DEPARTMENT OF ENGINEERING MANAGEMENT

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The Electric On-Demand Bus Routing Problem with Partial Charging and Nonlinear Functions

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Abstract

Electric vehicle routing problems (EVRPs) with recharging policy consider the limited range of electric vehicles and thus include intermediate visits to charging stations (CSs). In general, minimizing the resultant charging costs such as charging duration or charging amount are also part of the objective of EVRP. Accordingly, EVRPs have received considerable attention over the past years. Nevertheless, this type of problems in the domain of passenger transportation, a VRP variant, has been rarely studied in the literature, especially with time windows, a realistic nonlinear charging function or partial charging policy. Hence this research extends the existing work on EVRP to an On-Demand Bus Routing Problem (ODBRP) which transports passengers with bus station assignment (BSA). The resultant problem is the EODBRP. Specifically, each passenger can have more than one stations to board or alight, and they are assigned to the ones with the smallest increase in the total user ride time (URT). In EODBRP, frequent intermediate visits to CSs are considered. Moreover, nonlinear charging functions are in use and partial charging strategy is applied. To solve the EODBRP, a greedy insertion method with 'charging first, routing second' strategy is developed, followed by a large neighborhood search (LNS) which consists of local search (LS) operators to further improve the solution quality. Experimental data were generated by a realistic instance generator based on a real city map, and the corresponding results show that the proposed heuristic algorithm performs well in solving the EODBRP. Finally, sensitivity analyses with divergent parameters such as the temporal distributions of passengers and bus ranges may provide practical guidance.

Keywords: on-demand bus routing problem, electric vehicle, non-linear charging function, partial charging

1. Introduction

Electric vehicles (EV) have a steady increase in the automotive market in recent years, and one of the reasons is to reduce greenhouse gas emissions from land transportation (Zhou et al., 2015). In parallel, the study of EV has also emerged in the vehicle routing problem (e.g. Schneider et al.,

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2014; Hof et al., 2017; Schiffer and Walther, 2017), which is referred as '*electric vehicle routing problem*'(EVRP), where the EV-related technological constraints are taken into account. In particular, compared with gasoline- or diesel-powered vehicles, the driving range of EV is limited, as a result, charging or battery swapping is a necessary component in route planning. Thus the main objective of the studied EVRP is an overall efficient routing of visits to customers and CSs. Especially, the charging process of lithium-ion batteries still generates greenhouse gases significantly (Onn et al., 2018), which adds the importance to investigate efficient charging solutions. Specifically, the charging process of these batteries is non-linear, and research has shown that ignoring nonlinearity can cause the solutions inefficient or even infeasible (Pelletier et al., 2017; Montoya et al., 2017).

Despite of research in EVRP, considering EV in the context of passenger transportation such as the dial-a-ride problem (DARP) or other variants, however, has not been widely studied. To the best of our knowledge, only a few papers (Masmoudi et al., 2018; Bongiovanni et al., 2019; Sayarshad et al., 2020; Ma et al., 2021; Zhang et al., 2022) have investigated EDARP with focus on detouring to battery charging or swapping stations, with small vehicles of six seats or electric autonomous cars. While in practice, given the development of the fast charging technologies of electric buses, a mini bus or even a regular bus can be quickly charged from empty to 80% full within an hour or fully charged in around two hours, which provides possibility to apply EVs in passenger transportation, both in real life and research field. In fact, there are cities have largely replaced conventional buses by pure electric or hybrid buses, benefiting from the technologies of fast charging and enlarged battery capacity (He et al., 2020). Hence, this work investigates an On-Demand Bus Routing Problem (ODBRP) integrated with intermediate visits to CSs. Specifically, partial charging and nonlinear charging functions are adopted in order to make the model more realistic.

In this work, we propose a heuristic approach to solve the EODBRP. To start with, the term 'leg' is used to refer to the route of a bus between two visits of CSs. Then each request is inserted in the leg with the minimum increase of the total URT. Related modifications are thus routing among stations, as well as adapting CSs, including which CS to visit and how much to charge. Calculating the recharging times is mathematically modeled as a subproblem and solved by CPLEX, if the number of the legs within a route is larger than or equal to three (for fewer legs the optimal recharging time can be calculated without CPLEX). In this way, optimal recharging times can be achieved. In the second phase, a LNS with LS operators is devised to improve the solution quality. Particularly, two parameters within the LNS framework control how many requests to destroy, and the number of candidate solutions to choose the most promising one from for LS. The performance of the LNS framework will be tested.

In terms of experiments, a real city map with given stations and distance matrix is in use, and requests were generated by a realistic instance generator. Then the performance of the proposed algorithm was tested, including the parameter setting of the LNS. Next, the features of the best known solutions(BKSs) are analyzed and presented. In addition, the effect of bus station assignment (BSA) is tested among various instance sizes, which distinguishes the ODBRP from the DARP and as a result improves the overall efficiency of the solution. Moreover, experiments to compare the performance with conventional buses were conducted, i.e. the number of buses needed, among instances with various temporal distributions of passengers, as well as the use of mini or regular buses. Experimental results show that intermediate charging plays an important role of applying electric bus; BSA not only improves the total URT but also reduces the total charging times and thus contributes both to the service quality and the environment; both mini and regular electric buses can be qualified substitutes of conventional buses with robustness, among various passengers' temporal distributions.

The contributions of this work can be listed as follows: It extends the use of EV in the context of passenger transportation, with realistic charging functions and charging policy. Subsequently, it proposes and verifies a possible division of the integrated problem, as well as solves the subproblem to optimality and the entire problem efficiently. Moreover, the parameter setting of the LNS framework achieves a tradeoff between exploring more neighborhoods and maintaining the neat attributes of the best known solution, the solution quality is thus improved. Last but not least, it conducts extensive numerical experiments with realistic artificial instances, in order to assess the performance of the proposed method as well as the feasibility of applying electric buses with recharging in the ODBRP. The outcome of the experiments may provide practical guidance.

The remainder of this paper is organized as follows. Section 2 presents the related work in the research field. Section 3 formally introduces the EODBRP. Next, section 4 describes the solution method, i.e. the heuristic and the CPLEX subproblem, and section 5 presents the computational experiments. Finally, section 6 concludes the paper and discusses future research.

2. Literature Review

This section provides a brief review on recent and related literature. First EVRP is introduced. Next, since this study involves partial charging and nonlinear charging function, the relevant literature are reviewed. Subsequently, as a variant of '*Fixed Route Vehicle Charging Problem*'(FRVCP) is the subproblem of this study for charging decisions, we revise the papers including FRVCP. Finally, the relevant research on EDARP are presented.

Research on EVRPs originally started with the green vehicle routing problem (Green VRP) proposed by Erdoğan and Miller-Hooks, 2012. In the Green VRP, vehicles with alternative fuels will be fully charged at refill stations. Then Schneider et al., 2014 extended Green VRP to EVRP. More specifically, in their paper, an EVRP with time windows was solved using a hybrid heuristic of variable neighborhood search and tabu search.

In EVRP, the charging policy particularly can be divided into three categories: full recharging with linear recharging function (e.g. Goeke and Schneider, 2015; Hiermann et al., 2016), partial recharging with linear function (Ángel Felipe et al., 2014; Schiffer and Walther, 2017; Desaulniers et al., 2016) or partial recharging with nonlinear function (Montoya et al., 2017). In this study, partial charging policy with nonlinear function is adopted. Thus in the following, we briefly review the literature dealing with partial charging and nonlinear function, while for more comprehensive and recent reviews of EVRP, we refer to Kucukoglu et al., 2021; Erdelić and Carić, 2019.

2.1. Partial charging policy

Full recharging was considered initially in most EVRP literature. However, this can be time-consuming and not eco-friendly. On the contrary, partial charging can enable the feasibility to serve more passengers with narrow time windows, since vehicles can have a wide range of charging time, from minutes to hours, depending on the State-of-Charge (SoC) level, charging technology and battery capacity (Martínez-Lao et al., 2017). In partial recharging, it is natural to only charge the amount that is enough to cover the remaining route, with the range anxiety of which the range cannot cover the whole route (Sweda et al., 2017) still taken into account. The concept of allowing partial charging was considered in Ángel Felipe et al., 2014; Bruglieri et al., 2015. Then Desaulniers et al., 2016 adopted partial charging in an EVRP with time windows. In addition, Keskin and Çatay, 2016 adopted partial charging in an EVRP with time windows and provided the mathematical model. This problem was solved by adaptive large neighborhood search. Comparing with the full charge strategy, they concluded partial charge can save the total costs substantially. In terms of EVRP with time windows, Desaulniers et al., 2016 also solved a problem in this domain and concluded that allowing multiple visits to CSs as well as partial charging both help to reduce routing costs and the number of vehicles compared with single visit and with full charging. Exact branch-price-and-cut algorithms was used to solve this problem. Similarly, Schiffer and Walther, 2017 concluded the benefits of partial charging in an electric location routing problem with time window. Specifically, a partial charging strategy can reduce the total travel distance and the number of visits to CSs.

2.2. Nonlinear charging function

The charging procedure was initially modeled as a linear function. While in fact, lithium-ion batteries are often charged in constant-current constant voltage (CC-CV) phases: in the CC pahse, SoC increases linearly, while in the CV phase nonlinearly and the charging time is prolonged due to the drop of the current (Pelletier et al., 2017). The standard way to deal with the nonlinear function is to transform it into a piece-wise linear function.

Based on the piecewise linear and concave charging functions, Zündorf, 2014 developed a propagating algorithm to compute a battery-constrained route. Later, Montoya et al., 2017 fitted piecewise linear functions according to real-world data, and showed ignoring nonlinearity can lead to poor or even infeasible solutions. A hybrid metaheuristic combining an iterated local search and a heuristic concentration was developed to solve the EVRP with nonlinear charging functions. Later, based on the nonlinear charging function in Montoya et al., 2017, Froger et al., 2019 proposed an arc-based model tracking of the time and the state of charge. Besides, they also proposed a path-based model. Experimental results showed the two models outperformed the node-based model.

2.3. FRVCP

One of the most challenging issues when solving an EVRP and its variants is the charging decisions, especially with the partial charging policy and nonlinear functions, it is critical to decide when and how much to charge, since the charging decisions influence significantly the feasibility and quality of the solutions. Therefore, in terms of solution methods to position a CS in a route, some authors have studied the FRVCP as a subproblem of EVRP. In such problems, the customer sequence in a route is fixed while the position of CS and the amount to charge are adjusted. The FRVCP is a variant of the Fixed Route Vehicle Refueling Problem (FRVRP) which aims to minimize the refueling cost for a fixed route. The FRVRP is NP-hard (Suzuki, 2014), thus the FRVCP is also NP-hard, and various solution approaches have been proposed in literature.

