0 \in \Omega\} \quad (21)$$

where $R(t|\mathbf{x}_0)$ represents the probability of $\mathbf{X}(t)$ staying in Ω in $(0, t]$ if it is in Ω at the initial time.

According to SDP theory, $R(t|\mathbf{x}_0)$ satisfies the following KBE

$$\frac{\partial R}{\partial t} = \sum_i a_i(\mathbf{x}_0) \frac{\partial R}{\partial x_{i0}} + \sum_{i,j} \frac{1}{2} b_{ij}(\mathbf{x}_0) \frac{\partial^2 R}{\partial x_{i0} \partial x_{j0}} \quad (22)$$

The initial condition is

$$R(0|\mathbf{x}_0) = 1, \quad \mathbf{x}_0 \in \Omega \quad (23)$$

When Γ is an absorbing boundary, the boundary condition will be

$$R(t|\mathbf{x}_0) = 0, \quad \mathbf{x}_0 \in \Gamma \quad (24)$$

The detailed boundary conditions should be determined based on physical or engineering circumstances. In addition, Eq. 22 is a high-dimensional parabolic partial

differential equation and when a_i is much greater than b_{ij} , it is convection-dominated such that there will be a strong numerical oscillation. Thus a first-order upwind difference scheme is suggested [16].

With the result of solving Eq.22, the conditional failure probability can be obtained

$$P_f(t|\mathbf{x}_0) = 1 - R(t|\mathbf{x}_0) \quad (25)$$

Define $p(\tau|\mathbf{x}_0)$ as the probability density function of $P_f(t|\mathbf{x}_0)$, where τ is the time of $\mathbf{X}(t)$ firstly passing through Γ . Then $p(\tau|\mathbf{x}_0)$ can be obtained through the differential operation

$$p(\tau|\mathbf{x}_0) = \left. \frac{\partial P_f}{\partial t} \right|_{t=\tau} = - \left. \frac{\partial R}{\partial t} \right|_{t=\tau} \quad (26)$$

The mean first passage time (MFPT) or mean lifetime $E(\tau)$ can be expressed as

$$E(\tau) = \int_0^\infty \tau p(\tau|\mathbf{x}_0) d\tau \quad (27)$$

Solving Eq. 27 with the numerical method should make sure that $p(\tau|\mathbf{x}_0)$ reaches zero in a satisfactory way, i.e., the simulation time for solving Eq. 22 should be long enough. Generally, the initial state of $\mathbf{X}(t)$ is given and always in the safe domain in engineering. Thus the unconditional functions of $R(t|\mathbf{x}_0)$, $P_f(t|\mathbf{x}_0)$, $p(\tau|\mathbf{x}_0)$ and $E(\tau)$ share the same values with the conditional ones.

3 Framework of the approach and its demonstration

3.1 The framework

Being different from the Probit model, which is essentially an empirical binary-class regression, the theoretical structural reliability methods are introduced to determine the DPF. The framework of the approach mainly consists of three parts as shown in Fig. 1:

- (i) Establishing the lumped temperature model (LTM), which is usually an ordinary differential equation or equations describing the temperature profile of target equipment from the moment that the fire starts to attack and is customized with different target equipment;
- (ii) Pre-analysis of stable state (PASS) and calculation of the static DPF using the first-order reliability method (FORM) and response surface method (RSM), where PASS is applied to the established LTM to derive the limit state equation (LSE);
- (iii) Calculation of the dynamic DPF, mainly including three steps: firstly, convert the established LTM to a stochastic differential equation (SDE) to obtain the drift and diffusion coefficients; secondly, define a conditional reliability function based on first passage failure (FPF) theory, and derive the Kolmogorov backward equation (KBE) that the defined function satisfies; and thirdly, solve the derived KBE using the finite difference method (FDM) with some reasonable boundary conditions.