Montoya et al., 2016 solved an FRVCP with full charging and constant charging time. In their work, the FRVCP was formulated as a constrained shortest path problem and solved by the Pulse algorithm (Lozano and Medaglia, 2013). Similar approach was also applied in EVRP with time windows (Keskin and Çatay, 2018; Hiermann et al., 2019), electric location routing problem (Schiffer and Walther, 2018a,b). Next, some literature assumed an EV can visit at most one CS between any pair of non-CS vertices. Among them, an FRVCP within an EVRPTW with single charging technology and linear charging function were solved in Hiermann et al., 2016; Schiffer and Walther, 2018a; Hiermann et al., 2019. The difference is whether the full charging (Hiermann et al., 2016) or the partial charging (Schiffer and Walther, 2018a; Hiermann et al., 2019) was adopted. Exact algorithms were addressed to solve these problems. Meanwhile, Montoya et al., 2017 solved an FRVCP as a sub-problem of the EVRP with nonlinear charging functions, heterogeneous charging stations and the partial charging policy. A mixed integer linear programming (MILP) model was proposed and the problem was solved by a hybrid metaheuristic combining an iterated local search (ILS) and a heuristic concentration (HC). Based on the specific MILP formulation and the metaheuristic, Koç et al., 2019 solved an FRVCP in an EVRP with shared CSs and nonlinear charging. They proposed a multistart heuristic performing an adaptive large neighborhood search, together with the solution of mixed integer linear programs. Moreover, Baum et al., 2019 solved the FRVCP for EVs with realistic and heterogeneous CSs and battery swapping stations. A specific algorithm which combines different algorithmic techniques was developed and can solve this problem optimally on realistic inputs.

2.4. Extension in EDARP

A typical DARP includes operational constraints related to time windows, vehicles' capacities, maximum route duration and maximum ride time. Then the EDARP integrates classic DARP with the following features: routing to charging facilities, selecting charging stations, determining recharging times, as well as adapting the arrival and departure time at passengers' stations. In literature, Masmoudi et al., 2018 proposed an EDARP with battery swapping stations and a realistic energy consumption function. Three enhanced evolutionary variable neighborhood search algorithms were devised to solve this problem.

Besides, there are papers adopt electric autonomous cars which are more flexible to modify vehicles' routes, especially in real-time. Among them, Bongiovanni et al., 2019 proposed an electric autonomous DARP which covers detours to charging stations and recharge times. This problem is formulated as a 2 index and 3 index MILP, and solved by a Branch-and-Cut algorithm with new valid inequalities, with the objective to minimize a weighted sum of the total travel time and excess URT. Later, Sayarshad et al., 2020 devised a non-myopic dynamic routing problem of electric taxis with battery swapping stations, and proposed a formulation of traveling salesman problem with pickup and drop-off, including the battery capacity constraints. Then Ma et al., 2021 propose a location routing problem for the car-sharing system with autonomous electric vehicles, solved separately by genetic algorithm and GAMS. Recently, Zhang et al., 2022 proposed a routing problem of shared autonomous electric vehicles under uncertain travel time and uncertain service time. Branch-and-price algorithm was used to solve this problem.

To summarize, EVRP which takes charging decisions into account have drawn interest from academia. On the other hand, EDARP and its variants has not been widely studied. Thus applying EVs as well as considering charging decisions with partial strategy and nonlinear charging function distinguish this study from literature. In addition, this study proposes a unique variant of FRVCP with a 'charging first, routing second' strategy. Details will be explained in the following sections 3 and 4.

3. Problem Description

In this paper, we investigate the ODBRP using electric vehicles with non-linear charging function and the partial charging policy (EODBRP). Mathematically, EODBRP can be defined with a mixed integer programming (MIP) formulation, which consists two parts: the ODBRP with conventional vehicles introduced by Melis and Sörensen, 2022; in addition, the constraints concerning electric vehicles with nonlinear charging function and the partial charging policy are from an EVRP Montoya et al., 2017.

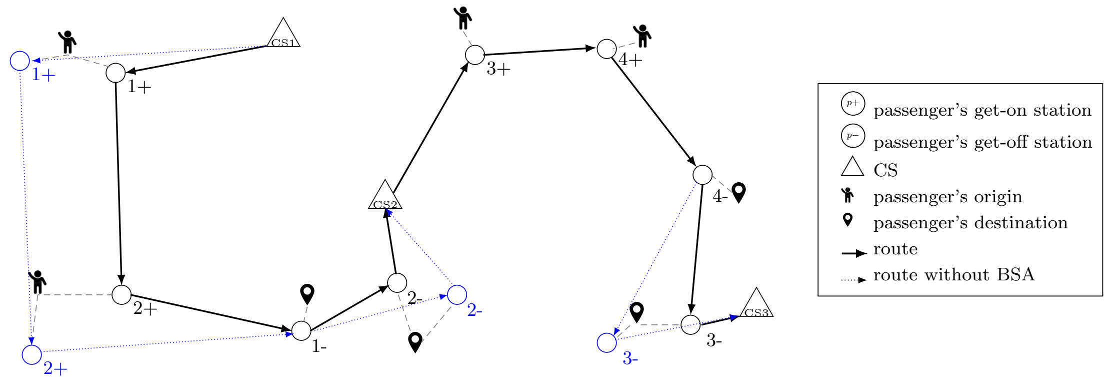

Figure 1: Illustrative example of EODBRP

3.1. The electric on-demand bus routing problem

A formal description of the EODBRP is as follows.

Let S denote the set of predefined stations for passengers to board and alight. Each station node $s \in S$ can be visited more than once (even by the same bus) or never. Let F be the set of CSs each with unlimited capacity where vehicles can be fully or partially charged, then F' the set of β copies of F. The use of F' is for the modeling convenience as each node $\in F'$ can be restricted to visit at most once. Next, let V be the set of nodes, i.e. $V = S \cup F'$. Let $G = (V, A)$ be a directed graph, where A is the set of arcs connecting any pair of vertices $\in V$. For any arc $(i, j) \in A$, the travel time is constant over time.

Passengers are geographically dispersed within a service area, and they send in travel requests in advance, possibly via a mobile application or website. The requests are assumed static and deterministic. For simplicity, we set each request corresponds to one passenger. Besides, passengers also specify their locations of origin and destination. Thus, let the sets of bus stations available for each passenger p's pickup/drop-off be noted a^u_{ps} and a^o_{ps} respectively. Then for each passenger the stations within a predefined walking distance to the origin or destination are stored in a^u_{ps} or a^o_{ps}. In our problem setting, at least one station is guaranteed for each passenger to respectively board and alight. If there are two or more stations, then bus station assignment (BSA), namely, which station is selected to board or alight among the sets, is a decision to be made by the algorithm. An illustrative example is in Fig 1. A bus departs from a CS, then visits passengers' stations as well as an intermediate CS, finally ends at a CS. Compared with the dotted lines where passengers are only picked up and dropped off at the closest stations to their origins and destinations, BSA reduces the distance both between passengers' stations and to/from CSs. BSA also allows for pooling of passengers at stops, so that the bus does not has to stop as frequently.

Passengers also indicate either their desired departure time or arrival time, then we implicitly calculate a hard time window for each passenger. In literature such as Cordeau and Laporte, 2003, a general formulation of time windows distinguishes inbound or outbound requests, as well as has separate time windows for pickup and drop-off, together with a constraint for the maximum ride time. However, in our model, each passenger p has one single hard time window of earliest departure and latest arrival $[e_p, l_p]$, thus the passenger has to be picked up and dropped off within $[e_p, l_p]$. The duration of each time window is set to $f \times t_{dir}$, i.e. $l_p - e_p = f \times t_{dir}$ where f is a constant, and t_{dir} is the direct travel time from the origin to the destination. The parameter f controls how strict the time windows are. A small value leads to a strict time window and vice versa. In this way, the maximum ride time for each passenger is restricted as well.

A homogeneous fleet of electric buses is dispatched, each with a finite capacity Q^{cap}, i.e. the limit of the number of passengers aboard simultaneously. All the buses have a battery of capacity Q (expressed in km). It is assumed that the EVs are equipped with a fully charged battery at the beginning of their route. Then feasible solutions should satisfy that the battery level when an EV arrives at and departs from any vertex is between 0 and Q. The details of the charging function will be explained separately in section 3.2.

Travel between any two nodes $\in V$ involves a travel time and a consumption of electricity. The travel time is proportional to the distance, given the travel speed is constant; similarly, the electricity consumption is assumed to be only proportional to the distance as well, regardless of the load, slope, etc. Thus, the triangular inequality holds for both the travel time and the electricity consumption.

The station sequence of each bus b is denoted SS_b, which records the stations (including CSs) that bus b has to visit in sequence. SS_b is one of the decisions made by the algorithm. Each station in SS_b is visited only when needed, i.e. at least one passenger needs to get on/off, or the bus charges a positive amount of electricity at a CS. Obviously, a visit to a charging station CS $\in F'$ is only allowed when no passengers are on board.

Each bus b's arrival and departure times at station $s \in SS_b$ (where $s \in V$) are respectively denoted as t^a_{bs} and t^d_{bs}. For station $s \in F'$, $t^d_{bs} = t^a_{bs} + \delta_s$, where δ_s denotes the charging time at $s \in F'$. Namely, the departure time from a CS is equal to the arrival time plus the charging time. For station $s \in S$, t^d_{bs} may be later than the arrival time t^a_{bs} as well, that is to say, buses are allowed to wait at station $s \in S$, even if there are passengers on board, under the condition that at least one passenger p is boarding at s and the arrival t^a_{bs} is prior to the earliest departure e_p. So the departure time t^d_{bs} may be later than the arrival time t^a_{bs}, and the difference is called *waiting time* t^w_{bs}, where $t^w_{bs} = t^d_{bs} - t^a_{bs}$. The reason to allow waiting is that it is a way to decrease the total URT, and thus indirectly increase the possibility to serve more passengers. Although it increases the URT of passengers aboard, it can reduce others'. On the other hand, if the holding policy does not apply, we assume passengers board and alight immediately when the bus arrives, and this service duration is negligible, namely t^d_{bs} is simply equal to t^a_{bs}.

The exact MIP formulation of EODBRP is in Appendix A. The aim of the EODBRP is to transport all the passengers with high service quality. Thus the objective function is minimizing the total URT. As for the service quality regarding the waiting time before passengers getting on board, it is considered guaranteed as long as the time windows are satisfied. Similarly, the service quality in terms of the walking distance is fulfilled implicitly when calculating a^u_{ps} and a^o_{ps} for every passenger p. In addition to the total URT, in the subproblem FRVCP, the total charging time is minimized for a given bus route, in order to minimize the impact on the environment, given the charging amount monotonically increases with charging time.

Finally, the decisions to be made in this EODBRP are summarized as follows: first, the passenger-bus assignment has to be performed, namely which bus serves which passenger; second, the bus station assignment, i.e. which station for each passenger to board and alight; finally, the station sequence of each bus as well as the arrival and departure time at each station, especially when, where and how much to charge.

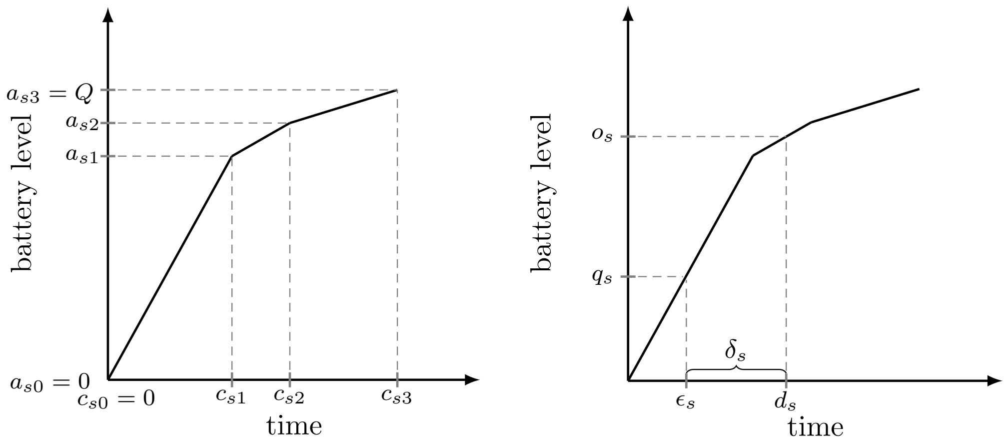

Figure 2: Piecewise linear charging function

3.2. Modeling of the nonlinear charging function

For the charging-related formulation, we adopt the model of electric vehicle introduced by Montoya et al., 2017, regardless of the constraints related to the VRP. That paper proposed an EVRP with nonlinear charging functions which were further approximated by piecewise linear functions.

In this study, we assume the CSs are identical, i.e. with the same charging function. Specifically, each CS $s \in F'$ is associated with a piecewise linear concave charging function $g_s(q_s, \delta_s)$. This function maps the charging level q_s and the charging time δ_s. More precisely, when a vehicle arrives at a CS s, the electricity level is represented by variable q_s, then the vehicle leaves s with the electricity level o_s, and the corresponding charging duration is δ_s as explained before this subsection. Moreover, in order to obtain the value of δ_s, let $\hat{g_s(\delta_s)}$ be the charging function when $q_s = 0$ and the battery is charged for δ_s time units, therefore the generalized δ_s when $q_s \geq 0$ and $o_s \geq q_s$ is calculated by the values of the inverse function as $\delta_s = \hat{g_s^{-1}(o_s)} - \hat{g_s^{-1}(q_s)}$.

As $g_s(q_s, \delta_s)$ is piecewise linear, the breakpoint of each piece is represented by $k \in \Gamma$. For instance, the first linear piece is bounded with breakpoints 0 and 1, while the second piece with 1 and 2, etc. Then c_{sk} and a_{sk} represent the charging time and the charging level for the breakpoint $k \in \Gamma$ of the CS $s \in F'$, where $\Gamma = \{0, 1, .., \gamma\}$ is the set of breakpoints of the piecewise linear function. Figure 2 is an illustration of $g_s(q_s, \delta_s)$ with breakpoints. ϵ_s and d_s are the mapped charging times according to the function. In other words, the electricity level q_s corresponds to the charging time point ϵ_s, while o_s corresponds to d_s. Naturally, variable $\delta_s = d_s - \epsilon_s$ represents the time spent at CS $s \in F'$. Variables z_{sk} and w_{sk} are equal to 1 if the charge level is between $a_{s,k-1}$ and a_{sk} , with $k \in \Gamma$ 0, when the EV arrives at and departs from CS $s \in F'$ respectively. Finally, variables α_{sk} and λ_{sk} are the coefficients of the breakpoint $k \in \Gamma$ in the piecewise linear approximation, when the EV arrives at and departs from CS $s \in F'$ respectively. These variables and the corresponding constraints are included in the exact MIP formulation of EODBRP in Appendix A.

4. Solution Method

Healy and Moll, 1995 demonstrated that the standard DARP is NP-hard. Since ODBRP is a generalization of DARP, we deduce that it is also NP-hard. Furthermore, EODBRP can be considered as a generalization of ODBRP, because ODBRPs are EODBRPs in the special case where the battery has enough capacity thus charging is not needed. Equivalently,EODBRP is NP-hard.

In order to develop an algorithm to solve the EODBRP efficiently, let us first qualitatively analyze the problem. In EODBRP, briefly, four main decisions need to be made. First, passenger-bus assignment needs to be made; second, the alternative bus stations of each passenger have to be selected; third, the sequences of the passengers to be picked up and dropped off have to be fixed for each bus; fourth, the schedule of each bus when to arrive at and depart from each station has to be settled; finally, in each route, whether charging is necessary has to be examined, if so, when, where, how many times to charge, as well as how much to charge have to be determined. To start with, the first three decisions are interdependent on each other, and they then determines the scheduling and the charging decisions. As the inter-dependency increases the complexity of the problem, we choose the 'charging first, routing second' policy. Besides, two parameters K and σ are introduced in the proposed LNS algorithm, trying to find better solutions more efficiently. The details are explained in 4.3.

Like numerous heuristics for VRPs and the variants, our approach consists of a greedy insertion and a subsequent heuristic to respectively construct the initial solution and further improve the solution quality. More specifically, for the second stage, a large neighborhood search (LNS) combined with local search (LS) was developed. In both stages, the algorithm makes routing and charging decisions. The former determines the sequence of the stations where each passenger being picked up as well as dropped off, while the latter chooses where and calculates how much to charge. The routing and charging decisions can be made either simultaneously or sequentially, and in this work, the option is charging first, routing second. In particular, multiple legs in each route are created, that is, each leg is bounded by two visits to the CSs. To make charging decisions, we solve the FRVCP which decides which CS to visit and how much to charge at this visit. Thus in the following subsections, we respectively describe the greedy insertion (4.1), the FRVCP (4.1) and the LNS (4.3).

Algorithm 1: solution method

Input: requests' locations and time windows, distance matrix, etc
Requests are ranked by e_p;
Empty legs are created as placeholders;
for *passenger* $p = 1,2,\ldots,P$ **do**
    for *positions in nonempty legs* **do**
        Assume p is inserted;
        Calculate the charging amount directly or by FRVCP;
    If at least a feasible position exists:
    Then insert p at the position w.r.t the least increase in the total URT;
    Else:
    for *empty legs* **do**
        Assume p is inserted;
        Calculate the charging amount directly or by FRVCP;
    If at least a feasible position exists:
    Then insert p at the position w.r.t the least increase in the total URT;
    Else:
    Assign p to a new bus;
$x \leftarrow x_0$; /*generated the initial solution, thus it is the best known solution so far*/
while *stopping criterion is not met* **do**
    LNS:
    for *round* $= 1,\ldots,\sigma$ **do**
        Remove randomly K requests from x;
        Repair operator generates x'';
        If $f(x'') < f(x')$:
        Then $x' \leftarrow x''$, i.e. pick the best solution x' from neighborhood $N(x)$;
    Apply LS on x';
    Move or not:
    If $f(x') < f(x)$ **Then** $x \leftarrow x'$;
Output: best known solution

4.1. Construct the initial solution

For the initialization phase, greedy insertion method is in use. The procedure to construct the initial solution is as follows. To start with, the requests are ranked in ascending order of their earliest departure time. In other words, the request with the smallest earliest departure is in the first of the request list. Next, empty legs as placeholders are created for each bus route. Then each request is removed from the request list and inserted in one of the non-empty legs of all the buses, in the position with the least increase of total URT that respects time windows, bus capacity and battery level. If r cannot be inserted in any non-empty legs due to any violation of these constraints, r is inserted into a new leg, and the station pair among its alternative get-on/off choices with the least increase of the total URT is chosen. The bus to serve r is the one with the smallest distance between r's boarding station and its previous passenger station. Similarly, if r cannot be inserted in any existing bus routes, a new bus is in use to serve this request. Again, the station pair with the least increase of the total URT is chosen. This insertion procedure is repeated until all requests are served.

If there is only one leg in a bus route, in other words, the bus only charges at the beginning and the end of its route, then the route is feasible as long as the battery level upon arrival at the CS at the end is nonnegative. The charging at the beginning is omitted, as we simply assume the battery has been fully charged overnight. The CS at the beginning of the route is the one closest to the first passenger station of the route. Similarly, the CS at the end is the one closest to the last passenger station.

If there are two legs, in other words, besides the CSs at the beginning and the end of the route mentioned before, the bus pays an intermediate visit to a CS. In this case, an additional requirement is, given the battery level upon arrival at the intermediate CS, and the maximum allowed charging time (i.e. the duration that can be postponed without any violation of time windows), the battery level upon leaving the intermediate CS is enough to reach the CS at the end, that is, the battery level is larger than or equal to 0 upon arrival at the final CS. Same as the one-leg case, the CSs at the beginning and the end are simply the closest ones to the first or last passenger stations. However, for the CS in the middle, it cannot be simply chosen as the one with the smallest sum of distance between its previous and next passenger stations, since this may cause one of the legs' range exceed the battery level. Instead, all the CSs are ranked according to the sum of distance, then the first one that respects the battery levels of two legs is chosen as the mid-route CS.

If there are three or more legs, the optimal total charging time of this bus route is not as straightforward as the former two cases. To solve this case, the rule proposed by Ángel Felipe et al., 2014 is usually adopted in literature within heuristic methods: when visiting a CS, charge the minimum amount of energy needed to make the route energy-feasible. Specifically, if it is the last CS before returning to the depot, the minimum amount is thus to cover the remaining route to the depot. Or if there is at least one more CSs downstream, charge the amount needed to reach the next CS. If reaching the next CS (or the depot) is impossible, the move is deemed infeasible.

However, the optimal charging time or amount may not be obtained by the rule mentioned above, especially in the EODBRP passengers have hard time windows, and the buffer also determines the maximum time can be spent at a CS. Hence in this study, the exact mathematical model is proposed and CPLEX is called to solve the subproblem to optimal, which will be explained in section 4.2.

4.2. mathematical formulation of FRVCP

The concept of the FRVCP is explained in Montoya et al., 2017. However, the difference in the problem setting of this work is that the positions of the intermediate CSs are fixed. However, which CS to visit each time is still need to be determined, since it cannot be simply set as the CS with the smallest total distance between its proceeding and succeeding stations. Nevertheless, we adopt the variables and related constraints used in their formulation, with the details as follows.

Let $i = \{0, 1, ..., n_r\}$ be the index of the CSs in a specific bus route, where 0 and n_r respectively corresponds to the CS at the beginning and the end of the route, while $1, 2, ..., n_{r-1}$ correspond to the intermediate CSs. As explained in section 3.2, all CSs $\in F'$ have the same piecewise linear function, where each piece is defined by the breakpoint set B. Specifically, piece k is between the breakpoint $k-1$ and k where $k \in B \setminus \{0\}$, with a slope denoted as ρ_k. Besides, each piece k is bounded by the battery levels a_{k-1} and a_k. In addition, since the positions of the CSs are determined, together with the CSs to visit (which CSs are chosen will be explained in the last paragraph of this subsection), the distance to travel within each leg (including the distance from the CS prior to the leg and the CS after the leg) is simply a constant that can be easily calculated. Let $d^p_{i,i+1}$ where

$i = \{0, 1, ..., n_r - 1\}$ denote the energy consumption of the route that only consists of passengers' stations between the CSs with id i and $i+1$. Similarly, let d_{ij} where $i = \{0, 1, ..., n_r - 1\}$ denote the energy consumption between the CS with id j and its succeeding passengers' station of leg i. Naturally, $d_{i-1,j}$ where $i = \{1, 1, ..., n_r - 1\}$ denote the energy consumption between the CS with id j and its preceding passengers' station of leg $i-1$. Thus the energy consumption is consistent among the entire route. Finally, let the buffer at the beginning of each leg be divided into two components, denoted w and b, respectively represent the slack that can be consumed without/with postponing the start time of the succeeding legs. The formulation of the buffer is the same as in literature Savelsbergh, 1992; Cordeau and Laporte, 2003.