Fig. 1. Scheme of the framework for determining the DPF

3.2 Demonstration using a vertical plate

3.2.1 Establishing the LTM

A vertical plate exposed to heat radiation from fire is used to demonstrate the approach. Assume the length and width of the plate are limitless, the fire causing the heat radiation on one side of the plate is stable, and the cold air on the other side is sufficient to dissipate heat at a constant temperature. Ignoring both the spatial distribution of plate temperature and the heat convection on two sides, the LTM can be expressed as

$$c_w \rho_w h_w \frac{dT_w}{dt} = (\alpha_w \varepsilon_f \sigma T_f^4 - \varepsilon_w \sigma T_w^4) - (\varepsilon_w \sigma T_w^4 - \alpha_w \varepsilon_a \sigma T_a^4) \quad (28)$$

where c_w , ρ_w , h_w , T_w , α_w and ε_w are the specific heat (in $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$), density (in $\text{kg} \cdot \text{m}^{-3}$), thickness (in m), average temperature (in K), radiant surface absorptivity, and emissivity of the plate, respectively. T_f and ε_f are the temperature (in K) and radiant surface emissivity of the fire, T_a and ε_a are the temperature (in K) and radiant surface emissivity of the cold air, and σ is the Boltzmann constant (in $5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$).

The left hand side of Eq. 28 represents the heat absorption rate per unit area of the plate, while the first two terms of the right hand side represent the heat radiation flux into the fire-side of the plate and the last two terms represent the heat flux out of the no-fire-side. Assume c_w , ρ_w , h_w , α_w , and ε_w are all independent of T_w . Then a more concise formula of Eq. 28 can be derived

$$\frac{dT_w}{dt} = A_1 T_w^4 + B_1 \quad (29)$$

where $A_1 = -\frac{2\varepsilon_w \sigma}{c_w \rho_w h_w}$, $B_1 = \frac{\alpha_w \varepsilon_f \sigma T_f^4 + \alpha_w \varepsilon_a \sigma T_a^4}{c_w \rho_w h_w}$, and both are constants.

Eq. 28 and Eq. 29 can be solved using the Runge-Kutta method [17] given the initial value of T_w . The solution offers a deterministic result for the temperature profile of

the plate. Fig. 2 shows an example calculated with the specific parameter values provided in Table 1. c_w takes the value of carbon steel of GB 20# heated from 20 °C to 500 °C [18]. ε_w takes the Birk [19] suggested value, and α_w approximately equals ε_w according to the Kirchhoff's law of thermal radiation. T_f also takes the Birk suggested value for pool fire and when for jet fire, the value is 1024 °C. Taking 1 for ε_f means the fire is treated as a blackbody. T_a and the initial value of T_w , T_{w0} , share the same value of ambient temperature. In Fig. 2, both curves tend to be stable, but the one for jet fire reaches stability more quickly and produces a higher stable temperature than the one for pool fire.

Table 1. Parameter values for LTM

Parameters	Values	Parameters	Values
c_w (J·kg ⁻¹ ·K ⁻¹)	569.3	T_f (°C)	816
ρ_w (kg·m ⁻³)	7850	ε_f	1
h_w (m)	0.016	T_a (°C)	20
α_w	0.9	ε_a	0.9
ε_w	0.9	T_{w0} (°C)	20

Fig. 2. Temperature profiles of the plate exposed to pool fire and jet fire

3.2.2 Stability and the static DPF

The LSE for applying FORM and RSM should be time-independent. Thus PASS is proposed to achieve the derivation of LSE from LTM. Traditionally, the various

material strengths and structural stresses are used to predict the failure of structures. Since the material strengths are uniform functions of temperature, it is feasible to substitute the material strengths and structural stresses with the corresponding temperatures to make a failure prediction in high temperature circumstances. PASS is the research on stable temperatures and time needed to achieve stability.