In terms of variables, let ϕ_i denote the battery level upon arrival at the CS with index i. Then let δ_{ik} denote the charging amount when the charging process finishes on the piece between breakpoints $k-1$ and k, at the CS with index i. Further, let μ_{ik} denote the battery level when the charging process finishes on the piece between breakpoints $k-1$ and k, at the CS with index i. Finally, we introduce the variables x_i to indicate whether the start time of the succeeding legs are affected. Then the mixed integer programming (MIP) formulation is as follows.

$$\min \sum_{i=\{1,2,...n_r-1\}} \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \frac{\delta_{ijk}}{\rho_k} \quad (1)$$

s.t.

$$\phi_1 = Q - d^p_{01} - \sum_{j \in F} \epsilon_{0j} d_{0j} - \sum_{j \in F} \epsilon_{1j} d_{1j} \quad (2)$$

$$\phi_i = \mu_{i-1} - d^p_{i-1,i} - \sum_{j \in F} \epsilon_{i-1,j} d_{i-1,j} - \sum_{j \in F} \epsilon_{ij} d_{ij} \qquad \forall i \in \{1, 2, ..., n_r\} \quad (3)$$

$$\mu_{ij1} = \phi_i + \delta_{ij1} \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F \quad (4)$$

$$\mu_{ijk} = \mu_{ij,k-1} + \delta_{ijk} \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0, 1\} \quad (5)$$

$$\mu_{ijk} \geq a_{j,k-1} \theta_{ijk} \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0, 1\} \quad (6)$$

$$\mu_{ijk} \leq a_{jk} \theta_{ijk} + (1 - \theta_{ijk}) Q \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (7)$$

$$\delta_{ijk} \leq \theta_{ijk} Q \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (8)$$

$$\theta_{ijk} \leq \epsilon_{ij} \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (9)$$

$$\sum_{j \in F} \epsilon_{ij} \leq 1 \qquad \forall i \in \{1, 2, ..., n_r\} \quad (10)$$

$$x_i \geq \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \frac{\delta_{ijk}}{\rho_k} - w_i \qquad \forall i \in \{1, 2, ..., n_r - 1\} \quad (11)$$

$$\sum_{j \in F} \sum_{k \in B \setminus \{0\}} \frac{\delta_{ijk}}{\rho_k} \leq b_i + w_i - \sum_{n=1,...,i-1} x_n \qquad \forall i \in \{1, 2, ..., n_r - 1\} \quad (12)$$

$$\phi_i \geq 0 \qquad \forall i \in \{1, 2, ..., n_r\} \quad (13)$$

$$\delta_{ijk} \geq 0 \qquad \forall i \in \{1, 2, ..., n_r\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (14)$$

$$\mu_{ijk} \geq 0 \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (15)$$

$$\theta_{ijk} \in \{0, 1\} \qquad \forall i \in \{1, 2, ..., n_r - 1\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (16)$$

$$\epsilon_{ij} \in \{0, 1\} \qquad \forall i \in \{1, 2, ..., n_r\}, \forall j \in F \quad (17)$$

$$x_i \geq 0 \qquad \forall i \in \{1, 2, ..., n_r - 1\} \quad (18)$$

The objective function (1) seeks to minimize the total charging time of all visits to the intermediate CSs. Constraints (2) and (3) are the consistency of the battery level upon arrival at each CS, namely the battery level when arriving at this CS is equal to the battery level after charging at the last CS minus the energy consumption (simply the distance). Constraints (4) and (5) impose the battery level after charging equals to the battery level before charging plus the total amount charged within each segment of the charging function. Constraints (6) and (7) impose if the battery is charged within a segment, the battery level after charging is within the bounded values a_{k-1} and a_k. Constraints (8) ensures that a battery can be only charged at the selected segment of the charging function. Constraints (9) impose that charging is allowed only if the CS is visited. Constraints (10) forbid a bus visiting two or more CSs simultaneously. Constraints (11) use variables x to represent whether the charging time affects the succeeding legs as explained before. If the charging time at i is smaller or equal to w_i, then the succeeding legs' maximum charging time are not affected; otherwise they are reduced by x_i. Then constraints (12) impose the charging time at i does not exceed the allowed slack minus the reduced values by the previous legs. Finally, constraints (13) - (18) set the range of the decision variables.

In terms of choosing the CSs to visit, naturally the CSs at the beginning and the end are still the closest ones as before. For the CSs in the middle, the principle is as the two-leg case while more complicated. Instead of the rank of one single CS, all the combinations of the mid-route CSs are ranked according to the total distance from/to CSs. Correspondingly, the first combination that respects the battery levels is chosen. Apart from this, the CSs can be chosen by CPLEX as well, as in the mathematical model j is the index of CS and which CS to visit is to be determined.

To accelerate the procedure of the FRVCP, a strategy that eliminates infeasible solutions is adopted before sending them to the CPLEX, given the premise that the piecewise linear charging function is concave (i.e., $\rho_{k-1} \geq \rho_k, \forall k \in \Gamma \backslash \{0, \gamma\}$), in other words, the first segment has the fastest charging rate. Besides, since the maximum buffer at each position is known, regardless the impact of the former legs (i.e. $b_i + w_i$ in constraints (12)), if $\frac{b_i + w_i}{\rho_0}$ is less than the travel distance of leg i, this solution can be eliminated directly. Nevertheless, this is only a rough strategy, and more sophisticated strategies can be developed in future work.

4.3. LNS

In order to solve the EODBRP, a large neighborhood search (LNS) (Pisinger and Ropke, 2019) is developed. In the LNS framework, solutions are partially destroyed and reconstructed in each iteration. In recent years, LNS has shown its effectiveness in solving the DARPs (Li et al., 2016; Belhaiza, 2019; Gschwind and Drexl, 2019).

4.3.1. Destroy operator

In the LNS framework, solutions need to be destroyed and then rebuilt, in order to escape local optima. In this regard, a destroy operator is needed. In our algorithm, at each iteration the destroy operator removes a certain number (represented by K) of requests from the best known solution. The requests are randomly chosen while the total number K is predetermined and constant. More specifically, when requests are removed and not served by any bus, the buses can possibly have fewer stations to visit and larger buffers, thus can leave room for new solutions. With regard to K, it can be varied strategically, based on the fact if K is too large, the preferable properties resulting in the best known solution are also significantly eliminated, then the new solutions will be likely of low quality; on the other hand, a too small K results in a too small neighborhood and the searching procedure can be thus slow. Hence in the section 5.2 different values of K are tested.

4.3.2. Repair operator

The principle of the repair operator is similar to the greedy initial solution algorithm, i.e. insert the unassigned requests into the current solution, each at the best position with the minimum increase of the objective value, respecting the constraints of time windows and bus capacity. However, there are several differences with the initial solution algorithm. First, the requests in the repair operator are inserted in the inverse order of being removed, namely the request first removed is inserted at last. In this way a new solution can be possibly generated; otherwise, the original solution is likely to be reached again. In addition, the destroy-repair operator is repeated σ times resulting in maximum σ different candidate solutions, and the best one in terms of the objective value is chosen as the current solution for the LS to further improve. The intention of choosing among σ candidate solutions is to try to find a tradeoff between searching in a larger neighborhood and maintaining the neat attributes of the best known solution. In such manner a more promising current solution is possibly made. Divergent values of σ are tested with the combination of different values of K in the destroy operator in section 5.2.

In the destroy-repair cycle, infeasible solutions can occur. If a new solution fails at serving all the passengers due to the violation of time windows, bus capacity or charging, this solution will not be considered as a candidate solution but simply discarded. Thus only feasible solutions will be kept as candidates for further possible improvement by the LS. Note due to infeasible solutions, the actual size of the candidate pool can be smaller than σ.

Same as in the initial stage, in the repair operator, the charging decisions are adjusted once a route has been modified by the routing operators. More specifically, the same procedure of solving the FRVCP in the initial stage is triggered in order to find the best charging solution for each route.

4.3.3. Local Search

LS is included in the LNS framework and applied after a new feasible constructive solution is found by the repair operator, thus the quality of this new solution can be possibly improved by making small changes. This is done by a reinsert operator. Specifically, each time the repair operator generates the best new solution among σ candidates, reinsert is applied to this solution by trying to remove and reinsert each request sequentially. Each request is inserted into the position with the minimum objective value, without violating any constraint. If there is no better position among all buses in terms of the objective value, the request remains in its original position. The LS repeats until no improvement is found within the last round where it examines all the requests in sequence.

4.3.4. Acceptance criterion

In each iteration of the LNS framework, each time a new feasible solution is generated, its objective value is compared to that of the best known solution. If the value improves, the new solution is accepted and it replaces the best known solution in the next iteration of the LNS.

4.3.5. Stopping criterion

After the initial solution is computed, the loop of the LNS algorithm starts. The algorithm terminates if a predetermined runtime, or a round limit without improvement (will be specified in section 5) is reached.

5. Computational Experiments

In this section, the experimental data and computational results are presented. To begin with, the instance generation is explained in subsection 5.1, then the parameters in the LNS framework are tested in subsection 5.2. Next, the properties of the BKSs are analyzed in subsection 5.3, followed by the analysis of BSA in subsection 5.4. Then whether the local search operator contributes to the solution quality is investigated in subsection 5.5. Last but not least, the comparison with conventional buses are presented in subsection 5.6.

5.1. Instance generation

The instances were generated based on the map of Antwerp, Belgium. The city's area is 204.51 km^2 with 10 randomly located clusters, each has a maximum area $4km^2(2km \times 2km)$. Figure 3 shows the city map and road network with clusters (in red). In order to make realistic instances, real stations on the map within the clusters are in use for passengers to board and alight. The total number of stations are 215, with the average distance between neighbors 500 meters. In addition, passengers' trips can be either intra- or inter-clusters. For detailed instances' generation, we refer to Queiroz et al., 2022.

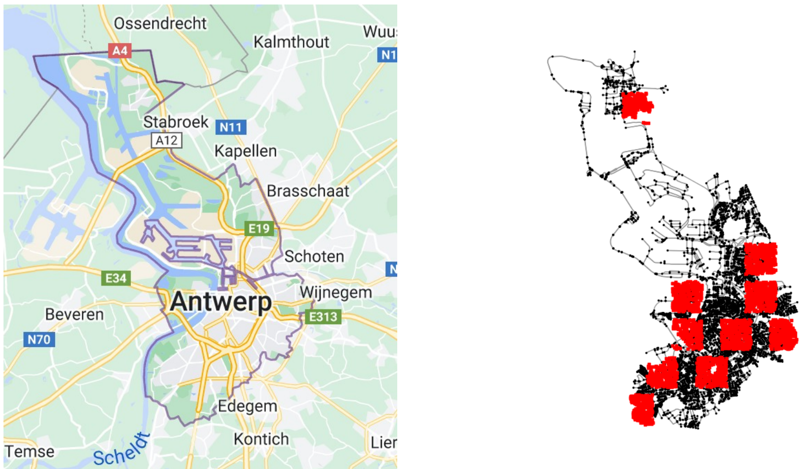

Figure 3: map and clusters' locations

The number of passengers ranges among small, medium to large, specifically, 100, 200, ..., 900, 1000, 1500, 2000, 3000. One instance of each and there are in total 13 instances. Topology of the clusters and stations, as well as the generation principle of passengers remain the same among all instances.

The travel speed is 30km/h and assumed constant everywhere for simplicity. The operation time is from 6am to 6pm (12 hours). Passengers' time windows are evenly distributed within the operation time. The time window for each passenger is set to 1.5 $\times$ direct ride, where 1.5 is chosen intuitively, thus the time windows are neither too strict nor too loose, which is in line with our definition of on-demand bus, a tradeoff between the public transport and a direct ride.

Each request can have at most two stations respectively to board and alight.

Three different types of buses are used in this study for the variant experiments. We choose buses on the market for the sake of a realistic formulation of the charging function and battery capacity. They are the electric minibus Karsan e-Jest with single or double batteries (`https://www.karsan.com/en/jest-electric-highlights`), and a transit bus BYD K7MER (`https://en.byd.com/bus/k7mer`). The detailed parameters of the bus seats and maximum ranges are listed in table 1. The capacity of the bus is equal to the number of passenger seats written in the brochure, as standing is not considered in this work, due to the consideration of service quality and the unknown impact on the energy consumption rate.

On the other hand, the exact charging functions with fast charging mode are not specified in the brochures. Instead, we calculated them based on the regulation presented by Montoya et al., 2017, i.e., the charging rate differs among the three segments of a piecewise linear charging function (see Fig. 2): in the first segment, 0.7 time units per 1 unit of charge from 0 to 85% (excluding) of battery capacity; in the second segment, 1.5 time units per 1 unit of charge from 85% (including) to 95% (excluding) of battery capacity; in the third segment, 5 time units per 1 unit charge from 95 percent (including) to 100 percent (including) of battery capacity. The piecewise linear charging function is merely an approximation of the real-world charging function, with relative absolute errors ranging from 0.9% to 1.9% (Montoya et al., 2017), thus can be considered as a precise approximation.

Based on the calculation above, the detailed parameters of the charging function are also provided in table 1. Note the battery capacity is usually represented with kWh, while in this study, as the energy consumption is assumed to be proportional to the traveled distance, we replace kWh by the maximum range (km) given in the brochure, and the regulation described above still holds.

Table 1: bus parameters

bus	seats	break points of time (min)			break points of amount (km)		
		c_{s1}	c_{s2}	c_{s3}	a_{s1}	a_{s2}	a_{s3}
mini (single)	12	58.44	73.17	97.72	89.25	99.75	105
mini (double)	12	69.06	86.375	115.2	178.5	199.5	210
regular	20	72	90	120	267.75	299.25	315

All code was written in C++ (Visual Studio 2017) and all tests were performed on a computer with an Intel® Core^{TM} i7-8850H 2.60Ghz processor, 16GB RAM and Windows 10 system.

5.2. parameter setting

The first experiment evaluates the impact of different K and σ on the performance of the LNS, as illustrated in section 4.3. The double-battery mini bus is in use for this experiment. The values of K are 2, 5 and 10 requests despite the instances' size. The minimum is set to 2 since 1 request at a time is identical to the LS operator in our algorithm. Similarly, σ is chosen from 4 values: 1, 5, 10 and 15. The minimum value 1 means without selection. Since each time the destroy operator chooses the requests randomly, the best known solution varies as well. To obtain an adequate estimate of the performance of the LNS, each instance with each combination of K and σ was run 10 times, and the resultant average value, standard deviation, min, max and quartiles were computed. Each time the LNS algorithm was forced to terminate when the duration reaches 0.1 $\times$ instance size seconds. For example, the instance with 100 passengers is allowed to run 10 seconds, while with 1000 passengers the algorithm stops once the duration reaches 100 seconds. The runtime was limited due to the total number of combinations, also because the combination that achieves the steepest descent within a short time is preferred. For each instance, the combination of each instance that performs the best is listed in table 2, and the frequency of different combinations that perform the best is shown in Fig 4.

Despite of the limited number of instances and randomness, there are several observations. First, no K or σ is optimal for all instances. However, a smaller K usually performs better, K = 2 more frequently reaches a lower total URT than K = 5 or 10. The reason is that the BKS is destroyed less thus the preferable features are more likely to be maintained if K is small. Similar conclusion is reached in Galarza Montenegro et al., 2021. Last but not least, although the frequencies of reaching the lowest total URT when $\sigma = 5$, 10 or 15 are comparable, $\sigma = 1$, i.e. without selection of the LNS solutions to apply the LS, is less likely to be optimal.

In addition, a method for testing the robustness of the combinations is proposed, in order to prevent selecting a combination that performs the best for some instances but poorly for others. The rank of each combination among all the instances is recorded. In particular, if a combination outperforms the others for one instance, it receives a score of 1. On the other hand, it receives a score of 12 if it performs the worst out of 12 combinations, and so on. The results are shown in Fig 5. Despite no single combination dominates the others, the previous findings are confirmed: a smaller K is preferable, and $\sigma = 1$ is the worst for various Ks, while the disparity when σ varies among 5, 10 and 15 is not obvious.

Table 2: Results of K and σ

instance	100	200	300	400	500	600	700	800	900	1000	1500	2000	3000
K	10	5	2	2	2	2	2	2	5	2	2	5	2
σ	15	15	10	15	5	5	5	15	10	5	10	10	1

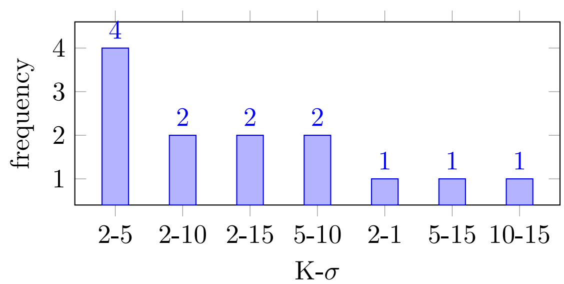

Figure 4: Results of K and σ

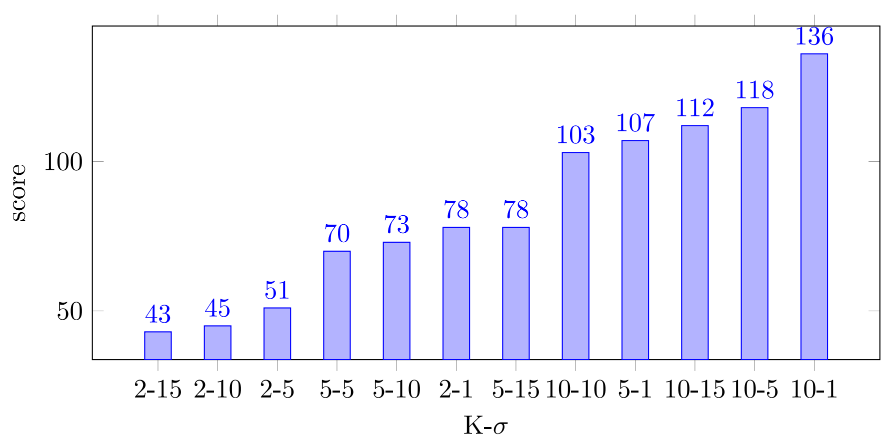

Figure 5: Results of K and σ

5.3. properties of BKSs

The properties of the BKSs found in our study are analyzed in this subsection, in order to provide readers with possibly useful information in developing new solution methods for the EODBRP with intermediate charging. The single- and double-battery mini bus are in use for the experiment in this subsection. First, once the combination of K and σ (K = 2 and $\sigma = 15$) that performs the best on average is found in the last subsection, it is fixed for all the experiments. In other words, the best combination given by the double-battery mini bus are in use for all the instances with the single-battery bus directly, despite the combination do not necessarily hold for every case. However,the LNS does not stop when the runtime reaches 0.1 $\times$ instance size, but if the BKS has not been improved after 1000 consecutive rounds.

Subsequently, the corresponding solutions are analyzed as below. The solutions with the double-battery bus are presented first in subsection 5.3.1, followed by the single-battery case in subsection 5.3.2.

5.3.1. BKSs with the double-battery buses

In this subsection, the BKSs of the EODBRP with the double-battery buses are presented and analyzed. Specifically, the average URT of each passenger, as well as the average charging time and amount of each charged bus, are presented in table 3 and 4. Same as in the last subsection, all the results are the average of 10 separate solutions.

The results of average URT in table 3 show that the URT does not have an obvious trend of increasing with instance sizes, either with electric or conventional buses. Next, the last column of table 3 is the redundant URT compared with conventional buses. A distinct gap exists between electric and conventional buses, with the average value of all instances equal to 25.58%. This is due to EVs need to make a detour and spend excessive time to the CSs and to be charged, despite the detours are done with empty buses, this still occupies considerably the operation time serving passengers. This adds further constraints to our problem. In particular, having EVs instead of conventional vehicles is in some ways equivalent to having tighter time windows for the passengers: this indirectly affect the solution quality.

instance	**average URT (minute)**				**conv (%)**
	ave	**std**	**min**	**max**	
100	13.02	0.0002	11.73	14.08	25.59
200	13.22	0.0058	12.56	13.84	15.18
300	14.45	0.0038	13.88	15.65	33.12
400	13.93	0.0017	13.62	14.49	23.76
500	15.32	0.0026	14.33	16.12	29.41
600	15.74	0.0001	15.67	15.84	35.62
700	16.24	0.0017	15.10	16.73	30.76
800	16.14	0.0009	15.47	16.85	39.97
900	14.59	0.0009	13.32	14.90	16.27
1000	14.31	0.0008	13.20	14.83	21.55
1500	15.17	0.0001	15.03	15.66	25.83
2000	16.68	0.0000	16.58	16.79	25.66
3000	17.72	0.0000	17.72	17.73	9.82

Table 3: average URT of each passenger, double batteries

instance	charging time (minute)				charging amount (km)				slope
	ave	std	min	max	ave	std	min	max	
100	13.11	2.63	9.19	18.03	21.17	6.81	11.03	33.89	1.62
200	26.21	0.03	26.14	26.26	35.48	0.08	35.31	35.62	1.35
300	15.38	0.25	13.75	16.89	39.75	0.64	35.53	43.66	2.58
400	23.95	0.28	23.69	24.06	61.91	0.72	61.23	62.20	2.58
500	16.34	0.13	15.66	17.75	42.22	0.33	40.48	45.87	2.58
600	20.20	0.13	18.41	21.06	52.21	0.34	47.58	54.44	2.58
700	15.63	0.08	14.93	16.83	40.40	0.20	38.58	43.51	2.58
800	22.93	0.10	22.35	24.51	59.28	0.26	57.77	63.34	2.58
900	19.45	0.06	18.72	20.33	50.26	0.16	48.38	52.53	2.58
1000	16.99	0.04	16.22	17.36	43.92	0.10	41.93	44.88	2.58
1500	17.48	0.01	17.47	17.51	45.18	0.02	45.16	45.25	2.58
2000	13.44	0.02	12.90	13.82	34.74	0.05	33.34	35.72	2.58
3000	24.87	0.01	24.57	24.97	64.28	0.02	63.51	64.54	2.58

Table 4: average charging time and amount of the charged buses, double batteries

Next, our BKSs are made up of 355 routes in total, and the fraction of which that include intermediate charging are presented in Fig. 6. The data show that 73.2% of the routes in the BKSs contain an intermediate CS. This percentage vary among instances, with a slight increasing trend when the instances are larger. The results show intermediate charging and its solution quality are essential for decent EODBRP solutions, especially in these instances the number of buses was set as the smallest values found by the initial solution (which will be explained in section 5.6).

Thirdly, the number of intermediate charging per route is calculated. Despite charging more than once is allowed, in the solution with double-battery buses, the 73.2% buses that charge in mid-route only charge once. Given the data in table 4, each bus on average spends 18.92 minutes to charge the amount covering 45.45 km's ride.

Finally, we analyze the segment(s) within which the energy is recovered through mid-route charging. According to the calculated slope (the charging amount divided by charging time) in table 4, most buses only charge in the first segment, i.e., the one with the fastest charging rate. This is reasonable since when visiting CSs, the objective is to minimize the charging time, and only until the battery level is as low as it is in the first segment CSs are visited. In addition, whether each bus is fully or partially charged is recorded as well, and it shows all the mid-route charges are partial, this proves the efficiency and importance of the partial charging policy compared with full charging.