In Eq. 28, there is an ultimate value for T_w because the heat radiation flux into the fire-side decreases with T_w while the heat flux out of the no-fire-side increases. When the right hand side of Eq. 28 reaches zero, T_w will never change and the system holds stable. Define the stable temperature as T_s (in K) and assume $\frac{dT_w}{dt} = 0$. According to Eq. 29, T_s can be expressed as

$$T_s = \left(-\frac{B_1}{A_1}\right)^{0.25} \quad (30)$$

It is obvious that T_s is independent of c_w , ρ_w and h_w for the simplification of $\frac{B_1}{A_1}$. Define the time needed to achieve stability as t_{2s} (in s). It can be obtained through the integral operation

$$t_{2s} = \int_{T_{w0}}^{T_s} (A_1 T_w^4 + B_1)^{-1} dT_w \quad (31)$$

Eq. 31 can be solved with the Simpson formula [17]. An example calculated with the parameter values provided in Table 1 shows that it takes 1722.5 s to reach a T_s of 643.8 °C and when $T_f = 1204$ °C, it takes 770.9 s to reach a T_s of 969.4 °C. Namely, compared with Fig. 2, there is some numerical error.

Note the critical temperature of the material of the vertical plate as T_{cr} (in K). Because T_w increases monotonically with time and T_s is the maximum of T_w , the comparison of T_s and T_{cr} could form a LSE and predict the failure of the plate. The formed LSE

is the following

$$g(T_{cr}, T_s) = T_{cr} - T_s = T_{cr} - \left(-\frac{B_1}{A_1}\right)^{0.25} = 0 \quad (32)$$

where $g(T_{cr}, T_s)$ is the performance function.

Define the static DPF as P_{sd} . It can be expressed as

$$P_{sd} = 1 - P(T_{cr} - T_s > 0) \quad (33)$$

Then P_{sd} can be obtained applying FORM and RSM to Eq. 32. An example calculated with the specific values of distribution parameters of random variables provided in Table 2 gets a result of P_{sd} of 0.4148. Taking 650 °C as the mean value of T_{cr} is to make sure T_s is close to T_{cr} such that the result is sensitive. Only the correlation coefficient between α_w and ε_w is considered.

Table 2. Distribution parameter values of random variables for LSE

Random variables	Means, $\mu(\cdot)$	Standard deviations, $\sigma(\cdot)$	Distribution patterns	Correlation coefficients, $\rho(\cdot)$
α_w	0.9	0.05/3	Normal	$\rho(\alpha_w, \varepsilon_w) = 0.8$
ε_w	0.9	0.05/3	Normal	
T_f (°C)	816	100/3	Normal	
T_a (°C)	20	7/3	Normal	
ε_a	0.9	0.05/3	Normal	
T_{cr} (°C)	650	20/3	Normal	

Fig. 3 partly shows the sensitivities of random variables to P_{sd} . Each analysis is obtained by fixing the distribution parameter values of the sensible random variable at 10 equidistant discrete points in a reasonable interval as shown in legend of Fig. 3, and always $\varepsilon_f = 1$. It is clear that the means of T_f and T_{cr} impact P_{sd} greatly. Both are very easy to make P_{sd} approaching 1.

Fig. 3. Sensitivities of random variables to P_{sd}

3.2.3 The dynamic DPF

The flame temperature of the fire, T_f , is actually a stochastic process $T_f(t)$. It could be considered as the excitation of the system. For Eq. 29, the system can be modeled as a one-dimensional SDP with the response process $T_w(t)$ and the excitation process $B_1(t)$, where $B_1(t)$ is a function of $T_f(t)$. Assume $T_f(t)$ is a Gaussian white noise

$$T_f(t) = E[T_f(t)] + N(t) \quad (34)$$

where $E[T_f(t)]$ is the mean of $T_f(t)$ and will be a constant if $T_f(t)$ is a stationary process, and $N(t)$ is a zero-mean Gaussian white noise with an intensity of $2D$.

Similarly, $B_1(t)$ can be expressed as

$$B_1(t) = E[B_1(t)] + W(t) \quad (35)$$

where $W(t)$ is also a zero-mean Gaussian white noise but with an intensity of $2E$, and the relationship between $2E$ and $2D$ is the following

$$\begin{cases} 2E = \left(\frac{\Delta B_1}{3}\right)^2 \\ \Delta B_1 = \frac{1}{2} \frac{\alpha_w \epsilon_f \sigma}{c_w \rho_w h_w} \{ [E(T_f) + \Delta T_f]^4 - [E(T_f) - \Delta T_f]^4 \} \\ 2D = \left(\frac{\Delta T_f}{3}\right)^2 \end{cases} \quad (36)$$

where ΔB_1 and ΔT_f are the deviations of $B_1(t)$ and $T_f(t)$, respectively, and both

$2E$ and $2D$ take the values of one-third of their deviations based on engineering experiences.

Then according to SDP theory, the Ito SDE of Eq. 29 can be expressed as

$$dT_w = \{A_1 T_w^4 + E[B_1(t)]\}dt + \sqrt{2E}dB(t) \quad (37)$$

The drift coefficient $a(T_w, t)$ and diffusion coefficient $b(T_w, t)$ are

$$a(T_w, t) = A_1 T_w^4 + E[B_1(t)] \quad (38)$$

$$b(T_w, t) = 2E \quad (39)$$

Assume $T_f(t)$ is a stationary process, i.e., $E[T_f(t)]$ is a constant. Then both $a(T_w, t)$ and $b(T_w, t)$ will not explicitly contain t , and $T_w(t)$ will be time-homogeneous. Define the FPF of the vertical plate occurs when $T_w(t)$ crosses T_{cr} at least once in $(0, t]$. Then the conditional reliability function $R(t|T_{w0})$ can be defined as

$$R(t|T_{w0}) = P\{T_w(s) < T_{cr}, s \in (0, t] | T_w(0) = T_{w0} < T_{cr}\} \quad (40)$$

And it satisfies the following KBE

$$\frac{\partial R}{\partial t} = a(T_{w0}) \frac{\partial R}{\partial T_{w0}} + \frac{1}{2} b(T_{w0}) \frac{\partial^2 R}{\partial T_{w0}^2} \quad (41)$$

The initial and boundary conditions are

$$R(0|T_{w0}) = 1, \quad T_{w0} < T_{cr} \quad (42)$$

$$R(t|T_{w0}) = 0, \quad T_{w0} = T_{cr} \quad (43)$$

$$R(t|T_{w0}) = \text{finite}, \quad T_{w0} = 0 \quad (44)$$

Theoretically, there is only the generalized solution for Eq. 41 ~ Eq. 44 because Eq. 42 and Eq. 43 are incompatible at the corner point of T_{cr} . However, the numerical solution still exists. In Eq. 44, ‘finite’ should be replaced with the reliability value of 1 for the unreachable temperature of 0 K according to the third law of thermodynamics. A

proposed difference scheme for solving Eq. 41 ~ Eq. 44 is the following

$$\begin{cases} \frac{u_j^{n+1} - u_j^n}{\Delta t} = a(x_j) \frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{1}{2} b(x_j) \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}, & a(x_j) \leq 0 \\ \frac{u_j^{n+1} - u_j^n}{\Delta t} = a(x_j) \frac{u_{j+1}^n - u_j^n}{\Delta x} + \frac{1}{2} b(x_j) \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}, & a(x_j) > 0 \end{cases} \quad (45)$$

where Δt and Δx are time and space step size, n and j are index numbers for time and T_{w0} , and u_j^n is the numerical solution at the grid point of (n, j) .

Define the dynamic DPF as P_{dd} . Then it can be obtained as follows

$$P_{dd} = 1 - R(t) \quad (46)$$

To perform a calculation, 816 ± 56 °C for $T_f(t)$ as shown in Fig. 4, and 427 °C for T_{cr} , which Birk suggested as the limit value of plate temperature [19], are taken firstly. Other parameters share the same values provided in Table 1. The time and space step size are 1 s and 2 °C. The simulation time is 2400 s. The result is shown in Fig. 5. It is clear that there is an obvious rising time interval (RTI) for P_{dd} and when T_{w0} increases, the width of RTI becomes smaller as well as the starting point of RTI becomes closer to zero.