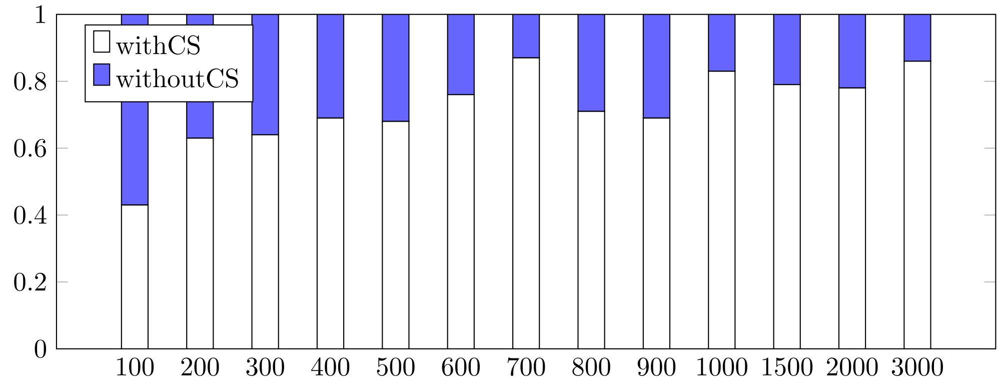

Figure 6: Percentage of the routes with/without an intermediate CS by instance size, double batteries

5.3.2. BKSs with the single-battery buses

In this subsection, the BKSs of the EODBRP with the single-battery buses are presented and analyzed. Same as the last subsection, the average URT of each passenger, as well as the average charging time and amount of each charged bus, are presented in table 5 and 6. All the results are the average of 10 separate solutions as well.

The results of average URT in table 5 show that despite the URT is not obviously correlated to the instance size, the URT of the EODBRP with single-battery buses is generally even larger compared to the double-battery buses. Specifically, the average gap between electric and conventional buses of all instances is 39.32%. Correspondingly, the charging time and amount drastically increase, i.e. each bus on average spends 50.32 minutes to charge the amount covering 73.06km's ride. The reasons are the range of the single-battery bus is only half of the double-battery one, plus the charging rates are substantially lower as well. In particular, the charging rate of the first segment of the double-battery bus is $\frac{178.5}{69.06} \approx 2.58$, while the single-battery one is only $\frac{89.25}{58.44} \approx 1.53$.

The BKSs consist 459 routes in total, and the fraction of which that include intermediate charging are presented in Fig. 7. In the double-battery case, certain buses charge once while the others do not charge. However, in the single-battery case, all buses charge at least once in mid-route. In particular, 88.9% of the routes in the BKSs contain an intermediate CS, while the 11.1% contain two. The results once again show intermediate charging and its solution quality are essential for the EODBRP.

Finally, the calculated slope in table 6 indicates the segment(s) within which the energy is recovered through mid-route charging. Compared with the double-battery case, it is more common that buses not only charge in the first segment, but also the slower segment(s). Since if the frequency of mid-route charging increases, the problem is more complicated to solve, while the positions to insert CSs are not a decision of the FRVCP in this work. In other words, optimizing the positions to insert CSs together with minimizing the charging time is not explicitly included in the objective function, thus the charging solution may not be optimal, despite the charging amount calculated by the CPLEX model with given CSs is optimal. Thus it is one limitation of this work. Nevertheless, none of the buses is fully charged according to the records, thus it can be once again concluded that partial charging provides more flexibility and efficiency than full charging.

instance	average URT (minute)				conv (%)
	ave	std	min	max	
100	17.77	0.0195	16.77	18.29	71.42
200	13.66	0.0190	12.57	14.81	19.05
300	16.13	0.0226	13.97	17.68	46.96
400	14.42	0.0048	13.05	15.25	28.09
500	18.44	0.0007	18.17	18.63	55.85
600	17.33	0.0017	16.59	17.57	51.88
700	18.72	0.0011	18.39	19.87	53.01
800	16.74	0.0035	15.23	17.96	43.71
900	15.60	0.0020	14.34	15.89	25.92
1000	15.29	0.0007	14.83	15.51	32.26
1500	16.76	0.0002	16.54	16.79	32.95
2000	17.24	0.0004	16.87	17.48	29.90
3000	19.39	0.0001	19.13	19.60	20.17

Table 5: average URT of each passenger, single battery

instance	charging time (minute)				charging amount (km)				slope
	ave	std	min	max	ave	std	min	max	
100	11.13	0.0036	11.11	11.20	16.99	0.0042	16.92	17.06	1.53
200	36.95	0.1938	34.05	38.43	56.43	0.1443	51.97	58.69	1.53
300	61.91	0.0709	60.44	63.11	84.99	0.0923	83.07	86.54	1.37
400	38.79	0.0254	37.86	39.53	59.22	0.0714	57.81	60.34	1.53
500	66.29	0.0221	65.53	66.55	99.75	0.0827	97.37	101.18	1.50
600	60.40	0.1708	55.31	63.49	88.12	0.1312	81.20	92.02	1.46
700	59.80	0.0286	58.87	60.56	86.02	0.0630	83.88	87.45	1.44
800	55.23	0.0417	52.97	57.27	81.05	0.0200	80.46	82.33	1.47
900	46.28	0.0193	45.24	46.92	69.33	0.0386	67.98	71.12	1.50
1000	52.32	0.0212	50.89	52.96	78.07	0.0484	76.15	80.17	1.49
1500	54.88	0.0308	53.20	56.77	72.04	0.0419	69.28	74.04	1.31
2000	46.76	0.0109	45.77	47.55	71.39	0.0166	69.89	72.61	1.53
3000	63.37	0.0017	63.17	63.52	86.34	0.0027	86.04	86.60	1.36

Table 6: average charging time and amount of the charged buses, single battery

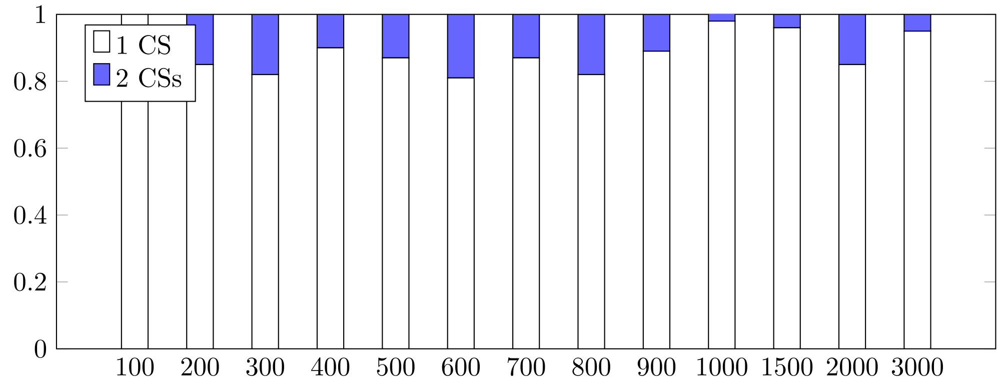

Figure 7: Percentage of the routes charging once or twice in mid-route by instance size

5.4. Effect of bus station assignment

Since the inclusion of BSA distinguishes the ODBRP from DARP in essence, the effect of BSA is investigated in this subsection. As explained in section 3, when BSA is considered, passengers can be assigned to alternative stations for boarding and alighting, and the number of alternates are limited to two in our experimental setting. On the contrary, when BSA is excluded, the standard way in the DARP literature is assigning passengers to the closest stations, or even transporting passengers door-to-door. Since the locations of stations are predefined and buses are only allowed to dwell at the stations in the ODBRP, when BSA is excluded, passengers are simply assigned to the closest stations in terms of their origins and destinations. According to the experimental results in Melis and Sörensen, 2022, excluding BSA leads to a significant increase in the objective value (the total URT), when conventional buses are in use. Thus in this study, apart from the average URT, we further compare the impact of BSA on the total charging time and amount. In addition, we vary in each instance the percentage of passengers who have alternative stations, namely 0, 50% and 90% respectively. The maximum is set to 90%, as we assume in real life it might not be the case that every passenger or every station has alternates nearby. Then for the case 50%, 40% passengers were chosen randomly to remove their alternative stations. Finally for the case 0, simply no passenger has alternative stations. To analyze the effect of BSA, the instances with 1000, 2000 and 3000 passengers were chosen for experiments. Same as before, each instance was run 10 times with $0.1 \times$ size seconds. The average URT, total charging time and amount are respectively in Fig 8 (Note the y-axis starts approximately at 6 instead of 0), 9 and 10. The observations are, first, BSA can not only reduce the URT, but also the charging time and amount, thus benefit the environment. Next, the more passengers with alternative stations, the more effective BSA is. Nevertheless, each passenger has at most two stations in this study, more stations can be worth investigating for future research.

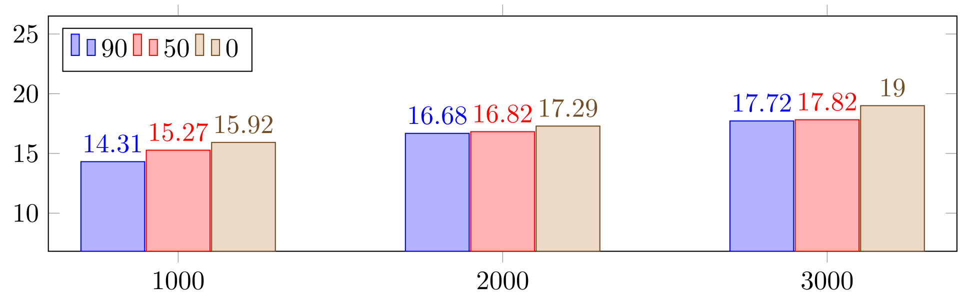

Figure 8: BSA - average URT (min)

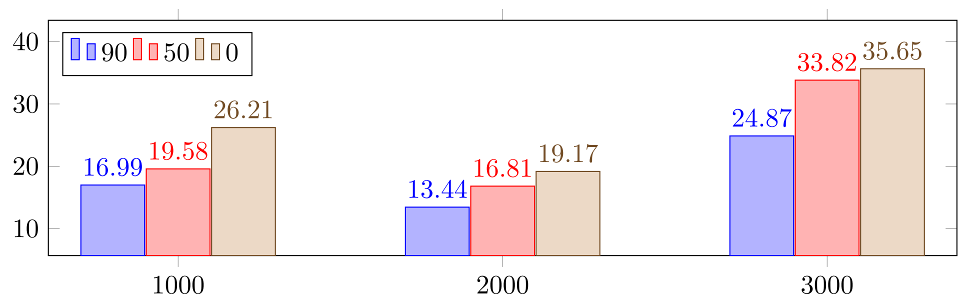

Figure 9: BSA - average charging time per charged bus (min)

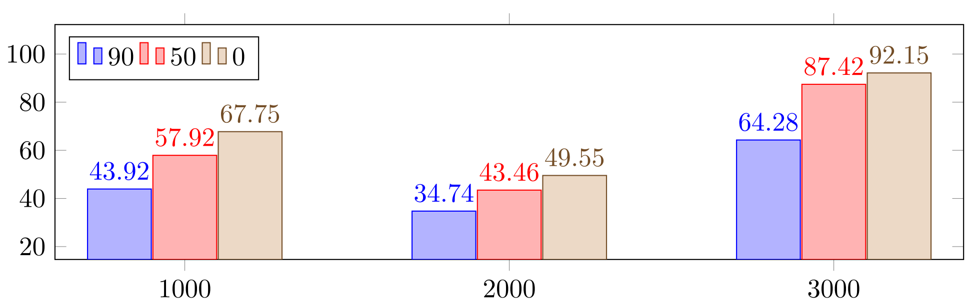

Figure 10: BSA - average charging amount per charged bus (km)

5.5. Influence of the local search operator

The LNS algorithm is combined with a local search operator which sequentially removes and reinserts each request, and the operator's influence on the solution quality is investigated in this subsection. The average runtime of LNS with LS where the BKS has not been improved within 1000 consecutive rounds is set for the LNS algorithm without LS. Three instances are chosen for this test: 1000, 2000, 3000. The double-battery mini bus is used. Each instance is run 10 times and the average URT, charging time and amount are calculated and presented in Fig 11, where the values are the percentage relative to the solution solved by the LNS with the LS operator. It can be seen the local search operator benefits the solution quality by reducing not only the average URT, but also the charging time and amount. This effect holds for variant instance sizes.

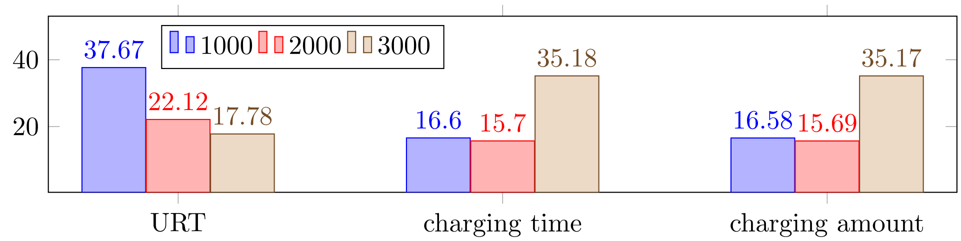

Figure 11: Solution quality without LS (%)

5.6. comparison with conventional buses

In this subsection, we explicitly compare the number of needed buses with conventional vehicles. For the case of conventional buses, since the maximum range is 360km given our operation time (12 hours) and the constant speed (30km/h), refueling is simply not considered. Thus the number of buses needed is only constrained by the time windows and the bus capacity.

The number of either conventional or electric buses is calculated solely by constructing the initial solution, while the LNS does not contribute to reduce the number of buses. This is a limitation of the study, as a more sophisticated algorithm can lead to more accurate numbers of buses.

Once we obtain the number of needed conventional buses, the same number is set for electric bus. If the solution is feasible, then the number of electric buses needed is found; otherwise, an empty bus is added and the procedure repeats.

In order to perform sensitivity analysis of the number of buses, the passengers' temporal distribution and the bus's maximum range are varied, as well as are described in section 5.6.1 and 5.6.2 respectively.

5.6.1. temporal distribution of passengers

In the former instances, the passengers' earliest departure times and thus the time windows were randomly generated. However, in this subsection, the temporal distribution is changed to more realistic. In particular, duration from 7 am to 9 am, and from 4 pm to 6 pm are set as peak hours where more passengers belong to. In the experiments, we intuitively set that 70% passengers belong to the peak hours, while the rest 30% passengers are evenly scattered in the other hours of the day. Therefore, for each instance, it has 2 temporal distributions, and in total 26 instances. For each instance size, we compare the two passenger distributions. Note for the two temporal distributions, for each passenger, only the time window is (possibly) different, while the stations to get on/off remain the same. The results with even distribution or peak hours are shown in Fig 12 and 13, where the numbers correspond to electric vehicles. The results show the number of electric buses are the same as the conventional buses for most instances, while for certain instances one more bus is needed. In addition, the distribution with peak hours needs significantly more buses, which also results in a smaller gap between conventional and electric buses, that is, only 1 instance needs one more electric bus, while for the case of even distribution, the number is 5.

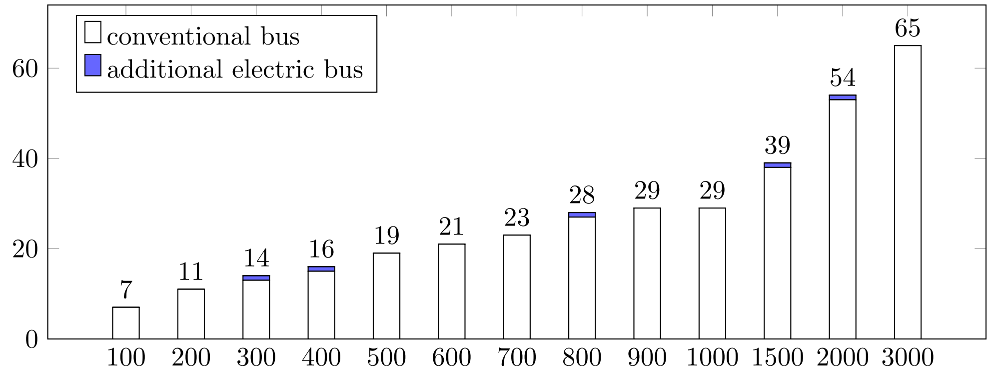

Figure 12: number of conventional and electric buses

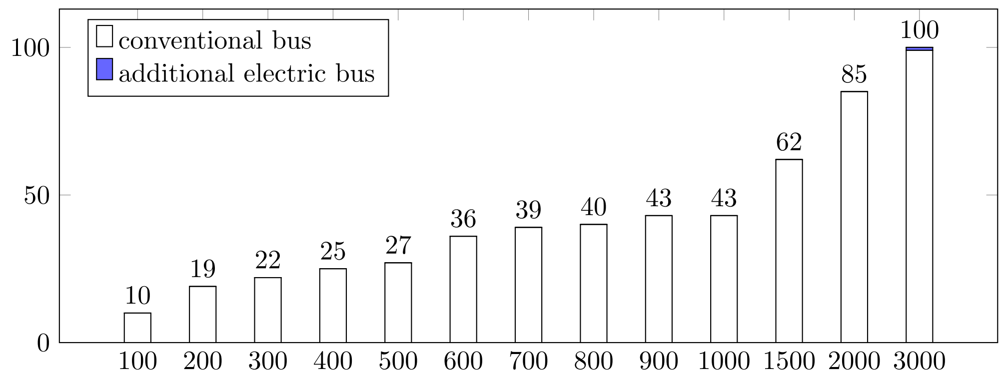

Figure 13: number of conventional and electric buses with peak hours

5.6.2. bus range

In this subsection, we investigate the impact of bus range and capacity on the number of needed buses. As the results of the double-battery mini bus have been presented in the last subsection 5.6.1, here only the results of the single-battery mini bus and the regular bus are listed, in order to evaluate the relationship of the bus range and the gap between EV and conventional buses.

The additional single-battery mini buses or regular buses with the two temporal distributions are shown in table 7. Comparing the single- and double-battery buses, the numbers of single-battery bus increase significantly, while the numbers of double-battery bus are generally the same as the conventional bus. Similar to the reason of the surge in the charging time and amount explained in section 5.3.2, the increased number of buses is also due to the smaller range of the single-battery bus plus and the lower charging rates.

For the regular bus, despite the increased bus capacity compared with the mini one, the number of buses do not decrease, due to the density of requests is small, this is one of the limitations of this study. Nevertheless, since the maximum range is significantly larger, the number of electric buses are the same as conventional bus for all the instances and two temporal distributions.

instances	conv	mini	reg	conv, ph	mini, ph	reg, ph
100	7	1	0	10	0	0
200	11	2	0	19	0	0
300	13	4	0	22	0	0
400	15	5	0	25	0	0
500	19	4	0	27	1	0
600	21	5	0	36	0	0
700	23	7	0	39	0	0
800	27	7	0	40	1	0
900	29	8	0	43	0	0
1000	29	11	0	43	1	0
1500	38	14	0	62	0	0
2000	53	14	0	85	0	0
3000	65	27	0	99	3	0

Table 7: additional single-battery mini buses and regular buses

6. Conclusion

In this work, we have investigated an on-demand bus routing problem with detours to charging stations. The charging functions were modeled as piecewise linear, and partial charging is allowed at CSs. To solve the problem we adopted a 'charging first, routing second' strategy, then proposed a hybrid heuristic with LNS and a subproblem FRVCP that can be solved by CPLEX. The LNS consists of parameter settings and LS, and both contribute to the solution quality. The subproblem FRVCP optimizes the charging decisions (where and how much to charge) given the fixed sequence of passengers' stations. Realistic instances based on a real city map were generated. The experimental results show the proposed algorithm is able to deliver high-quality solutions. Then the analysis of the solutions concludes that good solutions tend to partially charge in mid-route, and employ the linear segment of the charging function. Subsequently, the bus station assignment of ODBRP not only reduces the URT, but also the total charging time and charging amount. Finally, our results conclude both electric mini and regular buses can substitute conventional buses under variant passenger distributions.

Possible future research directions include a more realistic energy consumption function that may consider road slope, speed, temperature or the number of passengers aboard. Another interesting direction could be to develop more sophisticated heuristics or exact methods, together with speedup techniques that can exclude quickly infeasible solutions. Furthermore, capacitated charging stations and queuing is worth study. Last but not least, the comparison between battery charging and swapping stations as well as location routing problem could also be an interesting research topic.

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Appendix A. Mathematical model

Table A.8: Variables and parameters of the EODBRP

x_{snb}	1 if the n-th station of bus b is bus station s and 0 otherwise
y^u_{pnb}	1 if passenger p is picked up at the n-th station of bus b and 0 otherwise
y^o_{pnb}	1 if passenger p is dropped off at the n-th station of bus b and 0 otherwise
q^{cap}_{nb}	net number of passengers picked up (or dropped off) at the n-th station of bus b
t^a_{nb}	arrival time of bus b at its n-th station
t^d_{nb}	departure time of bus b at its n-th station
T_p	user ride time of passenger p
B	the fleet of buses
P	the set of transportation requests, $\|P\|$ denotes the number of requests
S	the set of bus stations
Q^{cap}	capacity of bus
a^u_{ps}	1 if passenger p can be assigned to station s for pick-up
a^o_{ps}	1 if passenger p can be assigned to station s for drop-off
e_p	earliest pick-up time for passenger p
l_p	latest drop-off time for passenger p
$TT_{ss'}$	travel time between station s and station s'
u_{nb}	battery level at the n-th stop of bus b
$e_{ss'}$	electricity consumption from s to s', where s, s' $\in$ V
Q	battery capacity
q_s	battery level upon arriving at s $\in$ F'
o_s	battery level upon departing from s $\in$ F'
ϵ_s	the corresponding charging time of q_s
d_s	the corresponding charging time of o_s
δ_s	the time spent at s $\in$ F'
$z_{s\gamma}$	equal to 1 if the battery level is between $a_{s,\gamma-1}$ and $a_{s,\gamma}$, where $\gamma \in \Gamma\backslash\{0\}$, upon arriving at $\in$ F'
$w_{s\gamma}$	equal to 1 if the battery level is between $a_{s,\gamma-1}$ and $a_{s,\gamma}$, where $\gamma \in \Gamma\backslash\{0\}$, upon departing from $\in$ F'
$\alpha_{s\gamma}$	coefficients of the break point $\gamma \in B\backslash\{0\}$ upon arriving at $\in$ F'
$\lambda_{s\gamma}$	coefficients of the break point $\gamma \in B\backslash\{0\}$ upon departing from $\in$ F'