Fig. 4. Stationary excitation process of $T_f(t) = 816 \pm 56$ °C

Fig. 5. 3D view of P_{dd} when $T_f(t) = 816 \pm 56$ °C and $T_{cr} = 427$ °C

Fig. 6 shows the sensitivity of $E[T_f(t)]$ to P_{dd} specifying $T_{w0} = 21$ °C. A temperature of 1000 °C is taken instead of 1204 °C for jet fire because there is a strong numerical oscillation solving KBE (out of the scope of the paper). Assume a condition of $10^{-6} \leq P_{dd} \leq 0.9999$ to RTI. Then the width of RTI reduces from 199 s to 46 s and the starting point of RTI advances from 357 s to 209 s as $E[T_f(t)]$ increases from 816 °C to 1000 °C.

Fig. 7 shows the sensitivity of ΔT_f to P_{dd} . Since the influence of ΔT_f is relatively small, ΔP_{dd} , which is an increase of P_{dd} obtained when $\Delta T_f = 56$ °C, is analyzed. The shape of $\Delta P_{dd}(t)$ is approximately a single cycle sine wave and its amplitude becomes larger when ΔT_f increases. The cycle length corresponds to RTI and ΔT_f has little effect on it. In addition, the increase of $E[T_f(t)]$ could decrease the cycle length and increase the amplitude. Simply calculating indicates that the order of magnitude of ΔP_{dd} per unit ΔT_f is $10^{-5} \sim 10^{-4}$.

Fig. 6. Sensitivity of $E[T_f(t)]$ to P_{dd} specifying $T_{w0} = 21$ °C

Fig. 7. Sensitivity of ΔT_f to P_{dd} when: (a) $E[T_f(t)] = 816$ °C; and (b) $E[T_f(t)] = 1000$ °C

Fig. 8 shows the result of MFPT, $E(\tau)$, vs. T_{w0} . In the general interval of T_{w0} , the fluctuation of $E(\tau)$ will not exceed 60 s. T_{cr} influences $E(\tau)$ greatly. When $E[T_f(t)] = 816$ °C, approximately 1.6 s will be increased per unit T_{cr} and when $E[T_f(t)] = 1000$ °C, the increment will be approximately 0.7 s. In addition, the result indicates that ΔT_f nearly makes no difference to $E(\tau)$. The reason should be that the diffusion coefficient is too small compared to the drift coefficient.

Fig. 8. $E(\tau)$ vs. T_{w0} when $\Delta T_f = 56^{\circ}\text{C}$

3.3 Interpretations and comparison with the Probit model

In the framework of the approach, the static and dynamic DPF are distinguished for their different principles and application. They offer complementary perspectives to state the same problem. Comparatively, the dynamic DPF is closer to reality because both the fire excitation and the temperature response of equipment are stochastic processes. However, its entire procedure is time-expensive. The static DPF is essentially a special case of the dynamic one focusing on a specific time interval or duration in which the system holds stable. Both of them could be supplement of the Probit model for quantitative risk assessment of domino effect, and also could be beneficial to inherently safer design of target equipment. Table 3 lists a detailed comparison of the three methods in their application.