The objective function is to minimize total URT. Constraints (A.2) enforce the fact that a bus can only stop at one station at the same time. Constraints (A.3) make sure stations that the positions used in the bus route are used consecutively and start at the first position. Constraints (A.4) and (A.5) respectively enforce a bus to stop at one station if and only if at least one passenger uses it either to board or alight. Constraints (A.6) and (A.7) respectively impose that a station is designated to a passenger to board/alight only if the station belongs to the passenger, i.e. it is within the predefined walking distance. Constraints (A.9) impose for any two consecutive stations, the arrival time at the later station is (larger than or) equal to the departure time at the previous one plus the travel time. Constraints (A.10) guarantee the departure time at a passenger's pickup station is greater than or equal to the earliest allowed value. Correspondingly, constraints (A.9) guarantee the arrival time at a passenger's drop-off station is smaller or equal to the latest allowed value. Constraints (A.11) impose that the pickup station precedes the corresponding drop-off one for any passengers. Constraints (A.12) enforce each passenger gets on and gets off the same bus. Constraints (A.13) make sure each passenger is served at most once. Together with constraints (A.18), every request is served once and only once. Constraints (A.14) forbid two consecutive stations be the same. Constraints (A.15) calculate the net capacity at each station, which is equal to the number of passengers getting on minus the one of getting off. Consequently, constraints (A.16) forbid the violation of bus capacity. Constraints (A.17) calculate the URT of each passenger, i.e. the arrival time at the get-off station minus the departure time at the get-on station. Constraints (A.53) make sure the electricity level at any node is larger than or equal to 0. Constraints (A.19) and (A.20) make the electricity level equal to o_s when departing from the CS s. Constraints (A.21) compute the electricity level at the first station after leaving the depot. Constraints (A.22) - (A.25) track the battery level of each node. Constraints (A.26) define the relationship of the battery levels when arriving at and departing from the CS. Constraints (A.27) - (A.33) define the battery level and its corresponding charging time from the charging function, when a bus arrives at the CS. Similarly, constraints (A.34) - (A.40) define the counterparts when a bus departs from the CS. Constraints (A.41) calculate the time spent at each copy of the CS. Constraints (A.42) and (A.43) impose the relationship of the departure and arrival time at a node. Constraints (A.44) forbid visits to the CS consecutively. Constraints (A.45) and (A.46) ensure the copies of the CS i are visited in order. Constraints (A.54) - (A.56) define the domain of the variables. Constraints (A.47) - (A.48) enforce that a bus can visit the CS only if no passengers aboard. Constraints (A.49) - (A.52) define the range for each variable.

$$\min \text{URT} = \sum_{p \in P} T_p \tag{A.1}$$

s.t.

$$\sum_{s \in V} x_{snb} \leq 1 \quad \forall n \in N, b \in B \tag{A.2}$$

$$\sum_{s \in V} (x_{snb} - x_{s(n+1)b}) \geq 0 \quad \forall n \in N, b \in B \tag{A.3}$$

$$M \sum_{s \in S} x_{snb} - \sum_{p \in P} (y^u_{pnb} + y^o_{pnb}) \geq 0 \quad \forall n \in N, b \in B \tag{A.4}$$

$$\sum_{s \in S} x_{snb} - \sum_{p \in P} (y^u_{pnb} + y^o_{pnb}) \leq 0 \quad \forall n \in N, b \in B \tag{A.5}$$

$$x_{snb} + y^u_{pnb} - a^u_{ps} \leq 1 \quad \forall s \in S, n \in N, p \in P, b \in B \tag{A.6}$$

$$x_{snb} + y^o_{pnb} - a^o_{ps} \leq 1 \quad \forall s \in S, n \in N, p \in P, b \in B \tag{A.7}$$

$$t^a_{(n+1)b} - t^d_{nb} - TT_{ss'} + (x_{snb} + x_{s'(n+1)b} - 2)(-M) \geq 0 \quad \forall s, s' \in V \mid s \neq s', n \in N, b \in B \tag{A.8}$$

$$t^d_{nb} - e_p + (y^u_{pnb} - 1)(-M) \geq 0 \quad \forall p \in P, n \in N, b \in B \tag{A.9}$$

$$t^a_{nb} - l_p + (y^o_{pnb} - 1)M \leq 0 \quad \forall p \in P, n \in N, b \in B \tag{A.10}$$

$$\sum_{n \in N} (n y^u_{pnb} - n y^o_{pnb}) \leq 0 \quad \forall p \in P, b \in B \tag{A.11}$$

$$\sum_{n \in N} (y^u_{pnb} - y^o_{pnb}) = 0 \quad \forall p \in P, b \in B \tag{A.12}$$

$$\sum_{b \in B} \sum_{n \in N} y^u_{pnb} \leq 1 \quad \forall p \in P \tag{A.13}$$

$$x_{snb} + x_{s(n+1)b} \leq 1 \quad \forall s, n, b \tag{A.14}$$

$$\sum_{p \in P} (y^u_{pnb} - y^o_{pnb}) - q^{cap}_{nb} = 0 \quad \forall n \in N, b \in B \tag{A.15}$$

$$\sum_{n' \leq n} q^{cap}_{nb} \leq Q^{cap} \quad \forall n, n' \in N \mid n \geq n', b \in B \tag{A.16}$$

$$T_p + (2 - y^o_{pn'b} - y^u_{pnb})M - t^a_{n'b} + t^d_{nb} \geq 0 \quad \forall n, n' \in N \mid n' > n, p \in P, b \in B \tag{A.17}$$

$$\sum_{p \in P} \sum_{b \in B} \sum_{n \in N} y^u_{pnb} = |P| \tag{A.18}$$

$$u_{nb} \geq o_s + (\sum_{s \in F'} x_{snb} - 1)M \quad \forall s \in F', n \in N, b \in B \tag{A.19}$$

$$u_{nb} \leq o_s + (1 - \sum_{s \in F'} x_{snb})M \quad \forall s \in F', n \in N, b \in B \tag{A.20}$$

$$u_{0b} = Q - \sum_{s \in S} e_{0s} x_{s0b} \quad \forall b \in B \tag{A.21}$$

$$u_{nb} - u_{(n+1)b} \leq (e_{ss'} - Q)(x_{snb} + x_{s'(n+1)b} - 1) + Q \quad \forall s \in V, s' \in S, s \neq s' \tag{A.22}$$

$$u_{nb} - u_{(n+1)b} \geq (e_{ss'} + Q)(x_{snb} + x_{s'(n+1)b} - 1) - Q \quad \forall s \in V, s' \in S, s \neq s' \tag{A.23}$$

$$u_s - q_{s'} \leq (e_{ss'} - Q)(x_{snb} + x_{s'(n+1)b} - 1) + Q \quad \forall s \in V, s' \in F', s \neq s' \tag{A.24}$$

$$u_s - q_{s'} \geq (e_{ss'} + Q)(x_{snb} + x_{s'(n+1)b} - 1) - Q \quad \forall s \in V, s' \in F', s \neq s' \tag{A.25}$$

$$q_s \leq o_s \quad \forall s \in F' \tag{A.26}$$

$$q_s = \sum_{k \in \Gamma} \alpha_{sk} a_{sk} \quad \forall s \in F' \tag{A.27}$$

$$\epsilon_s = \sum_{k \in \Gamma} \alpha_{sk} c_{sk} \quad \forall s \in F' \tag{A.28}$$

$$\sum_{k \in \Gamma} \alpha_{sk} = \sum_{k \in \Gamma \setminus \{0\}} z_{sk} \quad \forall s \in F' \tag{A.29}$$

$$\sum_{k \in \Gamma \setminus \{0\}} z_{sk} = \sum_{n \in N} \sum_{b \in B} x_{snb} \quad \forall s \in F' \tag{A.30}$$

$$\alpha_{s0} \leq z_{s1} \quad \forall s \in F' \tag{A.31}$$

$$\alpha_{sk} \leq z_{sk} + z_{s(k+1)} \quad \forall s \in F', k \in \Gamma \setminus \{0, \gamma\} \tag{A.32}$$

$$\alpha_{s\gamma} \leq z_{s\gamma} \quad \forall s \in F' \tag{A.33}$$

$$o_s = \sum_{k \in \Gamma} \lambda_{sk} a_{sk} \quad \forall s \in F' \tag{A.34}$$

$$d_s = \sum_{k \in \Gamma} \lambda_{sk} c_{sk} \quad \forall s \in F' \tag{A.35}$$

$$\sum\nolimits_{k\in\Gamma}\lambda_{sk}=\sum\nolimits_{k\in\Gamma\backslash\{0\}}w_{sk} \qquad \forall s\in F' \qquad (A.36)$$

$$\sum\nolimits_{k\in\Gamma\backslash\{0\}}w_{sk}=\sum\nolimits_{n\in N}\sum\nolimits_{b\in B}x_{snb} \qquad \forall s\in F' \qquad (A.37)$$

$$\lambda_{s0}\leq w_{s1} \qquad \forall s\in F' \qquad (A.38)$$

$$\lambda_{sk}\leq w_{sk}+w_{s(k+1)} \qquad \forall s\in F',k\in\Gamma\backslash\{0,\gamma\} \qquad (A.39)$$

$$\lambda_{s\gamma}\leq w_{s\gamma} \qquad \forall s\in F' \qquad (A.40)$$

$$\delta_s=d_s-\epsilon_s \qquad \forall s\in F' \qquad (A.41)$$

$$t^a_{nb}\geq t^d_{nb} \qquad \forall n\in N,b\in B \qquad (A.42)$$

$$t^a_{nb}\geq t^d_{nb}+\delta_s+(x_{snb}-1)M \qquad \forall s\in F',n\in N,b\in B \qquad (A.43)$$

$$x_{snb}+x_{s'(n+1)b}\leq 1 \qquad \forall s\in F' \qquad (A.44)$$

$$t^a_{nb}\sum\nolimits_{n\in N}\sum\nolimits_{b\in B}x_{s'nb}\geq t^a_{nb}\sum\nolimits_{n\in N}\sum\nolimits_{b\in B}x_{snb} \qquad \forall s,s'\in F',s\leq s' \qquad (A.45)$$

$$\sum\nolimits_{n\in N}\sum\nolimits_{b\in B}x_{snb}\geq\sum\nolimits_{n\in N}\sum\nolimits_{b\in B}x_{s'nb} \qquad \forall s,s'\in F',s\leq s' \qquad (A.46)$$

$$q^{cap}_{nb}\geq\sum\nolimits_{s\in F'}(x_{snb}-1)M \qquad \forall n\in N,b\in B \qquad (A.47)$$

$$q^{cap}_{nb}\leq\sum\nolimits_{s\in F'}(x_{snb}-1)(-M) \qquad \forall n\in N,b\in B \qquad (A.48)$$

$$x_{snb}\in\{0,1\} \qquad \forall s\in S,n\in N,b\in B \qquad (A.49)$$

$$y^u_{pnb}\in\{0,1\} \qquad \forall p\in P,n\in N,b\in B \qquad (A.50)$$

$$y^o_{pnb}\in\{0,1\} \qquad \forall p\in P,n\in N,b\in B \qquad (A.51)$$

$$q^{cap}_{nb}\in\mathbb{Z} \qquad \forall n\in N,b\in B \qquad (A.52)$$

$$u_{nb}\geq 0 \qquad \forall n\in N,b\in B \qquad (A.53)$$

$$z_{sk}\in\{0,1\},w_{sk}\in\{0,1\} \qquad \forall s\in F',k\in\Gamma\backslash\{0\} \qquad (A.54)$$

$$\alpha_{sk}\geq 0,\lambda_{sk}\geq 0 \qquad \forall s\in F',k\in\Gamma \qquad (A.55)$$

$$q_s\geq 0,o_s\geq 0,\epsilon_s\geq 0,d_s\geq 0,\delta_s\geq 0 \qquad \forall s\in F' \qquad (A.56)$$