Table 3. Comparison of the three methods in their application

Methods	Principles	Strengths	Weaknesses
The Probit model	<ul style="list-style-type: none"> • Empirical statistics • Binary-class regression and generalized linear model 	<ul style="list-style-type: none"> • Light and concise • Emergency is considered • No modeling of target equipment 	<ul style="list-style-type: none"> • Only randomness of tte is presented • Not precise and customized to inherently safer design of target equipment and layout optimization of plants
The	<ul style="list-style-type: none"> • Theoretical 	<ul style="list-style-type: none"> • Lighter than the dynamic DPF 	<ul style="list-style-type: none"> • Establishing the LTM is time-

static	analysis	<ul style="list-style-type: none"> • More parameters are considered and variability presentation of parameters are comprehensive • More precise to equipment-related safety design 	consuming
DPF	<ul style="list-style-type: none"> • Limit state and/or stress-strength interference • FORM and RSM 		<ul style="list-style-type: none"> • Stability conditions should be satisfied
The	• Theoretical	<ul style="list-style-type: none"> • Closest to reality • Address on the dynamic process and MFPT can be estimated 	<ul style="list-style-type: none"> • Establishing the LTM and solving the KBE are both time-consuming
dynamic	analysis		
DPF	<ul style="list-style-type: none"> • SDP and FPF 		

4 Case study of a horizontal LPG tank

4.1 The LTM and its validation

A two-node LTM considering safety valve action is developed and validated with Anderson et al.' experiment [6]. The specific parameters and values are provided and sketched in Fig. 9.

Fig. 9. Sketch of the tank exposed to pool fire

Fig. 10 shows the results of the LTM and the experiment. Node 67, 68 and 24 in Fig. 10(a) represent the inner dry wall temperature of the experiment, and Node 114, 116 and 48 represent the liquid temperature. It shows that before 24.5 min (1470 s), being

the damage time of the experiment, the T_M curve agrees well with the experiment result while the T_{uw} curve is about 50 °C higher but shares the same trend. The difference is partly because of the inconsistent parameter values between the LTM and the experiment, and a higher value of T_{uw} is conservative from the safety perspective. The trends of the vapor pressure profiles in Fig. 10(b) are corresponding. However, the LTM curve does not draw the initial fluctuation caused by safety valve action and after 1100 s, the calculated vapor pressure still increases while the experiment's holds stable. This error should be from the safety valve modeling. As shown in Fig. 10(c), the cross point of the circumferential stress curve and the tensile strength curve is at 1426 s, indicating that the tank will take 23.8 min to achieve the defined failure or damage, which is conservative compared to 24.5 min of the experiment.

Fig. 10. Profiles of: (a) temperature; (b) vapor pressure p_v ; and (c) circumferential stress σ_θ and material tensile strength S

4.2 The static DPF

Two stable temperatures of 657.46 °C for T_{uw} and 47.51 °C for T_M can be obtained applying PASS to the two-node LTM with the parameter values provided in Fig. 9. Then applying FORM and RSM to the derived LSE with the specific values of distribution parameters of random variables provided in Table 4 leads to a result of P_{sd} of 2.6175×10^{-11} . In this paper, only the randomness of tank geometry, safety valve and fire are considered. All of them are mutually independent.

Table 4. Distribution parameter values of random variables for LSE of the tank

Random variables	Means	Standard deviations	Distribution patterns
V (m ³)	127	2/3	Normal
R (m)	1.51	0.05/3	Normal
h_w (m)	0.0159	0.001/3	Normal
$A_C \times 10^4$ (m ²)	50.6	5/3	Normal
F_C	0.86	0.1/3	Normal
T_f (°C)	816	56/3	Normal

Fig. 11 shows the sensitivities of random variables to P_{sd} . The influences of tank geometry are small as shown in Fig. 11(a). The wall thickness could make a difference in an order of magnitude of $10^{-16} \sim 10^{-4}$, while the influences of the tank volume and radius are less than 10^{-8} . As shown in Fig. 11(b), the influences of safety valve and fire are very great in the order of magnitude of 10^0 , i.e., both are very easy to make P_{sd} approaching 1.

Fig. 11. Sensitivities to P_{sd} of: (a) tank geometry; and (b) safety valve and fire

4.3 The dynamic DPF

Define that the FPF of the horizontal LPG tank exposed to pool fire occurs when either $T_{uw}(t)$ crosses T_{cr} , or $T_M(t)$ crosses T_{cg} , at least once in $(0, t]$, where T_{cr} is the critical temperature of the tank wall material relying on both T_{uw} and T_M , i.e., T_{cr} is related to both the decreased tensile strength and the increased internal pressure, and T_{cg} is the critical temperature of propane. Then a two-dimensional KBE can be derived, and the actual computational domain is a closed surface made up of the lines of $T_{uw0} = 0$, $T_{M0} = 0$ and T_{cr} curve sketched in Fig. 12. The two endpoints of T_{cr} curve are obtained assuming $T_{M0} = T_{cg} = 369.83$ K and $\sigma_\theta(T_{M0}) = S(T_{uw0}) = 0$ MPa, respectively. In addition, the modeled SDP is not time-homogeneous such that the stochastic averaging method is employed [15].

Fig. 12. Sketch of the computational domain for KBE of the tank

Taking the simulation time as 2400 s, the grid quantity of $T_{uw0} \times T_{M0} \times t$ as $118 \times 74 \times 481$ and ΔT_f as 56 °C, leads to a result of P_{dd} shown in Fig. 13. There is also a RTI for P_{dd} , and T_{uw0} and T_{M0} could impact the width and starting point of RTI greatly. Fig. 13(b) clearly shows the P_{dd} trend on boundaries as time increases.

Fig. 13. (a) 4D slice of P_{dd} specifying $t = 1426$ s, $T_{uw0} = 20$ °C and $T_{M0} = 20$ °C; (b) sequences of 3D view of P_{dd} specifying $t = 300$ s, 600 s, 1200 s, and 1500 s; and (c) 3D view of P_{dd} specifying $T_{M0} = 26$ °C and $T_{uw0} = 28$ °C

Fig. 14 shows the sensitivity of ΔT_f to P_{dd} , where ΔP_{dd} is the increase of P_{dd} specifying $T_{M0} = 26$ °C and $T_{uw0} = 28$ °C when $T_f(t) = 816 \pm 56$ °C. Relatively, the influence of ΔT_f is small. The order of magnitude of ΔP_{dd} per unit ΔT_f is 10^{-5} .

The shape of $\Delta P_{dd}(t)$ is approximately half a cycle length of sine wave, which is different from the vertical plate's. The reason should be that the simulation time is not long enough to make sure that P_{dd} is approaching 1 sufficiently, as shown in Fig. 14(a).

Fig. 14. (a) 2D view of P_{dd} specifying $T_{M0} = 26\text{ }^{\circ}\text{C}$ and $T_{uw0} = 28\text{ }^{\circ}\text{C}$; and (b) sensitivity of ΔT_f to P_{dd}

Fig. 15 shows the result of MFPT. When the initial temperature increases approximately from $0\text{ }^{\circ}\text{C}$ to $35\text{ }^{\circ}\text{C}$, $E(\tau)$ will decrease from 1569 s to 1114 s. The dominant factor is the initial LPG temperature, which means that the failure or damage in an overheated state is more sensitive than the one in a maximum tensile stress.

Fig. 15. (a) 3D view of $E(\tau)$; and (b) 2D view of $E(\tau)$

5 Conclusions

An innovative framework for determining the damage probability of equipment

exposed to fire (DPF) was proposed in which the static DPF and dynamic DPF were distinguished. The static one was obtained through the first-order reliability method and response surface method, while the dynamic one was based on stochastic diffusion process and first passage failure theories. They offered complementary perspectives to state the same problem.

The demonstration of a vertical plate and case study of a horizontal LPG tank showed the feasibility and availability of the proposed approach. Furthermore, interpretations and comparison with the Probit model were presented in detail. Inherently safer design of target equipment and layout optimization of plants could be elaborated by making the static and dynamic DPF as low as possible, for which the Probit model is unsuitable. Moreover, the estimated mean first passage time of the dynamic DPF could be addressed as a reference index for the emergency plan.

Acknowledgments

The research was supported by the National Natural Science Foundation of China (No. 21576102).

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